Warm pseudoscalar inflation

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In this paper, we introduce an action for the warm inflation model with direct coupling between the pseudoscalar field and massless $SU(2)$ gauge fields. The potential of the inflaton is protected against the thermal corrections in a thermal bath of gauge fields even with strong direct interaction between the inflaton and light fields. The dissipation parameter of this model is approximately constant in the high-dissipative regime. In this regime, the model is compatible with observational data and non-Gaussianity is in the order of the Hubble slow-roll parameter $\frac{\epsilon_\phi}{1+Q}$ even in the $f < M_p$ limit.

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I. THE SETUP AND MOTIVATION

We can show that the inflation epoch occurs at the finite temperature $T > H$. The model with interaction between the inflaton and other (mainly) light fields is named warm inflation (WI) [1,2]. Out of the warm inflation condition, i.e., $T < H$, the nearly thermal bath of light particles cannot be sustained because of the accelerated expansion of the Universe. In the WI model, there is an energy exchange between the inflaton field and other light fields during the slow-roll epoch, which leads to an additional friction term in the equation of motion (EOM) of the inflaton scalar field at the background level:

$$\ddot{\phi} + 3H\dot{\phi} + \Upsilon\dot{\phi} + \frac{dV}{d\phi} = 0. \tag{1}$$

On the other hand, when the light fields thermalize, the evolution of radiation energy density is presented by

$$\dot{\rho}_R + 4H\rho_R = \Upsilon\dot{\phi}^2. \tag{2}$$

Transferring the energy density of the inflaton field into the cosmic plasma may sustain the condition $T \geq H$ during inflation. In the slow-roll regime of radiation fluid,

$$\dot{\rho}_R \ll 4H\rho_R \Rightarrow \rho_R = \frac{\Upsilon\dot{\phi}^2}{4H} = \frac{\pi^2}{30} g_* T^4, \tag{3}$$

we can find

$$\frac{T}{H} = \left(\frac{\Upsilon}{H}\frac{\dot{\phi}^2}{H^4}\frac{15}{2g_*\pi^2}\right)^{\frac{1}{4}}. \tag{4}$$

If we have $\dot{\phi} \gg H^2$, even for weak dissipation regime ($\Upsilon < H$), the main condition of warm inflation ($T > H$) can be sustained in the slow-roll regime [$\dot{\phi} \ll (V(\phi))^{\frac{1}{2}} \simeq HM_p$]. Therefore, the presence of the dissipative effect may lead to WI rather than supercold inflation. In the above equations, g_* is the relativistic degree of freedom (DOF) and the source of radiation varies adiabatically when $\Upsilon = \Upsilon(\phi, T)$. The slow-roll conditions in the WI model are modified,

$$\epsilon_\phi \; \eta_\phi \ll 1 + Q, \tag{5}$$

where

$$\epsilon_\phi = \frac{1}{2} M_p^2 \left(\frac{V'}{V}\right)^2 \quad \eta_\phi = M_p^2 \left(\frac{V''}{V}\right) \quad Q = \frac{\Upsilon}{3H} \tag{6}$$

[3–5]. These new slow-roll conditions show that the additional friction term $\Upsilon\dot{\phi}$ in modified EOM can alleviate the required flatness of the potential in the slow-roll regime. Using EOM (1) of the thermalized inflaton, we can find

$$\frac{\rho_R}{V(\phi)} \simeq \frac{1}{2}\frac{\epsilon_\phi}{1+Q}\frac{Q}{1+Q}, \tag{7}$$

where

$$\ddot{\phi} < 3H(1+Q)\dot{\phi}, \tag{8}$$

which presents $\rho_R < V(\phi)$ during the accelerated expansion epoch of the Universe's evolution (slow-roll inflation). But the radiation energy density can smoothly become the dominant component at the end of inflation, where $\epsilon_\phi \sim 1 + Q$ with $Q \gg 1$, without the need for a separate reheating epoch. Dissipation effects also modify the growth of inflaton fluctuations with a distinctive imprint on the primordial spectrum.

II. PROBLEMS OF WARM INFLATION

It was shown that the idea of WI is not easy to present as a concrete model [6]. An inflation field with direct coupling to light fields has problems. For example, Yukawa interaction $\lambda\phi\bar{\psi}\psi$ leads to fermions with mass $m_\psi = \lambda\phi$. Typically large inflaton values are needed in the slow-roll limit. But we want light fermions. On the other hand, when the small coupling λ is considered, the thermal bath condition $T > H$ may not be sustained. Direct coupling to light fields may add a large thermal correction to inflaton mass $\sim\lambda T$, which can stop the slow-roll condition $\dot{\phi}^2 \ll V(\phi)$ in the warm inflation regime $T > H$. In the literature, WI has mostly been explained by inflatons, which indirectly couple to light DOF through heavy intermediate fields [7,8]. Thermal correction of the inflaton mass is exponentially suppressed, but there is a new problem in this realization: The dissipation coefficient is suppressed by powers $(\frac{T}{M_m}) < 1$, where M_m is the mediator mass. Solving this problem implies that we need a large number of intermediate fields, which is required to hold a thermal bath for 50–60 e-folds of inflation. The case of the brane constructions can be used for WI realization, which is discussed in [9].

III. WARM INFLATION WITH A FEW LIGHT FIELDS

Recently a new idea of WI was published called "warm little inflaton" (WLI) [5]. In the WLI scenario, for the first time, WI may be realized by directly coupling pseudo-Nambu-Golden bosons (PNGBs) of broken gauge symmetry as an inflaton field to a few light fields. The critical point in WLI is that the Higgs boson is the PNGB of a broken gauge symmetry breaking which has a protected mass against large radiative corrections [10]. In this paper, we will introduce a new warm inflation model where the inflaton directly couples to light fields. In our scenario, Chern-Simons interaction between the inflaton field and non-Abelian gauge fields will be proposed:

$$\mathcal{L} = \sqrt{-g}\left[-\frac{R}{2} - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a - \frac{1}{2}\partial_\mu\partial^\mu\varphi - V(\varphi) - \frac{\varphi}{8M}F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a\right], \tag{9}$$

where M is the symmetry breaking scale, $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{a\mu\nu}$ is the dual field strength [11,12], and

$$F^a_{\mu\nu} = \frac{1}{ig}[D^a_\mu, D^a_\nu] \quad D^a_\mu = \partial_\mu - igA_\mu J^a \quad \mathrm{Tr}[J_a, J_b] = \frac{1}{2}\delta_{ab}. \tag{10}$$

The EOM for the gauge field strength tensor F in the presence of Chern-Simons-like coupling is presented as

$$(\delta^{ab}\nabla_\alpha - gf^{abc}A^c_\alpha)F^{a\alpha\beta} - \frac{\epsilon^{\mu\nu\beta\alpha}}{2M}\partial^{ab}_\alpha(\phi F^b_{\mu\nu}) = 0, \tag{11}$$

where ∇_α is the space-time covariant derivative of the inflaton field and $\phi = \langle\varphi\rangle$ is the thermal average of the inflaton field φ. A thermal system with the finite temperature T in our model is a cosmic expanding plasma which is composed of inflatons with an equation of state (EOS) $w = \frac{P}{\rho} < 0$ and gauge-light field particles with a radiation-like equation of state $w = \frac{P}{\rho} = \frac{1}{3}$ [13].

Light particles are quanta of the gauge field

$$\begin{aligned} A^a_i &= \int \frac{d^3k}{(2\pi)^3}\varepsilon^a_i(k,t)e^{ik^\mu x_\mu} \\ &= \int \frac{d^3k}{(2\pi)^3}\delta^a_i J_a \varepsilon(k,t)e^{ik^\mu x_\mu} \quad A_0 = 0, \end{aligned} \tag{12}$$

where J_a is a generator of $SU(2)$ with commutation relations

$$[J_a, J_b] = if_{abc}J_c, \tag{13}$$

with the $SU(2)$ structure function $f_{abc} = \epsilon_{abc}$. Averaging on quantum fields leads to a homogeneous and isotropic EOM of gauge fields (11),

$$\frac{\ddot{\Phi}}{a} + H\frac{\dot{\Phi}}{a} + 2g^2\frac{\Phi^3}{a^3} = \frac{g}{2M}\dot{\phi}\frac{\Phi^2}{a^2}, \tag{14}$$

where

$$\Phi(t)\delta^a_i = a(t)\psi(t)\delta^a_i = \langle A^a_i\rangle. \tag{15}$$

The background evolution of a warm pseudoscalar (WPS) field in an isotropic and homogeneous thermal bath, using (15) and action (9), is presented by

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = -\frac{1}{2Ma^3}g\partial_t(\Phi^3). \tag{16}$$

In the slow-roll limit of warm inflation $\frac{\ddot{\Phi}}{a} \ll H\frac{\dot{\Phi}}{a}$ (see the Appendix) and in the $g \ll 1$ limit, the equations (14),

$$\frac{\dot{\Phi}}{a} \simeq \frac{g\psi^2}{2MH}\dot{\phi}, \tag{17}$$

and (16),

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = -3H\frac{g\psi^2}{2MH}\frac{\dot{\Phi}}{a}, \tag{18}$$

lead to the EOM of warm inflation,

$$\ddot{\phi} + 3H(1+Q)\dot{\phi} + \frac{dV}{d\phi} = 0, \tag{19}$$

where $Q = (\frac{g\psi^2}{2MH})^2 > 1$, which is a function of background variables. In the high dissipative regime, the variation of background variables during the last 60 e-folds of inflation is very hard [19], so we have an approximately constant dissipation parameter Q. In our model, the main properties of warm inflation $[\frac{T}{H} > 1, \frac{\rho_r}{V(\phi)} < 1]$ can be sustained:

$$\frac{\rho_r}{V} = \frac{\epsilon_\phi}{1+Q}\frac{Q}{1+Q}, \tag{20}$$

where $Q > 1$ [20].

IV. PERTURBATION

In the warm inflation scenario, the curvature power spectrum is modified by the dissipation effect:

$$\Delta^2_{\mathcal{R}} = \frac{V_*(1+Q_*)^2}{24\pi^2 M_p^4 \epsilon_{\phi_*}}\left(1 + 2n_* + \frac{2\sqrt{3}\pi Q_*}{\sqrt{3+4\pi Q_*}}\frac{T_*}{H_*}\right)G(Q). \tag{21}$$

The first term is the power spectrum of quantum fluctuations which is introduced in the cold model of inflation. Another two terms are the modification of the power spectrum in the warm scenario of inflation and G is calculated numerically for temperature dependent dissipation parameters Q [5]. A well-known potential of pseudoscalar fields,

$$U(\phi) = m_\phi^2 f^2 \left(1 + \cos\left(\frac{\phi}{f}\right)\right), \tag{22}$$

is suitable for the inflation epoch, where $\frac{\phi}{f}$ is not close to π. We can study this potential in the context of the warm pseudoscalar field model. Leading terms of perturbation parameters in slow-roll and high dissipative $Q > 1$ limits are presented in Table I. In Figs. 1 and 2, the best fits of the model with observational data are presented for $Q > 1$ and $f < M_p$ limits.

TABLE I. Important perturbation parameters of pseudoscalar warm inflation in the high dissipative regime $Q > 1$ compared with observational data.

$Q > 1$	Theoretical amount	Constant parameter
$\Delta_{\mathcal{R}}$	$\Delta_{0\mathcal{R}}(\frac{U}{\epsilon_\phi})^{\frac{3}{2}}$	$\Delta_{0\mathcal{R}} = \frac{Q^{\frac{9}{4}}}{2\sqrt{2}\pi(2\pi M_p^2)^{\frac{3}{2}}}$
$n_s - 1$	$-\frac{1.5}{N} - \frac{4.5}{\alpha}$	$\alpha = \frac{2(1+Q)f^2}{M_p^2}$, $\alpha > N$
r	$r_0 \epsilon_\phi^{\frac{3}{4}} U^{\frac{1}{4}}$	$r_0 = \frac{16}{3\pi M_p^4 \Delta_{0\mathcal{R}}}$
$\sin(\frac{\phi_*}{2f})$	$A_0 \exp(-\frac{3}{\alpha}N)$	$A_0 = \cos(\frac{\sqrt{2}}{\alpha})$

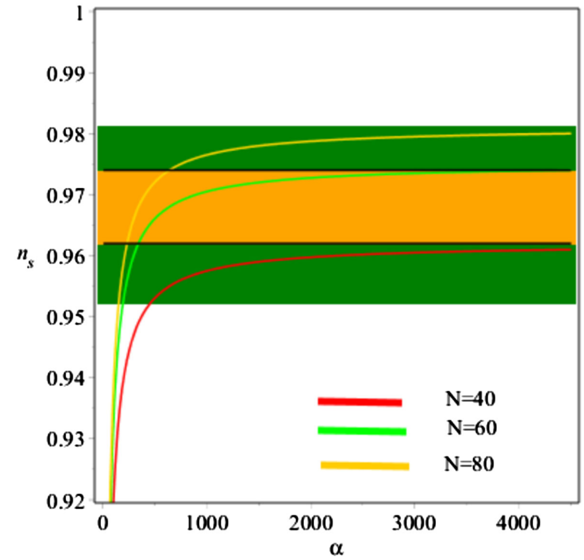

FIG. 1. Variation of n_s in terms of parameter α for three cases of numbers of e-folds.

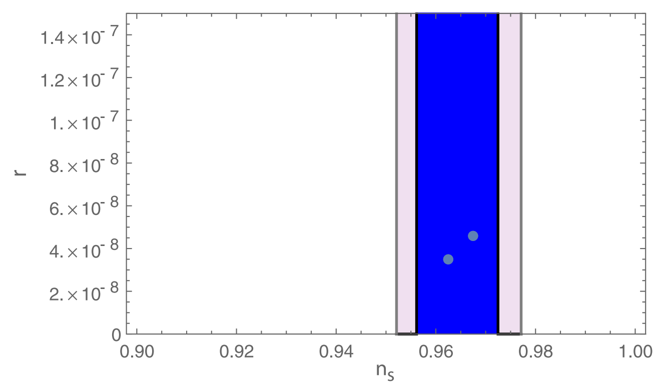

FIG. 2. (r, n_s) of our model for two cases $N = 60$ and $N = 50$ in the $1-\sigma$ confidence level of $r - n_s$ Planck results [22].

V. NON-GAUSSIANITY

Now we consider the non-Gaussianity of our model using δN formalism [21]. In this method, the perturbation theory of cosmology is studied by a quasihomogeneous, spatially flat, Friedman–Lemaitre–Robertson–Walker (FLRW) space-time with scale factor $a(t)$. The spatial part of the FLRW metric is presented by

$$g_{ij} = a^2(t)e^{2\zeta(t,\mathbf{x})}\gamma_{ij}(t,\mathbf{x}), \tag{23}$$

where the primordial curvature perturbation ζ is actually the perturbation of $\ln(a)$, which has an approximately Gaussian power spectrum, and γ_{ij} is the tensor perturbation. The nearly Gaussian term ζ, at least at second order, which has good accuracy, is presented by derivatives of the number of e-folds with respect to the inflaton field ϕ:

$$\zeta(t,\mathbf{x}) = N_\phi \delta\phi + \frac{1}{2}N_{\phi\phi}(\delta\phi)^2. \tag{24}$$

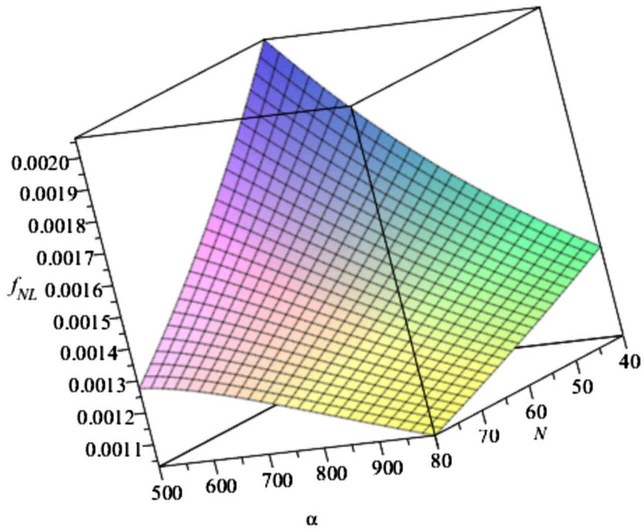

FIG. 3. Small values of non-Gaussianity are presented for reasonable values of α.

Perturbation of the inflaton field $\delta\phi$ in the warm inflation scenario is almost Gaussian, which leads to the scale independent nonlinear parameter $f_{\rm NL}$ as a function of derivatives of N with respect to the inflaton field [21]:

$$-\frac{3}{5}f_{\rm NL} = \frac{1}{2}\frac{N_{\phi\phi}}{N_\phi^2}, \tag{25}$$

where $N_\phi = -\frac{1}{M_p^2}\frac{U(1+Q)}{U_\phi}$. The nonlinear parameter of the non-Gaussianity parameter $f_{\rm NL}$ in the $Q \simeq$ const case is presented by

$$f_{\rm NL} = \frac{5}{6}\frac{1}{1+Q}(2\epsilon_\phi - \eta_\phi), \tag{26}$$

which is in order of the Hubble slow-roll parameters. For our potential (22), the level of non-Gaussianity is a function of the number of e-folds and parameter α:

$$f_{\rm NL} = \frac{5}{3\alpha\cos^2(\frac{\sqrt{2}}{\alpha})}\exp\left(-\frac{6}{\alpha}N\right). \tag{27}$$

Therefore, we have an insignificant non-Gaussianity of warm pseudoscalar inflation even with $f < M_p$. In Fig. 3, the variations of non-Gaussianity, during inflation, in terms of parameter α are presented.

VI. OBSERVATION CONSTRAINT

In Fig. 1, we show the spectral index in terms of parameter α for some amounts of the number of e-folds. The yellow region is the 1σ confidence level of the spectral index. For the $N = 60$ case, we have $450 < \alpha < 4500$ amounts, which agree with observational data. Using $\alpha = 600$ and two cases $N = 50$ and $N = 60$, we present two points in the 1σ confidence level of the $r - n_s$ graph (2), where the upper point is the case ($\alpha = 600$, $N = 50$) and the next is ($\alpha = 600$, $N = 60$), which are in good agreement with observational data with a very small amount of tensor-to-scalar ratio. Using CMB observational data, we found that the sub-Planckian limit of symmetry breaking parameter f is happened in high dissipative regime $Q > 1$. There are some methods to constrain these parameters (f, Q) in warm inflation [23,25,26,36]. We will use these methods for more exact constraints of our model parameters in future works. In a phase space of model parameters (α, N) we show that our model has small amounts of non-Gaussianity (3).

VII. DISCUSSION AND CONCLUSIONS

In this paper, we introduce a model of warm inflation with a direct connection between axion inflaton fields and light gauge fields. In the slow-roll limit, we have found approximately constant dissipation parameter Q in term of background parameters of the model. The model is in good agreement with observational data in the high dissipative regime $Q > 1$, which can resolve the swampland conjecture [27–32]. The small amounts of non-Gaussianity are found for the allowed phase space of the model parameters. We can compare our model with the famous Starobinsky cold inflation model with the asymptotic behavior of the effective potential as $V(\phi) \propto [1 - 2{\rm e}^{-B\phi/M_{\rm pl}} + \mathcal{O}({\rm e}^{-2B\phi/M_{\rm pl}})]$ in the slow-roll limit where the perturbation parameters are presented by [33–35]: $r \approx 8/B^2N^2$ and $n_s \approx 1–2/N$, where $B^2 = 2/3$. For $N = 60$ and $N = 50$, the Starobinsky perturbation parameters are $(n_s, r) \approx (0.967, 0.0033)$ and $(n_s, r) \approx (0.96, 0.0048)$, respectively, but our model has smaller values of a tensor-to-scalar ratio (see Fig. 2). On the other hand, the comparison with warm little inflation [5], which agrees with observation in the $Q < 1$ limit, shows that our result is in a better situation with the smaller value of the tensor-to-scalar ratio in $Q > 1$, which is needed to resolve the swampland conjecture [27–32]. Non-Gaussianity of our model (27) is also very small in comparison with other warm inflation models [21,36].

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APPENDIX: SLOW-ROLL CONDITION OF LIGHT FIELDS

In this section, we present the slow-roll condition of the effective radiation field Φ and EOMs by using energy conservation:

$$\dot{\rho} + 3H(\rho + P) = 0$$
$$\rho = \rho_\phi + \rho_r$$
$$P = P_\phi + P_r, \qquad (A1)$$

which leads to

$$\dot{\rho}_\phi + 3H(\rho_\phi + P_\phi) = -[\dot{\rho}_r + 3H(\rho_r + P_r)]. \qquad (A2)$$

Using the action of our model, we present

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$
$$P_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) \qquad (A3)$$

and

$$\rho_r = \frac{3}{2}\left(\frac{\dot{\Phi}^2}{a^2} + g^2\frac{\Phi^4}{a^4}\right)$$
$$P_r = \frac{1}{2}\left(\frac{\dot{\Phi}^2}{a^2} + g^2\frac{\Phi^4}{a^4}\right). \qquad (A4)$$

In the warm inflation scenario, the slow-roll limit of the radiation part is presented by $\dot{\rho}_r \ll 4H\rho_r$. Using the above equation, we present

$$\dot{\rho}_r = \frac{3\dot{\Phi}}{a}\left(\frac{\ddot{\Phi}}{a} - H\frac{\dot{\Phi}}{a} + \frac{2g^2\Phi^3}{a^3}\right) - \frac{6Hg^2\Phi^4}{a^4}. \qquad (A5)$$

In the slow-roll and $g \ll 1$ limits, we find the slow-roll condition for thermalizing the radiation field Φ as

$$\frac{\ddot{\Phi}}{a} \ll 3H\frac{\dot{\Phi}}{a}. \qquad (A6)$$

On the other hand, we can find the lhs of the scalar field and effective radiation field EOMs using energy conservation (A1):

$$\dot{\phi}\left(\ddot{\phi} + 3H\dot{\phi} + \frac{dV(\phi)}{d\phi}\right) = -\frac{3\dot{\Phi}}{a}\left(\frac{\ddot{\Phi}}{a} + H\frac{\dot{\Phi}}{a} + \frac{2g^2\Phi^3}{a^3}\right). \qquad (A7)$$

This relation agrees with the EOMs

$$\dot{\phi}\left[\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi}\right] = \left[-\frac{1}{a^3}\frac{g}{2M}\partial_t(\Phi^3)\right]\dot{\phi}$$
$$\frac{3\dot{\Phi}}{a}\left[\frac{\ddot{\Phi}}{a} + H\frac{\dot{\Phi}}{a} + 2g^2\frac{\Phi^3}{a^3}\right] = \left[\frac{g}{2M}\dot{\phi}\frac{\Phi^2}{a^2}\right]\frac{3\dot{\Phi}}{a}, \qquad (A8)$$

which come from the variation of the effective action. We note that the rhs's of the EOMs come from the Chern–Simons interaction in the action.

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