Addendum to "Impact of polarization observables and $B_c \to \tau\nu$ on new physics explanations of the $b \to c\tau\nu$ anomaly"

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In this addendum to Ref. [1], we update our results to include the recent measurement of $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ by the Belle Collaboration [2]: $\mathcal{R}(D)_{\text{Belle}} = 0.307 \pm 0.037 \pm 0.016$ and $\mathcal{R}(D^*)_{\text{Belle}} = 0.283 \pm 0.018 \pm 0.014$, resulting in the new HFLAV fit result $\mathcal{R}(D) = 0.340 \pm 0.027 \pm 0.013$, $\mathcal{R}(D^*) = 0.295 \pm 0.011 \pm 0.008$, exhibiting a 3.1σ tension with the Standard Model. We present the new fit results and update all figures, including the relevant new collider constraints. The updated prediction for $\mathcal{R}(\Lambda_c)$ from our sum rule reads $\mathcal{R}(\Lambda_c) = \mathcal{R}_{\text{SM}}(\Lambda_c)(1.15 \pm 0.04) = 0.38 \pm 0.01 \pm 0.01$. We also comment on theoretical predictions for the fragmentation function f_c of $b \to B_c$ and their implication on the constraint from $B_{u/c} \to \tau\nu$ data.

In this Addendum, we present an update of our article [1] in which we studied the impact of polarization observables and the bound on $\text{BR}(B_c \to \tau\nu)$ on new physics explanations of the $b \to c\tau\nu$ anomaly.

Our updated results incorporate the new experimental results for $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ measured by the Belle Collaboration [2]:

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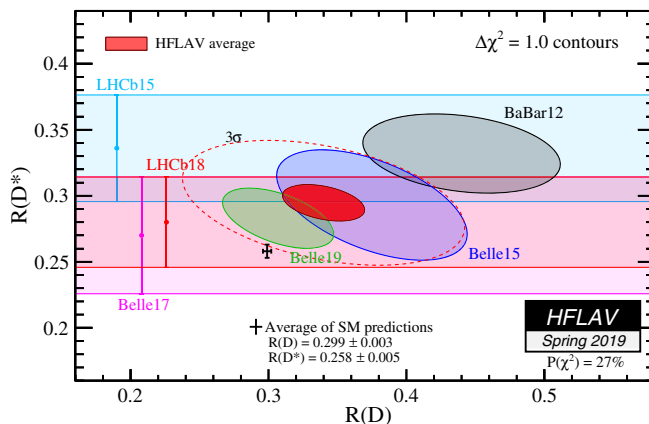[‖]ulrich.nierste@kit.edu
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FIG. 1. The green ellipse shows the result of the new measurement by the Belle Collaboration [2], while the red ellipse shows the new world average. The SM predictions are represented by the black bars. Figure taken from Ref. [3].

TABLE I. Updated fit results for the 1D hypotheses (hyp.) of Ref. [1], with the Wilson coefficients defined at the scale $\mu = 1$ TeV.

1D hyp.	best-fit	1σ range	2σ range	p-value (%)	pull$_{\rm SM}$	$\mathcal{R}(D)$	$\mathcal{R}(D^*)$	$F_L(D^*)$	$P_\tau(D^*)$	$P_\tau(D)$	$\mathcal{R}(\Lambda_c)$
C_V^L	0.07	[0.05, 0.09]	[0.04, 0.11]	44	4.0	0.347 $+0.2\sigma$	0.292 -0.2σ	0.46 -1.6σ	-0.49 -0.2σ	0.32	0.38
C_S^R	0.09	[0.06, 0.11]	[0.03, 0.14]	2.7	3.1	0.380 $+1.4\sigma$	0.260 -2.6σ	0.47 -1.5σ	0.46 -0.1σ	0.46	0.36
C_S^L	0.07	[0.04, 0.10]	[$-0.00, 0.13$]	0.26	2.1	0.364 $+0.8\sigma$	0.250 -3.3σ	0.45 -1.7σ	-0.51 -0.2σ	0.44	0.35
$C_S^L = 4C_T$	-0.03	[$-0.07, 0.01$]	[$-0.11, 0.04$]	0.04	0.7	0.278 -2.1σ	0.263 -2.3σ	0.46 -1.6σ	-0.47 -0.2σ	0.27	0.33

TABLE II. Updated fit results for the 2D hypotheses (hyp.) of Ref. [1], with the Wilson coefficients defined at the scale $\mu = 1$ TeV.

2D hyp.	best-fit	p-value (%)	pull$_{\rm SM}$	$\mathcal{R}(D)$	$\mathcal{R}(D^*)$	$F_L(D^*)$	$P_\tau(D^*)$	$P_\tau(D)$	$\mathcal{R}(\Lambda_c)$
$(C_V^L, C_S^L = -4C_T)$	$(0.10, -0.04)$	29.8	3.6	0.333 -0.2σ	0.297 $+0.2\sigma$	0.47 -1.5σ	-0.48 -0.2σ	0.25	0.38
$(C_S^R, C_S^L)\|_{60\%}$	$(0.29, -0.25)$ $(-0.16, -0.69)$	75.7	3.9	0.338 0.1σ	0.297 $+0.1\sigma$	0.54 -0.7σ	-0.27 $+0.2\sigma$	0.39	0.38
$(C_S^R, C_S^L)\|_{30\%}$	$(0.21, -0.15)$ $(-0.26, -0.61)$	30.9	3.6	0.353 $+0.4\sigma$	0.280 -1.1σ	0.51 -1.0σ	-0.35 0.0σ	0.42	0.37
$(C_S^R, C_S^L)\|_{10\%}$	$(0.11, -0.04)$ $(-0.37, -0.51)$	2.6	2.9	0.366 $+0.9\sigma$	0.263 -2.3σ	0.48 -1.4σ	-0.44 -0.1σ	0.44	0.36
(C_V^L, C_S^R)	$(0.08, -0.01)$	26.6	3.6	0.343 $+0.1\sigma$	0.294 -0.1σ	0.46 -1.6σ	-0.49 -0.2σ	0.31	0.38
$(\mathrm{Re}[C_S^L = 4C_T], \mathrm{Im}[C_S^L = 4C_T])\|_{60,30\%}$	$(-0.06, \pm0.31)$	25.0	3.6	0.339 0.0σ	0.295 0.0σ	0.45 -1.7σ	-0.41 -0.1σ	0.41	0.38
$(\mathrm{Re}[C_S^L = 4C_T], \mathrm{Im}[C_S^L = 4C_T])\|_{10\%}$	$(-0.03, \pm0.24)$	5.9	3.2	0.330 -0.3σ	0.275 -1.4σ	0.46 -1.6σ	-0.45 -0.1σ	0.38	0.36

$$\mathcal{R}(D)_{\rm Belle} = 0.307 \pm 0.037 \pm 0.016,$$

$$\mathcal{R}(D^*)_{\rm Belle} = 0.283 \pm 0.018 \pm 0.014. \qquad (1)$$

The first quoted error is statistical and the second one is systematic. The new measurement is consistent with the Standard Model (SM) predictions [3]

$$\mathcal{R}_{\rm SM}(D) = 0.299 \pm 0.003,$$

$$\mathcal{R}_{\rm SM}(D^*) = 0.258 \pm 0.005 \qquad (2)$$

at the 0.2σ and 1.1σ level, respectively.

Combining this with the previous measurements presented by the *BABAR*, Belle, and LHCb collaborations in Refs. [4–12], the HFLAV Collaboration [3] has determined the averages

$$\mathcal{R}(D) = 0.340 \pm 0.027 \pm 0.013,$$

$$\mathcal{R}(D^*) = 0.295 \pm 0.011 \pm 0.008, \qquad (3)$$

with an $\mathcal{R}(D)$–$\mathcal{R}(D^*)$ correlation of -0.38. The new world averages deviate from the SM at 1.4σ [$\mathcal{R}(D)$], 2.5σ [$\mathcal{R}(D^*)$], and 3.1σ [$\mathcal{R}(D)$–$\mathcal{R}(D^*)$ combination] [3]. This situation is shown in Fig. 1.

Including all four observables $\mathcal{R}(D), \mathcal{R}(D^*), P_\tau(D^*)$ and $F_L(D^*)$,[1] we find the new p-value of the two-sided test for the SM

$$p\text{-value}_{\rm SM} \sim 0.1\%, \qquad (4)$$

which corresponds to a 3.3σ tension, where we neglect the SM uncertainty. Note that our choice of the form factors was explained in Ref. [1], and we obtain the following central values of the SM predictions:

$$\mathcal{R}_{\rm SM}(D) = 0.301, \qquad \mathcal{R}_{\rm SM}(D^*) = 0.254,$$

$$P_{\tau,\rm SM}(D) = 0.32, \qquad P_{\tau,\rm SM}(D^*) = -0.49,$$

$$F_{L,\rm SM}(D^*) = 0.46, \qquad \mathcal{R}_{\rm SM}(\Lambda_c) = 0.33. \qquad (5)$$

All our fit results are based on these numbers.[2]

The authors of Ref. [15] deduced the stringent constraint $\mathrm{BR}(B_c \to \tau\nu) < 10\%$ from data on a mixed sample of $B_c^- \to \tau\nu_\tau$ and $B^- \to \tau\nu$ candidate events taken at the Z

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[1]The impact of the $F_L(D^*)$ measurement on new physics in $b \to c\tau\nu$ was previously considered in Refs. [13,14].

[2]On the other hand, based on the SM predictions in Eq. (2), we obtain p-value$_{\rm SM} \sim 0.2\%$ corresponding to a 3.1σ tension instead of Eq. (4).
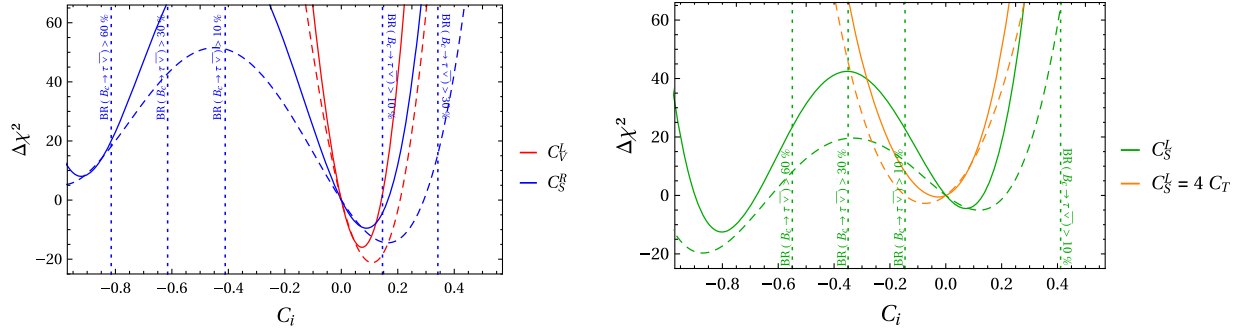
FIG. 2. $\Delta\chi^2$ of $\mathcal{R}(D)$, $\mathcal{R}(D^*)$, $P_\tau(D^*)$ and $F_L(D^*)$ for the four one-dimensional (1D) scenarios where $\mu = 1$ TeV. The dashed lines do not include the latest Belle results [2], while the solid lines include all data. The dotted vertical lines correspond to the limit on $C_S^{L,R}$ from BR($B_c \to \tau\nu$) assuming a maximal value of 10%, 30% or 60%. Best-fit points are not constrained from the 10% limit.
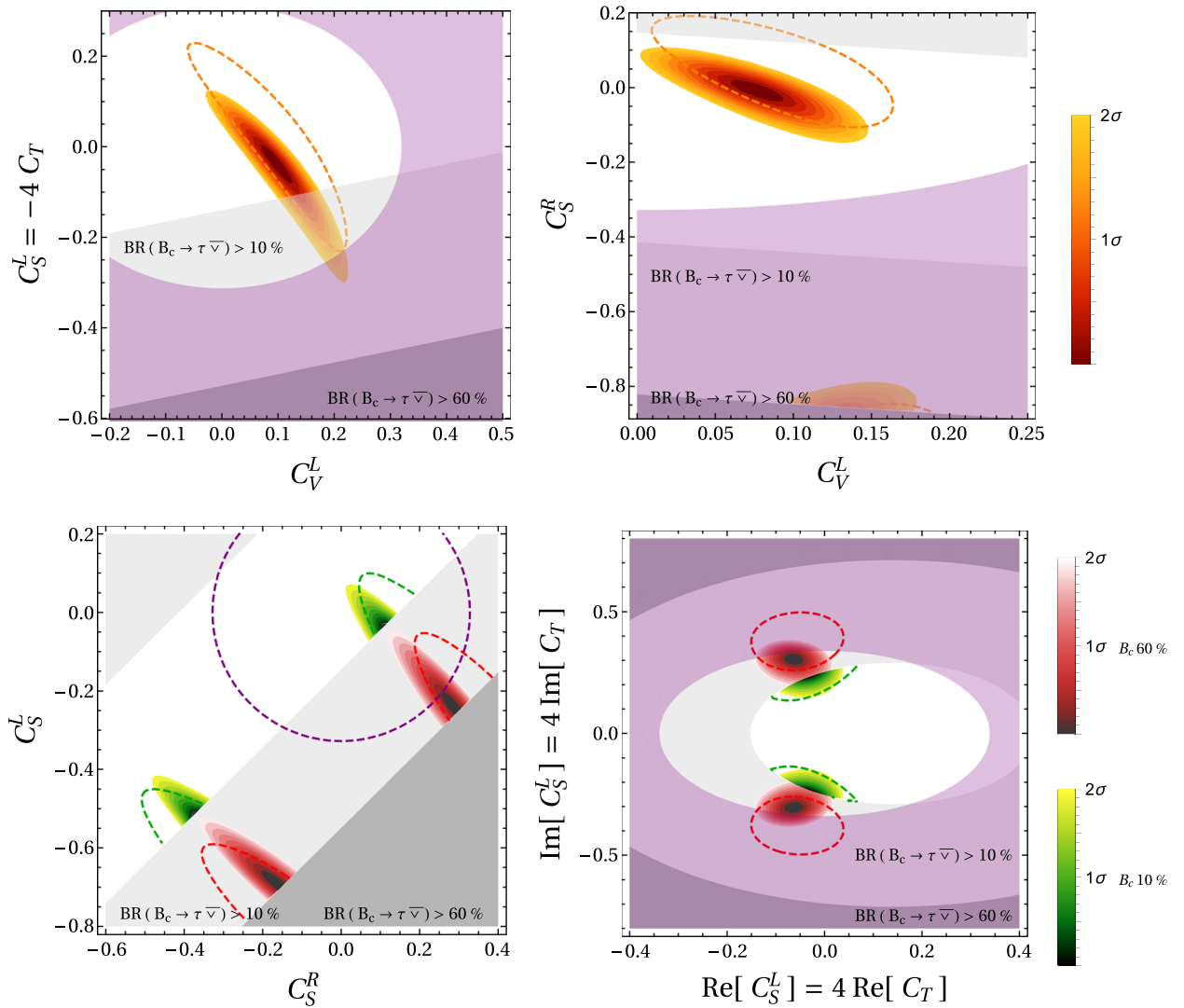

FIG. 3.   Updated results of the fits for the 2σ regions in the four 2D scenarios of Ref. [1], with Wilson coefficients given at the matching scale of 1 TeV. The dashed contours do not include the latest Belle results [2], while the shaded ellipses include all data. The current collider bounds in Eq. (9) exclude the purple shaded regions at the 2σ level. The dashed purple circle in the lower left plot indicates the collider constraint on the charged Higgs scenario (see text).
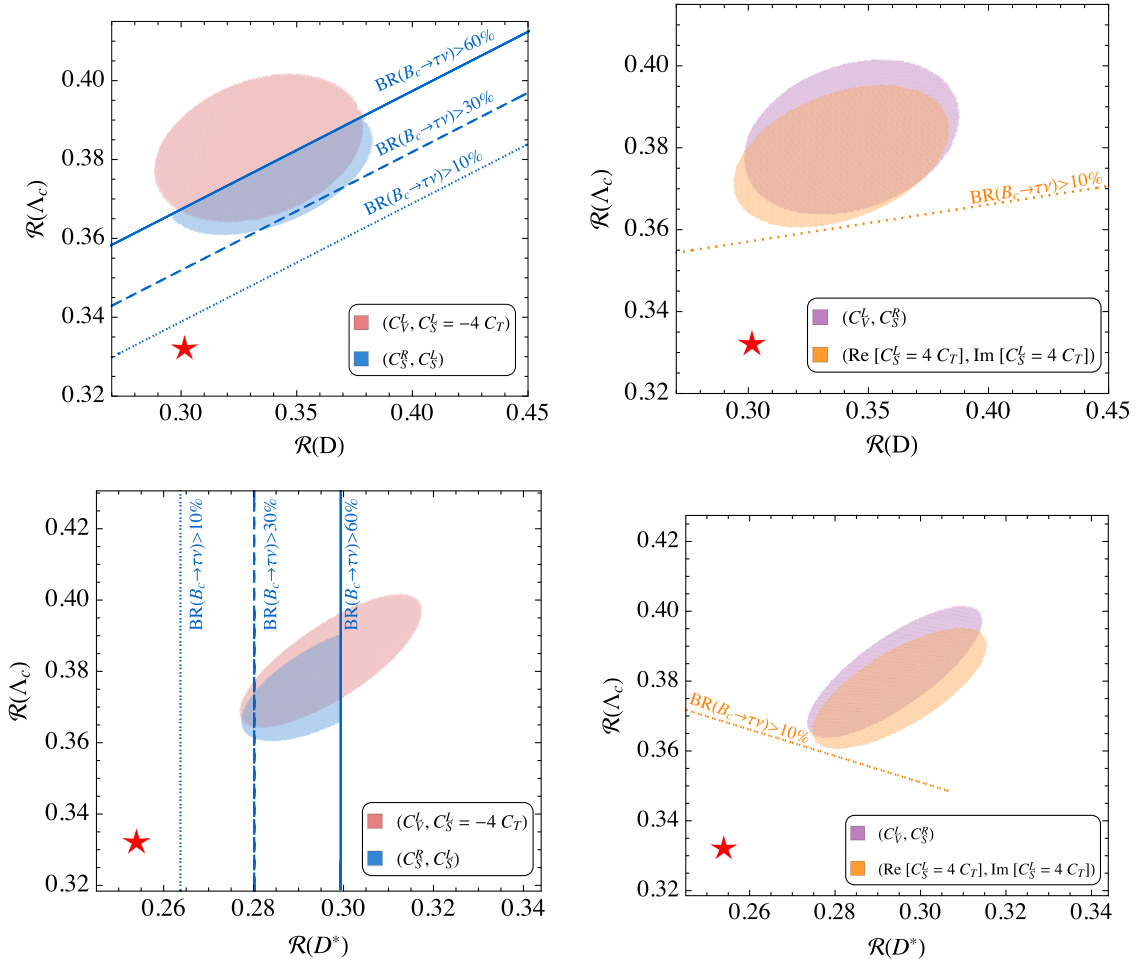
FIG. 4.   Preferred 1σ regions in the four two-dimensional scenarios in the $\mathcal{R}(D^{(*)})$–$\mathcal{R}(\Lambda_c)$ plane for $\mathrm{BR}(B_c \to \tau\nu_\tau) < 60\%$, updating Fig. 3 of Ref. [1].

peak in the LEP experiment. To this end, the fragmentation function f_c of $b \to B_c^-$ has been extracted from data accumulated at hadron colliders. For asymptotically large values of the transverse b momentum p_T, fragmentation functions are numbers which are independent of the kinematical variables and the b production mechanism. In Ref. [1], we pointed out that hadron collider data exhibit a sizable p_T dependence and pointed to production mechanisms beyond fragmentation (see also Ref. [16]). In Fig. 1 of Ref. [15], f_c/f_u was extracted from CMS and LHCb data. Using the world average of the $b \to B^-$ fragmentation function $f_u = 0.404(6)$ [17], we find that the result of Ref. [15] implies

$$2.1 \times 10^{-3} \lesssim f_c \lesssim 4.4 \times 10^{-3}. \tag{6}$$

If one instead uses a calculation of B_c^- production on the Z peak at e^+e^- colliders employing nonrelativistic quantum chromodynamics (NRQCD) at next-to-leading order [18,19] (see also Ref. [20]), one finds

$$f_c \sim 3 \times 10^{-4}, \tag{7}$$

with essentially the same estimate for $b \to B_c^{*-}$ fragmentation. If one further assumes that B_c^{*-} decays into final states with B_c^- with a branching ratio of 1,[3] then f_c effectively changes to

$$f_c \sim 6 \times 10^{-4}. \tag{8}$$

Therefore by comparing Eqs. (6) and (8), we conclude that the constraint on $\mathrm{BR}(B_c \to \tau\nu)$ derived in Ref. [15] is too stringent by a factor of 3 to 4. Taking into account the intrinsic uncertainties of the NRQCD calculation, the Z peak data cannot rule out our most conservative scenario which permits $\mathrm{BR}(B_c \to \tau\nu)$ to be as large as 60%.

Tables I and II update the respective tables in Ref. [1], showing the numerical results of the fit in the various one- (1D) and two-dimensional (2D) scenarios for the Wilson coefficients. The corresponding plots are shown in Figs. 2 and 3. In all cases, the best-fit points moved closer to the SM, with the biggest change being in the one-dimensional scalar scenarios.

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[3]While $B_c(2S)^-$ and $B_c^*(2S)^-$ have been observed through a transition of $B_c^{(*)}(2S)^- \to B_c^{(*)-}\pi^+\pi^-$ [21–23], no B_c^{*-} has been detected yet.
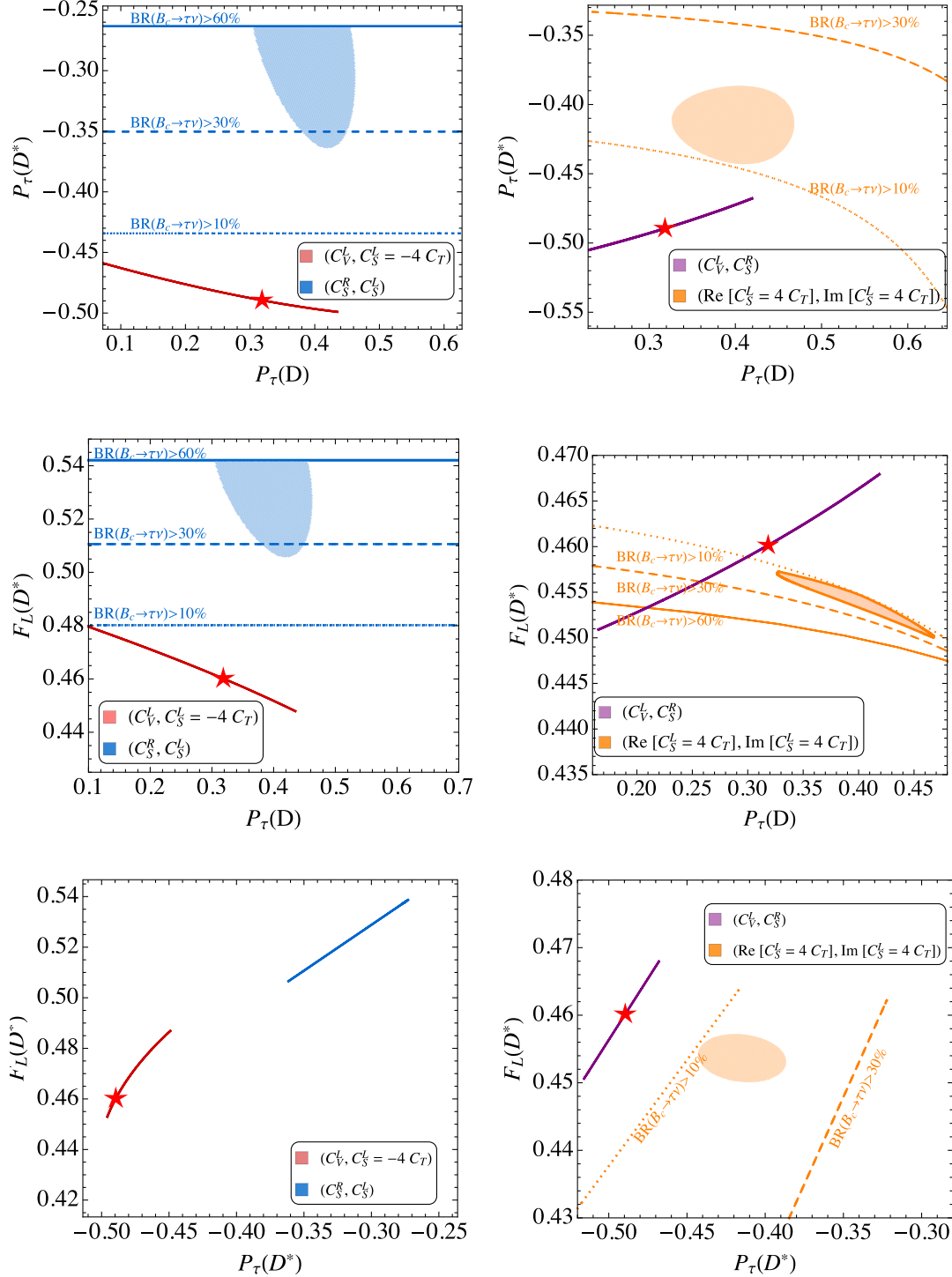
FIG. 5.   Pairwise correlations between the observables $P_\tau(D), P_\tau(D^*)$ and $F_L(D^*)$, updating Fig. 4 of Ref. [1].

In the C_S^R scenario, the best-fit point is hence no longer in tension with the aggressive $\mathrm{BR}(B_c \to \tau\nu) < 10\%$ bound.

The most general and powerful collider constraint on the $b \to c\tau\nu$ operators comes from high-p_T tails in mono-τ searches. Reference [24] investigated the constraints on the effective field theory (EFT) operators mediating $b \to c\tau\nu$. This EFT analysis is valid for certain leptoquark models

if the leptoquarks are sufficiently heavy.[4] The resulting 2σ upper bounds from the current collider data are [24]

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[4]Direct searches for leptoquarks coupled to third-generation quarks constrain their masses to roughly $m_{\mathrm{LQ}} > 1$ TeV [25,26]. These direct collider bounds significantly depend on the branching fractions of the leptoquarks.
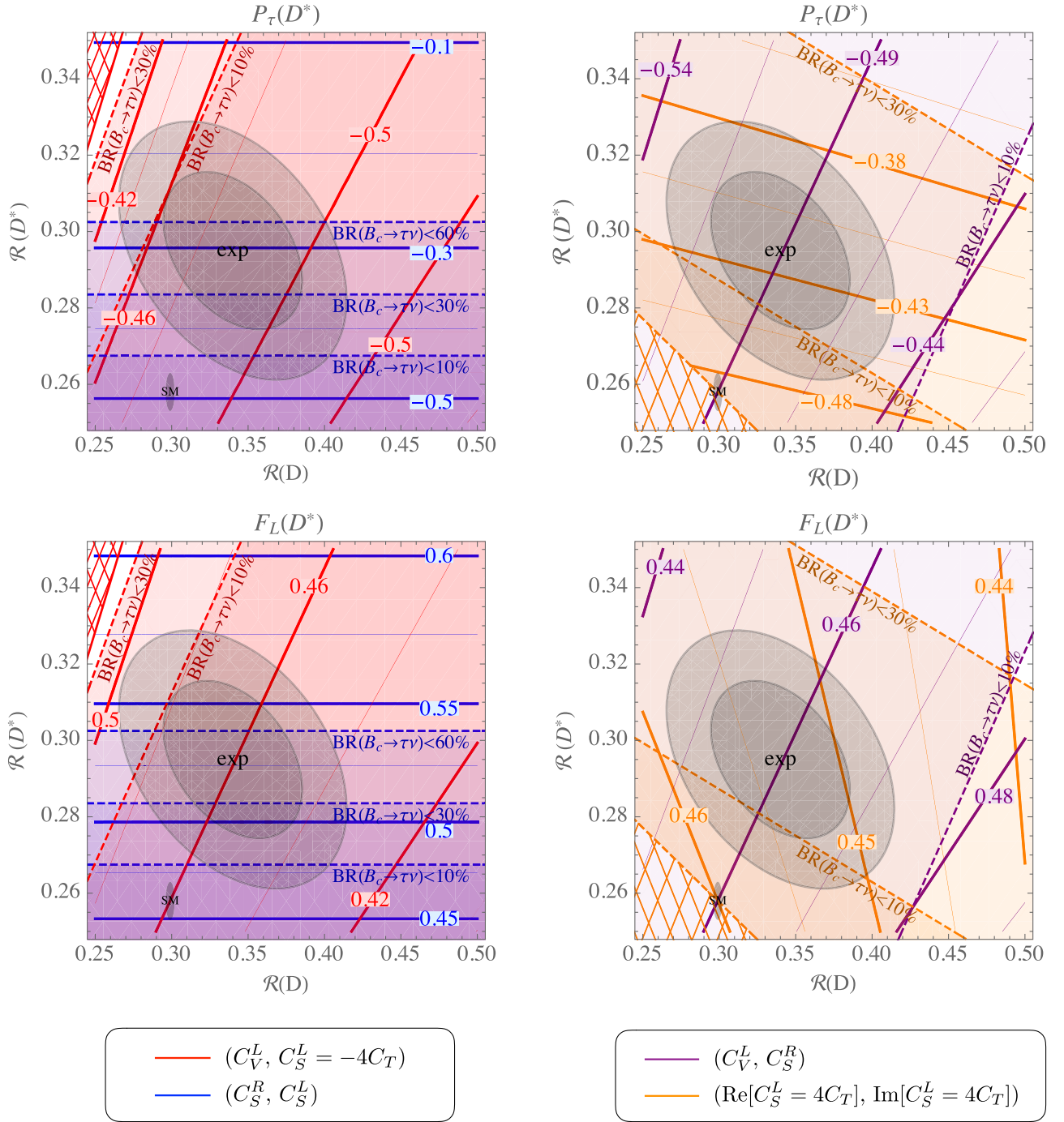
FIG. 6. Contour lines of the τ polarization and the longitudinal D^* polarization for the two-dimensional scenarios in the $\mathcal{R}(D)$–$\mathcal{R}(D^*)$ plane, updating Fig. 5 of Ref. [1].

$$|C_V^L| < 0.32, \qquad |C_S^{L(R)}| < 0.57, \qquad |C_T| < 0.16, \qquad (9)$$

at the scale $\mu = m_b$. In Fig. 3, we apply these collider bounds to the four two-dimensional scenarios, where we assume that interference between two different operators is suppressed. Note that in contrast to our findings in Ref. [1], the best-fit points in the complex $C_S^L = 4C_T$ scenario are no

longer in tension with the collider constraints. Scenarios with color-singlet s-channel mediators, like a charged scalar, require model-dependent studies beyond the EFT framework; see e. g. Refs. [27,28]. Hence, for the (C_S^R, C_S^L) scenario originating from the exchange of a charged Higgs boson, the collider bound is valid only in the heavy-mass limit, and we therefore indicate it by a dashed line.

Figure 4 shows the prediction for $\mathcal{R}(\Lambda_c)$ in the four two-dimensional scenarios, as functions of $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$, respectively. In Ref. [1], we obtained a sum rule

$$\frac{\mathcal{R}(\Lambda_c)}{\mathcal{R}_{\mathrm{SM}}(\Lambda_c)} \simeq 0.262 \frac{\mathcal{R}(D)}{\mathcal{R}_{\mathrm{SM}}(D)} + 0.738 \frac{\mathcal{R}(D^*)}{\mathcal{R}_{\mathrm{SM}}(D^*)}. \quad (10)$$

The decrease in $\mathcal{R}(D^{(*)})$ implied by the new Belle measurement leads to a decreased prediction for $\mathcal{R}(\Lambda_c)$ through our sum rule [1]

$$\begin{aligned}\mathcal{R}(\Lambda_c) &= \mathcal{R}_{\mathrm{SM}}(\Lambda_c)(1.15 \pm 0.04)\\ &= 0.38 \pm 0.01 \pm 0.01,\end{aligned} \quad (11)$$
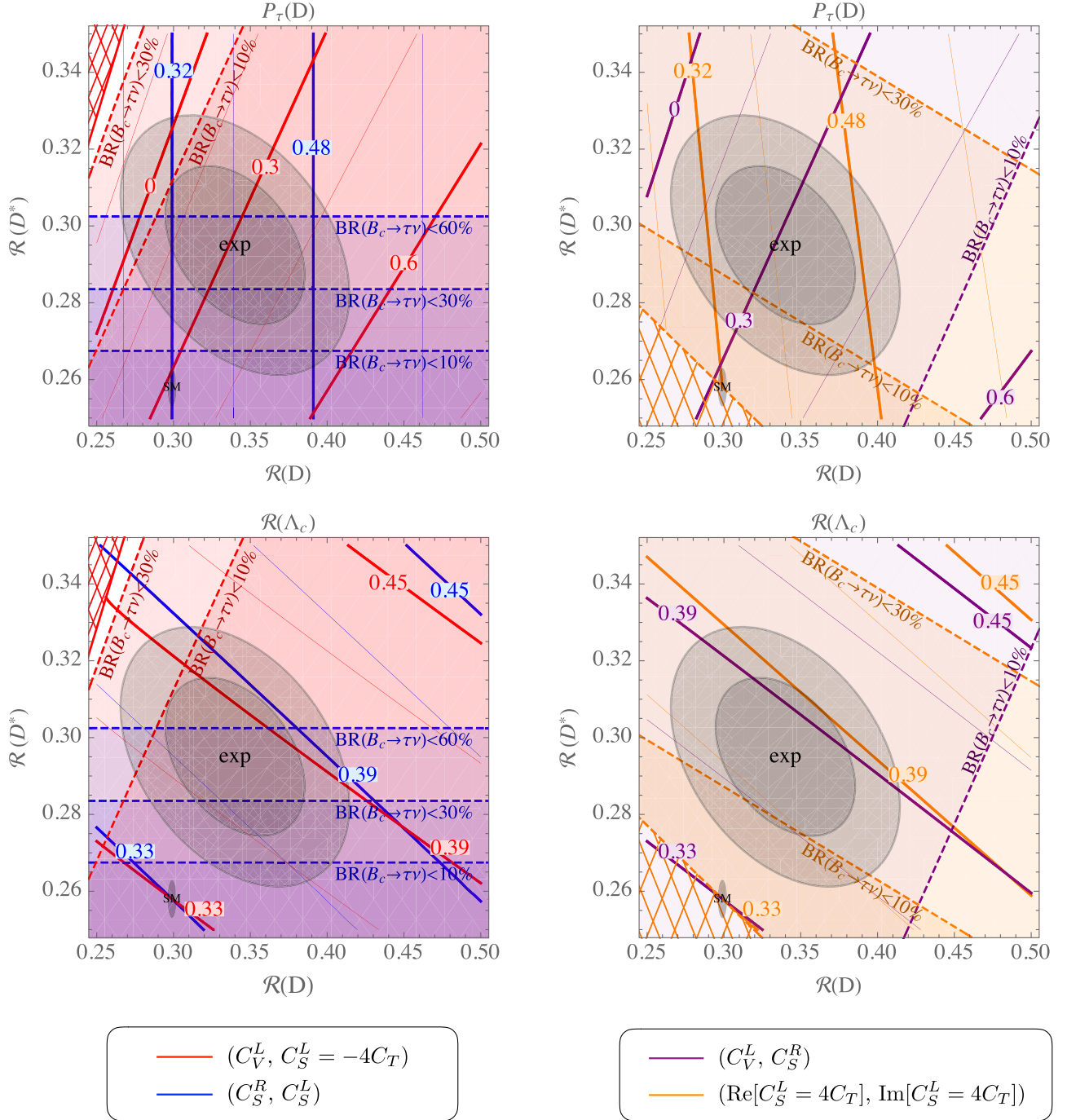

FIG. 7.   Contour lines of $P_\tau(D)$ and $\mathcal{R}(\Lambda_c)$ for the two-dimensional scenarios in the $\mathcal{R}(D)$–$\mathcal{R}(D^*)$ plane, updating Fig. 6 of Ref. [1].

where the first error arises from the experimental uncertainty of $\mathcal{R}(D^{(*)})$, while the second error comes from the form factors. This model-independent relation between $\mathcal{R}(D)$, $\mathcal{R}(D^*)$, and $\mathcal{R}(\Lambda_c)$ originates from heavy-quark symmetry: in the heavy-quark limit the inclusive $b \to c\tau\nu$ rate is saturated by the sum of $B \to D\tau\nu$ and $B \to D^*\tau\nu$ in the mesonic case, and by $\Lambda_b \to \Lambda_c\tau\nu$ in the baryonic case [29]. We have checked that the sum rule in Eq. (10) also holds for new physics scenarios with right-handed neutrinos, although they are not considered in our analysis.

As shown in Fig. 5, the pairwise correlations between the polarization observables $P_\tau(D)$, $P_\tau(D^*)$, and $F_L(D^*)$ are still distinct for the various two-dimensional scenarios. In order to fully exploit their potential, besides better measurements more precise theoretical predictions for the $B \to D$ and $B \to D^*$ form factors are also necessary.

Figures 6 and 7 show the contour lines of the polarization observables $P_\tau(D)$, $P_\tau(D^*)$, and $F_L(D^*)$ and the ratio $\mathcal{R}(\Lambda_c)$ in the $\mathcal{R}(D)$–$\mathcal{R}(D^*)$ plane. In these plots only the position of the experimentally preferred region for $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$ has been changed with respect to the version shown in Figs. 5 and 6 of Ref. [1].

In conclusion, we have updated our fit results for the $b \to c\tau\nu$ anomaly to include the recent data by the Belle Collaboration [2]. The predictions for polarization observables from the fit significantly depend on the Wilson coefficient scenario. Therefore, by accurately probing their correlations at the ongoing Belle II experiment [30], one can in principle distinguish between different new physics models. To exploit their full discriminatory power, however, more precise predictions of the relevant form factors

are also necessary. Furthermore we revisited the constraint on $\mathrm{BR}(B_c \to \tau\nu_\tau)$ from LEP data at the Z peak, focusing on the theoretical predictions for the fragmentation of a b quark into a B_c meson, and concluded that our most conservative scenario $\mathrm{BR}(B_c \to \tau\nu_\tau) < 60\%$ is not excluded at present. Moreover, reevaluating our sum rule connecting $\mathcal{R}(\Lambda_c)$ with $\mathcal{R}(D^{(*)})$, we predicted an enhancement of $\mathcal{R}(\Lambda_c)$ of $(15 \pm 4)\%$ with respect to its SM value model independently, which serves as a good experimental cross-check of the $b \to c\tau\nu$ anomaly.

ACKNOWLEDGEMENT

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