Baryogenesis from flavon decays

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Many popular attempts to explain the observed patterns of fermion masses involve a flavon field. Such weakly coupled scalar fields tend to dominate the energy density of the Universe before they decay. If the flavon decay happens close to the electroweak transition, the right-handed electrons stay out of equilibrium until the sphalerons shut off. We show that an asymmetry in the right-handed charged leptons produced in the decay of a flavon can explain the baryon asymmetry of the Universe.

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I. INTRODUCTION

There is more matter than antimatter in the Universe. This baryon asymmetry of the Universe (BAU) is measured to be [\[1,2\]](#page-4-0)

$$
\eta_{\text{obs}} \equiv \frac{n_B - n_{\bar{B}}}{s} \simeq 8 \times 10^{-11},\tag{1}
$$

where $n_{B(\bar{B})}$ is the (anti)baryon number density and s is the entropy density.

In order to explain the BAU, the three so called Sakharov conditions [\[3\]](#page-4-1) must be satisfied: (i) baryon (or lepton) number must be violated, (ii) C and CP must be violated and (iii) there must be a departure from thermal equilibrium.

Baryon number and CP are violated in the electroweak (EW) sector of the Standard Model (SM). However the CP violation in the SM is too small to account for the BAU. In addition, there is no out-of-equilibrium process in the SM. In order to produce the BAU, beyond-the-SM physics with additional degrees of freedom (d.o.f.) is needed. These d.o.f. can also solve other deficiencies in the SM. The most notable example is in leptogenesis [\[4\]](#page-4-2), where the right-handed neutrinos are the only required additional d.o.f. These right-handed neutrinos are part of the seesaw mechanism [\[5\]](#page-4-3), and thus provide an explanation for the smallness of neutrino masses. Through their out-ofequilibrium decays, the observed BAU can be obtained.

In this paper, we show that another d.o.f., the *flavon*, that conceivably plays an important role in understanding fermion masses, can also generate the observed BAU. In general, a flavon is a scalar particle whose vacuum expectation value (VEV) dictates the (Yukawa) couplings of the SM fermions, with the most popular example being the Froggatt–Nielsen field [\[6\]](#page-4-4). We show that the out-ofequilibrium decays of such a flavon can be the necessary beyond-the-SM physics that generates the observed BAU. The flavon decays may create a left-right charged lepton asymmetry. In a universe where the flavon dominates the energy density, the right-handed electrons may not come into chemical equilibrium, and an asymmetry in these righthanded electrons can be converted into a baryon number asymmetry by sphalerons.

II. BASIC MECHANISM

Consider the couplings of the SM charged leptons to an SM singlet scalar, the flavon S, charged under the flavor symmetry $U(1)_{FN}$,

$$
\mathcal{L} \supset y_0^{fg} \left(\frac{v_S + S}{\Lambda} \right)^{n_{fg}} \bar{e}_R^g \cdot \phi^* \cdot \mathcal{E}_L^f + \text{H.c.}
$$
 (2)

Here, ϕ denotes the electroweak Higgs doublet, ℓ_{L}^{j} is the fth-generation left-handed lepton doublet, and e_R^g repre-
sents the *o*th-generation right-handed charged lepton, v_a is sents the gth-generation right-handed charged lepton. v_S is the flavon VEV, with Λ being the cutoff scale of the flavor symmetry, y_0 an $\mathcal{O}(1)$ coupling constant and n_{fg} integers related to the Froggatt-Nielsen charges under $U(1)_{FN}$. In the following, we will restrict ourselves to the simplest version of the scenario, in which the flavon only couples to leptons. As a benchmark model, we will take

$$
y_0 \approx 1
$$
, $\varepsilon = \frac{v_S}{\Lambda} = 0.2$, $n_e = 9$, $n_\tau = 3$. (3)

The key observation in our scenario is that the flavon decays

$$
S \to \bar{e}_L + \phi + e_R, \qquad S^* \to e_L + \phi^* + \bar{e}_R, \qquad (4)
$$

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violate left-right symmetry, e.g., an S particle only decays into an antiparticle ℓ_L and particle e_R . Crucially, if there is an initial flavon asymmetry, say an excess of S over S^* , after the flavons decay there will be more left-handed after the flavons decay, there will be more left-handed antileptons than left-handed leptons. Note that the total lepton number, L, is conserved because the asymmetry in the left-handed leptons will be balanced by an equal and opposite asymmetry in the right-handed leptons. However, sphalerons will only act on left-handed leptons and partially convert this asymmetry into a baryon (B) asymmetry. This is similar to Dirac leptogenesis [\[7\]](#page-4-5), in which nonzero baryon number asymmetry is possible with $B - L$ conservation, as it utilizes the features that sphalerons only couple to left-handed fields. In the scenario discussed here, the left-right asymmetry originates from a primordial flavon-antiflavon asymmetry, which we will discuss below in more detail.

III. FLAVON ASYMMETRY

Our scenario requires the flavon to have a large asymmetry when it decays. This means that an asymmetry must be created and that flavon number must be conserved during the flavon oscillations. It has been pointed out in the context of the Affleck-Dine scenario [\[8\]](#page-4-6) that scalar field dynamics in the early Universe can exhibit these features. As explained in Sec. III.C of [\[9\],](#page-4-7) in supersymmetric theories the mass terms of scalar fields S preserve S number while interaction terms do not. These interactions may provide an "initial kick" (in the terminology of Ref. [\[10\]\)](#page-4-8) which leads to an angular momentum of the S field in the complex plane, i.e., an S-number asymmetry. In general, if the lowest-order interactions preserve flavon number and the higher-order terms do not, the latter can lead to an initial asymmetry that is preserved when the higher-order terms become unimportant. While this has been discussed mostly in supersymmetric scenarios [\[9\]](#page-4-7), such setups may also be a consequence of, say, a \mathbb{Z}_N symmetry. As explicitly shown in Ref. [\[10\],](#page-4-8) a large flavon asymmetry may originate from S^6/Λ^2 interactions if the flavon mass is of the order of the gravitino mass, $m_S ~ m_{3/2}$. For higher-dimensional terms, a large flavon asymmetry is even easier to produce [\[9\]](#page-4-7). That is, the Affleck-Dine mechanism provides us with a concrete example of a setting in which the large asymmetry is generated, yet there are other possibilities. The point of this paper, however, is that, due to the intricate dynamics of the chemical potential of the electrons and the sphaleron, an asymmetry of the S particles, which carry neither baryon nor lepton number, can explain the baryon number of the Universe. Note that the higher-dimensional terms become unimportant later when the S field performs coherent oscillations, such that the asymmetry persists until the flavon decays.

A reason for concern might be the origin of the U(1) symmetry that ensures flavon number conservation. At first sight, one may conclude that if $U(1)_{FN}$ gets broken spontaneously, one is left with two options: (i) $U(1)_{FN}$ is local, in which case the would-be Goldstone mode gets eaten, and one is left with a real scalar and no $U(1)_{S}$ that allows us to define the S-number; (ii) $U(1)_{FN}$ is global, in which case there is a Goldstone mode, and the situation is even worse. However, in explicit models the situation is often richer than that. For instance, in supersymmetric scenarios, the scalar fields are complex. In such models, one starts with more d.o.f. and the flavon is a complex linear combination of these fields, whose mass term preserves a U(1) symmetry. There are also nonsupersymmetric examples that have additional custodial symmetries (see e.g., Refs. [\[11,12\]](#page-4-9) for recent applications of these symmetries in dark matter model building). We will assume that for energy scales far below Λ an approximate $U(1)$ _s is preserved by the flavon potential, i.e.,

$$
\mathcal{V}_S = m^2 |S|^2 + \begin{pmatrix} S\text{-number-violating terms} \\ \text{suppressed by powers of } \Lambda \end{pmatrix}.
$$
 (5)

IV. FLAVON COSMOLOGY

By assumption, the flavon we consider is a weakly coupled scalar field. Hence it can perform coherent oscillations around the $T = 0$ minimum of its potential. In fact, thermal corrections to the potential will push the flavon away from its expectation value at $T = 0$ [\[13\]](#page-4-10). The energy density stored in these oscillations, ρ_s , only drops as a^{-3} whereas the energy density of radiation, ρ_{rad} , drops as a^{-4} , where a is the scale factor. It is expected that at a time t_{\ast} , corresponding to a temperature t_{\ast} , the energy density stored in flavon oscillations starts to dominate the radiation , corresponding to a temperature T_* , the energy density ored in flavon oscillations starts to dominate the radiation contribution. We define this point as

$$
\rho_S|_{T=T_*} = \rho_{\text{rad}}|_{T=T_*}.\tag{6}
$$

The flavon decays to SM leptons with a decay rate

$$
\Gamma_S \sim \frac{1}{\varepsilon} \frac{|n_\tau y_\tau|^2}{64\pi^3} \frac{m_S^3}{\Lambda^2}.
$$
 (7)

Most of the decay products thermalize with the radiation and contribute to the radiation density. However, as we will show below, the right-handed electrons might not come into chemical equilibrium before the sphalerons have switched off.

The evolution of the relevant energy densities is given by

$$
\frac{\mathrm{d}\rho_S}{\mathrm{d}t} + 3H\rho_S = -\Gamma_S\rho_S,\tag{8a}
$$

$$
\frac{\mathrm{d}\rho_{\text{rad}}}{\mathrm{d}t} + 4H\rho_{\text{rad}} = \Gamma_{\text{S}}\rho_{\text{S}},\tag{8b}
$$

where the Hubble rate is determined by the Friedmann equation

$$
H^{2} = \frac{8\pi}{3M_{\text{Pl}}^{2}} (\rho_{S} + \rho_{\text{rad}}),
$$
 (9)

with $M_{\text{Pl}} \simeq 1.2 \times 10^{19}$ GeV being the Planck mass.

We solve this set of equations numerically and show a benchmark scenario in Fig. [1.](#page-2-0) Analytical approximations for the two energy densities can be given at a time t before the flavon decays, $t_* < t < \tau \sim \Gamma_S^{-1}$, as

$$
\rho_S(t) \simeq \frac{M_{\rm Pl}^2}{6\pi t^2} e^{-\Gamma_S t},\tag{10a}
$$

$$
\rho_{\rm rad}(t) \simeq \frac{M_{\rm Pl}^2 t_*^{2/3}}{6\pi t^{8/3}} + \frac{\Gamma_S M_{\rm Pl}^2}{10\pi t}.
$$
 (10b)

The first term in ρ_{rad} is the initial radiation energy density and the second term is generated from flavon decays. This second term starts taking over ρ_{rad} long before Γ_S^{-1} .
We will show in the next section that BAIL c

We will show in the next section that BAU can be produced via flavon decays for $\Gamma_S \sim O(10^{-13}-10^{-17} \text{ GeV})$.
Thus, using Eq. (7) we will require the flavon mass to be Thus, using Eq. [\(7\)](#page-1-0) we will require the flavon mass to be

$$
\frac{m_S}{\text{TeV}} \sim \left(\frac{\Lambda}{10^9 \text{ GeV}}\right)^{\frac{2}{3}} \left(\frac{\Gamma_S}{10^{-15} \text{ GeV}}\right)^{\frac{1}{3}},\tag{11}
$$

such that Λ is below the Planck scale.

FIG. 1. Energy densities stored in flavon oscillations, ρ_S , and in radiation ρ_{rad} , with respect to temperature. As a benchmark scenario, it is assumed that ρ_S starts to dominate at $T_* = 100 \text{ TeV}$ and the flavon decay rate is $\Gamma = 10^{-15} \text{GeV}$ which 100 TeV and the flavon decay rate is $\Gamma_s = 10^{-15}$ GeV, which corresponds to a decay temperature $T_d \approx 10$ GeV. We also show the temperature when the EW sphalerons shut off, $T_{\text{EW}} \sim 160 \text{ GeV}.$

V. GENERATION AND NONEQUILIBRATION OF RIGHT-HANDED ELECTRONS

Among other leptons, the flavon decays into righthanded electrons with a branching fraction of $B_e \sim \left(\frac{n_e y_e}{n_{e y_e}}\right)^2 \sim 7.5 \times 10^{-7}$. Through its decays, the flavon asymmetry will 7.5×10^{-7} . Through its decays, the flavon asymmetry will get partially converted into an asymmetry in right-handed electrons. Like in leptogenesis [\[4\]](#page-4-2), this asymmetry is turned into a baryon asymmetry by sphalerons. Similarly to Dirac leptogenesis [\[7\],](#page-4-5) our scenario does not require $B - L$ violation. However, our scenario works both for Dirac and Majorana neutrinos.

Right-handed electrons equilibrate with the SM plasma mainly through their interactions with the Higgs boson and $2 \rightarrow 2$ scatterings. This equilibration rate has been recently calculated to be [\[14\]](#page-4-11)

$$
\Gamma_{LR} \simeq 10^{-2} y_e^2 T,\tag{12}
$$

which is larger by almost an order of magnitude than the initial estimate [\[15\]](#page-4-12) and by a factor of a few than a refined estimate [\[16\].](#page-4-13) Comparing this rate to the Hubble rate for a radiation-dominated universe, one finds that the righthanded electrons come into equilibrium at $T \sim 10^5$ GeV. Hence, in standard cosmology, any asymmetry in the righthanded electrons would be washed out long before the EW transition at $T \sim 160$ GeV. However, in a universe that is dominated by a flavon until temperatures around the electroweak scale, right-handed electrons may not equilibrate. (See Fig. [2](#page-2-1).)

We find the asymmetry in the number density of righthanded electrons after the flavon decays by solving the Boltzmann equation for $n_{\rm R} = n_{e_{\rm R}} - n_{\bar{e}_{\rm R}}$,

$$
\frac{\mathrm{d}n_{\mathrm{R}}}{\mathrm{d}t} = -3Hn_{\mathrm{R}} - \Gamma_{\mathrm{LR}}n_{\mathrm{R}} + B_e \Gamma_S n_S, \tag{13}
$$

FIG. 2. Comparison of the equilibration rate of right-handed electrons, Γ_{LR} , to the Hubble rates in a radiation-dominated universe, H_{rad} and in a universe with intermediate flavon domination, H_S . For this benchmark case the flavon energy density starts to dominate at $T_* = 10^7$ TeV and the flavon decays at $T_* \sim 10$ GeV at $T_d \simeq 10$ GeV.

FIG. 3. Right-handed electron asymmetry generated through asymmetric flavon decays for different values of the flavon decay rate Γ_s . The observed baryon asymmetry $\eta_{obs} \simeq 8 \times 10^{-11}$ is shown for reference.

where B_e is the flavon branching ratio to electrons. The asymmetry in the flavon number density, n_S , is parametrized through an initial flavon asymmetry, η_s , as

$$
n_S \equiv \eta_S \frac{\rho_S}{m_S}.\tag{14}
$$

We solve for n_R together with Eq. [\(8\)](#page-1-1). In Fig. [3](#page-3-0) we show the dimensionless quantity $\eta_R = \frac{n_R}{s}$, where s is the entropy
density for a henchmark scenario with density, for a benchmark scenario with

$$
m_S = 1 \text{ TeV}, \qquad \eta_S = 1, \qquad T_* = 100 \text{ TeV}.
$$
 (15)

As usual, the temperature is defined as the temperature of radiation, which may or may not dominate the energy density of the Universe.

The behavior of η_R can be understood qualitatively as follows. A right-handed electron asymmetry is produced through flavon decays and is proportional to the flavon number density. When the Hubble rate drops below the leftright equilibration rate, the asymmetry is washed out by SM interactions. The time when this washout starts depends on the flavon lifetime, $\tau = \Gamma_5^{-1}$, and is generally
before τ . A smaller Γ , can delay the equilibration until after before τ . A smaller Γ_S can delay the equilibration until after sphalerons shut off at $T_{EW} \sim 160$ GeV. However, this also means that a smaller number of flavons decay before T_{EW} . Hence, in general, there is a balance between the flavon lifetime and Γ_{LR} that describes the region where the right amount of right-handed electron asymmetry is produced before the sphalerons shut off.

VI. BARYON ASYMMETRY

In the SM $B + L$ number is violated by EW processes called sphalerons [\[17,18\].](#page-4-14) Sphalerons are unsuppressed above the EW transition temperature T_{EW} , with a thermal rate $\Gamma_{\text{soh}} \simeq (\alpha_2 T)^4$, while they are exponentially suppressed due to finite gauge boson masses after EW symmetry

FIG. 4. Baryon asymmetry ($\eta_{10} \equiv \eta_B \times 10^{10}$) of the Universe as a function of the flavon decay rate $\Gamma_{\rm S}$ for different values of the flavon mass m_S . As a benchmark scenario we take the initial flavon asymmetry $\eta_s = 1$. The observed baryon asymmetry $\eta_{\text{obs}} \simeq 8 \times 10^{-11}$ is shown for reference.

breaking. While sphalerons violate $B + L$, they conserve $B - L$. Hence a lepton asymmetry will be turned into a baryon asymmetry [\[4\]](#page-4-2). In our scenario this baryon asymmetry is given by [\[16\]](#page-4-13)

$$
\eta_B \equiv \frac{n_B}{s} \simeq \frac{198}{481} \frac{n_R}{s} \bigg|_{T_{\text{EW}}}.
$$
\n(16)

We numerically solve Eqs. [\(8\)](#page-1-1) and [\(13\)](#page-2-2) to find the electron asymmetry and the entropy of the Universe at $T_{\text{EW}} \simeq 160 \text{ GeV}$. Our results are shown in Figs. [3](#page-3-0) and [4.](#page-3-1) As can be seen in Fig. [4](#page-3-1), there is a large parameter space where asymmetric flavon decays produce the observed BAU. We comment on the dependence of η_B on the parameters of the model.

- (1) Our scenario is rather sensitive to the flavon decay rate. If the flavon decays too early, $T_d \gg T_{EW}$, righthanded electrons equilibrate and (essentially) no baryon asymmetry is generated. If the flavon decays too late, $T_d \ll T_{EW}$, the right-handed electron asymmetry is produced when sphalerons are inoperative, and again no baryon asymmetry emerges. We find that the observed baryon asymmetry is produced for 10^{-16} GeV $\lesssim \Gamma_s \lesssim 10^{-13}$ GeV.
- (2) The baryon asymmetry is virtually independent of the time t_* when the flavon domination starts as long
as $T \gg T_+$, T_{true} as $T_* \gg T_d$, T_{EW} .
As the flavon ma
- (3) As the flavon mass increases, its number density drops. This is because each flavon decay produces entropy, with the dilution factor being m_S/T_d . Hence the baryon asymmetry is inversely proportional to m_S . The flavon decay rate also depends on its mass. For given values of m_S and Γ_S , the appropriate flavon scale Λ can be found from Eq. [\(11\)](#page-2-3). We find that a flavon with $\mathcal{O}(TeV)$ mass can produce the observed BAU. This requires

 $\Lambda \sim \mathcal{O}(10^9 \text{ GeV})$. We note that for such a large flavon scale, constraints on the flavon mass are $O(100 \text{ GeV})$ [\[19\]](#page-4-15).

(4) The baryon asymmetry is directly proportional to the primordial flavon asymmetry η_s . We show our results in Fig. [4](#page-3-1) for the maximal value of $\eta_s = 1$. If we allow for a flavon as light as 200 GeV, we only require $\eta_s \gtrsim 10^{-2}$.

VII. CONCLUSIONS

In this paper, we have shown that the observed baryon asymmetry can be produced in flavon decays. This scenario requires the flavon to decay around the time of the electroweak transition and that the flavon carries an $\mathcal{O}(10^{-2} - 1)$ primordial asymmetry. The flavon mass is $\mathcal{O}(1-10 \text{ TeV})$. The viable parameter ranges are illustrated in Fig. [4](#page-3-1). The role of the flavon is twofold:

- (1) Its decays produce a left-right asymmetry in the lepton sector, the left-handed part of which is converted to a baryon asymmetry by sphalerons.
- (2) It dominates the Universe before the EW scale, thus increasing the Hubble rate and preventing the righthanded electrons from equilibrating.

Our scenario is rather constrained and thus predictive. We "lose" a significant amount of efficiency because our scalar is a flavon, i.e., it predominantly decays into the heavier generations. A more "efficient" version of this scenario would be to couple a scalar to light fermions only, in which case one may also consider the decay into up or even down quarks. Note that this scalar does not need to be the flavon, but any complex scalar field that satisfies the above properties could give these results. However, it is arguably an appealing aspect of our scenario that we do not have to introduce a new d.o.f. in order to explain the observed baryon asymmetry. Rather, a traditional flavon can do that without being tweaked.

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