


R_2 as a single leptoquark solution to $R_{D^{(*)}}$ and $R_{K^{(*)}}$

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We show that, up to plausible uncertainties in $\text{BR}(B_c \rightarrow \tau\nu)$, the R_2 leptoquark can simultaneously explain the observation of anomalies in $R_{K^{(*)}}$ and $R_{D^{(*)}}$ without requiring large couplings. The former is achieved via a small coupling to first generation leptons, which boosts the decay rate $\Gamma(\bar{B} \rightarrow \bar{K}^{(*)} e^+ e^-)$. Finally we motivate a neutrino mass model that includes the S_3 leptoquark, which can alleviate a mild tension with the most conservative limits on $\text{BR}(B_c \rightarrow \tau\nu)$.

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I. INTRODUCTION

There have recently been multiple independent anomalous measurements of semileptonic B decays that depart from standard model (SM) predictions. Rare decays into $D^{(*)}$ mesons show a discrepancy from SM predictions in *BABAR* [1,2], *Belle* [3–5], and *LHCb* [6,7] measurements of the lepton flavor universality (LFU) ratios. New results from *Belle* combined with measurements from *BABAR* and *LHCb* give [8]¹

$$\mathcal{R}_D = \frac{\Gamma(\bar{B} \rightarrow D\tau\bar{\nu})}{\Gamma(\bar{B} \rightarrow De/\mu\bar{\nu})} = \begin{cases} 0.299 \pm 0.003 & \text{SM [9]} \\ 0.335 \pm 0.031 & \text{observed [8]} \end{cases} \quad (1)$$

and

$$\mathcal{R}_{D^*} = \frac{\Gamma(\bar{B} \rightarrow D^*\tau\bar{\nu})}{\Gamma(\bar{B} \rightarrow D^*e/\mu\bar{\nu})} = \begin{cases} 0.258 \pm 0.005 & \text{SM [10]} \\ 0.298 \pm 0.015 & \text{observed [8]} \end{cases}. \quad (2)$$

When the correlation between the two observables is taken into account the significance of the anomaly is at the 3.1σ

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¹The best-fit value and error bars have been extracted from the figure on slide 9.

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level [8]. The SM calculation is reliable as it is largely insensitive to hadronic uncertainties, which cancel out in the ratios $R_{D^{(*)}}$.

LHCb has similarly found an intriguing deviation from LFU in the semileptonic B meson decays to $K^{(*)}$ mesons. The LFU ratios

$$R_{K^{(*)}} = \frac{\Gamma(\bar{B} \rightarrow \bar{K}^{(*)}\mu^+\mu^-)}{\Gamma(\bar{B} \rightarrow \bar{K}^{(*)}e^+e^-)} \quad (3)$$

provide a clean probe of new physics effects because hadronic uncertainties cancel out in the ratios as long as new physics effects are small [11–13]. *LHCb* measured the ratios for the dilepton invariant mass range $1.1 \text{ GeV}^2 < q^2 < 6 \text{ GeV}^2$. A combination of run I and run II from *LHCb* gives

$$R_K = \begin{cases} 1.0003 \pm 0.0001 & \text{SM [14]} \\ 0.846_{-0.054}^{+0.06}(\text{stat})_{-0.014}^{+0.016}(\text{sys}) & \text{observed [15]} \end{cases} \quad (4)$$

and

$$R_{K^*} = \begin{cases} 1.00 \pm 0.01 & \text{SM [16]} \\ 0.716_{-0.057}^{+0.070} & \text{observed [17, 18]} \end{cases} \quad (5)$$

where we combined the *LHCb* measurement [18] of R_{K^*} with the new *Belle* measurement [17] using the methods described in Ref. [19]. Experimental sensitivity to both of these anomalies is expected to improve by orders of magnitude over the next few years and make a potential confirmation of a departure from the SM imminent. The measurements are not just quantitatively different from the

SM but qualitatively so as well, because the SM has no notable violation of lepton flavor universality.

The most common explanation for these anomalies is to extend the SM by leptoquarks (see Refs. [20–38] for a leptoquark solution to the $R_{K^{(*)}}$ anomalies, Refs. [34, 39–45] for the $R_{D^{(*)}}$ anomalies and Refs. [34, 46–69] for simultaneous explanations). Vector leptoquarks have issues with ultraviolet (UV) completion and their tendency is to be heavy in UV complete models. Therefore it is attractive to consider scalar leptoquark solutions to these anomalies. To date the only known candidate that simultaneously explains both sets of anomalies is the S_1 leptoquark [46, 57, 70], but it only satisfies $R_{K^{(*)}}$ at $2 - \sigma$ [57]. In this work we show that the R_2 leptoquark can provide a simultaneous solution at $1 - \sigma$ consistent with all known constraints.

In addition to the anomalous measurements of $R_{D^{(*)}}$ and $R_{K^{(*)}}$, two other anomalies have generated interest: On the one hand, the value of the angular observable P'_5 [71, 72] and more generally the data of $b \rightarrow s\mu\bar{\mu}$ point to a deviation from the SM [73]. While these anomalies are intriguing they are currently less clean signals of new physics due to large hadronic uncertainties and the difficulty in estimating a signal for the P'_5 anomalies [72]. On the other hand, similar to the LFU ratios $R_{D^{(*)}}$ the LFU ratio $R_{J/\psi} = \Gamma(B_c^+ \rightarrow J/\psi\tau\nu)/\Gamma(B_c^+ \rightarrow J/\psi\mu\nu)$ points to a larger branching fraction to τ leptons compared to muons, but it is still consistent with the SM at $2 - \sigma$ due to the large error bars [74]. We therefore leave the consideration of such anomalies to future work.

The R_2 leptoquark has quantum numbers $(3, 2, 7/6)$ with respect to the SM gauge group $SU(3) \times SU(2) \times U(1)$ and has been proposed as a cause of the $R_{D^{(*)}}$ anomalies [10, 75, 76] with $O(1)$ couplings as well as the $R_{K^{(*)}}$ anomalies with very large couplings through a new contribution to the decay $b \rightarrow s\mu\bar{\mu}$ [77–81]. These operators are induced at the one-loop level and thus require undesirably large couplings with at least one coupling needing to be a lot larger than 1. We reopen the case of this leptoquark and find that a more promising route to what has previously been studied is to boost the denominator in Eq. (3), by allowing the leptoquark to couple to electrons.² As the relevant operator is generated at tree level, the required couplings are quite small. The deviations in the LFU ratios $R_{D^{(*)}}$ can be explained at the same time by introducing a coupling of the R_2 leptoquark to τ leptons. A mild tension with the theoretically inferred constraint on $\text{BR}(B_c \rightarrow \tau\nu)$ [83] can be resolved by the introduction of the S_3 leptoquark, which can be motivated within a radiative neutrino mass model.

The structure of this paper is as follows. In Sec. II we perform an effective field theory (EFT) analysis of the R_2

leptoquark. We then explain the most relevant constraints in Sec. III and show that R_2 can explain $R_{K^{(*)}}$ and $R_{D^{(*)}}$. In Sec. IV we introduce a minimal model for neutrino masses based on the R_2 and S_3 leptoquarks. Finally we conclude in Sec. V.

II. EFFECTIVE FIELD THEORY ANALYSIS FOR THE R_2 LEPTOQUARK

The $R_2 \sim (3, 2, 7/6)$ leptoquark is an electroweak doublet and couples to both left-handed and right-handed SM quarks and leptons. Its Yukawa couplings with SM fermions are

$$\mathcal{L}_{R_2} = -(Y_2)_{ab}\bar{u}_a R_2^\alpha \epsilon_{\alpha\beta} P_L L_b^\beta - (Y_4)_{ab}\bar{e}_a R_2^\dagger P_L Q_b + \text{H.c.} \quad (6)$$

We work in the basis, where the flavor eigenstates of down-type quarks and charged leptons coincide with their mass eigenstates. In particular, the component of R_2 with electric charge $2/3$ couples right-handed charged leptons to left-handed down-type quarks and right-handed up-type quarks to neutrinos and thus contributes to both $b \rightarrow s$ and the $b \rightarrow c$ processes.

For energies below the mass of the leptoquark, it is convenient to write an effective Lagrangian to capture the relevant contributions beyond the SM. Using the Warsaw basis [84] of the SM effective field theory (SMEFT), the relevant terms in the effective Lagrangian are

$$\begin{aligned} \mathcal{L} = & C_{abcd}^{qe} (\bar{Q}_a \gamma_\mu Q_b) (\bar{e}_c \gamma^\mu e_d) + C_{abcd}^{lu} (\bar{L}_a \gamma_\mu L_b) (\bar{u}_c \gamma^\mu u_d) \\ & + C_{abcd}^{lequ1} (\bar{L}_a^j e_b) \epsilon_{jk} (\bar{Q}_c^k u_d) \\ & + C_{abcd}^{lequ3} (\bar{L}_a^j \sigma_{\mu\nu} e_b) \epsilon_{jk} (Q_c^k \sigma^{\mu\nu} u_d), \end{aligned} \quad (7)$$

where $\sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu]$ with Wilson coefficients

$$\begin{aligned} C_{bdca}^{qe}(m_{R_2}) &= -\frac{(Y_4)_{ab}^* (Y_4)_{cd}}{2m_{R_2}^2} \\ C_{dbac}^{lu}(m_{R_2}) &= -\frac{(Y_2)_{ab} (Y_2)_{cd}^*}{2m_{R_2}^2} \end{aligned} \quad (8)$$

$$C_{bcda}^{lequ1}(m_{R_2}) = 4C_{bcda}^{lequ3}(m_{R_2}) = -\frac{(Y_2)_{ab} (Y_4)_{cd}}{2m_{R_2}^2}, \quad (9)$$

which are defined at the renormalization scale $\mu = m_{R_2}$, the mass of leptoquark R_2 . The vector Wilson coefficient C_{sbee}^{qe} contributes to $b \rightarrow see$ and thus modifies the LFU ratios $R_{K^{(*)}}$. This is illustrated in the left panel of Fig. 1. The blue-shaded region indicates the $1 - \sigma$ -allowed region for $R_{K^{(*)}}$. For a fixed leptoquark mass $m_{R_2} = 1$ TeV, the LFU ratio R_K decreases when increasing the magnitude of the Yukawa couplings $|(Y_4)_{es}(Y_4)_{eb}|$, thus increasing the magnitude of the Wilson coefficient C_{sbee}^{qe} .

²A model independent discussion of couplings to electrons and muons using effective field theory has been performed in Ref. [82].

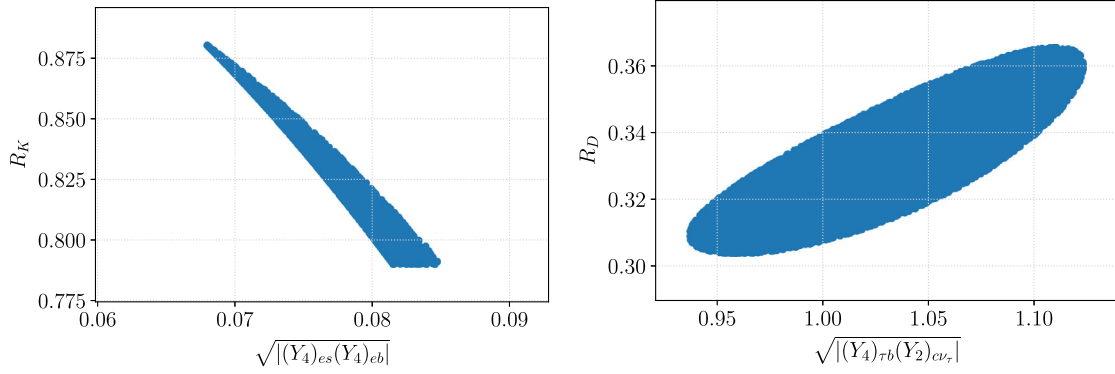


FIG. 1. Dependence of $R_{D^{(*)}}$ and $R_{K^{(*)}}$ on the relevant Yukawa couplings for fixed leptoquark mass $m_{R_2} = 1$ TeV.

Similarly the scalar and tensor Wilson coefficients $C_{\nu\tau bc}^{lequ1,3}$ contribute to $b \rightarrow c\tau\nu$ and thus modify the LFU ratios $R_{D^{(*)}}$. As the final state neutrino is not measured, there is a contribution from all three flavors. The coupling to $\nu_{e,\mu}$ is accompanied by couplings to e and μ respectively and hence there are additional constraints from lepton flavor violating processes. In order to avoid these additional constraints, we only consider couplings to ν_τ . The dependence of R_D to the magnitude of the Yukawa couplings $|(Y_2)_{c\nu_\tau}(Y_4)_{\tau b}|$ is illustrated in the right panel of Fig. 1. The blue-shaded region indicates the $1 - \sigma$ -allowed region for $R_{D^{(*)}}$. The Yukawa couplings $|(Y_2)_{c\nu_\tau}|$ and $|(Y_4)_{\tau b}|$ are generally of order 1 with $\sqrt{|(Y_2)_{c\nu_\tau}(Y_4)_{\tau b}|} \sim 1$ and thus typically larger than the Yukawa couplings required to explain $R_{K^{(*)}}$. As there is generally operator mixing, when evolving the Wilson coefficients from the scale of the leptoquark to the scale of the b -quark, the large Wilson coefficients $C_{\nu,\tau bc}^{lequ1,3}$ typically modify the result for $R_{K^{(*)}}$ and thus the interesting parameter range for the Yukawa

couplings $(Y_4)_{es}$ and $(Y_4)_{eb}$ differs, when attempting to explain both $R_{K^{(*)}}$ and $R_{D^{(*)}}$ simultaneously.

A minimal set of Yukawa couplings to accommodate a simultaneous solution to $R_{K^{(*)}}$ and $R_{D^{(*)}}$ is

$$Y_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & (Y_2)_{c\nu_\tau} \\ 0 & 0 & 0 \end{pmatrix}, \quad Y_4 = \begin{pmatrix} 0 & (Y_4)_{es} & (Y_4)_{eb} \\ 0 & 0 & 0 \\ 0 & 0 & (Y_4)_{\tau b} \end{pmatrix}, \quad (10)$$

which we focus on in the following.

Before discussing the phenomenology of the R_2 leptoquark we briefly make a connection to the operators in the commonly used operator basis in B -physics. We limit our discussion to the operators induced after integrating out the R_2 leptoquark. In the weak effective theory, after integrating out the Higgs, Z - and W -bosons and the top quark, the relevant operators in the effective Lagrangians governing $b \rightarrow sll$ and $b \rightarrow c\ell\nu$ decays are

$$\begin{aligned} \mathcal{L}_{sb\ell\ell} &= \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha_{em}}{4\pi} \sum_{\ell} [C_9^{\ell} (\bar{s}\gamma_{\mu} P_L b) (\bar{\ell}\gamma^{\mu} \ell) + C_{10}^{\ell} (\bar{s}\gamma_{\mu} P_L b) (\bar{\ell}\gamma^{\mu} \gamma_5 \ell)] \\ \mathcal{L}_{cb\ell\nu} &= -\frac{4G_F}{\sqrt{2}} V_{cb} \sum_{i,j} [C_V^{ij} (\bar{c}\gamma^{\mu} P_L b) (\bar{\ell}_i \gamma_{\mu} P_L \nu_j) + C_S^{ij} (\bar{c} P_L b) (\bar{\ell}_i P_L \nu_j) + C_T^{ij} (\bar{c}\sigma^{\mu\nu} P_L b) (\bar{\ell}_i \sigma_{\mu\nu} P_L \nu_j)], \end{aligned} \quad (11)$$

respectively, with the CKM mixing matrix elements V_{ij} . The Wilson coefficients in weak effective theory are related to the ones in SMEFT by

$$\begin{aligned} C_9^e &= C_{10}^e = \frac{\pi C_{bsee}^{qe}}{\sqrt{2} V_{tb} V_{ts}^* G_F \alpha_{em}} \\ C_S^{c\nu_\tau} &= 4C_T^{c\nu_\tau} = -\frac{C_{\nu,\tau bc}^{lequ1}}{2\sqrt{2} V_{cb} G_F}. \end{aligned} \quad (12)$$

In our numerical analysis we use the flavio package [85] for the renormalization group evolution of the Wilson

coefficients and the calculation of most processes. We vary the magnitude of the four Yukawa couplings over the range consistent with perturbativity and the explanation of the $R_{D^{(*)}}$ and $R_{K^{(*)}}$ anomalies at $1 - \sigma$ and their phases over the whole allowed range $[0, 2\pi]$ while fixing the mass $m_{R_2} = 1$ TeV.

III. EXPERIMENTAL CONSTRAINTS AND THE VIABLE PARAMETER SPACE

In this section we first summarize the most significant constraints on the couplings of the R_2 leptoquark in

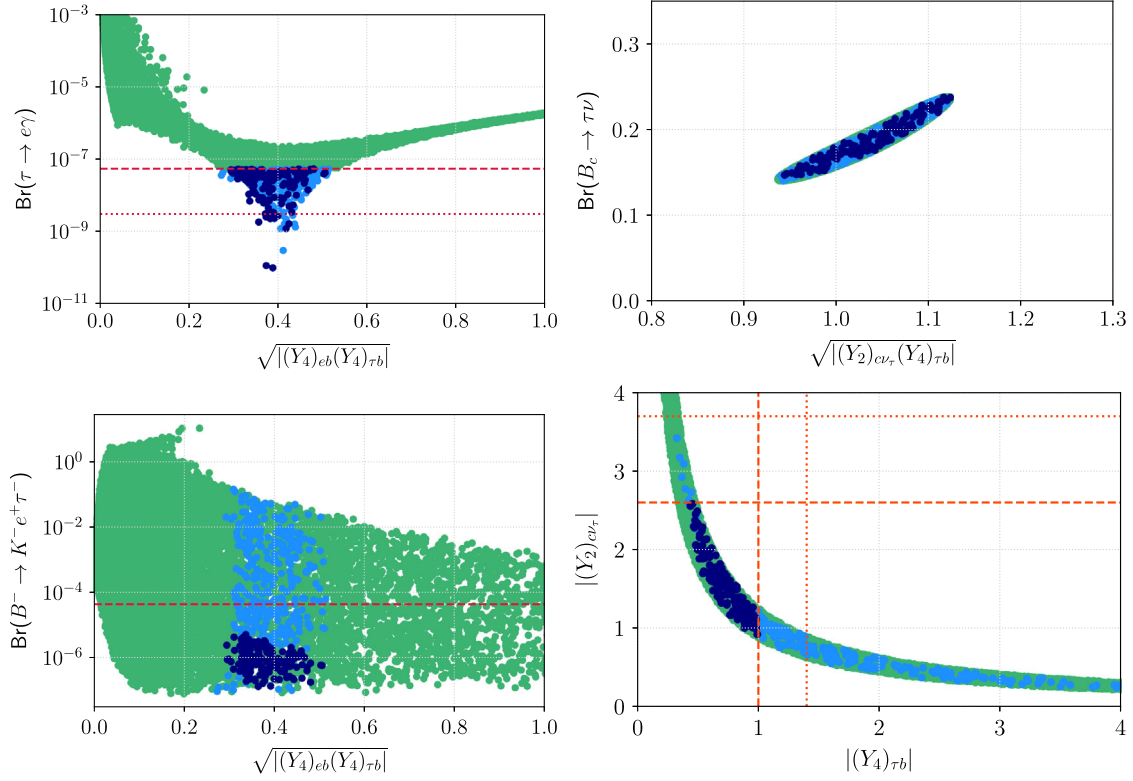


FIG. 2. Points that simultaneously explain $R_{D^{(*)}}$ and $R_{K^{(*)}}$ at $1 - \sigma$ with observables $\text{BR}(\tau^- \rightarrow e^- \gamma)$, $\text{BR}(B_c \rightarrow \tau \nu)$, $\text{BR}(B^- \rightarrow K^- e^+ \tau^-)$ shown in the top left, top right, and bottom left panels, respectively. The bottom right panel shows the relative size of the largest Yukawa couplings. The orange dashed (dotted) gridlines indicate the contributions to the $Z\tau\tau$ coupling at the level of the $1 - \sigma$ ($2 - \sigma$) experimental error. All points explain $R_{K^{(*)}}$ and $R_{D^{(*)}}$ at the $1 - \sigma$ level. Dark blue points satisfy all constraints. Light blue points satisfy strict limits on $\text{BR}(\tau^- \rightarrow e^- \gamma)$ but are excluded by other constraints. In the top right we show the three most stringent bounds on $\text{BR}(B_c \rightarrow \tau \nu)$ inferred from different theoretical considerations [83,86,87] and note that only the most stringent one rules out the scenario.

Sec. III A, followed by a discussion of the viable parameter space in Sec. III G.

A. Constraints

We show the impact of the most relevant constraints on the parameter space satisfying a $1 - \sigma$ simultaneous solution for $R_{K^{(*)}}$ and $R_{D^{(*)}}$ in Fig. 2. The four most stringent constraints are posited by the decays $\tau \rightarrow e\gamma$, $B^+ \rightarrow K^+ \tau^+ e^-$, $Z \rightarrow \tau\tau$, and $B_c \rightarrow \tau\nu$.

B. $\tau^- \rightarrow e^- \gamma$

The radiative lepton flavor violating decay $\tau^- \rightarrow e^- \gamma$ occurs at loop level. Its branching ratio takes the form [88]

$$\frac{\text{BR}(\tau^- \rightarrow e^- \gamma)}{\text{BR}(\tau^- \rightarrow e^- \nu_\tau \bar{\nu}_e)} \simeq \frac{27\alpha_{em}}{256\pi G_F^2 m_{R_2}^4} \left| (Y_4^* Y_4^T)_{\tau e} - \frac{4}{3} \sum_{q=u,c,t} (Y_2)_{q\tau} (Y_4 V^\dagger)_{eq} \frac{m_q}{m_\tau} \right|^2 \times \left(1 - \ln \frac{m_q^2}{m_{R_2}^2} \right)^2 \quad (13)$$

in the limit of vanishing final state electron mass and to leading order in the quark masses in the loop. The branching ratio for the SM purely leptonic τ decay is $\text{BR}(\tau \rightarrow e \nu_\tau \bar{\nu}_e) = 0.178$. In the numerical scan, we use the exact expression and impose the current limit on the branching ratio $\text{BR}(\tau^- \rightarrow e^- \gamma) < 5.4 \times 10^{-8}$ obtained by HFLAV [89]. The HFLAV limit is less stringent than the limit quoted in the PDG, because it combines the BABAR result [90] with the less stringent Belle result [91] while the PDG [92] relies only on the former. The combined limit is less aggressive as Belle saw a small excess of this process (see Table 319 in Ref. [89]). Irrespective of whether the Belle result is included or not, the simultaneous explanation of both $R_{K^{(*)}}$ and $R_{D^{(*)}}$ is viable.

In the top left panel of Fig. 2 we show the branching ratio vs $(|(Y_4)_{eb}(Y_4)_{tb}|)^{1/2}$. For large couplings $|(Y_4)_{eb}(Y_4)_{tb}|$, the branching ratio is dominated by the first term and thus increases for increasing Yukawa couplings. For small $(Y_4)_{tb}$, the Yukawa coupling $(Y_2)_{e\nu_\tau}$ becomes large in order to explain $R_{D^{(*)}}$ as shown in the bottom right plot and thus the second term in Eq. (13) dominates, which explains the increasing branching ratio for small

$|(Y_4)_{eb}(Y_4)_{\tau b}|$. The Belle II experiment [93] is expected to improve the sensitivity to $\tau \rightarrow e\gamma$ by more than 1 order of magnitude to 3×10^{-9} (indicated by a dotted red line) and thus probe a large part of the remaining parameter space.

C. $B^+ \rightarrow K^+\tau^+e^-$

Another constraint on the $\tau - e$ flavor violating processes originates from the semileptonic lepton flavor violating B decay $B^+ \rightarrow K^+\tau^+e^-$. Its branching ratio satisfies $\text{BR}(B^+ \rightarrow K^+\tau^+e^-) < 1.5 \times 10^{-5}$ [92]. The R_2 leptoquark induces the vector operator

$$C_{sb\tau e}^{qe} = -\frac{(Y_4^*)_{es}(Y_4)_{\tau b}}{2m_{R_2}^2}, \quad (14)$$

which contributes to $B^+ \rightarrow K^+\tau^+e^-$ and thus constrains the simultaneous explanation of $R_{K^{(*)}}$ and $R_{D^{(*)}}$. As we demonstrate in the bottom left panel of Fig. 2, it provides a moderate constraint on the parameter space that simultaneously explains $R_{K^{(*)}}$ and $R_{D^{(*)}}$. The region excluded by $B^+ \rightarrow K^+\tau^+e^-$ is also excluded by $\tau \rightarrow e\gamma$.

D. Z decays

The R_2 leptoquark also contributes to several Z-boson decay processes. In particular, its contribution to $Z \rightarrow \tau\tau$ is significant due to the large couplings to τ leptons. Approximate expressions for the left-handed and right-handed couplings of the Z-boson to τ leptons

$$\begin{aligned} \text{Re}(\delta g_L^\tau) \simeq & \frac{|(Y_4)_{\tau b}|^2}{16\pi^2} \left\{ -\frac{3}{2}x_t[1 + \ln x_t] \right. \\ & \left. + x_Z \left[\frac{23}{12} + \left(\frac{128}{9} + 8 \ln x_t - \frac{1}{3} \ln x_Z \right) \sin^2 \theta_W \right] \right\} \end{aligned} \quad (15)$$

$$\begin{aligned} \text{Re}(\delta g_R^\tau) \simeq & \frac{|(Y_2)_{\nu\tau}|^2}{16\pi^2} x_Z \left[\frac{1}{12} - \frac{1}{2} \ln x_Z \right. \\ & \left. + \left(\frac{1}{18} + \frac{2}{3} \ln x_Z \right) \sin^2 \theta_W \right] \end{aligned} \quad (16)$$

in terms of the Weinberg angle θ_W and the ratios $x_t = (m_t/m_{R_2})^2$ and $x_Z = (m_Z/m_{R_2})^2$ are obtained by expanding the expressions given in Ref. [94] to leading order in x_t , x_Z and the quark mixing angles by taking $V_{tb} \simeq 1$.

The LEP experiments measured the Z-boson couplings precisely [95] with $1 - \sigma$ uncertainties of $|\text{Re}(\delta g_L^\tau)| < 5.8 \times 10^{-4}$ for couplings to left-handed τ leptons and $|\text{Re}(\delta g_R^\tau)| \leq 6.2 \times 10^{-4}$ for right-handed τ leptons. This translates to a constraint on the magnitude of the Yukawa couplings $|(Y_4)_{\tau b}|$ and $|(Y_2)_{\nu\tau}|$ of $|(Y_4)_{\tau b}| \lesssim 1.0(1.4)$ and $|(Y_2)_{\nu\tau}| \lesssim 2.6(3.7)$ using $1 - \sigma$ ($2 - \sigma$) experimental uncertainties, respectively. This is indicated in the bottom

right panel of Fig. 2 as orange dashed (dotted) lines. The dark blue points in the numerical scan do not lead to any correction larger than the $1 - \sigma$ experimental uncertainties. In reality, a full global fit to all electroweak observables is needed to impose a reliable constraint, and it is probable that significantly larger deviations to effective Z couplings can be accommodated. We leave such a work to the future and comment here that even our pessimistic approach does not rule out our model.

E. $B_c \rightarrow \tau\nu$

The R_2 leptoquark contributes to $B_c \rightarrow \tau\nu$ via the same couplings relevant to $R_{D^{(*)}}$, since the same scalar operator contributes to both $R_{D^{(*)}}$ and $B_c \rightarrow \tau\nu$. In the top right panel of Fig. 2 we show the prediction for $\text{BR}(B_c \rightarrow \tau\nu)$. We find branching ratios between 15% and 23% for the region of parameter space that explains both $R_{D^{(*)}}$ and $R_{K^{(*)}}$ at $1 - \sigma$. Thus limits on this process pose a direct constraint on the explanation of $R_{D^{(*)}}$.

Several groups inferred limits on $\text{BR}(B_c \rightarrow \tau\nu) < [0.1, 0.6]$ [52,83,86,96,97] via different theoretical arguments. In the top right panel of Fig. 2 we indicate the three most stringent theoretically argued upper limits: 10% [83], 20% [87], and 30% [86], which are indicated by a dotted, dash-dotted, and dashed red line, respectively. In particular, Ref. [83] found that the branching ratio can be at most 10%, which is in tension with the viable parameter space of the R_2 leptoquark explanation of $R_{D^{(*)}}$. However, there is some controversy over this constraint that relies on the probability that a bottom quark hadronizes with a charmed quark that has not yet been measured at the LHC. There is currently an ongoing discussion on how the probability at the LHC can be inferred from LEP measurements and Monte Carlo simulations, where the authors of the first paper [83,87] advocate values in the range 10%–20% compared to some recent papers that advocate liberal constraints in the range 39%–60% [96,97]. The large range of values comes from the fact that the limit is quite sensitive to the ratio of hadronization probabilities f_c/f_u , that is, the probability of hadronizing with a charm or up quark, respectively. Figure 5 in Ref. [97], in particular, shows this dependency where a bound of 10% corresponds to $f_c/f_u \sim 4 \times 10^{-3}$ and the most liberal bound of 60% corresponds to a ratio that is a factor of 5 smaller, $f_c/f_u \sim 8 \times 10^{-4}$. Reference [97] use PYTHIA8 [98,99], which relies on experimental data to tune hadronization, to estimate the hadronization fraction and derive a fairly liberal lower bound of 39%. However, they and others [87] express skepticism of the true lower bound. We remain agnostic with regards to this constraint and show our prediction for this branching ratio in the top right panel of Fig. 2.

Even the most stringent constraint of 10% may be avoided by extending the model with a S_3 leptoquark, as we discuss in Sec. IV.

F. Other constraints

Apart from the discussed constraints we also studied possible constraints from several other processes and we briefly summarize the results. The limits obtained from lepton flavor violating semileptonic τ decays, $\tau \rightarrow eP$ with a pseudoscalar meson $P = K, \pi$, are always substantially weaker than the limit from $\tau \rightarrow e\gamma$ and thus we do not report them here. Furthermore, we considered leptonic meson decays, in particular, $B_s \rightarrow ee$, $D_0 \rightarrow ee$ and $D_s \rightarrow e\nu$, using flavio and as expected neither of them provides a relevant constraint. As the couplings required for an explanation of $R_{K^{(*)}}$ are small, the contribution to $B_s \rightarrow ee$ is suppressed. The dominant contribution to $D_s \rightarrow e\nu$ is controlled by $(Y_2)_{c\nu_\tau}(Y_4)_{es}$. While $(Y_4)_{es}$ is small, $(Y_2)_{c\nu_\tau}$ is constrained by its contribution to $\tau \rightarrow e\gamma$. Moreover, the contribution to the LFU ratios $R_D^{\mu/e} \equiv \Gamma(B \rightarrow D\mu\mu)/\Gamma(B \rightarrow Dee)$ and $R_{D^*}^{e/\mu} \equiv \Gamma(B \rightarrow D^*ee)/\Gamma(B \rightarrow D^*\mu\mu)$ are generally small, because the couplings $(Y_4)_{es}$ and $(Y_4)_{eb}$ that are responsible for explaining $R_{K^{(*)}}$ are small.

Let us turn our attention to $B_s - \bar{B}_s$ mixing. Matching the full theory with the R_2 leptoquark to SMEFT induces an effective four-quark interaction in SMEFT,

$$\mathcal{L} = -\frac{(Y_4^\dagger Y_4)_{ij}^2}{128\pi^2 m_{R_2}^2} (\bar{Q}_i \gamma_\mu Q_j) (\bar{Q}_i \gamma^\mu Q_j). \quad (17)$$

In particular, this four-quark interaction induces a new contribution to $B_s - \bar{B}_s$ mixing, which can be parametrized by

$$\mathcal{L} = C_{VLL,SM}^{bsbs} (\bar{s}\gamma^\mu P_L b) (\bar{s}\gamma_\mu P_L b) \quad C_{VLL,R_2}^{bsbs} = -\frac{(Y_4^\dagger Y_4)_{sb}^2}{128\pi^2 m_{R_2}^2} \quad (18)$$

in weak effective theory. It interferes with the SM contribution (see, e.g., [100])

$$C_{VLL,SM}^{bsbs} = \frac{G_F^2 m_W^2}{4\pi^2} (V_{ib}^* V_{ts})^2 S_0(m_t^2/m_W^2), \quad (19)$$

where S_0 is the Inami-Lim function [101]

$$S_0(x) = \frac{x^3 - 11x^2 + 4x}{4(x-1)^2} - \frac{3x^3}{2(x-1)^3} \ln x. \quad (20)$$

The contribution of R_2 to C_{VLL}^{bsbs} can be expressed in terms of the Wilson coefficient C_9^e . A simple order of magnitude estimate shows that C_{VLL,R_2}^{bsbs} is several orders of magnitude smaller as the SM contribution for the interesting R_2 leptoquark mass range

$$\left| \frac{C_{VLL,R_2}^{bsbs}}{C_{VLL,SM}^{bsbs}} \right| \simeq \left(\frac{\alpha_{em} m_{R_2}}{2\pi m_W} C_9^e \right)^2. \quad (21)$$

We independently checked the contribution to $B_s - \bar{B}_s$ mixing using flavio with the same result.

Finally, as the deviation in the ratios $R_K^{(*)}$ originates entirely from a modification to the branching fraction of $B \rightarrow K^{(*)} e^+ e^-$ one may wonder whether this deviation is consistent with measurements of the branching ratio itself. Under the given assumptions taking the 1σ boundaries (central value) the experimental value implies an increase of the binned differential branching ratio $\langle \frac{d\text{Br}(B^+ \rightarrow K^+ e^+ e^-)}{dq^2} \rangle \times (1.1 \text{ GeV}^2 < q^2 < 6.0 \text{ GeV}^2)$ by 9%–21% (15%) and similarly for $B \rightarrow K^* e^+ e^-$ by 21%–36% (28%).

While the experimental error has been reduced to the 10% level [15] from 15% to 30% in earlier measurements [102–105], there remains a large theory error due to the uncertainties in the hadronic matrix elements [85,106–109]. For $B^+ \rightarrow K^+ e^+ e^-$ the error in the different individual theory calculations is of the order 16%–34%. Moreover, the central values differ: To give an example, flavio [85] predicts $(3.49 \pm 0.53) \times 10^{-8} \text{ GeV}^{-2}$ for $1 \text{ GeV}^2 < q^2 < 6 \text{ GeV}^2$, while Ref. [109] obtains $(4.38^{+0.62}_{-0.57} \pm 0.28) \times 10^{-8} \text{ GeV}^{-2}$ with a central value that is increased by 25%. Using flavio we find for the allowed parameter space after imposing all constraints $[3.99, 4.42] \times 10^{-8} \text{ GeV}^{-2}$, which is consistent with the experimental measurement given the large theory uncertainties in $B \rightarrow K e^+ e^-$. A similar argument applies for $B \rightarrow K^* e^+ e^-$. Flavio predicts $\langle \frac{d\text{Br}(B^0 \rightarrow K^* e^+ e^-)}{dq^2} \rangle (1.1 \text{ GeV}^2 < q^2 < 6 \text{ GeV}^2) = (4.77 \pm 0.71) \times 10^{-8} \text{ GeV}^{-2}$, which corresponds to a theory uncertainty of 15%, while Ref. [110] quotes errors between 25% and 100% depending on the assumptions on the distribution of nuisance parameters. Using flavio we find branching ratios in the range $[6.09, 7.02] \times 10^{-8} \text{ GeV}^{-2}$ after imposing all constraints, which corresponds to an increase of 28%–47% compared to the SM prediction of flavio and thus larger than the SM uncertainty quoted by flavio, but well below the conservative uncertainty estimate in Ref. [110].

In summary, the treatment of the hadronic effects in the theoretical predictions for the branching ratios $B \rightarrow K^{(*)} e^+ e^-$ is still the subject of considerable debate, as we illustrated above by referring to the literature, and thus currently $R_{K^{(*)}}$ can be explained by a correction to the semileptonic decay $B \rightarrow K^{(*)} e^+ e^-$.

G. Viable parameter space

We show the viable parameter space in Fig. 3 and the bottom right panel of Fig. 2. For a fixed leptoquark mass of $m_{R_2} = 1 \text{ TeV}$, the bottom right panel of Fig. 2 shows that an aggressive constraint from Z decays restricts two of the Yukawa couplings to the range $|(Y_4)_{\tau b}| \in [0.44, 1.0]$ and $|(Y_2)_{c\nu_\tau}| \in [1.0, 2.6]$, respectively. All quoted ranges are approximate and are only intended to give an indication. The product is also constrained by the need to explain $R_{D^{(*)}}$ at the $1 - \sigma$ level to the range $|(Y_2)_{c\nu_\tau}(Y_4)_{\tau b}| \in [0.88, 1.3]$. The product is almost purely imaginary with $\arg((Y_2)_{c\nu_\tau} \times (Y_4)_{\tau b}) \in \pm[0.45, 0.54]\pi$, irrespective of the experimental constraints, which confirms previous findings [10,75,76].

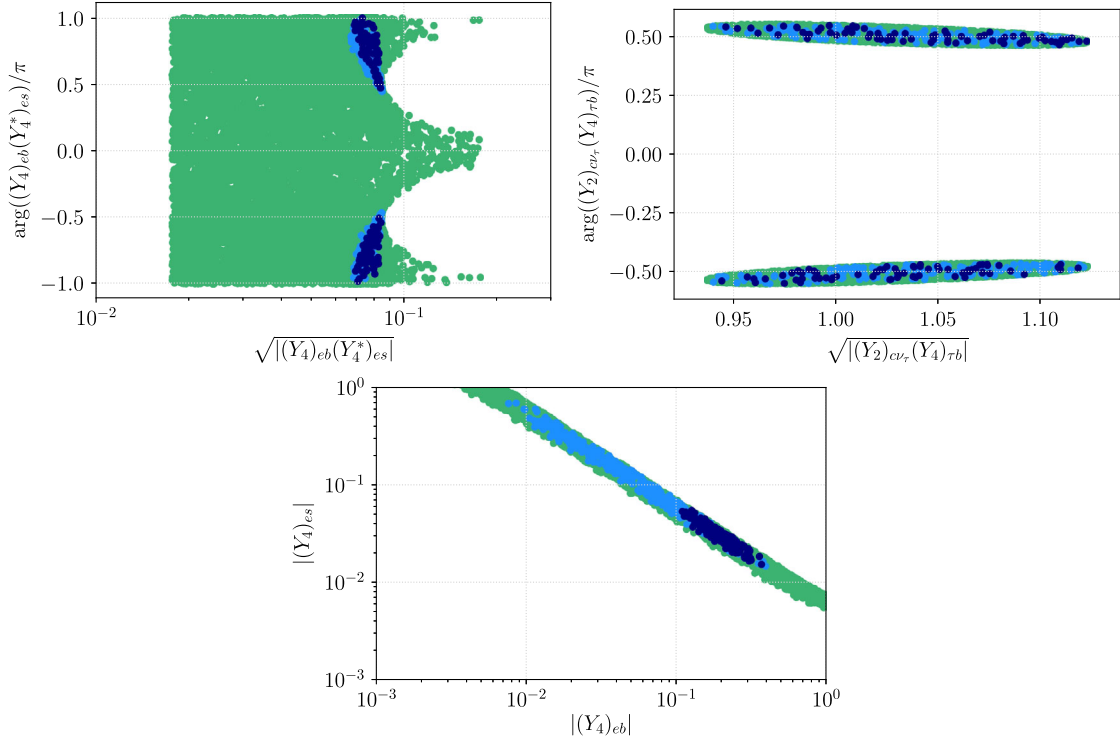


FIG. 3. Relevant parameter range of the Yukawa couplings. The top left panel shows the phase vs the magnitude for $(Y_4)_{eb}(Y_4^*)_{es}$ and the top right panel the phase vs the magnitude of $(Y_2)_{cv\tau}(Y_4)_{tb}$. In the bottom panel we plot the absolute values of the Yukawa couplings entering $R_{K^{(*)}}$ against each other. All points explain $R_{K^{(*)}}$ and $R_{D^{(*)}}$ at the $1 - \sigma$ level. Dark blue points satisfy all experimental constraints. Light blue points satisfy strict limits on $\tau^- \rightarrow e^- \gamma$ but are excluded by other constraints.

The other two Yukawa couplings are generally smaller with $|(Y_4)_{eb}| \in [0.11, 0.37]$, $|(Y_4)_{es}| \in [0.015, 0.055]$. Their product constrained to the narrow range $|(Y_4)_{eb}(Y_4)_{es}| \in [0.0047, 0.0070]$ after imposing all experimental constraints. This is shown in the bottom panel of Fig. 3. For the points that satisfy all experimental constraints, the real part of the product is generally negative; the argument is weakly constrained to the range $\arg((Y_4)_{eb}(Y_4^*)_{es}) \in \pm[0.47, 1.0]$. This implies that the Wilson coefficient C_{sbee}^{qe} is generally positive. The absolute value of the product is by contrast confined to a narrow range $\sqrt{|(Y_4)_{eb}(Y_4^*)_{es}|} \in [4.7, 7.0] \times 10^{-3}$. The hierarchy $|(Y_4)_{es}| \ll |(Y_4)_{eb}|$ can be understood as follows. The constraint from Z-boson decays constrains the coupling $(Y_4)_{tb} \lesssim 1.0$ and thus the coupling $(Y_2)_{cv\tau}$ has to be larger than 1 in order to explain $R_{D^{(*)}}$. This in turn leads to a stronger constraint on $|(Y_2)_{cv\tau}|$ from $\tau \rightarrow e\gamma$, because the suppression from the ratio m_s/m_τ is compensated by the large logarithm, $\ln(m_s^2/m_{R_2}^2)$.

Finally, we obtain a prediction for the branching ratios of the decays $t \rightarrow c\ell_i^+\ell_j^-$ [111], $\text{BR}(t \rightarrow \ell_i^+\ell_j^- + c) \simeq \frac{1.3}{48\pi^2} |e_{ij}^{RL}|^2$, where e_{ij}^{RL} is the Wilson coefficient of the effective operator

$$\mathcal{L} = -2\sqrt{2}G_F e_{ij}^{RL} [\bar{e}_i \gamma_\mu P_R e_j][\bar{c} \gamma^\mu P_L t]. \quad (22)$$

The R_2 leptoquark especially generates the Wilson coefficients

$$\begin{aligned} \epsilon_{te}^{RL} &= \frac{((Y_4^*)_{es} V_{cs}^* + (Y_4^*)_{eb} V_{cb}^*)(Y_4)_{tb} V_{tb}}{4\sqrt{2}G_F m_{R_2}^2}, \\ \epsilon_{e\tau}^{RL} &= \frac{(Y_4)_{tb} V_{cb}^* ((Y_4)_{es} V_{ts} + (Y_4)_{eb} V_{tb})}{4\sqrt{2}G_F m_{R_2}^2}. \end{aligned} \quad (23)$$

We find that the decays $t \rightarrow c\tau^\pm e^\mp$ are generally tiny with branching ratios

$$\text{BR}(t \rightarrow c\tau^+ e^-) \lesssim 2 \times 10^{-9} \quad \text{BR}(t \rightarrow c\tau^- e^+) \lesssim 8 \times 10^{-11} \quad (24)$$

for the parameter space, which explains both $R_{K^{(*)}}$ and $R_{D^{(*)}}$ at the $1 - \sigma$ level with a leptoquark mass $m_{R_2} = 1$ TeV and thus we do not expect any signal at the LHC experiments.

IV. ALLEVIATING THE (POSSIBLE) TENSION WITH $\text{BR}(B_c \rightarrow \nu)$ VIA A NEUTRINO MASS MODEL

It is well known that the $S_3 \sim (\bar{3}, 3, 1/3)$ leptoquark³ with the Yukawa interaction

³The collider phenomenology of an S_3 leptoquark has been studied in Ref. [112].

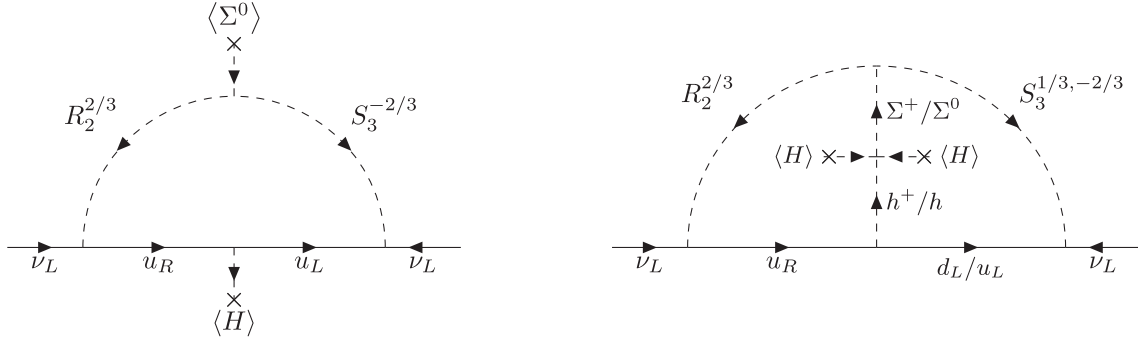


FIG. 4. Neutrino mass from leptoquarks in the loop. The superscripts q of R_2^q and S_3^q denote the electromagnetic charge of the different components of R_2 and S_3 , respectively.

$$\mathcal{L}_{S3} = -(Y_S)_{ab} \overline{Q}_{ai}^c P_L L_{bk} S_{3jl} e^{ij} e^{kl} + \text{H.c.} \quad (25)$$

and mass m_S contributes to the Wilson coefficients

$$C_{dbca}^{lq1} = \frac{3(Y_S)_{ab}(Y_S^*)_{cd}}{8m_S^2} \quad C_{dbca}^{lq3} = \frac{(Y_S)_{ab}(Y_S^*)_{cd}}{8m_S^2} \quad (26)$$

of the vector operators [113–115]

$$\mathcal{L} = C_{abcd}^{lq1} (\bar{L}_a \gamma_\mu L_b) (\bar{Q}_c \gamma^\mu Q_d) + C_{abcd}^{lq3} (\bar{L}_a \gamma_\mu \tau^I L_b) (\bar{Q}_c \gamma^\mu \tau^I Q_d), \quad (27)$$

which can help alleviate the possible tension with $\text{BR}(B_c \rightarrow \tau\nu)$ at the cost of a contribution to the decay $B \rightarrow K\nu\nu$. Such a model involving two leptoquarks can be motivated by a neutrino mass model. In this section we sketch out how this is possible leaving a detailed analysis to future work.

A. Neutrino masses

Just extending the SM with R_2 and S_3 leptoquarks is not sufficient to generate nonzero neutrino masses.⁴ To keep our model minimal we extend our two leptoquark extension of the SM by a single particle, which is a $\text{SU}(2)_L$ quadruplet with quantum numbers $\Sigma \sim (1, 4, \frac{3}{2})$. Then neutrino masses are generated at the one-loop level as shown in the left panel of Fig. 4. There is also a two-loop contribution that is shown on the right.

If the two-loop diagram can be neglected, neutrino masses are approximately given by their one-loop contribution⁵

⁴See Ref. [116] for a discussion of different possibilities in the context of a grand unified theory.

⁵Further details we relegate to the Appendix.

$$(m_\nu)_{ij} \simeq \frac{1}{16\pi^2} \frac{\mu \langle \Sigma^0 \rangle}{m_{R_2}^2 - m_S^2} (Y_2)_{ki} \left\{ m_k \left[F\left(\frac{m_{R_2}^2}{m_k^2}\right) - F\left(\frac{m_S^2}{m_k^2}\right) \right] V_{kl}^* \right\} (Y_S)_{lj} + (i \leftrightarrow j) \quad (28)$$

in the limit of small mixing between R_2 and S_3 leptoquarks, which is generated by the trilinear potential term $\mu \Sigma^{*ijk} R_{2i} S_{3jk}$. The Yukawa coupling matrices are defined in the basis where charged lepton and down-type quark mass matrices are diagonal. Thus the loop diagram with up-type quarks in the loop is proportional to the CKM mixing matrix element V_{kl}^* . Roman indices i, j, k, l indicate flavor, m_k the up-type quark mass, m_{R_2}, m_S are R_2 and S_3 leptoquark masses, respectively, and $\langle \Sigma^0 \rangle$ the vacuum expectation value of the neutral component of Σ . The loop function $F(x)$ is defined as

$$F(x) = \frac{x \ln x}{1-x}. \quad (29)$$

The more general expression for a general mixing angle between the R_2 and S_3 leptoquarks is given in Appendix. The two-loop contribution features a similar flavor structure.

V. CONCLUSION

We demonstrate that the $R_2 \sim (3, 2, 7/6)$ leptoquark is a new single particle candidate for explaining the anomalous lepton-flavor-universality ratios $R_{K^{(*)}}$ and $R_{D^{(*)}}$. There is possibly a mild tension with the theoretically derived limit on the branching ratio $\text{BR}(B_c \rightarrow \tau\nu)$. Since we require the branching ratio of $B_c \rightarrow \tau\nu$ to be within a relatively narrow range, the viability of the R_2 leptoquark as a single particle solution to these anomalies is a directly falsifiable scenario. Another promising probe of the viable parameter space of our model is $\text{BR}(\tau \rightarrow e\gamma)$, where the projected sensitivity for Belle II is expected to improve by an order of magnitude [93]. Another intriguing possibility is whether the large imaginary couplings needed to explain $R_{D^{(*)}}$ leave a permanent neutron electric dipole moment that is detectable in future experiments [117].

The tension with the disputed aggressive limit on $\text{BR}(B_c \rightarrow \tau\nu)$ can be alleviated through the introduction of a S_3 leptoquark, which can be motivated by a neutrino mass model as discussed in Sec. A. This suggests that even if future analysis indeed rules out the R_2 leptoquark as a single leptoquark solution to anomalous B decays, it still can play a substantial role in an extended model.

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APPENDIX: LEPTOQUARK MIXING AND NEUTRINO MASSES

The relevant terms in the scalar potential $V = V_0 + V_1$ are

$$V_0 = \sum_{\substack{H, R_2, S_3, \\ \Sigma \in \mathbf{x}}} \left((-1)^{q_x} \mu_x^2 |\mathbf{x}|^2 + \frac{\lambda_x}{2} |\mathbf{x}|^4 \right) + \sum_{\substack{H, R_2, S_3, \\ \Sigma \in \{\mathbf{x} < \mathbf{y}\}}} \lambda_{xy} |\mathbf{x}|^2 |\mathbf{y}|^2 \quad (\text{A1})$$

$$V_1 = \mu \Sigma^{*ijk} R_{2i} S_{3jk} + \lambda_{3\Sigma H} \Sigma^{*ijk} H_i H_j H_k + \lambda_{2H33} R_2^{*i} S_{3ij} S_{3kl} H_m \epsilon^{jk} \epsilon^{lm} + \text{H.c.}, \quad (\text{A2})$$

$$m_{S_{1,2}}^2 = \frac{\mu_R^2 + \mu_S^2 + \frac{v_H^2}{2} (\lambda_{HR} + \lambda_{HS}) + \frac{v_\Sigma^2}{2} (\lambda_{R\Sigma} + \lambda_{S\Sigma})}{2} \pm \frac{1}{2} \sqrt{\left[\mu_R^2 - \mu_S^2 + \frac{v_H^2}{2} (\lambda_{HR} - \lambda_{HS}) + \frac{v_\Sigma^2}{2} (\lambda_{R\Sigma} - \lambda_{S\Sigma}) \right]^2 + 4\mu^2 \frac{v_\Sigma^2}{2}}. \quad (\text{A7})$$

For small μ and thus small mixing, the square of the masses m_{R_2} and m_S in the main part of the text can be identified with the diagonal elements of the scalar mass matrix M_S^2 , $m_{R_2}^2 = \mu_R^2 + \lambda_{HR} v_H^2/2 + \lambda_{R\Sigma} v_\Sigma^2/2$ and $m_S^2 = \mu_S^2 + \lambda_{HS} v_H^2/2 + \lambda_{S\Sigma} v_\Sigma^2/2$.

where $(-1)^{q_x}$ is -1 for H and Σ and $+1$ for the other scalar fields. Thus the general form for neutrino masses at one-loop order is given by

$$(m_\nu)_{mn} = \frac{1}{16\pi^2} (U_s^\dagger)_{R_2 s_i} (Y_2)_{km} \left[m_k F \left(\frac{m_{S_i}^2}{m_k^2} \right) V_{kl}^* \right]^{kl} \times (Y_S)_{ln} (U_s)_{s_i s_3} + (m \leftrightarrow n). \quad (\text{A3})$$

The mixing between $R_2^{2/3}$ and $S_3^{*2/3}$ is generated by $\langle \Sigma^0 \rangle$ and is obtained by diagonalizing the charge $2/3$ leptoquark mass matrix, which is given in the $(R_2^{2/3}, S_3^{*2/3})$ basis

$$M_S^2 = \begin{pmatrix} \mu_R^2 + \lambda_{HR} \frac{v_H^2}{2} + \lambda_{R\Sigma} \frac{v_\Sigma^2}{2} & \mu \frac{v_\Sigma}{\sqrt{2}} \\ \mu \frac{v_\Sigma}{\sqrt{2}} & \mu_S^2 + \lambda_{HS} \frac{v_H^2}{2} + \lambda_{S\Sigma} \frac{v_\Sigma^2}{2} \end{pmatrix} = U_S^T \text{Diag}(m_{S_1}^2, m_{S_2}^2) U_S \quad (\text{A4})$$

with $v_H = \langle H^0 \rangle / \sqrt{2}$ and $v_\Sigma = \langle \Sigma^0 \rangle / \sqrt{2}$, the masses m_{S_i} and the 2×2 unitary mixing matrix U_S which defines the mass eigenstates S_i in terms of the flavor eigenstates

$$\begin{pmatrix} S_1 \\ S_2 \end{pmatrix} = U_S \begin{pmatrix} R_2^{2/3} \\ S_3^{*2/3} \end{pmatrix} \quad U_S = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}. \quad (\text{A5})$$

A straightforward calculation results in the following expressions for the rotation angle and the masses

$$\tan(2\theta) = \frac{\sqrt{2} \mu v_\Sigma}{\mu_R^2 - \mu_S^2 + \frac{v_H^2}{2} (\lambda_{HR} - \lambda_{HS}) + \frac{v_\Sigma^2}{2} (\lambda_{R\Sigma} - \lambda_{S\Sigma})} \quad (\text{A6})$$

[1] J. P. Lees *et al.* (BABAR Collaboration), *Phys. Rev. Lett.* **109**, 101802 (2012).
 [2] J. P. Lees *et al.* (BABAR Collaboration), *Phys. Rev. D* **88**, 072012 (2013).

[3] M. Huschle *et al.* (Belle Collaboration), *Phys. Rev. D* **92**, 072014 (2015).
 [4] Y. Sato *et al.* (Belle Collaboration), *Phys. Rev. D* **94**, 072007 (2016).

- [5] S. Hirose *et al.* (Belle Collaboration), *Phys. Rev. Lett.* **118**, 211801 (2017).
- [6] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **115**, 111803 (2015); **115**, 159901(E) (2015).
- [7] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. D* **97**, 072013 (2018).
- [8] G. Caria (Belle Collaboration), talk given at Moriond (2019).
- [9] J. A. Bailey *et al.* (MILC Collaboration), *Phys. Rev. D* **92**, 034506 (2015).
- [10] M. Tanaka and R. Watanabe, *Phys. Rev. D* **87**, 034028 (2013).
- [11] G. Hiller and F. Kruger, *Phys. Rev. D* **69**, 074020 (2004).
- [12] B. Capdevila, S. Descotes-Genon, J. Matias, and J. Virto, *J. High Energy Phys.* **10** (2016) 075.
- [13] B. Capdevila, A. Crivellin, S. Descotes-Genon, J. Matias, and J. Virto, *J. High Energy Phys.* **01** (2018) 093.
- [14] C. Bobeth, G. Hiller, and G. Piranishvili, *J. High Energy Phys.* **12** (2007) 040.
- [15] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **122**, 191801 (2019).
- [16] M. Bordone, G. Isidori, and A. Pattori, *Eur. Phys. J. C* **76**, 440 (2016).
- [17] M. Prim (Belle Collaboration), talk given at Moriond (2019).
- [18] R. Aaij *et al.* (LHCb Collaboration), *J. High Energy Phys.* **08** (2017) 055.
- [19] R. Barlow, [arXiv:physics/0306138](https://arxiv.org/abs/physics/0306138).
- [20] A. J. Buras, J. Girrbach-Noe, C. Niehoff, and D. M. Straub, *J. High Energy Phys.* **02** (2015) 184.
- [21] B. Gripaios, M. Nardecchia, and S. A. Renner, *J. High Energy Phys.* **05** (2015) 006.
- [22] H. Päs and E. Schumacher, *Phys. Rev. D* **92**, 114025 (2015).
- [23] R. Barbieri, C. W. Murphy, and F. Senia, *Eur. Phys. J. C* **77**, 8 (2017).
- [24] M. Duraisamy, S. Sahoo, and R. Mohanta, *Phys. Rev. D* **95**, 035022 (2017).
- [25] O. Sumensari, *Proc. Sci.*, EPS-HEP2017 (2017) 245.
- [26] D. Aloni, A. Dery, C. Frugiuele, and Y. Nir, *J. High Energy Phys.* **11** (2017) 109.
- [27] O. Sumensari, in *Proceedings, 52nd Rencontres de Moriond on Electroweak Interactions and Unified Theories: La Thuile, Italy* (2017), pp. 445–448, ISBN: 9791096879021.
- [28] G. Hiller and I. Nisandzic, *Phys. Rev. D* **96**, 035003 (2017).
- [29] G. D’Amico, M. Nardecchia, P. Panci, F. Sannino, A. Strumia, R. Torre, and A. Urbano, *J. High Energy Phys.* **09** (2017) 010.
- [30] J. M. Cline, *Phys. Rev. D* **97**, 015013 (2018).
- [31] S.-Y. Guo, Z.-L. Han, B. Li, Y. Liao, and X.-D. Ma, *Nucl. Phys.* **B928**, 435 (2018).
- [32] A. Crivellin, D. Müller, A. Signer, and Y. Ulrich, *Phys. Rev. D* **97**, 015019 (2018).
- [33] A. K. Alok, B. Bhattacharya, A. Datta, D. Kumar, J. Kumar, and D. London, *Phys. Rev. D* **96**, 095009 (2017).
- [34] A. Angelescu, D. Bečirević, D. A. Faroughy, and O. Sumensari, *J. High Energy Phys.* **10** (2018) 183.
- [35] I. De Medeiros Varzielas and S. F. King, *Phys. Rev. D* **99**, 095029 (2019).
- [36] I. de Medeiros Varzielas and J. Talbert, *Eur. Phys. J. C* **79**, 536 (2019).
- [37] J.-H. Sheng, R.-M. Wang, and Y.-D. Yang, *Int. J. Theor. Phys.* **58**, 480 (2019).
- [38] S. Balaji, R. Foot, and M. A. Schmidt, *Phys. Rev. D* **99**, 015029 (2019).
- [39] M. Freytsis, Z. Ligeti, and J. T. Ruderman, *Phys. Rev. D* **92**, 054018 (2015).
- [40] A. Biswas, D. K. Ghosh, A. Shaw, and S. K. Patra, *Int. J. Mod. Phys. A* **34**, 1950112 (2019).
- [41] J. Zhang, Y. Zhang, Q. Zeng, and R. Sun, *Eur. Phys. J. C* **79**, 164 (2019).
- [42] U. Aydemir, T. Mandal, and S. Mitra, [arXiv:1902.08108](https://arxiv.org/abs/1902.08108).
- [43] T. Mandal, S. Mitra, and S. Raz, *Phys. Rev. D* **99**, 055028 (2019).
- [44] S. Bansal, R. M. Capdevilla, and C. Kolda, *Phys. Rev. D* **99**, 035047 (2019).
- [45] S. Iguro, T. Kitahara, Y. Omura, R. Watanabe, and K. Yamamoto, *J. High Energy Phys.* **02** (2019) 194.
- [46] M. Bauer and M. Neubert, *Phys. Rev. Lett.* **116**, 141802 (2016).
- [47] S. Fajfer and N. Košnik, *Phys. Lett. B* **755**, 270 (2016).
- [48] B. Bhattacharya, A. Datta, J.-P. Guévin, D. London, and R. Watanabe, *J. High Energy Phys.* **01** (2017) 015.
- [49] S. Sahoo, R. Mohanta, and A. K. Giri, *Phys. Rev. D* **95**, 035027 (2017).
- [50] D. Bečirević, S. Fajfer, N. Košnik, and O. Sumensari, *Phys. Rev. D* **94**, 115021 (2016).
- [51] D. Bečirević, N. Košnik, O. Sumensari, and R. Z. Funchal, *J. High Energy Phys.* **11** (2016) 035.
- [52] X.-Q. Li, Y.-D. Yang, and X. Zhang, *J. High Energy Phys.* **08** (2016) 054.
- [53] B. Chauhan and B. Kindra, [arXiv:1709.09989](https://arxiv.org/abs/1709.09989).
- [54] L. Calibbi, A. Crivellin, and T. Li, *Phys. Rev. D* **98**, 115002 (2018).
- [55] L. Di Luzio, A. Greljo, and M. Nardecchia, *Phys. Rev. D* **96**, 115011 (2017).
- [56] D. Buttazzo, A. Greljo, G. Isidori, and D. Marzocca, *J. High Energy Phys.* **11** (2017) 044.
- [57] Y. Cai, J. Gargalionis, M. A. Schmidt, and R. R. Volkas, *J. High Energy Phys.* **10** (2017) 047.
- [58] A. Crivellin, D. Müller, and T. Ota, *J. High Energy Phys.* **09** (2017) 040.
- [59] D. Müller, *EPJ Web Conf.* **179**, 01015 (2018).
- [60] C. Cornella, J. Fuentes-Martin, and G. Isidori, *J. High Energy Phys.* **07** (2019) 168.
- [61] L. Da Rold and F. Lamagna, *J. High Energy Phys.* **03** (2019) 135.
- [62] M. Schmaltz and Y.-M. Zhong, *J. High Energy Phys.* **01** (2019) 132.
- [63] B. Fornal, S. A. Gadam, and B. Grinstein, *Phys. Rev. D* **99**, 055025 (2019).
- [64] N. Assad, B. Fornal, and B. Grinstein, *Phys. Lett. B* **777**, 324 (2018).
- [65] M. Blanke and A. Crivellin, *Phys. Rev. Lett.* **121**, 011801 (2018).
- [66] D. Bečirević, I. Doršner, S. Fajfer, N. Košnik, D. A. Faroughy, and O. Sumensari, *Phys. Rev. D* **98**, 055003 (2018).

- [67] A. Azatov, D. Bardhan, D. Ghosh, F. Sgarlata, and E. Venturini, *J. High Energy Phys.* **11** (2018) 187.
- [68] A. Azatov, D. Barducci, D. Ghosh, D. Marzocca, and L. Ubaldi, *J. High Energy Phys.* **10** (2018) 092.
- [69] Z.-R. Huang, Y. Li, C.-D. Lu, M. A. Paracha, and C. Wang, *Phys. Rev. D* **98**, 095018 (2018).
- [70] O. Popov and G. A. White, *Nucl. Phys.* **B923**, 324 (2017).
- [71] S. Descotes-Genon, J. Matias, and J. Virto, *Phys. Rev. D* **88**, 074002 (2013).
- [72] D. Guadagnoli, *Mod. Phys. Lett. A* **32**, 1730006 (2017).
- [73] S. Descotes-Genon, L. Hofer, J. Matias, and J. Virto, *J. High Energy Phys.* **06** (2016) 092.
- [74] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **120**, 121801 (2018).
- [75] I. Doršner, S. Fajfer, N. Košnik, and I. Nišandžić, *J. High Energy Phys.* **11** (2013) 084.
- [76] Y. Sakaki, M. Tanaka, A. Tayduganov, and R. Watanabe, *Phys. Rev. D* **88**, 094012 (2013).
- [77] S. Sahoo and R. Mohanta, *Phys. Rev. D* **91**, 094019 (2015).
- [78] C.-H. Chen, T. Nomura, and H. Okada, *Phys. Rev. D* **94**, 115005 (2016).
- [79] U. K. Dey, D. Kar, M. Mitra, M. Spannowsky, and A. C. Vincent, *Phys. Rev. D* **98**, 035014 (2018).
- [80] D. Bečirević and O. Sumensari, *J. High Energy Phys.* **08** (2017) 104.
- [81] B. Chauhan, B. Kindra, and A. Narang, *Phys. Rev. D* **97**, 095007 (2018).
- [82] G. Hiller and M. Schmaltz, *J. High Energy Phys.* **02** (2015) 055.
- [83] A. G. Akeroyd and C.-H. Chen, *Phys. Rev. D* **96**, 075011 (2017).
- [84] B. Grzadkowski, M. Iskrzynski, M. Misiak, and J. Rosiek, *J. High Energy Phys.* **10** (2010) 085.
- [85] D. M. Straub, arXiv:1810.08132.
- [86] R. Alonso, B. Grinstein, and J. M. Camalich, *Phys. Rev. Lett.* **118**, 081802 (2017).
- [87] A. Akeroyd (private communication).
- [88] L. Lavoura, *Eur. Phys. J. C* **29**, 191 (2003).
- [89] Y. Amhis *et al.* (HFLAV Collaboration), *Eur. Phys. J. C* **77**, 895 (2017).
- [90] B. Aubert *et al.* (BABAR), *Phys. Rev. Lett.* **104**, 021802 (2010).
- [91] K. Hayasaka *et al.* (Belle), *Phys. Lett. B* **666**, 16 (2008).
- [92] M. Tanabashi *et al.* (Particle Data Group), *Phys. Rev. D* **98**, 030001 (2018).
- [93] W. Altmannshofer *et al.* (Belle-II Collaboration), arXiv:1808.10567.
- [94] P. Aman, D. Becirevic, F. Mescia, and O. Sumensari, *J. High Energy Phys.* **02** (2019) 109.
- [95] S. Schael *et al.* (ALEPH, DELPHI, L3, OPAL, SLD Collaborations, LEP Electroweak Working Group, SLD Electroweak Group, SLD Heavy flavor Group), *Phys. Rep.* **427**, 257 (2006).
- [96] M. Blanke, A. Crivellin, S. de Boer, M. Moscati, U. Nierste, I. Nišandžić, and T. Kitahara, *Phys. Rev. D* **99**, 075006 (2019).
- [97] D. Bardhan and D. Ghosh, *Phys. Rev. D* **100**, 011701 (2019).
- [98] T. Sjöstrand, S. Mrenna, and P. Z. Skands, *J. High Energy Phys.* **05** (2006) 026.
- [99] T. Sjöstrand, S. Ask, J. R. Christiansen, R. Corke, N. Desai, P. Ilten, S. Mrenna, S. Prestel, C. O. Rasmussen, and P. Z. Skands, *Comput. Phys. Commun.* **191**, 159 (2015).
- [100] R. Fleischer, in *High-Energy Physics. Proceedings, 4th Latin American CERN-CLAF School, Vina del Mar, Chile, 2007* (2008), pp. 105–157, <https://doi.org/10.5170/CERN-2008-004>.
- [101] T. Inami and C. S. Lim, *Prog. Theor. Phys.* **65**, 297 (1981); **65**, 1772(E) (1981).
- [102] T. Aaltonen *et al.* (CDF Collaboration), *Phys. Rev. Lett.* **107**, 201802 (2011).
- [103] J. P. Lees *et al.* (BABAR), *Phys. Rev. D* **86**, 032012 (2012).
- [104] R. Aaij *et al.* (LHCb Collaboration), *J. High Energy Phys.* **06** (2014) 133.
- [105] R. Aaij *et al.* (LHCb 133), *J. High Energy Phys.* **10** (2015) 034.
- [106] J. A. Bailey *et al.* (Fermilab Lattice and MILC Collaborations), *Phys. Rev. Lett.* **115**, 152002 (2015).
- [107] D. Du, A. X. El-Khadra, S. Gottlieb, A. S. Kronfeld, J. Laiho, E. Lunghi, R. S. Van de Water, and R. Zhou, *Phys. Rev. D* **93**, 034005 (2016).
- [108] C. Bouchard, G. P. Lepage, C. Monahan, H. Na, and J. Shigemitsu (HPQCD Collaboration), *Phys. Rev. Lett.* **111**, 162002 (2013); **112**, 149902(E) (2014).
- [109] A. Khodjamirian and A. V. Rusov, *J. High Energy Phys.* **08** (2017) 112.
- [110] S. Jäger and J. M. Camalich, *Phys. Rev. D* **93**, 014028 (2016).
- [111] S. Davidson, M. L. Mangano, S. Perries, and V. Sordini, *Eur. Phys. J. C* **75**, 450 (2015).
- [112] E. Alvarez, L. Da Rold, A. Juste, M. Szewc, and T. V. Schroeder, *J. High Energy Phys.* **12** (2018) 027.
- [113] I. Doršner, S. Fajfer, D. A. Faroughy, and N. Košnik, *J. High Energy Phys.* **10** (2017) 188.
- [114] C.-H. Chen, T. Nomura, and H. Okada, *Phys. Lett. B* **774**, 456 (2017).
- [115] D. Das, C. Hati, G. Kumar, and N. Mahajan, *Phys. Rev. D* **94**, 055034 (2016).
- [116] I. Doršner, S. Fajfer, and N. Košnik, *Eur. Phys. J. C* **77**, 417 (2017).
- [117] W. Dekens, J. de Vries, M. Jung, and K. K. Vos, *J. High Energy Phys.* **01** (2019) 069.