Color-allowed bottom baryon to s-wave and p-wave charmed baryon nonleptonic decays

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We study color allowed bottom baryon to s-wave and p-wave charmed baryon nonleptonic decays in this work. The charmed baryons include spin-1/2 and spin-3/2 states. Explicitly, we consider $\Lambda_b \to \Lambda_c^{(*,**)} M^-$, $\Xi_b \to \Xi_c^{(**)} M^-$, and $\Omega_b \to \Omega_c^{(*,**)} M^-$ decays with $M = \pi, K, \rho, K^*, a_1, D, D_s, D^*, D_s^*$, $\Lambda_c^{(*,**)} = \Lambda_c, \Lambda_c(2595), \Lambda_c(2625), \Lambda_c(2765), \Lambda_c(2940)$, $\Xi_c^{(**)} = \Xi_c, \Xi_c(2790), \Xi_c(2815)$ and $\Omega_c^{(*,**)} = \Omega_c, \Omega_c(2770), \Omega_c(3050), \Omega_c(3090), \Omega_c(3120)$. There are six types of transitions, namely, (i) $\mathcal{B}_b(\bar{\mathbf{3}}_\mathbf{f}, 1/2^+)$ to $\mathcal{B}_c(\bar{\mathbf{3}}_\mathbf{f}, 1/2^+)$, (ii) $\mathcal{B}_b(\mathbf{6}_\mathbf{f}, 1/2^+)$ to $\mathcal{B}_c(\mathbf{6}_\mathbf{f}, 1/2^+)$, (iii) $\mathcal{B}_b(\mathbf{6}_\mathbf{f}, 1/2^+)$ to $\mathcal{B}_c(\mathbf{6}_\mathbf{f}, 3/2^+)$, (iv) $\mathcal{B}_b(\mathbf{6}_\mathbf{f}, 1/2^+)$ to $\mathcal{B}_c(\mathbf{6}_\mathbf{f}, 3/2^-)$, (v) $\mathcal{B}_b(\bar{\mathbf{3}}_\mathbf{f}, 1/2^+)$ to $\mathcal{B}_c(\bar{\mathbf{3}}_\mathbf{f}, 1/2^-)$, and (vi) $\mathcal{B}_b(\bar{\mathbf{3}}_\mathbf{f}, 1/2^+)$ to $\mathcal{B}_c(\bar{\mathbf{3}}_\mathbf{f}, 3/2^-)$ transitions. Types (i) to (iii) involve spin 1/2 and 3/2 s-wave charmed baryons, while types (iv) to (vi) involve spin 1/2 and 3/2 p-wave charmed baryons. The light diquarks are spectating in these transitions. The transition form factors are calculated in the light-front quark model approach. All of the form factors in the $1/2 \to 1/2$ and $1/2 \to 3/2$ transitions are extracted, and they are found to reasonably satisfy the relations obtained in the heavy quark limit, as we are using heavy but finite m_b and m_c. Using naïve factorization, decay rates and up-down asymmetries of the above modes are predicted and can be checked experimentally. We compare our results to data and other theoretical predictions. The study on these decay modes may shed light on the quantum numbers of $\Lambda_c(2765)$, $\Lambda_c(2940)\Omega_c(3050)$, $\Omega_c(3090)$, and $\Omega_c(3120)$.

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I. INTRODUCTION

There are some experimental progresses in the charmed baryon sector recently. In year 2017 LHCb discovered $\Lambda_c(2864)$ and five Ω_c states, namely $\Omega_c(3000)^0$, $\Omega_c(3050)^0$, $\Omega_c(3066)^0$, $\Omega_c(3090)^0$, and $\Omega_c(3120)^0$ [1,2]. The first four states among the five newly discovered Ω_c were confirmed by Belle later [3]. In Table I the mass spectra, decay widths and quantum numbers of charmed baryons observed up to now are summarized. Note that 16 out of 40 charmed baryons have unspecified quantum numbers, while the quantum numbers of the rests are determined with different levels of certainty [4,5]. Those with unspecified quantum numbers include $\Lambda_c(2765)^+$, $\Sigma_c(2800)^{++,+,0}$, $\Xi_c(2930)^0$, $\Xi_c(2970)^{+,0}$, $\Xi_c(3055)^+$, $\Xi_c(3080)^{+,0}$, $\Xi_c(3123)^+$ and the above mentioned five Ω_c states. Various suggestions on the quantum numbers of the newly discovered Ω_c states were put forward after the discovery, see e.g., [6–13]. Note that even for the one with specified quantum number in PDG, it is still room for different assignments. For example, two possible quantum numbers of $\Lambda_c(2940)^+$ were proposed. LHCb and PDG preferred the $\frac{3}{2}^-$ quantum number, but it is not certain [1,4], while a $\frac{1}{2}^-$ state was advocated by the authors of Ref. [6]. It is timely, of great interest and importance to identify the quantum numbers of these states and to study their properties.

The study of bottom baryon to charmed baryon weak decays may shed light on the quantum numbers of some of the charmed baryons. Up to now only several color allowed $\Lambda_b \to \Lambda_c P$ decay rates were measured. These include the rates of $\Lambda_b \to \Lambda_c \pi^-$, $\Lambda_c K^-$, $\Lambda_c D^-$ and $\Lambda_c D_s^-$ decays, which were reported by LHCb several years ago [17–19]. It is not unrealistic to expect that further progress on the experimental side, either from LHCb, from Belle II or from elsewhere, will occur soon. In [16] we studied the color allowed $\Lambda_b^0 \to \Lambda_c^{(*,**)} M$, $\Xi_b \to \Xi_c^{(**)} M$ and $\Omega_b \to \Omega_c^{(*)} M$ decays with $M = \pi, K, \rho, K^*, a_1, D, D_s, D^*, D_s^*$, and $\Lambda_c^{(*,**)} = \Lambda_c, \Lambda_c(2595), \Lambda_c(2765), \Lambda_c(2940)$, $\Xi_c^{(**)} = \Xi_c, \Xi_c(2790)$ and $\Omega_c^{(*)} = \Omega_c, \Omega_c(3090)$, which are spin-1/2 charm baryons. In this work we will extend the study to

TABLE I. Mass spectra, widths (in units of MeV) and quantum numbers of charmed baryons are summarized. Experimental values of masses and widths and J^P are taken from the Particle Data Group (PDG) [4,5]. Note that $S_{[qq]}$, L_k, L_K, and J_l are the spin of the diquark $[qq]$, the orbital angular momentum between the light quarks, the orbital angular momentum of the $Q-[qq]$ system and the total angular momentum of the light degree of freedom, respectively. See [14–16] for more details.

State	J^P	n	(L_K, L_k)	$S^P_{[qq]}$	$J^{P_\ell}_\ell$	Mass	Width	Decay modes
Λ_c^+	$\frac{1}{2}^+$	1	(0,0)	0^+	0^+	2286.46 ± 0.14		Weak
$\Lambda_c(2595)^+$	$\frac{1}{2}^-$	1	(1,0)	0^+	1^-	2592.25 ± 0.28	2.6 ± 0.6	$\Lambda_c\pi\pi, \Sigma_c\pi$
$\Lambda_c(2625)^+$	$\frac{3}{2}^-$	1	(1,0)	0^+	1^-	2628.11 ± 0.19	<0.97	$\Lambda_c\pi\pi, \Sigma_c\pi$
$\Lambda_c(2765)^+$	$?^?$	?	?	?	?	2766.6 ± 2.4	50	$\Sigma_c\pi, \Lambda_c\pi\pi$
$\Lambda_c(2860)^+$	$\frac{3}{2}^+$	1	(2,0)	0^+	2^+	$2856.1^{+2.3}_{-6.0}$	68^{+12}_{-22}	$\Sigma_c^{(*)}\pi, D^0p, D^+n$
$\Lambda_c(2880)^+$	$\frac{5}{2}^+$	1	(2,0)	0^+	2^+	2881.63 ± 0.24	$5.6^{+0.8}_{-0.6}$	$\Sigma_c^{(*)}\pi, \Lambda_c\pi\pi, D^0p$
$\Lambda_c(2940)^+$	$\frac{3}{2}^-$	2	(1,0)	0^+	1^-	$2939.6^{+1.3}_{-1.5}$	20^{+6}_{-5}	$\Sigma_c^{(*)}\pi, \Lambda_c\pi\pi, D^0p$
$\Sigma_c(2455)^{++}$	$\frac{1}{2}^+$	1	(0,0)	1^+	1^+	2453.97 ± 0.14	$1.89^{+0.09}_{-0.18}$	$\Lambda_c\pi$
$\Sigma_c(2455)^+$	$\frac{1}{2}^+$	1	(0,0)	1^+	1^+	2452.9 ± 0.4	<4.6	$\Lambda_c\pi$
$\Sigma_c(2455)^0$	$\frac{1}{2}^+$	1	(0,0)	1^+	1^+	2453.75 ± 0.14	$1.83^{+0.11}_{-0.19}$	$\Lambda_c\pi$
$\Sigma_c(2520)^{++}$	$\frac{3}{2}^+$	1	(0,0)	1^+	1^+	$2518.41^{+0.21}_{-0.19}$	$14.78^{+0.30}_{-0.40}$	$\Lambda_c\pi$
$\Sigma_c(2520)^+$	$\frac{3}{2}^+$	1	(0,0)	1^+	1^+	2517.5 ± 2.3	<17	$\Lambda_c\pi$
$\Sigma_c(2520)^0$	$\frac{3}{2}^+$	1	(0,0)	1^+	1^+	2518.48 ± 0.20	$15.3^{+0.4}_{-0.5}$	$\Lambda_c\pi$
$\Sigma_c(2800)^{++}$	$?^?$	?	?	?	?	2801^{+4}_{-6}	75^{+22}_{-17}	$\Lambda_c\pi, \Sigma_c^{(*)}\pi, \Lambda_c\pi\pi$
$\Sigma_c(2800)^+$	$?^?$	?	?	?	?	2792^{+14}_{-5}	62^{+60}_{-40}	$\Lambda_c\pi, \Sigma_c^{(*)}\pi, \Lambda_c\pi\pi$
$\Sigma_c(2800)^0$	$?^?$	?	?	?	?	2806^{+5}_{-7}	72^{+22}_{-15}	$\Lambda_c\pi, \Sigma_c^{(*)}\pi, \Lambda_c\pi\pi$
Ξ_c^+	$\frac{1}{2}^+$	1	(0,0)	0^+	0^+	2467.87 ± 0.30		Weak
Ξ_c^0	$\frac{1}{2}^+$	1	(0,0)	0^+	0^+	$2470.87^{+0.28}_{-0.31}$		Weak
$\Xi_c'^+$	$\frac{1}{2}^+$	1	(0,0)	1^+	1^+	2577.4 ± 1.2		$\Xi_c\gamma$
$\Xi_c'^0$	$\frac{1}{2}^+$	1	(0,0)	1^+	1^+	2578.8 ± 0.5		$\Xi_c\gamma$
$\Xi_c(2645)^+$	$\frac{3}{2}^+$	1	(0,0)	1^+	1^+	2645.53 ± 0.31	2.14 ± 0.19	$\Xi_c\pi$
$\Xi_c(2645)^0$	$\frac{3}{2}^+$	1	(0,0)	1^+	1^+	2646.32 ± 0.31	2.35 ± 0.22	$\Xi_c\pi$
$\Xi_c(2790)^+$	$\frac{1}{2}^-$	1	(1,0)	0^+	1^-	2792.0 ± 0.5	8.9 ± 1.0	$\Xi_c'\pi$
$\Xi_c(2790)^0$	$\frac{1}{2}^-$	1	(1,0)	0^+	1^-	2792.8 ± 1.2	10.0 ± 1.1	$\Xi_c'\pi$
$\Xi_c(2815)^+$	$\frac{3}{2}^-$	1	(1,0)	0^+	1^-	2816.67 ± 0.31	2.43 ± 0.26	$\Xi_c^*\pi, \Xi_c\pi\pi, \Xi_c'\pi$
$\Xi_c(2815)^0$	$\frac{3}{2}^-$	1	(1,0)	0^+	1^-	2820.22 ± 0.32	2.54 ± 0.25	$\Xi_c^*\pi, \Xi_c\pi\pi, \Xi_c'\pi$
$\Xi_c(2930)^0$	$?^?$	?	?	?	?	2931 ± 6	36 ± 13	$\Lambda_c\bar{K}$
$\Xi_c(2970)^+$	$?^?$	?	?	?	?	2969.4 ± 0.8	$20.9^{+2.4}_{-3.5}$	$\Sigma_c\bar{K}, \Lambda_c\bar{K}\pi, \Xi_c\pi\pi$
$\Xi_c(2970)^0$	$?^?$	?	?	?	?	2967.8 ± 0.8	$28.1^{+3.4}_{-4.0}$	$\Sigma_c\bar{K}, \Lambda_c\bar{K}\pi, \Xi_c\pi\pi$
$\Xi_c(3055)^+$	$?^?$	?	?	?	?	3055.9 ± 0.4	7.8 ± 1.9	$\Sigma_c\bar{K}, \Lambda_c\bar{K}\pi, D\Lambda$
$\Xi_c(3080)^+$	$?^?$	?	?	?	?	3077.2 ± 0.4	3.6 ± 1.1	$\Sigma_c\bar{K}, \Lambda_c\bar{K}\pi, D\Lambda$
$\Xi_c(3080)^0$	$?^?$	?	?	?	?	3079.9 ± 1.4	5.6 ± 2.2	$\Sigma_c\bar{K}, \Lambda_c\bar{K}\pi, D\Lambda$
$\Xi_c(3123)^+$	$?^?$	?	?	?	?	3122.9 ± 1.3	4 ± 4	$\Sigma_c^*\bar{K}, \Lambda_c\bar{K}\pi$
Ω_c^0	$\frac{1}{2}^+$	1	(0,0)	1^+	1^+	2695.2 ± 1.7		weak
$\Omega_c(2770)^0$	$\frac{3}{2}^+$	1	(0,0)	1^+	1^+	2765.9 ± 2.0		$\Omega_c\gamma$
$\Omega_c(3000)^0$	$?^?$	?	?	?	?	3000.4 ± 0.4	4.5 ± 0.7	$\Xi_c\bar{K}$
$\Omega_c(3050)^0$	$?^?$	?	?	?	?	3050.2 ± 0.33	<1.2	$\Xi_c\bar{K}$
$\Omega_c(3065)^0$	$?^?$	?	?	?	?	3065.6 ± 0.4	3.5 ± 0.4	$\Xi_c\bar{K}$
$\Omega_c(3090)^0$	$?^?$	?	?	?	?	3090.2 ± 0.7	8.7 ± 1.3	$\Xi_c^{(\prime)}\bar{K}$
$\Omega_c(3120)^0$	$?^?$	?	?	?	?	3119.1 ± 1.0	<2.6	$\Xi_c^{(\prime)}\bar{K}$

TABLE II. Configurations of s-wave and p-wave singly charmed baryons are shown. The angular momenta are defined as $\vec{S}_{qq}$, $\vec{S}_{[qq]} \equiv \vec{L}_k + \vec{S}_{qq}$, $\vec{J}_l \equiv \vec{S}_{[qq]} + \vec{L}_K$ and $\vec{J} \equiv \vec{J}_l + \vec{S}_Q$, which are the angular momenta of the light quark pair (without the relative orbital angular momentum), the whole diquark system, the light-degree of freedom, and the whole baryon, respectively. The quantum number assignments of these states are from Table I and [16], while those with (†) are taken from [6]. States with $L_K = 0$ and 1 correspond to s-wave and p-wave states, respectively.

n	L_K	L_k	flavor	S_{qq}	$S^P_{[qq]}$	J^P_l	J^P	$\mathcal{B}_c$
1	0	0	$\bar{\mathbf{3}}_\mathbf{f}$	0	0^+	0^+	$\frac{1}{2}^+$	Λ_c^+, $\Xi_c^{+,0}$
2	0	0	$\bar{\mathbf{3}}_\mathbf{f}$	0	0^+	0^+	$\frac{1}{2}^+$	$\Lambda_c(2765)^+(\dagger)$
1	0	0	$\mathbf{6}_\mathbf{f}$	1	1^+	1^+	$\frac{1}{2}^+$	$\Sigma_c(2455)^{++,+,0}$, $\Xi_c'^{+,0}$, Ω_c^0
2	0	0	$\mathbf{6}_\mathbf{f}$	1	1^+	1^+	$\frac{1}{2}^+$	$\Xi_c'(2970)^{+,0}(\dagger)$, $\Omega_c(3090)^0(\dagger)$
1	0	0	$\mathbf{6}_\mathbf{f}$	1	1^+	1^+	$\frac{3}{2}^+$	$\Sigma_c(2520)^{++,+,0}$, $\Xi_c(2645)^{+,0}$, $\Omega_c(2770)^0$
2	0	0	$\mathbf{6}_\mathbf{f}$	1	1^+	1^+	$\frac{3}{2}^+$	$\Omega_c(3120)^0(\dagger)$
1	1	0	$\bar{\mathbf{3}}_\mathbf{f}$	0	0^+	1^-	$\frac{1}{2}^-$	$\Lambda_c(2595)^+$, $\Xi_c(2790)^{+,0}$
2	1	0	$\bar{\mathbf{3}}_\mathbf{f}$	0	0^+	1^-	$\frac{1}{2}^-$	$\Lambda_c(2940)^+(\dagger)$
1	1	0	$\bar{\mathbf{3}}_\mathbf{f}$	0	0^+	1^-	$\frac{3}{2}^-$	$\Lambda_c(2625)^+$, $\Xi_c(2815)^{+,0}$
2	1	0	$\bar{\mathbf{3}}_\mathbf{f}$	0	0^+	1^-	$\frac{3}{2}^-$	$\Lambda_c(2940)^+$
1	1	0	$\mathbf{6}_\mathbf{f}$	1	1^+	2^-	$\frac{3}{2}^-$	$\Sigma_c(2800)^{++,+,0}(\dagger)$, $\Xi_c'(2930)^{+,0}(\dagger)$, $\Omega_c(3050)^0(\dagger)$
1	1	0	$\mathbf{6}_\mathbf{f}$	1	1^+	2^-	$\frac{5}{2}^-$	$\Omega_c(3066)^0(\dagger)$

s-wave and p-wave charmed baryons up to spin-3/2 states, these include $\Lambda_c^{(*,**)} = \Lambda_c, \Lambda_c(2595), \Lambda_c(2625), \Lambda_c(2765)$, $\Lambda_c(2940)$(spin-1/2), $\Lambda_c(2940)$(spin-3/2), $\Xi_c^{(**)} = \Xi_c$, $\Xi_c(2790), \Xi_c(2815)$ and $\Omega_c^{(*,**)} = \Omega_c, \Omega_c(2770), \Omega_c(3050)$, $\Omega_c(3090), \Omega_c(3120)$.

In Table II configurations of s-wave and p-wave charmed baryons are shown. We consider the color allowed nonleptonic two body weak decays, where the light quarks are spectating in the processes. These transitions are straightforward and easier to study. There are six types of $\mathcal{B}_b \to \mathcal{B}_c$ transitions to be studied in this work, namely,

TABLE III. Various bottom baryon to s-wave and p-wave charmed baryon transitions are shown. There are six transition types. Types (i)-(iii) involve s-wave states, the corresponding transitions are (i) $\mathcal{B}_b(\bar{\mathbf{3}}_\mathbf{f}, 1/2^+) \to \mathcal{B}_c(\bar{\mathbf{3}}_\mathbf{f}, 1/2^+)$, (ii) $\mathcal{B}_b(\mathbf{6}_\mathbf{f}, 1/2^+) \to \mathcal{B}_c(\mathbf{6}_\mathbf{f}, 1/2^+)$ and (iii) $\mathcal{B}_b(\mathbf{6}_\mathbf{f}, 1/2^+) \to \mathcal{B}_c(\mathbf{6}_\mathbf{f}, 3/2^+)$ transitions. Types (iv)–(vi) involve p-wave baryons in the final states, the corresponding transitions are (iv) $\mathcal{B}_b(\mathbf{6}_\mathbf{f}, 1/2^+) \to \mathcal{B}_c(\mathbf{6}_\mathbf{f}, 3/2^-)$, (v) $\mathcal{B}_b(\bar{\mathbf{3}}_\mathbf{f}, 1/2^+) \to \mathcal{B}_c(\bar{\mathbf{3}}_\mathbf{f}, 1/2^-)$ and (vi) $\mathcal{B}_b(\bar{\mathbf{3}}_\mathbf{f}, 1/2^+) \to \mathcal{B}_c(\bar{\mathbf{3}}_\mathbf{f}, 3/2^-)$ transitions. Note that types (i), (v) and (vi) transitions involve scalar diquarks, while type (ii), (iii) and (iv) transitions involve axial-vector diquarks. These diquarks are spectating in the transitions. See Tables II for the quantum number assignments for charmed baryons. Note that the asterisks denote the transitions where the final state charmed baryons are radial excited. The two possibilities of quantum numbers for $\Lambda_c(2940)$, namely, a $\mathcal{B}_c(\bar{\mathbf{3}}_\mathbf{f}, 1/2^-)$ state [6] or a $\mathcal{B}_c(\bar{\mathbf{3}}_\mathbf{f}, 3/2^-)$ state [1], will be considered in this work.

Type	$(n, L_K, S^P_{[qq]}, J^P_l, J^P)_b \to (n', L'_K, S^P_{[qq]}, J'^P_l, J'^P)_c$	$\mathcal{B}_b \to \mathcal{B}_c$
(i)	$(1, 0, 0^+, 0^+, \frac{1}{2}^+) \to (1, 0, 0^+, 0^+, \frac{1}{2}^+)$	$\Lambda_b^0 \to \Lambda_c^+$, $\Xi_b^{0(-)} \to \Xi_c^{+(0)}$
(i)*	$(1, 0, 0^+, 0^+, \frac{1}{2}^+) \to (2, 0, 0^+, 0^+, \frac{1}{2}^+)$	$\Lambda_b^0 \to \Lambda_c(2765)^+(\dagger)$
(ii)	$(1, 0, 1^+, 1^+, \frac{1}{2}^+) \to (1, 0, 1^+, 1^+, \frac{1}{2}^+)$	$\Omega_b^- \to \Omega_c^0$
(ii)*	$(1, 0, 1^+, 1^+, \frac{1}{2}^+) \to (2, 0, 1^+, 1^+, \frac{1}{2}^+)$	$\Omega_b^- \to \Omega_c(3090)^0(\dagger)$
(iii)	$(1, 0, 1^+, 1^+, \frac{1}{2}^+) \to (1, 0, 1^+, 1^+, \frac{3}{2}^+)$	$\Omega_b^- \to \Omega_c(2770)^0$
(iii)*	$(1, 0, 1^+, 1^+, \frac{1}{2}^+) \to (2, 0, 1^+, 1^+, \frac{3}{2}^+)$	$\Omega_b^- \to \Omega_c(3120)^0(\dagger)$
(iv)	$(1, 0, 1^+, 1^+, \frac{1}{2}^+) \to (1, 1, 1^+, 2^-, \frac{3}{2}^-)$	$\Omega_b^- \to \Omega_c(3050)^0(\dagger)$
(v)	$(1, 0, 0^+, 0^+, \frac{1}{2}^+) \to (1, 1, 0^+, 1^-, \frac{1}{2}^-)$	$\Lambda_b^0 \to \Lambda_c(2595)^+$, $\Xi_b^{0(-)} \to \Xi_c(2790)^{+(0)}$
(v)*	$(1, 0, 0^+, 0^+, \frac{1}{2}^+) \to (2, 1, 0^+, 1^-, \frac{1}{2}^-)$	$\Lambda_b^0 \to \Lambda_c(2940)^+(\dagger)$
(vi)	$(1, 0, 0^+, 0^+, \frac{1}{2}^+) \to (1, 1, 0^+, 1^-, \frac{3}{2}^-)$	$\Lambda_b^0 \to \Lambda_c(2625)^+$, $\Xi_b^{0(-)} \to \Xi_c(2815)^{+(0)}$
(vi)*	$(1, 0, 0^+, 0^+, \frac{1}{2}^+) \to (2, 1, 0^+, 1^-, \frac{3}{2}^-)$	$\Lambda_b^0 \to \Lambda_c(2940)^+$

(i) $\mathcal{B}_b(\bar{\mathbf{3}}_{\mathbf{f}}, 1/2^+)$ to $\mathcal{B}_c(\bar{\mathbf{3}}_{\mathbf{f}}, 1/2^+)$, (ii) $\mathcal{B}_b(\mathbf{6}_{\mathbf{f}}, 1/2^+)$ to $\mathcal{B}_c(\mathbf{6}_{\mathbf{f}}, 1/2^+)$, (iii) $\mathcal{B}_b(\mathbf{6}_{\mathbf{f}}, 1/2^+)$ to $\mathcal{B}_c(\mathbf{6}_{\mathbf{f}}, 3/2^+)$, (iv) $\mathcal{B}_b(\mathbf{6}_{\mathbf{f}}, 1/2^+)$ to $\mathcal{B}_c(\mathbf{6}_{\mathbf{f}}, 3/2^-)$, (v) $\mathcal{B}_b(\bar{\mathbf{3}}_{\mathbf{f}}, 1/2^+)$ to $\mathcal{B}_c(\bar{\mathbf{3}}_{\mathbf{f}}, 1/2^-)$, and (vi) $\mathcal{B}_b(\bar{\mathbf{3}}_{\mathbf{f}}, 1/2^+)$ to $\mathcal{B}_c(\bar{\mathbf{3}}_{\mathbf{f}}, 3/2^-)$ transitions. They are summarized in Table III, where more information of the decaying particles and the final states are given. Types (i) to (iii) involve spin 1/2 and 3/2 s-wave charmed baryons, while types (iv) to (vi) involve spin 1/2 and 3/2 p-wave charmed baryons. Since there are two possible quantum number assignments for $\Lambda_c(2940)$, namely a radial excited p-wave spin-1/2 or a spin-3/2 state, it will be useful to consider both possibilities and compare the predictions on rates and so on. The transition form factors are calculated in the light-front quark model approach. For some other studies one is referred to [20–36]. We will compare our results to those obtained in other works.

The analysis and the scope of this work is improved and enlarged compared to a previous study [16] in several aspects. All of the form factors in the $1/2 \to 1/2$ and $1/2 \to 3/2$ transitions are extracted, while $1/2 \to 3/2$ transitions were not considered and only 2/3 of the $1/2 \to 1/2$ form factors were extracted in [16]. It is useful to note that in the heavy quark (HQ) limit, the $\mathcal{B}_b \to \mathcal{B}_c$ transition matrix elements with s-wave and p-wave $\mathcal{B}_c$ baryon form factors have simple behavior [22,37–41]. Form factors are usually related in the heavy quark limit. We will compare the form factors obtained in this work with the relations in HQ limit. Although some deviations are expected as we are using heavy but finite m_b and m_c, it is still interesting to see how well the form factors exhibiting the patterns required by heavy quark symmetry (HQS).

The layout of this paper is as following. In Sec. II we shall work out the formulas of form factors for various bottom baryon to s-wave and p-wave charmed baryon transitions in the light-front quark model approach. In Sec. III the numerical results of $\mathcal{B}_b \to \mathcal{B}_c$ transition form factors, decay rates and up-down asymmetries of $\mathcal{B}_b \to \mathcal{B}_c M$ decays will be presented. In Sec. IV we give our conclusions. Appendixes A and B are prepared to give some details of the light-front quark model and the derivations of the vertex functions, while some formulas involving kinematics are collected in Appendix C.

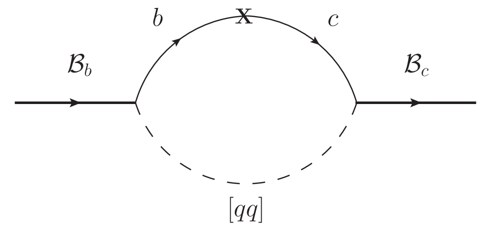

FIG. 1. Feynman diagram for a typical $\mathcal{B}_b \to \mathcal{B}_c$ transition, where the scalar or axial-vector diquark $[qq]$ is denoted by a dashed line and the vertex corresponding to the $\bar{c}\gamma^\mu(1-\gamma_5)b$ current is denoted by X. The orbital angular momentum of the $Q-[qq]$ system can be s-wave or p-wave.

II. OBTAINING FORM FACTORS IN THE LIGHT-FRONT APPROACH

A. $\mathcal{B}_b(1/2) \to \mathcal{B}_c(1/2)$ and $\mathcal{B}_b(1/2) \to \mathcal{B}_c(3/2)$ weak transitions

The Feynman diagram for a typical $\mathcal{B}_b \to \mathcal{B}_c$ transition, is shown in Fig. 1. For the $\mathcal{B}_b(1/2^+) \to \mathcal{B}_c(1/2^+)$ transition, the matrix elements of $\bar{c}\gamma_\mu b$ and $\bar{c}\gamma_\mu\gamma_5 b$ currents can be parametrized as

$$\begin{aligned}
\langle \mathcal{B}_c(P', J_z')|\bar{c}\gamma_\mu b|\mathcal{B}_b(P, J_z)\rangle &= \bar{u}(P', J_z')\left[f_1^V(q^2)\gamma_\mu + i\frac{f_2^V(q^2)}{M+M'}\sigma_{\mu\nu}q^\nu + \frac{f_3^V(q^2)}{M+M'}q_\mu\right]u(P, J_z),\\
\langle \mathcal{B}_c(P', J_z')|\bar{c}\gamma_\mu\gamma_5 b|\mathcal{B}_b(P, J_z)\rangle &= \bar{u}(P', J_z')\left[g_1^A(q^2)\gamma_\mu + i\frac{g_2^A(q^2)}{M-M'}\sigma_{\mu\nu}q^\nu + \frac{g_3^A(q^2)}{M-M'}q_\mu\right]\gamma_5 u(P, J_z),
\end{aligned} \tag{1}$$

with $q \equiv P - P'$. We find that it is more convenient to use $g_{2,3}/(M-M')$ instead of $g_{2,3}/(M+M')$ in the above matrix element. Note that these parametrization are different from those used in [16]. Similarly, for the $\mathcal{B}_b(1/2^+) \to \mathcal{B}_c(1/2^-)$ transition, we have

$$\begin{aligned}
\langle \mathcal{B}_c(P', J_z')|\bar{c}\gamma_\mu b|\mathcal{B}_b(P, J_z)\rangle &= \bar{u}(P', J_z')\left[g_1^V(q^2)\gamma_\mu + i\frac{g_2^V(q^2)}{M-M'}\sigma_{\mu\nu}q^\nu + \frac{g_3^V(q^2)}{M-M'}q_\mu\right]\gamma_5 u(P, J_z),\\
\langle \mathcal{B}_c(P', J_z')|\bar{c}\gamma_\mu\gamma_5 b|\mathcal{B}_b(P, J_z)\rangle &= \bar{u}(P', J_z')\left[f_1^A(q^2)\gamma_\mu + i\frac{f_2^A(q^2)}{M+M'}\sigma_{\mu\nu}q^\nu + \frac{f_3^A(q^2)}{M+M'}q_\mu\right]u(P, J_z).
\end{aligned} \tag{2}$$

For the $\mathcal{B}_b(1/2^+) \to \mathcal{B}_c(3/2^+)$ transition, the V^μ and A^μ matrix elements can be parametrized as

$$\langle\mathcal{B}_c(P',J'_z)|\bar{c}\gamma_\mu b|\mathcal{B}_b(P,J_z)\rangle = \bar{u}^\nu(P',J'_z)\left[\bar{f}_1^V(q^2)g_{\nu\mu}+\frac{\bar{f}_2^V(q^2)}{M}P_\nu\gamma_\mu+\frac{\bar{f}_3^V(q^2)}{MM'}P_\nu P'_\mu+\frac{\bar{f}_4^V(q^2)}{M^2}P_\nu P_\mu\right]\gamma_5 u(P,J_z),$$

$$\langle\mathcal{B}_c(P',J'_z)|\bar{c}\gamma_\mu\gamma_5 b|\mathcal{B}_b(P,J_z)\rangle = \bar{u}^\nu(P',J'_z)\left[\bar{g}_1^A(q^2)g_{\nu\mu}+\frac{\bar{g}_2^A(q^2)}{M}P_\nu\gamma_\mu+\frac{\bar{g}_3^A(q^2)}{MM''}P_\nu P'_\mu+\frac{\bar{g}_4^A(q^2)}{M^2}P_\nu P_\mu\right]u(P,J_z), \tag{3}$$

while for the $\mathcal{B}_b(1/2^+)\to\mathcal{B}_c(3/2^-)$ transition, we have

$$\langle\mathcal{B}_c(P',J'_z)|\bar{c}\gamma_\mu b|\mathcal{B}_b(P,J_z)\rangle = \bar{u}^\nu(P',J'_z)\left[\bar{g}_1^V(q^2)g_{\nu\mu}+\frac{\bar{g}_2^V(q^2)}{M}P_\nu\gamma_\mu+\frac{\bar{g}_3^V(q^2)}{MM'}P_\nu P'_\mu+\frac{\bar{g}_4^V(q^2)}{M^2}P_\nu P_\mu\right]u(P,J_z),$$

$$\langle\mathcal{B}_c(P',J'_z)|\bar{c}\gamma_\mu\gamma_5 b|\mathcal{B}_b(P,J_z)\rangle = \bar{u}^\nu(P',J'_z)\left[f_1^A(q^2)\bar{g}_{\nu\mu}+\frac{\bar{f}_2^A(q^2)}{M}P_\nu\gamma_\mu+\frac{\bar{f}_3^A(q^2)}{MM''}P_\nu P'_\mu+\frac{\bar{f}_4^A(q^2)}{M^2}P_\nu P_\mu\right]\gamma_5 u(P,J_z). \tag{4}$$

Using the light-front quark model, the general expression for a $\mathcal{B}_b(1/2)\to\mathcal{B}_c(1/2)$ [$\mathcal{B}_b(1/2)\to\mathcal{B}_c(3/2)$] transition matrix element is given by

$$\langle\mathcal{B}_c(P',J'_z)|\bar{c}\gamma^\mu(1-\gamma_5)b|\mathcal{B}_b(P,J_z)\rangle = \int\frac{dp_2^+d^2p_{2\perp}}{2(2\pi)^3}\frac{\phi'^*_{n'L'_K}(\{x'\},\{k'_\perp\})\phi_{1L_K}(\{x\},\{k_\perp\})}{2\sqrt{p_1^+p_1'^+(p_1\cdot\bar{P}+m_1M_0)(p'_1\cdot\bar{P}'+m'_1M'_0)}}$$
$$\times\,\bar{u}^{(\mu)}(\bar{P}',J'_z)\bar{\Gamma}_{(\mu)L'_KS_{[qq]}J'_l}(\not{p}'_1+m'_1)\gamma^\mu(1-\gamma_5)(\not{p}_1+m_1)\Gamma_{L_KS_{[qq]}J_l}u(\bar{P},J_z), \tag{5}$$

where the diquark is spectating in the transition and the kinematics of the constituents are

$$p_i^{(\prime)+}=x_i^{(\prime)}P^{(\prime)+},\qquad p_{i\perp}^{(\prime)}=x_i^{(\prime)}\vec{P}_\perp^{(\prime)}+\vec{k}_{i\perp}^{(\prime)},\qquad 1-\sum_{i=1}^2 x_i^{(\prime)}=\sum_{i=1}^2\vec{k}_{i\perp}^{(\prime)}=0,$$
$$(p_1-p'_1)^+=q^+,\qquad(\vec{p}_1-\vec{p}'_1)_\perp=\vec{q}_\perp,\qquad p_2=p'_2,\,p_i^{(\prime)2}=m_i^{(\prime)2}. \tag{6}$$

In Eq. (5) $\bar{\Gamma}^\mu$ denotes $\gamma_0\Gamma^{\dagger\mu}\gamma_0$ with the vertex functions $\Gamma^{(\mu)}_{L_KS_{[qq]}J_l}$ given by:

$$\Gamma_{s00}(p_1,p_2)=1,$$
$$\Gamma_{s11}(p_1,p_2,\lambda_2)=\frac{\gamma_5}{\sqrt{3}}\left(\not{\varepsilon}^*_{LF}(p_2,\lambda_2)-\frac{M_0+m_1+m_2}{\bar{P}\cdot p_2+m_2M_0}\varepsilon^*_{LF}(p_2,\lambda_2)\cdot\bar{P}\right),$$
$$\Gamma_{p01}(p_1,p_2)=\frac{\gamma_5}{2\sqrt{3}}\left(\not{p}_1-\not{p}_2-\frac{m_1^2-m_2^2}{M_0}\right),$$
$$\Gamma^\mu_{p01}(p_1,p_2)=-\frac{1}{2}(p_1-p_2)^\mu,$$
$$\Gamma^\mu_{s11}(p_1,p_2,\lambda_2)=-\left(\varepsilon^{*\mu}_{LF}(p_2,\lambda_2)-\frac{p_2^\mu}{\bar{P}\cdot p_2+m_2M_0}\varepsilon^*_{LF}(p_2,\lambda_2)\cdot\bar{P}\right),$$
$$\Gamma^\mu_{p12}(p_1,p_2,\lambda_2)=-\frac{1}{2\sqrt{10}}\gamma_5\left[\left(\varepsilon^{*\mu}_{LF}(p_2,\lambda_2)+(p_1-p_2)^\mu\frac{\varepsilon^*_{LF}(p_2,\lambda_2)\cdot\bar{P}}{\bar{P}\cdot p_2+M_0m_2}\right)\left(\not{p}_1-\not{p}_2-\frac{m_1^2-m_2^2}{M_0}\right)\right.$$
$$\left.+(p_1-p_2)^\mu\left(\not{\varepsilon}^*_{LF}(p_2,\lambda_2)-\frac{\varepsilon^*_{LF}(p_2,\lambda_2)\cdot\bar{P}}{M_0}\right)\right], \tag{7}$$

for baryon states with a $S_2 = 0$ or $S_2 = 1$ diquark. Note that the vertex functions Γ_{s00}, Γ_{s11} and Γ_{p01} are taken from [16], while Γ^μs are new and the derivations can be found in Appendixes A and B. For the wave function, we have

$$\phi_{nL_K} \equiv \sqrt{\frac{dk_{2z}}{dx_2}}\varphi_{nL_K}. \tag{8}$$

We shall use a Gaussian-like wave function in this work: [16,42,43]

$$\begin{aligned}
\varphi_{n=1,L_K=s}(\vec{k},\beta) &= 4\left(\frac{\pi}{\beta^2}\right)^{\frac{3}{4}}\exp\left(-\frac{k_z^2+k_\perp^2}{2\beta^2}\right),\\
\varphi_{n=1,L_K=p}(\vec{k},\beta) &= \sqrt{\frac{2}{\beta^2}}\varphi_{n=1}(\vec{k},\beta),\\
\varphi_{n=2,L_K=s}(\vec{k},\beta) &= \sqrt{\frac{3}{2}}\left(1-\frac{2}{3}\frac{\vec{k}^2}{\beta^2}\right)\varphi_{n=1}(\vec{k},\beta),\\
\varphi_{n=2,L_K=p}(\vec{k},\beta) &= \sqrt{\frac{5}{2}}\left(1-\frac{2}{5}\frac{\vec{k}^2}{\beta^2}\right)\varphi_{n=1,L_K=p}(\vec{k},\beta).
\end{aligned}\tag{9}$$

One is referred to Appendix A for more details.

We shall extend the formulas in [16,44,45] to project out all the form factors from the $\mathcal{B}_b(1/2^+) \to \mathcal{B}_c(1/2^\pm)$ and $\mathcal{B}_b(1/2^+) \to \mathcal{B}_c(3/2^\pm)$ transition matrix elements shown in Eq. (5). As in [16,42,44,45], we consider the $q^+ = 0$, $\vec{q}_\perp \neq \vec{0}$ case. By applying the following identities to Eq. (1) for V^+, A^+, $\vec{q}_\perp \cdot \vec{V}$, $\vec{q}_\perp \cdot \vec{A}$, $\vec{n}_\perp \cdot \vec{V}$ and $\vec{n}_\perp \cdot \vec{A}$,

$$\begin{aligned}
&\frac{\bar{u}(P,J_z)\gamma^+u(P',J'_z)}{2\sqrt{P^+P'^+}} = \delta_{J_zJ'_z}, \qquad i\frac{\bar{u}(P,J_z)\sigma^{+\nu}a_{\perp\nu}u(P',J'_z)}{2\sqrt{P^+P'^+}} = (\vec{\sigma}\cdot\vec{a}_\perp\sigma^3)_{J_zJ'_z},\\
&\frac{\bar{u}(P,J_z)\gamma^+\gamma_5u(P',J'_z)}{2\sqrt{P^+P'^+}} = (\sigma^3)_{J_zJ'_z}, \qquad i\frac{\bar{u}(P,J_z)\sigma^{+\nu}a_{\perp\nu}\gamma_5u(P',J'_z)}{2\sqrt{P^+P'^+}} = (\vec{\sigma}\cdot\vec{a}_\perp)_{J_zJ'_z},
\end{aligned}\tag{10}$$

with $a_\perp = (0, 0, \vec{a}_\perp)$, where $\vec{a}_\perp$ is an arbitrary $2-d$ vector, and the following identities to Eq. (5),

$$\begin{aligned}
\sum_{J_z,J'_z}u(\bar{P},J_z)\delta_{J_zJ'_z}\bar{u}(\bar{P}',J'_z) &= \frac{1}{2\sqrt{P^+P'^+}}(\bar{\not{P}}+M_0)\gamma^+(\bar{\not{P}}'+M'_0),\\
\sum_{J_z,J'_z}u(\bar{P},J_z)(\vec{\sigma}\cdot\vec{a}_\perp\sigma^3)_{J_zJ'_z}\bar{u}(\bar{P}',J'_z) &= \frac{i}{2\sqrt{P^+P'^+}}(\bar{\not{P}}+M_0)\sigma^{+\nu}a_{\perp\nu}(\bar{\not{P}}'+M'_0),\\
\sum_{J_z,J'_z}u(\bar{P},J_z)(\sigma^3)_{J_zJ'_z}\bar{u}(\bar{P}',J'_z) &= \frac{1}{2\sqrt{P^+P'^+}}(\bar{\not{P}}+M_0)\gamma^+\gamma_5(\bar{\not{P}}'+M'_0),\\
\sum_{J_z,J'_z}u(\bar{P},J_z)(\vec{\sigma}\cdot\vec{a}_\perp)_{J_zJ'_z}\bar{u}(\bar{P}',J'_z) &= \frac{i}{2\sqrt{P^+P'^+}}(\bar{\not{P}}+M_0)\sigma^{+\nu}a_{\perp\nu}\gamma_5(\bar{\not{P}}'+M'_0),
\end{aligned}\tag{11}$$

we obtain

$$
\begin{aligned}
f_1^V(q^2) &= \int \frac{dp_2^+ d^2p_{2\perp}}{2(2\pi)^3} \frac{\phi'^*_{n'L'_K}(\{x'\},\{k'_\perp\})\phi_{1L_K}(\{x\},\{k_\perp\})}{16P^+P'^+\sqrt{p_1'^+p_1^+(p'_1\cdot\bar{P}'+m'_1M'_0)(p_1\cdot\bar{P}+m_1M_0)}} \\
&\quad\times \mathrm{Tr}[(\bar{\not{P}}+M_0)\gamma^+(\bar{\not{P}}'+M'_0)\bar{\Gamma}_{L'_KS_{[qq]}J'_l}(\not{p}'_1+m'_1)\gamma^+(\not{p}_1+m_1)\Gamma_{L_KS_{[qq]}J_l}], \\
\frac{f_2^V(q^2)q^2}{M+M'} &= i\int \frac{dp_2^+ d^2p_{2\perp}}{2(2\pi)^3} \frac{\phi'^*_{n'L'_K}(\{x'\},\{k'_\perp\})\phi_{1L_K}(\{x\},\{k_\perp\})}{16P^+P'^+\sqrt{p_1'^+p_1^+(p'_1\cdot\bar{P}'+m'_1M'_0)(p_1\cdot\bar{P}+m_1M_0)}} \\
&\quad\times \mathrm{Tr}[(\bar{\not{P}}+M_0)\sigma^{+\nu}q_{\perp\nu}(\bar{\not{P}}'+M'_0)\bar{\Gamma}_{L'_KS_{[qq]}J'_l}(\not{p}'_1+m'_1)\gamma^+(\not{p}_1+m_1)\Gamma_{L_KS_{[qq]}J_l}], \\
f_3^V(q^2)-f_1^V(q^2) &= \int \frac{dp_2^+ d^2p_{2\perp}}{2(2\pi)^3} \frac{\phi'^*_{n'L'_K}(\{x'\},\{k'_\perp\})\phi_{1L_K}(\{x\},\{k_\perp\})}{8q^2\sqrt{P^+P'^+}\sqrt{p_1'^+p_1^+(p'_1\cdot\bar{P}'+m'_1M'_0)(p_1\cdot\bar{P}+m_1M_0)}} \\
&\quad\times \mathrm{Tr}[(\bar{\not{P}}+M_0)\gamma^+(\bar{\not{P}}'+M'_0)\bar{\Gamma}_{L'_KS_{[qq]}J'_l}(\not{p}'_1+m'_1)\not{q}_\perp(\not{p}_1+m_1)\Gamma_{L_KS_{[qq]}J_l}], \\
f_1^V(q^2)-f_2^V(q^2) &= \frac{-i}{M-M'}\int \frac{dp_2^+ d^2p_{2\perp}}{2(2\pi)^3} \frac{\phi'^*_{n'L'_K}(\{x'\},\{k'_\perp\})\phi_{1L_K}(\{x\},\{k_\perp\})}{8\sqrt{P^+P'^+}\sqrt{p_1'^+p_1^+(p'_1\cdot\bar{P}'+m'_1M'_0)(p_1\cdot\bar{P}+m_1M_0)}} \\
&\quad\times \mathrm{Tr}[(\bar{\not{P}}+M_0)\sigma^{+\nu}n_{\perp\nu}(\bar{\not{P}}'+M'_0)\bar{\Gamma}_{L'_KS_{[qq]}J'_l}(\not{p}'_1+m'_1)\not{n}_\perp(\not{p}_1+m_1)\Gamma_{L_KS_{[qq]}J_l}], \\
g_1^A(q^2) &= \int \frac{dp_2^+ d^2p_{2\perp}}{2(2\pi)^3} \frac{\phi'^*_{n'L'_K}(\{x'\},\{k'_\perp\})\phi_{1L_K}(\{x\},\{k_\perp\})}{16P^+P'^+\sqrt{p_1'^+p_1^+(p'_1\cdot\bar{P}'+m'_1M'_0)(p_1\cdot\bar{P}+m_1M_0)}} \\
&\quad\times \mathrm{Tr}[(\bar{\not{P}}+M_0)\gamma^+\gamma_5(\bar{\not{P}}'+M'_0)\bar{\Gamma}_{L'_KS_{[qq]}J'_l}(\not{p}'_1+m'_1)\gamma^+\gamma_5(\not{p}_1+m_1)\Gamma_{L_KS_{[qq]}J_l}], \\
\frac{g_2^A(q^2)q^2}{M-M'} &= -i\int \frac{dp_2^+ d^2p_{2\perp}}{2(2\pi)^3} \frac{\phi'^*_{n'L'_K}(\{x'\},\{k'_\perp\})\phi_{1L_K}(\{x\},\{k_\perp\})}{16P^+P'^+\sqrt{p_1'^+p_1^+(p'_1\cdot\bar{P}'+m'_1M'_0)(p_1\cdot\bar{P}+m_1M_0)}} \\
&\quad\times \mathrm{Tr}[(\bar{\not{P}}+M_0)\sigma^{+\nu}q_{\perp\nu}\gamma_5(\bar{\not{P}}'+M'_0)\bar{\Gamma}_{L'_KS_{[qq]}J'_l}(\not{p}'_1+m'_1)\gamma^+\gamma_5(\not{p}_1+m_1)\Gamma_{L_KS_{[qq]}J_l}], \\
g_1^A(q^2)+g_3^A(q^2) &= -\int \frac{dp_2^+ d^2p_{2\perp}}{2(2\pi)^3} \frac{\phi'^*_{n'L'_K}(\{x'\},\{k'_\perp\})\phi_{1L_K}(\{x\},\{k_\perp\})}{8q^2\sqrt{P^+P'^+}\sqrt{p_1'^+p_1^+(p'_1\cdot\bar{P}'+m'_1M'_0)(p_1\cdot\bar{P}+m_1M_0)}} \\
&\quad\times \mathrm{Tr}[(\bar{\not{P}}+M_0)\gamma^+\gamma_5(\bar{\not{P}}'+M'_0)\bar{\Gamma}_{L'_KS_{[qq]}J'_l}(\not{p}'_1+m'_1)\not{q}_\perp\gamma_5(\not{p}_1+m_1)\Gamma_{L_KS_{[qq]}J_l}], \\
g_1^A(q^2)+g_2^A(q^2) &= \frac{-i}{M+M'}\int \frac{dp_2^+ d^2p_{2\perp}}{2(2\pi)^3} \frac{\phi'^*_{n'L'_K}(\{x'\},\{k'_\perp\})\phi_{1L_K}(\{x\},\{k_\perp\})}{8\sqrt{P^+P'^+}\sqrt{p_1'^+p_1^+(p'_1\cdot\bar{P}'+m'_1M'_0)(p_1\cdot\bar{P}+m_1M_0)}} \\
&\quad\times \mathrm{Tr}[(\bar{\not{P}}+M_0)\sigma^{+\nu}n_{\perp\nu}\gamma_5(\bar{\not{P}}'+M'_0)\bar{\Gamma}_{L'_KS_{[qq]}J'_l}(\not{p}'_1+m'_1)\not{n}_\perp\gamma_5(\not{p}_1+m_1)\Gamma_{L_KS_{[qq]}J_l}], \qquad (12)
\end{aligned}
$$

with $q_\perp \equiv (0,0,\vec{q}_\perp)$, $n_\perp \equiv (0,0,\vec{n}_\perp)$, $n_\perp^2 = -1$ and $\vec{n}_\perp\cdot\vec{q}_\perp = 0$. In Ref. [16,44,45] only $f_{1,2}$ and $g_{1,2}$ can be extracted. Now we can extract all form factors. Note that the above equations are over constraining in determining f_i and g_i. There are consistency relations needed to be satisfied. The equations involving f_1, f_2 and f_1-f_2 need to be consistent and likewise for those involving g_1, g_2 and g_1+g_2.

Similarly, for $\mathcal{B}_b(1/2^+)\to\mathcal{B}_c(1/2^-)$ transition, we have

$$
\begin{aligned}
f_1^A(q^2) &= \int \frac{dp_2^+ d^2p_{2\perp}}{2(2\pi)^3} \frac{\phi'^*_{n'L'_K}(\{x'\},\{k'_\perp\})\phi_{1L_K}(\{x\},\{k_\perp\})}{16P^+P'^+\sqrt{p_1'^+p_1^+(p'_1\cdot\bar{P}'+m'_1M'_0)(p_1\cdot\bar{P}+m_1M_0)}} \\
&\quad\times \mathrm{Tr}[(\bar{\not{P}}+M_0)\gamma^+(\bar{\not{P}}'+M'_0)\bar{\Gamma}_{L'_KS_{[qq]}J'_l}(\not{p}'_1+m'_1)\gamma^+\gamma_5(\not{p}_1+m_1)\Gamma_{L_KS_{[qq]}J_l}], \\
\frac{f_2^A(q^2)q^2}{M+M'} &= i\int \frac{dp_2^+ d^2p_{2\perp}}{2(2\pi)^3} \frac{\phi'^*_{n'L'_K}(\{x'\},\{k'_\perp\})\phi_{1L_K}(\{x\},\{k_\perp\})}{16P^+P'^+\sqrt{p_1'^+p_1^+(p'_1\cdot\bar{P}'+m'_1M'_0)(p_1\cdot\bar{P}+m_1M_0)}} \\
&\quad\times \mathrm{Tr}[(\bar{\not{P}}+M_0)\sigma^{+\nu}q_{\perp\nu}(\bar{\not{P}}'+M'_0)\bar{\Gamma}_{L'_KS_{[qq]}J'_l}(\not{p}'_1+m'_1)\gamma^+\gamma_5(\not{p}_1+m_1)\Gamma_{L_KS_{[qq]}J_l}], \\
f_3^A(q^2)-f_1^A(q^2) &= \int \frac{dp_2^+ d^2p_{2\perp}}{2(2\pi)^3} \frac{\phi'^*_{n'L'_K}(\{x'\},\{k'_\perp\})\phi_{1L_K}(\{x\},\{k_\perp\})}{8q^2\sqrt{P^+P'^+}\sqrt{p_1'^+p_1^+(p'_1\cdot\bar{P}'+m'_1M'_0)(p_1\cdot\bar{P}+m_1M_0)}} \\
&\quad\times \mathrm{Tr}[(\bar{\not{P}}+M_0)\gamma^+(\bar{\not{P}}'+M'_0)\bar{\Gamma}_{L'_KS_{[qq]}J'_l}(\not{p}'_1+m'_1)\not{q}_\perp\gamma_5(\not{p}_1+m_1)\Gamma_{L_KS_{[qq]}J_l}],
\end{aligned}
$$

$$f_1^A(q^2) - f_2^A(q^2) = \frac{-i}{M-M'}\int \frac{dp_2^+ d^2p_{2\perp}}{2(2\pi)^3} \frac{\phi'^*_{n'L'_K}(\{x'\},\{k'_\perp\})\phi_{1L_K}(\{x\},\{k_\perp\})}{8\sqrt{P^+P'^+}\sqrt{p_1'^+p_1^+(p'_1\cdot\bar P' + m'_1M'_0)(p_1\cdot\bar P + m_1M_0)}}$$
$$\times \mathrm{Tr}[(\bar{P\!\!\!/} + M_0)\sigma^{+\nu}n_{\perp\nu}(\bar{P\!\!\!/}' + M'_0)\bar\Gamma_{L'_KS_{[qq]}J'_l}(p\!\!\!/'_1 + m'_1)n\!\!\!/_\perp\gamma_5(p\!\!\!/_1 + m_1)\Gamma_{L_KS_{[qq]}J_l}],$$

$$g_1^V(q^2) = \int \frac{dp_2^+ d^2p_{2\perp}}{2(2\pi)^3} \frac{\phi'^*_{n'L'_K}(\{x'\},\{k'_\perp\})\phi_{1L_K}(\{x\},\{k_\perp\})}{16P^+P'^+\sqrt{p_1'^+p_1^+(p'_1\cdot\bar P' + m'_1M'_0)(p_1\cdot\bar P + m_1M_0)}}$$
$$\times \mathrm{Tr}[(\bar{P\!\!\!/} + M_0)\gamma^+\gamma_5(\bar{P\!\!\!/}' + M'_0)\bar\Gamma_{L'_KS_{[qq]}J'_l}(p\!\!\!/'_1 + m'_1)\gamma^+(p\!\!\!/_1 + m_1)\Gamma_{L_KS_{[qq]}J_l}],$$

$$\frac{g_2^V(q^2)q^2}{M-M'} = -i\int \frac{dp_2^+ d^2p_{2\perp}}{2(2\pi)^3} \frac{\phi'^*_{n'L'_K}(\{x'\},\{k'_\perp\})\phi_{1L_K}(\{x\},\{k_\perp\})}{16P^+P'^+\sqrt{p_1'^+p_1^+(p'_1\cdot\bar P' + m'_1M'_0)(p_1\cdot\bar P + m_1M_0)}}$$
$$\times \mathrm{Tr}[(\bar{P\!\!\!/} + M_0)\sigma^{+\nu}q_{\perp\nu}\gamma_5(\bar{P\!\!\!/}' + M'_0)\bar\Gamma_{L'_KS_{[qq]}J'_l}(p\!\!\!/'_1 + m'_1)\gamma^+(p\!\!\!/_1 + m_1)\Gamma_{L_KS_{[qq]}J_l}],$$

$$g_1^V(q^2) + g_3^V(q^2) = -\int \frac{dp_2^+ d^2p_{2\perp}}{2(2\pi)^3} \frac{\phi'^*_{n'L'_K}(\{x'\},\{k'_\perp\})\phi_{1L_K}(\{x\},\{k_\perp\})}{8q^2\sqrt{P^+P'^+}\sqrt{p_1'^+p_1^+(p'_1\cdot\bar P' + m'_1M'_0)(p_1\cdot\bar P + m_1M_0)}}$$
$$\times \mathrm{Tr}[(\bar{P\!\!\!/} + M_0)\gamma^+\gamma_5(\bar{P\!\!\!/}' + M'_0)\bar\Gamma_{L'_KS_{[qq]}J'_l}(p\!\!\!/'_1 + m'_1)q\!\!\!/_\perp\gamma_5(p\!\!\!/_1 + m_1)\Gamma_{L_KS_{[qq]}J_l}],$$

$$g_1^V(q^2) + g_2^V(q^2) = \frac{-i}{M+M'}\int \frac{dp_2^+ d^2p_{2\perp}}{2(2\pi)^3} \frac{\phi'^*_{n'L'_K}(\{x'\},\{k'_\perp\})\phi_{1L_K}(\{x\},\{k_\perp\})}{8\sqrt{P^+P'^+}\sqrt{p_1'^+p_1^+(p'_1\cdot\bar P' + m'_1M'_0)(p_1\cdot\bar P + m_1M_0)}}$$
$$\times \mathrm{Tr}[(\bar{P\!\!\!/} + M_0)\sigma^{+\nu}n_{\perp\nu}\gamma_5(\bar{P\!\!\!/}' + M'_0)\bar\Gamma_{L'_KS_{[qq]}J'_l}(p\!\!\!/'_1 + m'_1)n\!\!\!/_\perp(p\!\!\!/_1 + m_1)\Gamma_{L_KS_{[qq]}J_l}], \tag{13}$$

with $q_\perp \equiv (0,0,\vec q_\perp)$, $n_\perp \equiv (0,0,\vec n_\perp)$, $n_\perp^2 = -1$ and $\vec n_\perp\cdot\vec q_\perp = 0$.

The following identities are useful in extracting $1/2 \to 3/2$ transition form factors,

$$\delta_{S_zS'_z} = \sqrt{\frac{3}{2}}\frac{M'}{P'^+}\frac{\bar u(P',S'_z)\gamma^+u^+(P,S_z)}{2\sqrt{P^+P'^+}},$$
$$(\vec\sigma\cdot\vec a_\perp\sigma^3)_{S'_zS_z} = \sqrt{\frac{3}{2}}\frac{M'}{P'^+}i\frac{\bar u(P,S_z)\sigma^{+\nu}a_{\perp\nu}u^+(P',S'_z)}{2\sqrt{P^+P'^+}},$$
$$(\sigma^3)_{S_zS'_z} = \sqrt{\frac{3}{2}}\frac{M'}{P'^+}\frac{\bar u(P,S_z)\gamma^+\gamma_5u^+(P',S'_z)}{2\sqrt{P^+P'^+}},$$
$$(\vec\sigma\cdot\vec a_\perp)_{S_zS'_z} = \sqrt{\frac{3}{2}}\frac{M'_0}{P'^+}i\frac{\bar u(P,S_z)\sigma^{+\nu}a_{\perp\nu}\gamma_5u^+(P',S'_z)}{2\sqrt{P^+P'^+}}, \tag{14}$$

and[1]

$$\sum_{S_z,S'_z=\pm1/2} u(\bar P,S_z)(\sigma^3)_{S_zS'_z}\bar u^\mu(\bar P',S'_z) = \frac{\sqrt3M'_0}{2\sqrt2P'^+\sqrt{P^+P'^+}}(\bar{P\!\!\!/} + M_0)\gamma^+\gamma_5\mathcal{P}^{+\mu}(\bar P'),$$
$$\sum_{S_z,S'_z=\pm1/2} u(\bar P,S_z)(\sigma_\perp\cdot\vec a_\perp)_{S_zS'_z}\bar u^\mu(\bar P',S'_z) = \frac{\sqrt3M'_0i}{2\sqrt2P'^+\sqrt{P^+P'^+}}(\bar{P\!\!\!/} + M_0)\sigma^{+\nu}a_{\perp\nu}\gamma_5\mathcal{P}^{+\mu}(\bar P'),$$
$$\sum_{S_z,S'_z=\pm1/2} u(\bar P,S_z)\delta_{S_zS'_z}\bar u^\mu(\bar P',S'_z) = \frac{\sqrt3M'_0}{2\sqrt2P'^+\sqrt{P^+P'^+}}(\bar{P\!\!\!/} + M_0)\gamma^+\mathcal{P}^{+\mu}(\bar P'),$$
$$\sum_{S_z,S'_z=\pm1/2} u^\mu(\bar P,S_z)(\vec\sigma_\perp\cdot\vec a_\perp\sigma^3)_{S_zS'_z}\bar u^\mu(\bar P',S'_z) = \frac{\sqrt3M'_0i}{2\sqrt2P'^+\sqrt{P^+P'^+}}(\bar{P\!\!\!/} + M_0)\sigma^{+\nu}a_{\perp\nu}\mathcal{P}^{+\mu(\bar P')}, \tag{15}$$

with (see, e.g., [46])

[1]Note that since we have $u^+(\bar P',\pm3/2) = u(\bar P',\pm1/2)\varepsilon^+_{LF}(\pm1) = 0$, see Eq. (B3), we can easily promote the sum in $S'_z = -1/2, +1/2$ to $J'_z = -3/2, -1/2, 1/2, 3/2$ and, consequently, the convenient full polarization sum formula, Eq. (16) can be used.

$$\mathcal{P}_{\mu\nu}(\bar{P}') \equiv \sum_{J'_z=-3/2}^{3/2} u_\mu(\bar{P}', J'_z)\bar{u}_\nu(\bar{P}', J'_z) = -(\bar{\not{P}}' + M'_0)\left(G_{\mu\nu}(\bar{P}') - \frac{1}{3}G_{\mu\sigma}(\bar{P}')G_{\nu\lambda}(\bar{P}')\gamma^\sigma\gamma^\lambda\right), \tag{16}$$

where $G_{\mu\nu}(\bar{P}')$ is defined as

$$G_{\mu\nu}(\bar{P}') \equiv g_{\mu\nu} - \frac{\bar{P}'_\mu \bar{P}'_\nu}{M'^2_0}. \tag{17}$$

We apply Eq. (14) to Eq. (3) for V^+, A^+, $\vec{q}_\perp \cdot \vec{V}$, $\vec{q}_\perp \cdot \vec{A}$, $\vec{n}_\perp \cdot \vec{V}$, and $\vec{n}_\perp \cdot \vec{A}$, and apply Eq. (15) to Eq. (5), and obtain, for the $\mathcal{B}_b(1/2^+) \to \mathcal{B}_c(3/2^+)$ transition,

$$\begin{aligned}
\mathcal{F}_1^V(q^2) &= \int \frac{dx_2 d^2k_{2\perp}}{2(2\pi)^3}\frac{3M'_0}{P'^+}\frac{\phi'^*_{n'L'_K}(\{x'\},\{k'_\perp\})\phi_{1L_K}(\{x\},\{k_\perp\})}{8P^+P'^+\sqrt{[(m_1+x_1M_0)^2+k^2_{1\perp}][(m'_1+x_1M'_0)^2+k'^2_{1\perp}]}}\\
&\quad\times \mathrm{Tr}[(\bar{\not{P}}+M_0)\gamma^+\gamma_5\mathcal{P}^{+\mu}\bar{\Gamma}_{\mu L'_K S_{[qq]}J'_l}(\not{p}'_1+m'_1)\gamma^+(\not{p}_1+m_1)\Gamma_{L_K S_{[qq]}J_l}],\\
\mathcal{F}_2^V(q^2) &= -i\int \frac{dx_2 d^2k_{2\perp}}{2(2\pi)^3}\frac{3M'_0}{P'^+}\frac{\phi'^*_{n'L'_K}(\{x'\},\{k'_\perp\})\phi_{1L_K}(\{x\},\{k_\perp\})}{8P^+P'^+\sqrt{[(m_1+x_1M_0)^2+k^2_{1\perp}][(m'_1+x_1M'_0)^2+k'^2_{1\perp}]}}\\
&\quad\times \mathrm{Tr}[(\bar{\not{P}}+M_0)\sigma^{+\nu}q_{\perp\nu}\gamma_5\mathcal{P}^{+\mu}\bar{\Gamma}_{\mu L'_K S_{[qq]}J'_l}(\not{p}'_1+m'_1)\gamma^+(\not{p}_1+m_1)\Gamma_{L_K S_{[qq]}J_l}],\\
\mathcal{F}_3^V(q^2) &= \int \frac{dx_2 d^2k_{2\perp}}{2(2\pi)^3}\frac{3M'_0}{P'^+}\frac{\phi'^*_{n'L'_K}(\{x'\},\{k'_\perp\})\phi_{1L_K}(\{x\},\{k_\perp\})}{4\sqrt{P^+P'^+}\sqrt{[(m_1+x_1M_0)^2+k^2_{1\perp}][(m'_1+x_1M'_0)^2+k'^2_{1\perp}]}}\\
&\quad\times \mathrm{Tr}[(\bar{\not{P}}+M_0)\gamma^+\gamma_5\mathcal{P}^{+\mu}\bar{\Gamma}_{\mu L'_K S_{[qq]}J'_l}(\not{p}'_1+m'_1)\not{q}_\perp(\not{p}_1+m_1)\Gamma_{L_K S_{[qq]}J_l}],\\
\mathcal{F}_4^V(q^2) &= -i\int \frac{dx_2 d^2k_{2\perp}}{2(2\pi)^3}\frac{3M'_0}{P'^+}\frac{\phi'^*_{n'L'_K}(\{x'\},\{k'_\perp\})\phi_{1L_K}(\{x\},\{k_\perp\})}{4\sqrt{P^+P'^+}\sqrt{[(m_1+x_1M_0)^2+k^2_{1\perp}][(m'_1+x_1M'_0)^2+k'^2_{1\perp}]}}\\
&\quad\times \mathrm{Tr}[(\bar{\not{P}}+M_0)\sigma^{+\nu}q_{\perp\nu}\gamma_5\mathcal{P}^{+\mu}\bar{\Gamma}_{\mu L'_K S_{[qq]}J'_l}(\not{p}'_1+m'_1)\not{n}_\perp(\not{p}_1+m_1)\Gamma_{L_K S_{[qq]}J_l}],\\
\mathcal{G}_1^A(q^2) &= \int \frac{dx_2 d^2k_{2\perp}}{2(2\pi)^3}\frac{3M'_0}{P'^+}\frac{\phi'^*_{n'L'_K}(\{x'\},\{k'_\perp\})\phi_{1L_K}(\{x\},\{k_\perp\})}{8P^+P'^+\sqrt{[(m_1+x_1M_0)^2+k^2_{1\perp}][(m'_1+x_1M'_0)^2+k'^2_{1\perp}]}}\\
&\quad\times \mathrm{Tr}[(\bar{\not{P}}+M_0)\gamma^+\mathcal{P}^{+\mu}\bar{\Gamma}_{\mu L'_K S_{[qq]}J'_l}(\not{p}'_1+m'_1)\gamma^+\gamma_5(\not{p}_1+m_1)\Gamma_{L_K S_{[qq]}J_l}],\\
\mathcal{G}_2^A(q^2) &= i\int \frac{dx_2 d^2k_{2\perp}}{2(2\pi)^3}\frac{3M'_0}{P'^+}\frac{\phi'^*_{n'L'_K}(\{x'\},\{k'_\perp\})\phi_{1L_K}(\{x\},\{k_\perp\})}{8P^+P'^+\sqrt{[(m_1+x_1M_0)^2+k^2_{1\perp}][(m'_1+x_1M'_0)^2+k'^2_{1\perp}]}}\\
&\quad\times \mathrm{Tr}[(\bar{\not{P}}+M_0)\sigma^{+\nu}q_{\perp\nu}\mathcal{P}^{+\mu}\bar{\Gamma}_{\mu L'_K S_{[qq]}J'_l}(\not{p}'_1+m'_1)\gamma^+\gamma_5(\not{p}_1+m_1)\Gamma_{L_K S_{[qq]}J_l}].\\
\mathcal{G}_3^A(q^2) &= \int \frac{dx_2 d^2k_{2\perp}}{2(2\pi)^3}\frac{3M'_0}{P'^+}\frac{\phi'^*_{n'L'_K}(\{x'\},\{k'_\perp\})\phi_{1L_K}(\{x\},\{k_\perp\})}{4\sqrt{P^+P'^+}\sqrt{[(m_1+x_1M_0)^2+k^2_{1\perp}][(m'_1+x_1M'_0)^2+k'^2_{1\perp}]}}\\
&\quad\times \mathrm{Tr}[(\bar{\not{P}}+M_0)\gamma^+\mathcal{P}^{+\mu}\bar{\Gamma}_{\mu L'_K S_{[qq]}J'_l}(\not{p}'_1+m'_1)\not{q}_\perp\gamma_5(\not{p}_1+m_1)\Gamma_{L_K S_{[qq]}J_l}],\\
\mathcal{G}_4^A(q^2) &= i\int \frac{dx_2 d^2k_{2\perp}}{2(2\pi)^3}\frac{3M'_0}{P'^+}\frac{\phi'^*_{n'L'_K}(\{x'\},\{k'_\perp\})\phi_{1L_K}(\{x\},\{k_\perp\})}{8P^+P'^+\sqrt{[(m_1+x_1M_0)^2+k^2_{1\perp}][(m'_1+x_1M'_0)^2+k'^2_{1\perp}]}}\\
&\quad\times \mathrm{Tr}[(\bar{\not{P}}+M_0)\sigma^{+\nu}n_{\perp\nu}\mathcal{P}^{+\mu}\bar{\Gamma}_{\mu L'_K S_{[qq]}J'_l}(\not{p}'_1+m'_1)\not{n}_\perp\gamma_5(\not{p}_1+m_1)\Gamma_{L_K S_{[qq]}J_l}],
\end{aligned} \tag{18}$$

with $q_\perp \equiv (0,0,\vec{q}_\perp)$, $n_\perp \equiv (0,0,\vec{n}_\perp)$, $n_\perp^2 = -1$, $\vec{n}_\perp \cdot \vec{q}_\perp = 0$ and

$$\mathcal{F}_1^V(q^2) \equiv \frac{M'-M}{M'}\left\{\bar{f}_1^V(q^2) - \left(M+M'+\frac{q^2}{(M'-M)}\right)\frac{\bar{f}_2^V(q^2)}{M} + \frac{1}{2}\left(M^2-M'^2-q^2\frac{2M'-M}{M'-M}\right)\left(\frac{\bar{f}_3^V(q^2)}{MM'}+\frac{\bar{f}_4^V(q^2)}{M^2}\right)\right\},$$
$$\mathcal{F}_2^V(q^2) \equiv \frac{q^2}{M'}\left\{\bar{f}_1^V(q^2) - \frac{M'}{M}\bar{f}_2^V(q^2) + \frac{1}{2}(M^2+M'M-2M'^2-q^2)\left(\frac{\bar{f}_3^V(q^2)}{MM'}+\frac{\bar{f}_4^V(q^2)}{M^2}\right)\right\},$$
$$\mathcal{F}_3^V(q^2) \equiv \frac{q^2}{M'}\left\{(2M-3M')\bar{f}_1^V(q^2) + (q^2-(M-2M')(M+M'))\frac{\bar{f}_2^V(q^2)}{M} + [(M-M')^2(M+M')-q^2(M-2M')]\frac{\bar{f}_3^V(q^2)}{MM'}\right\},$$
$$\mathcal{F}_4^V(q^2) \equiv (M'-M)\left\{\bar{f}_1^V(q^2) + \left((M+M')^2-q^2\frac{M+2M'}{(M'-M)}\right)\frac{\bar{f}_2^V(q^2)}{MM'}\right\},$$
$$\mathcal{G}_1^A(q^2) \equiv \frac{M'+M}{M'}\left\{\bar{g}_1^A(q^2) + \left(M-M'-\frac{q^2}{(M'+M)}\right)\frac{\bar{g}_2^A(q^2)}{M} + \frac{1}{2}\left(M^2-M'^2-q^2\frac{2M'+M}{M'+M}\right)\left(\frac{\bar{g}_3^A(q^2)}{MM'}+\frac{\bar{g}_4^A(q^2)}{M^2}\right)\right\},$$
$$\mathcal{G}_2^A(q^2) \equiv \frac{q^2}{M'}\left\{\bar{g}_1^A(q^2) - \frac{M'}{M}\bar{g}_2^A(q^2) + \frac{1}{2}(M^2-M'M-2M'^2-q^2)\left(\frac{\bar{g}_3^A(q^2)}{MM'}+\frac{\bar{g}_4^A(q^2)}{M^2}\right)\right\},$$
$$\mathcal{G}_3^A(q^2) \equiv \frac{q^2}{M'}\left\{-(2M+3M')\bar{f}_1^V(q^2) + (q^2-(M+2M')(M-M'))\frac{\bar{f}_2^V(q^2)}{M} + [-(M+M')^2(M-M')+q^2(M+2M')]\frac{\bar{f}_3^V(q^2)}{MM'}\right\}.$$
$$\mathcal{G}_4^A(q^2) \equiv (M'+M)\left\{\bar{g}_1^A(q^2) + \left(-(M'-M)^2+q^2\frac{M-2M'}{(M'+M)}\right)\frac{\bar{g}_2^A(q^2)}{MM'}\right\}. \tag{19}$$

Similarly, for $\mathcal{B}_b(1/2^+) \to \mathcal{B}_c(3/2^-)$ transition, we have

$$\mathcal{F}_1^A(q^2) = \int \frac{dx_2 d^2k_{2\perp}}{2(2\pi)^3}\frac{3M_0'}{P'^+}\frac{\phi'^*_{n'L_K'}(\{x'\},\{k'_\perp\})\phi_{1L_K}(\{x\},\{k_\perp\})}{8P^+P'^+\sqrt{[(m_1+x_1M_0)^2+k_{1\perp}^2][(m_1'+x_1M_0')^2+k_{1\perp}'^2]}}$$
$$\times \mathrm{Tr}[(\bar{\not{P}}+M_0)\gamma^+\gamma_5\mathcal{P}^{+\mu}\bar{\Gamma}_{\mu L_K' S_{[qq]}J_l'}(\not{p}_1'+m_1')\gamma^+\gamma_5(\not{p}_1+m_1)\Gamma_{L_K S_{[qq]}J_l}],$$
$$\mathcal{F}_2^A(q^2) = -i\int \frac{dx_2 d^2k_{2\perp}}{2(2\pi)^3}\frac{3M_0'}{P'^+}\frac{\phi'^*_{n'L_K'}(\{x'\},\{k'_\perp\})\phi_{1L_K}(\{x\},\{k_\perp\})}{8P^+P'^+\sqrt{[(m_1+x_1M_0)^2+k_{1\perp}^2][(m_1'+x_1M_0')^2+k_{1\perp}'^2]}}$$
$$\times \mathrm{Tr}[(\bar{\not{P}}+M_0)\sigma^{+\nu}q_{\perp\nu}\gamma_5\mathcal{P}^{+\mu}\bar{\Gamma}_{\mu L_K' S_{[qq]}J_l'}(\not{p}_1'+m_1')\gamma^+\gamma_5(\not{p}_1+m_1)\Gamma_{L_K S_{[qq]}J_l}],$$
$$\mathcal{F}_3^A(q^2) = \int \frac{dx_2 d^2k_{2\perp}}{2(2\pi)^3}\frac{3M_0'}{P'^+}\frac{\phi'^*_{n'L_K'}(\{x'\},\{k'_\perp\})\phi_{1L_K}(\{x\},\{k_\perp\})}{4\sqrt{P^+P'^+}\sqrt{[(m_1+x_1M_0)^2+k_{1\perp}^2][(m_1'+x_1M_0')^2+k_{1\perp}'^2]}}$$
$$\times \mathrm{Tr}[(\bar{\not{P}}+M_0)\gamma^+\gamma_5\mathcal{P}^{+\mu}\bar{\Gamma}_{\mu L_K' S_{[qq]}J_l'}(\not{p}_1'+m_1')\not{q}_\perp\gamma_5(\not{p}_1+m_1)\Gamma_{L_K S_{[qq]}J_l}],$$
$$\mathcal{F}_4^A(q^2) = -i\int \frac{dx_2 d^2k_{2\perp}}{2(2\pi)^3}\frac{3M_0'}{P'^+}\frac{\phi'^*_{n'L_K'}(\{x'\},\{k'_\perp\})\phi_{1L_K}(\{x\},\{k_\perp\})}{4\sqrt{P^+P'^+}\sqrt{[(m_1+x_1M_0)^2+k_{1\perp}^2][(m_1'+x_1M_0')^2+k_{1\perp}'^2]}}$$
$$\times \mathrm{Tr}[(\bar{\not{P}}+M_0)\sigma^{+\nu}q_{\perp\nu}\gamma_5\mathcal{P}^{+\mu}\bar{\Gamma}_{\mu L_K' S_{[qq]}J_l'}(\not{p}_1'+m_1')\not{P}_\perp\gamma_5(\not{p}_1+m_1)\Gamma_{L_K S_{[qq]}J_l}],$$
$$\mathcal{G}_1^V(q^2) = \int \frac{dx_2 d^2k_{2\perp}}{2(2\pi)^3}\frac{3M_0'}{P'^+}\frac{\phi'^*_{n'L_K'}(\{x'\},\{k'_\perp\})\phi_{1L_K}(\{x\},\{k_\perp\})}{8P^+P'^+\sqrt{[(m_1+x_1M_0)^2+k_{1\perp}^2][(m_1'+x_1M_0')^2+k_{1\perp}'^2]}}$$
$$\times \mathrm{Tr}[(\bar{\not{P}}+M_0)\gamma^+\mathcal{P}^{+\mu}\bar{\Gamma}_{\mu L_K' S_{[qq]}J_l'}(\not{p}_1'+m_1')\gamma^+(\not{p}_1+m_1)\Gamma_{L_K S_{[qq]}J_l}],$$
$$\mathcal{G}_2^V(q^2) = i\int \frac{dx_2 d^2k_{2\perp}}{2(2\pi)^3}\frac{3M_0'}{P'^+}\frac{\phi'^*_{n'L_K'}(\{x'\},\{k'_\perp\})\phi_{1L_K}(\{x\},\{k_\perp\})}{8P^+P'^+\sqrt{[(m_1+x_1M_0)^2+k_{1\perp}^2][(m_1'+x_1M_0')^2+k_{1\perp}'^2]}}$$
$$\times \mathrm{Tr}[(\bar{\not{P}}+M_0)\sigma^{+\nu}q_{\perp\nu}\mathcal{P}^{+\mu}\bar{\Gamma}_{\mu L_K' S_{[qq]}J_l'}(\not{p}_1'+m_1')\gamma^+(\not{p}_1+m_1)\Gamma_{L_K S_{[qq]}J_l}].$$

$$\mathcal{G}_3^V(q^2) = \int \frac{dx_2 d^2k_{2\perp}}{2(2\pi)^3} \frac{3M'_0}{P'^+} \frac{\phi'^*_{n'L'_K}(\{x'\},\{k'_\perp\})\phi_{1L_K}(\{x\},\{k_\perp\})}{4\sqrt{P^+P'^+}\sqrt{[(m_1+x_1M_0)^2+k_{1\perp}^2][(m'_1+x_1M'_0)^2+k'^2_{1\perp}]}}$$
$$\times \mathrm{Tr}[(\bar{\not{P}}+M_0)\gamma^+\mathcal{P}^{+\mu}\bar{\Gamma}_{\mu L'_K S_{[qq]}J'_l}(\not{p}'_1+m'_1)\not{q}_\perp(\not{p}_1+m_1)\Gamma_{L_K S_{[qq]}J_l}],$$
$$\mathcal{G}_4^V(q^2) = i\int \frac{dx_2 d^2k_{2\perp}}{2(2\pi)^3} \frac{3M'_0}{P'^+} \frac{\phi'^*_{n'L'_K}(\{x'\},\{k'_\perp\})\phi_{1L_K}(\{x\},\{k_\perp\})}{8P^+P'^+\sqrt{[(m_1+x_1M_0)^2+k_{1\perp}^2][(m'_1+x_1M'_0)^2+k'^2_{1\perp}]}}$$
$$\times \mathrm{Tr}[(\bar{\not{P}}+M_0)\sigma^{+\nu}n_{\perp\nu}\mathcal{P}^{+\mu}\bar{\Gamma}_{\mu L'_K S_{[qq]}J'_l}(\not{p}'_1+m'_1)\not{n}_\perp(\not{p}_1+m_1)\Gamma_{L_K S_{[qq]}J_l}], \tag{20}$$

with $q_\perp \equiv (0,0,\vec{q}_\perp)$, $n_\perp \equiv (0,0,\vec{n}_\perp)$, $n_\perp^2 = -1$ and $\vec{n}_\perp \cdot \vec{q}_\perp = 0$. The expressions of $\mathcal{F}_i^A$ and $\mathcal{G}_i^V$ in terms of f_i^A and g_i^V are similar to those of $\mathcal{F}_i^V$ and $\mathcal{G}_i^A$ in Eq. (19), by with V and A exchanged.

One can solve for $\bar{f}_i$ and $\bar{g}_i$ once F_i and G_i are known. It should be noted that there are relations in the $q^2 \to 0$ limit:

$$\lim_{q^2\to 0}\left\{\mathcal{F}_4^{V,A}(q^2)-(M+2M')\mathcal{F}_1^{V,A}(q^2)+(M'^2-M^2)\frac{\mathcal{F}_2^{V,A}(q^2)}{q^2}\right\}=0,$$
$$\lim_{q^2\to 0}\left\{\mathcal{G}_4^{A,V}(q^2)+(M-2M')\mathcal{G}_1^{A,V}(q^2)+(M'^2-M^2)\frac{\mathcal{G}_2^{A,V}(q^2)}{q^2}\right\}=0. \tag{21}$$

These are the consistency relations in the $1/2 \to 3/2$ case. In fact, they ensure the resulting form factors $\bar{f}_i$ and $\bar{g}_i$ to be finite in the $q^2 \to 0$ limit, i.e., $\lim_{q^2\to 0} q^2\bar{f}_i(q^2) = \lim_{q^2\to 0} q^2\bar{g}_i(q^2) = 0$.[2] Furthermore, we will expect

$$\lim_{q^2\to 0}\bar{f}_i(q^2) = \lim_{q^2\to 0}\frac{d}{dq^2}[q^2\bar{f}_i(q^2)],$$
$$\lim_{q^2\to 0}\bar{g}_i(q^2) = \lim_{q^2\to 0}\frac{d}{dq^2}[q^2\bar{g}_i(q^2)], \tag{23}$$

to hold.

Most of the traces in Eqs. (12), (13), (18), and (20) are rather complicated to work out. We find that it is convenient to work in the $\vec{P}_\perp = \vec{0}$ frame and all of the traces can be obtained with the help of the FeynCalc program [47,48]. The final expressions of the form factors can be obtained using the kinematics of light-front quantities collected in Appendix A. We found that all of the P^+ and P'^+ factors in these equations cancel out in the final expressions as they should.

B. Form factors for $\mathcal{B}_b(\bar{\mathbf{3}}_\mathbf{f},1/2^+) \to \mathcal{B}_c(\bar{\mathbf{3}}_\mathbf{f},1/2^+)$ transition [type (i)]

The $\mathcal{B}_b(\bar{\mathbf{3}}_\mathbf{f},1/2^+) \to \mathcal{B}_c(\bar{\mathbf{3}}_\mathbf{f},1/2^+)$ transitions involve initial states in $(n,L_K,S^P_{[qq]},J^P_l,J^P)_b = (1,0,0^+,0^+,\frac{1}{2}^+)$ configuration and final states in $(n',L'_K,S^P_{[qq]},J'^P_l,J'^P)_c = (n',0,0^+,0^+,\frac{1}{2}^+)$ configurations (with $n'=1,2$). This type of transition consists of $\Lambda_b^0 \to \Lambda_c^+$, $\Xi_b^{0(-)} \to \Xi_c^{+(0)}$ and $\Lambda_b^0 \to \Lambda_c(2765)^+$ transitions. In these transitions the spectating diquarks are scalar diquarks $[ud]$, $[us]$ and $[ds]$. To obtain form factors f_i^V and g_i^A, we use Eq. (12) with $\Gamma_{L_K S_{[qq]}J_l} = \Gamma_{s00}(p_1,p_2)$, $\bar{\Gamma}_{L'_K S_{[qq]}J'_l} = \bar{\Gamma}_{s00}(p'_1,p_2)$, which are given in Eq. (7), and $n' = 1$, 2.

C. Form factors for $\mathcal{B}_b(\mathbf{6}_\mathbf{f},1/2^+) \to \mathcal{B}_c(\mathbf{6}_\mathbf{f},1/2^+)$ transition [type (ii)]

The $\mathcal{B}_b(\mathbf{6}_\mathbf{f},1/2^+) \to \mathcal{B}_c(\mathbf{6}_\mathbf{f},1/2^+)$ transitions involve initial states in $(n,L_K,S^P_{[qq]},J^P_l,J^P)_b = (1,0,1^+,1^+,\frac{1}{2}^+)$ configuration and final states in $(n',L'_K,S^P_{[qq]},J'^P_l,J'^P)_c = (n',0,1^+,1^+,\frac{1}{2}^+)$ configurations (with $n'=1,2$). This type of transition consists of $\Omega_b^- \to \Omega_c^0$ and $\Omega_b^- \to \Omega_c(3090)^0$ transitions. In these transitions the spectating diquarks are axial-vector diquarks $[ss]$. To obtain form factors f_i^V and g_i^A, we use Eq. (12) with $\Gamma_{L_K S_{[qq]}J_l} = \Gamma_{s11}(p_1,p_2,\lambda_2)$,

[2] To see this we denote equations of $\mathcal{F}_i$ in Eq. (19) as

$$\begin{pmatrix}\mathcal{F}_1\\ \mathcal{F}_2/q^2\\ \mathcal{F}_3/q^2\\ \mathcal{F}_4\end{pmatrix} = A\begin{pmatrix}\bar{f}_1\\ \bar{f}_2\\ \bar{f}_3\\ \bar{f}_4\end{pmatrix}, \tag{22}$$

where A is a 4×4 matrix with elements correspond to the coefficients of $\bar{f}_i$ in Eq. (19). It is well known that $\bar{f}_i$ can be obtained by acting the inverse matrix $A^{-1} = adj(A)/|A|$ on $(\mathcal{F}_1,\mathcal{F}_2/q^2,\mathcal{F}_3/q^2,\mathcal{F}_4)^T$, where $adj(A)$ is the adjugate matrix of A. For q^2 approaches 0 the determinant of A, $|A| = \mathcal{O}(q^2)$, approaches 0, which seems to lead to diverging $\bar{f}_i(0)$ as the denominator of $[adj(A)\cdot(\mathcal{F}_1,\mathcal{F}_2/q^2,\mathcal{F}_3/q^2,\mathcal{F}_4)^T]/|A|$ is approaching 0, but it can be shown that the numerator, $adj(A)\cdot(\mathcal{F}_1,\mathcal{F}_2/q^2,\mathcal{F}_3/q^2,\mathcal{F}_4)^T$, is proportional to the l.h.s. of first equation in Eq. (21) for small q^2 and also approaches 0. Hence, with the help of the first equation in Eq. (21) the limit of $\bar{f}_i(q^2)$ for q^2 approaching 0 can be obtained by using the L'Hôpital's rule and we find that finite $\bar{f}_i(0)$ can be obtained in this way. Similar argument holds for the $\mathcal{G}_i$, $\bar{g}_i$ case.

$\bar{\Gamma}_{L'_K S_{[qq]} J'_l} = \bar{\Gamma}_{s11}(p'_1, p_2, \lambda_2)$, which are given in Eq. (7), and $n' = 1, 2$. Note that one needs to sum over the axial-vector diquark polarization (λ_2).

D. Form factors for $\mathcal{B}_b(\mathbf{6_f}, 1/2^+) \to \mathcal{B}_c(\mathbf{6_f}, 3/2^+)$ transition [type (iii)]

The $\mathcal{B}_b(\mathbf{6_f}, 1/2^+) \to \mathcal{B}_c(\mathbf{6_f}, 3/2^+)$ transitions involve initial states in $(n, L_K, S^P_{[qq]}, J^P_l, J^P)_b = (1, 0, 1^+, 1^+, \frac{1}{2}^+)$ configuration and final states in $(n', L'_K, S^P_{[qq]}, J'^P_l, J'^P)_c = (n', 0, 1^+, 1^+, \frac{3}{2}^+)$ configurations (with $n' = 1, 2$). This type of transition consists of $\Omega_b^- \to \Omega_c(2770)^0$ and $\Omega_b^- \to \Omega_c(3119)^0$ transitions. In these transitions the spectating diquarks are axial-vector diquarks $[ss]$. To obtain form factors $\bar{f}^V_i$ and $\bar{g}^A_i$, we use Eq. (18) with $\Gamma_{L_K S_{[qq]} J_l} = \Gamma_{s11}(p_1, p_2, \lambda_2)$, $\bar{\Gamma}^\mu_{L'_K S_{[qq]} J'_l} = \bar{\Gamma}^\mu_{s11}(p'_1, p_2, \lambda_2)$, which are given in Eq. (7), and $n' = 1, 2$. Note that one needs to sum over the axial-vector diquark polarization (λ_2).

E. Form factors for $\mathcal{B}_b(\mathbf{6_f}, 1/2^+) \to \mathcal{B}_c(\mathbf{6_f}, 3/2^-)$ transition [type (iv)]

The $\mathcal{B}_b(\mathbf{6_f}, 1/2^+) \to \mathcal{B}_c(\mathbf{6_f}, 3/2^-)$ transitions involve initial states in $(n, L_K, S^P_{[qq]}, J^P_l, J^P)_b = (1, 0, 1^+, 1^+, \frac{1}{2}^+)$ configuration and final states in $(n', L'_K, S^P_{[qq]}, J'^P_l, J'^P)_c = (1, 1, 1^+, 1^+, \frac{3}{2}^+)$ configurations. This type of transition consists of $\Omega_b^- \to \Omega_c(3050)^0$ transition. We follow Ref. [6] to consider $\Omega_c(3050)^0$ as a p-wave state. In these transitions the spectating diquarks are axial-vector diquarks $[ss]$. To obtain form factors $\bar{f}^A_i$ and $\bar{g}^V_i$, we use Eq. (20) with $\Gamma_{L_K S_{[qq]} J_l} = \Gamma_{s11}(p_1, p_2, \lambda_2)$, $\bar{\Gamma}^\mu_{L'_K S_{[qq]} J'_l} = \bar{\Gamma}^\mu_{p12}(p'_1, p_2, \lambda_2)$, which are given in Eq. (7), and $n' = 1$. Note that one needs to sum over the axial-vector diquark polarization (λ_2).

F. Form factors for $\mathcal{B}_b(\bar{\mathbf{3}}_\mathbf{f}, 1/2^+) \to \mathcal{B}_c(\bar{\mathbf{3}}_\mathbf{f}, 1/2^-)$ transition [type (v)]

The $\mathcal{B}_b(\bar{\mathbf{3}}_\mathbf{f}, 1/2^+) \to \mathcal{B}_c(\bar{\mathbf{3}}_\mathbf{f}, 1/2^-)$ transitions involve initial states in $(n, L_K, S^P_{[qq]}, J^P_l, J^P)_b = (1, 0, 0^+, 0^+, \frac{1}{2}^+)$ configuration and final states in $(n', L'_K, S^P_{[qq]}, J'^P_l, J'^P)_c = (n', 1, 0^+, 1^-, \frac{1}{2}^-)$ configurations (with $n' = 1, 2$). This type of transition consists of $\Lambda_b^0 \to \Lambda_c(2595)^+$, $\Xi_b^{0(-)} \to \Xi_c(2790)^{+(0)}$ and $\Lambda_b^0 \to \Lambda_c(2940)^+$ transitions. In these transitions the spectating diquarks are scalar diquarks $[ud]$, $[us]$ and $[ds]$. To obtain form factors f^A_i and g^V_i, we use Eq. (13) with $\Gamma_{L_K S_{[qq]} J_l} = \Gamma_{s00}(p_1, p_2)$, $\bar{\Gamma}_{L'_K S_{[qq]} J'_l} = \bar{\Gamma}_{p10}(p'_1, p_2)$, which are given in Eq. (7), and $n' = 1, 2$.

G. Form factors for $\mathcal{B}_b(\bar{\mathbf{3}}_\mathbf{f}, 1/2^+) \to \mathcal{B}_c(\bar{\mathbf{3}}_\mathbf{f}, 3/2^-)$ transition [type (vi)]

The $\mathcal{B}_b(\bar{\mathbf{3}}_\mathbf{f}, 1/2^+) \to \mathcal{B}_c(\bar{\mathbf{3}}_\mathbf{f}, 3/2^-)$ transitions involve initial states in $(n, L_K, S^P_{[qq]}, J^P_l, J^P)_b = (1, 0, 0^+, 0^+, \frac{1}{2}^+)$ configuration and final states in $(n', L'_K, S^P_{[qq]}, J'^P_l, J'^P)_c = (n', 1, 0^+, 1^-, \frac{3}{2}^-)$ configurations with $n' = 1, 2$. This type of transition consists of $\Lambda_b^0 \to \Lambda_c(2625)^+$, $\Lambda_c(2940)^+$ and $\Xi_b^{0(-)} \to \Xi_c(2815)^{+(0)}$ transitions. We follow LHCb and PDG [1,4] to take $\Lambda_c(2940)$ as a spin-3/2 particle. To obtain form factors $\bar{f}^A_i$ and $\bar{g}^V_i$, we use Eq. (20) with $\Gamma_{L_K S_{[qq]} J_l} = \Gamma_{s00}(p_1, p_2)$, $\bar{\Gamma}^\mu_{L'_K S_{[qq]} J'_l} = \bar{\Gamma}^\mu_{p01}(p'_1, p_2)$, which are given in Eq. (7), and $n' = 1, 2$.

H. Heavy quark limit

It is useful to note that in the heavy quark (HQ) limit, baryon form factors have simple behavior [37–39]. The $\mathcal{B}_b \to \mathcal{B}_c$ transition matrix elements with s-wave and p-wave $\mathcal{B}_c$ can be expressed as [22,37–40][3]

$$
\begin{aligned}
\langle \mathcal{B}_c(\bar{3}_f, 1/2^+)(v')|j^\mu_{V-A}|\mathcal{B}_b(\bar{\mathbf{3}}_f, 1/2^+)(v)\rangle &= \zeta(\omega)\bar{u}(v')\gamma^\mu(1-\gamma_5)u(v), \\
\langle \mathcal{B}_c(\bar{3}_f, 1/2^-)(v')|j^\mu_{V-A}|\mathcal{B}_b(\bar{\mathbf{3}}_f, 1/2^+)(v)\rangle &= \frac{\sigma(\omega)}{\sqrt{3}}\bar{u}(v')\gamma_5(\not{v} + v\cdot v')\gamma^\mu(1-\gamma_5)u(v), \\
\langle \mathcal{B}_c(\bar{3}_f, 3/2^-)(v')|j^\mu_{V-A}|\mathcal{B}_b(\bar{\mathbf{3}}_f, 1/2^+)(v)\rangle &= -\sigma(\omega)\bar{u}_\nu(v')v^\nu\gamma^\mu(1-\gamma_5)u(v), \\
\langle \mathcal{B}_c(\mathbf{6}_f, 1/2^+)(v')|j^\mu_{V-A}|\mathcal{B}_b(\mathbf{6}_f, 1/2^+)(v)\rangle &= -\frac{1}{3}(g^{\rho\sigma}\xi_1 - v^\rho v'^\sigma\xi_2)\times\bar{u}(v')(\gamma_\rho - v'_\rho)\gamma^\mu(1-\gamma_5)(\gamma_\sigma - v_\sigma)u(v), \\
\langle \mathcal{B}_c(\mathbf{6}_f, 3/2^+)(v')|j^\mu_{V-A}|\mathcal{B}_b(\mathbf{6}_f, 1/2^+)(v)\rangle &= -\frac{1}{\sqrt{3}}(g^{\rho\sigma}\xi_1 - v^\rho v'^\sigma\xi_2)\bar{u}_\rho(v')\gamma^\mu(1-\gamma_5)(\gamma_\sigma + v_\sigma)\gamma_5 u(v), \\
\langle \mathcal{B}_c(\mathbf{6}_f, 3/2^-)(v')|j^\mu_{V-A}|\mathcal{B}_b(\mathbf{6}_f, 1/2^+)(v)\rangle &= -\frac{1}{\sqrt{30}}[g^{\rho\sigma}\xi_5 + (v-v')^\rho(v-v')^\sigma\xi_6][\bar{u}\cdot v(\gamma_\rho - v'_\rho) + \bar{u}_\rho(\not{v} - \omega))] \\
&\quad\times\gamma^\mu(1-\gamma_5)(\gamma_\sigma - v_\sigma)u(v), \qquad (24)
\end{aligned}
$$

[3]Note that in the $\mathcal{B}_b(1/2^+) \to \mathcal{B}_c(3/2^\pm)$ transition matrix elements, we apply overall minus signs to match our sign convention, which follow from the sign convention of the Clebsch-Gordon coefficients, see Eq. (A6).

which imply, in the type (i) $\mathcal{B}_b(\bar{\mathbf{3}}_f, 1/2^+) \to \mathcal{B}_c(\bar{\mathbf{3}}_f, 1/2^+)$ transition,

$$f_1^V(\bar{\mathbf{3}}_f) = g_1^A(\bar{\mathbf{3}}_f) = \zeta(\omega), \qquad f_{2,3}^V(\bar{\mathbf{3}}_f) = g_{2,3}^A(\bar{\mathbf{3}}_f) = 0; \tag{25}$$

in the type (ii) $\mathcal{B}_b(\mathbf{6}_f, 1/2^+) \to \mathcal{B}_c(\mathbf{6}_f, 1/2^+)$ transition,

$$\begin{aligned}
f_1^V(\mathbf{6}_f) &= \frac{1}{3}[(2-\omega)\xi_1 + (1-\omega)^2\xi_2] + \frac{1}{3}\frac{M^2+M'^2}{MM'}[\xi_1 + (1-\omega)\xi_2],\\
f_2^V(\mathbf{6}_f) &= \frac{1}{3}\frac{(M+M')^2}{MM'}[\xi_1 + (1-\omega)\xi_2], \qquad f_3^V(\mathbf{6}_f) = -\frac{1}{3}\frac{M^2-M'^2}{MM'}[\xi_1 + (1-\omega)\xi_2],\\
g_1^A(\mathbf{6}_f) &= \frac{1}{3}[-(2+\omega)\xi_1 + (1+\omega)^2\xi_2] + \frac{1}{3}\frac{M^2+M'^2}{MM'}[\xi_1 - (1+\omega)\xi_2],\\
g_2^A(\mathbf{6}_f) &= -\frac{1}{3}\frac{(M-M')^2}{MM'}[\xi_1 - (1+\omega)\xi_2], \qquad g_3^A(\mathbf{6}_f) = \frac{1}{3}\frac{M^2-M'^2}{MM'}[\xi_1 - (1+\omega)\xi_2];
\end{aligned} \tag{26}$$

in the type (iii) $\mathcal{B}_b(\mathbf{6}_f, 1/2^+) \to \mathcal{B}_c(\mathbf{6}_f, 3/2^+)$ transition,

$$\begin{aligned}
&\bar{f}_1^V(\mathbf{6}_f) = -\bar{g}_1^A(\mathbf{6}_f) = -\frac{2}{\sqrt{3}}\xi_1(\omega), \qquad \bar{f}_3^V(\mathbf{6}_f) = -\bar{g}_3^A(\mathbf{6}_f) = +\frac{2}{\sqrt{3}}\xi_2(\omega), \qquad \bar{f}_4^V(\mathbf{6}_f) = \bar{g}_4^A(\mathbf{6}_f) = 0,\\
&\bar{f}_2^V(\mathbf{6}_f) = -\frac{1}{\sqrt{3}}[\xi_1 + (1-\omega)\xi_2(\omega)], \qquad \bar{g}_2^A(\mathbf{6}_f) = -\frac{1}{\sqrt{3}}[\xi_1 - (1+\omega)\xi_2(\omega)];
\end{aligned} \tag{27}$$

in the type (iv) $\mathcal{B}_b(\mathbf{6}_f, 1/2^+) \to \mathcal{B}_c(\mathbf{6}_f, 3/2^-)$ transition,

$$\begin{aligned}
\bar{f}_1^A(\mathbf{6}_f) &= -\frac{2}{\sqrt{30}}(1+\omega)\xi_5(\omega), \qquad \bar{f}_2^A(\mathbf{6}_f) = -\frac{2}{\sqrt{30}}[(1+\omega)\xi_5(\omega) + (1-\omega^2)\xi_6(\omega)],\\
\bar{f}_3^A(\mathbf{6}_f) &= -\frac{2}{\sqrt{30}}[\xi_5(\omega) - 2(1+\omega)\xi_6(\omega)], \qquad \bar{f}_4^A(\mathbf{6}_f) = \frac{4}{\sqrt{30}}[\xi_5(\omega) - (1+\omega)\xi_6(\omega)],\\
\bar{g}_1^V(\mathbf{6}_f) &= -\frac{2}{\sqrt{30}}(1-\omega)\xi_5(\omega), \qquad \bar{g}_2^V(\mathbf{6}_f) = \frac{1}{\sqrt{30}}[(1-2\omega)\xi_5(\omega) - 2(1-\omega^2)\xi_6(\omega)],\\
\bar{g}_3^V(\mathbf{6}_f) &= \frac{2}{\sqrt{30}}[\xi_5(\omega) + 2(1-\omega)\xi_6(\omega)], \qquad \bar{g}_4^V(\mathbf{6}_f) = \frac{4}{\sqrt{30}}[\xi_5(\omega) + (1-\omega)\xi_6(\omega)];
\end{aligned} \tag{28}$$

in the type (v) $\mathcal{B}_b(\bar{\mathbf{3}}_f, 1/2^+) \to \mathcal{B}_c(\bar{\mathbf{3}}_f, 1/2^-)$ transition,

$$\begin{aligned}
f_1^A(\bar{\mathbf{3}}_f) &= g_1^V(\bar{\mathbf{3}}_f) = \left(\omega - \frac{M'}{M}\right)\frac{\sigma(\omega)}{\sqrt{3}},\\
f_2^A(\bar{\mathbf{3}}_f) &= f_3^A(\bar{\mathbf{3}}_f) = -\frac{M+M'}{M}\frac{\sigma(\omega)}{\sqrt{3}}, \qquad g_2^V(\bar{\mathbf{3}}_f) = g_3^V(\bar{\mathbf{3}}_f) = -\frac{M-M'}{M}\frac{\sigma(\omega)}{\sqrt{3}};
\end{aligned} \tag{29}$$

and in the type (vi) $\mathcal{B}_b(\bar{\mathbf{3}}_f, 1/2^+) \to \mathcal{B}_c(\bar{\mathbf{3}}_f, 3/2^-)$ transition,

$$\bar{f}_2^A(\bar{\mathbf{3}}_f) = \bar{g}_2^V(\bar{\mathbf{3}}_f) = \sigma(\omega), \qquad \bar{f}_{1,3,4}^A(\bar{\mathbf{3}}_f) = \bar{g}_{1,34}^V(\bar{\mathbf{3}}_f) = 0. \tag{30}$$

For low lying $\mathcal{B}_c$ states, the following normalizations are applied,

$$\zeta(1) = 1, \qquad \xi_1(1) = 1, \tag{31}$$

and it has been shown that in the large N_c limit, one has [41]

$$\xi_1(\omega) = (1+\omega)\xi_2(\omega) = \zeta(\omega). \tag{32}$$

TABLE IV. The input parameters $m^S_{[qq']}$, $m^A_{[qq']}$, m_q and β's appearing in the Gaussian-type wave function (9) (in units of GeV). The superscript S and A denote scalar and axial vector diquarks, respectively.

$m^S_{[ud]}$	$m^S_{[us],[ds]}$	$m^A_{[ss]}$	m_b	m_c	$\beta(\Lambda_b)$	$\beta(\Xi_b^{0,-})$
0.65	0.86	1.10	4.44	1.42	0.750	0.850
$\beta(\Omega_b)$	$\beta(\Lambda_c)$	$\beta[\Lambda_c(2595)]$	$\beta[\Lambda_c(2625)]$	$\beta[\Lambda_c(2765)]$	$\beta[\Lambda_c(2940,\frac{1}{2}^-)]$	$\beta[\Lambda_c(2940,\frac{3}{2}^-)]$
0.900	0.345	0.350	0.450	0.345	0.350	0.450
$\beta(\Xi_c^{+,0})$	$\beta[\Xi_c^{+,0}(2790)]$	$\beta[\Xi_c^{+,0}(2815)]$	$\beta(\Omega_c)$	$\beta[\Omega_c(2770)]$	$\beta[\Omega_c(3050)]$	$\beta[\Omega_c(3090)]$
0.370	0.365	0.550	0.300	0.370	0.420	0.300
$\beta[\Omega_c(3120)]$						
0.370						

These imply very simple and specify relations of form factors at $q^2 = q^2_{\rm max}$ (or $\omega = 1$), namely

$$f_1^V(\bar{\mathbf{3}}_f) = g_1^A(\bar{\mathbf{3}}_f) = 1, \qquad f_{2,3}^V(\bar{\mathbf{3}}_f) = g_{2,3}^A(\bar{\mathbf{3}}_f) = 0, \quad (33)$$

in the type (i) $\mathcal{B}_b(\bar{\mathbf{3}}_f, 1/2^+) \to \mathcal{B}_c(\bar{\mathbf{3}}_f, 1/2^+)$ transition;

$$f_1^V(\mathbf{6}_f) = \frac{1}{3} + \frac{1}{3}\frac{M^2+M'^2}{MM'}, \qquad f_2^V(\mathbf{6}_f) = \frac{1}{3}\frac{(M+M')^2}{MM'},$$
$$f_3^V(\mathbf{6}_f) = -\frac{1}{3}\frac{M^2-M'^2}{MM'},$$
$$g_1^A(\mathbf{6}_f) = -\frac{1}{3}, \qquad g_2^A(\mathbf{6}_f) = g_3^A(\mathbf{6}_f) = 0, \quad (34)$$

in the type (ii) $\mathcal{B}_b(\mathbf{6}_f, 1/2^+) \to \mathcal{B}_c(\mathbf{6}_f, 1/2^+)$ transition; and

$$-\bar{f}_1^V(\mathbf{6}_f) = \bar{g}_1^A(\mathbf{6}_f) = -2\bar{f}_2^V(\mathbf{6}_f) = 2\bar{f}_3^V(\mathbf{6}_f)$$
$$= -2\bar{g}_3^A(\mathbf{6}_f) = \frac{2}{\sqrt{3}},$$
$$\bar{g}_2^A(\mathbf{6}_f) = \bar{f}_4^V(\mathbf{6}_f) = \bar{g}_4^A(\mathbf{6}_f) = 0, \quad (35)$$

in the type (iii) $\mathcal{B}_b(\mathbf{6}_f, 1/2^+) \to \mathcal{B}_c(\mathbf{6}_f, 3/2^+)$ transition. Furthermore, for other transitions at $q^2 = q^2_{\rm max}$, we have

$$-\bar{f}_1^A(\mathbf{6}_f) = -\bar{f}_2^A(\mathbf{6}_f) = -4\bar{g}_2^V(\mathbf{6}_f) = 2\bar{g}_3^V(\mathbf{6}_f)$$
$$= \bar{g}_4^V(\mathbf{6}_f) = \frac{4}{\sqrt{30}}\xi_5(1),$$
$$\bar{f}_3^A(\mathbf{6}_f) = -\frac{2}{\sqrt{30}}[\xi_5(1) - 4\xi_6(1)],$$
$$\bar{f}_4^A(\mathbf{6}_f) = \frac{4}{\sqrt{30}}[\xi_5(1) - 2\xi_6(1)], \qquad \bar{g}_1^V(\mathbf{6}_f) = 0, \quad (36)$$

in the type (iv) $\mathcal{B}_b(\mathbf{6}_f, 1/2^+) \to \mathcal{B}_c(\mathbf{6}_f, 3/2^-)$ transition;

$$f_1^A(\bar{\mathbf{3}}_f) = g_1^V(\bar{\mathbf{3}}_f) = -g_2^V(\bar{\mathbf{3}}_f) = -g_3^V(\bar{\mathbf{3}}_f) = \left(\frac{M-M'}{M}\right)\frac{\sigma(1)}{\sqrt{3}},$$
$$f_2^A(\bar{\mathbf{3}}_f) = f_3^A(\bar{\mathbf{3}}_f) = -\frac{M+M'}{M}\frac{\sigma(1)}{\sqrt{3}}, \quad (37)$$

in the type (v) $\mathcal{B}_b(\bar{\mathbf{3}}_f, 1/2^+) \to \mathcal{B}_c(\bar{\mathbf{3}}_f, 1/2^-)$ transition; and

$$\bar{f}_2^A(\bar{\mathbf{3}}_f) = \bar{g}_2^V(\bar{\mathbf{3}}_f) = \sigma(1), \quad \bar{f}_{1,3,4}^A(\bar{\mathbf{3}}_f) = \bar{g}_{1,34}^V(\bar{\mathbf{3}}_f) = 0, \quad (38)$$

in the type (vi) $\mathcal{B}_b(\bar{\mathbf{3}}_f, 1/2^+) \to \mathcal{B}_c(\bar{\mathbf{3}}_f, 3/2^-)$ transition.

Although obtaining these Isgur-Wise functions is beyond the scope of this work, the above relations on form factors can still be useful. Indeed, we expect our form factors to roughly exhibit the above patterns, since we have large but finite $m_{b,c}$.

III. NUMERICAL RESULTS

In this section we will present the numerical results of all the relevant $\mathcal{B}_b \to \mathcal{B}_c$ transition form factors. We will give predictions on the decay rates and up-down asymmetries of various $\Lambda_b \to \Lambda_c^{(*,**)} M^-$, $\Xi_b \to \Xi_c^{(**)} M^-$ and $\Omega_b \to \Omega_c^{(*)} M^-$ decays using naïve factorization.

A. $\mathcal{B}_b \to \mathcal{B}_c$ form factors

In Table IV we summarize the input parameters $m_{[qq']}$, m_q and β. Note that the constituent quark and diquark masses are close to but smaller than those in Ref. [49]. For the diquark masses, we use $m^S_{[ud]}$ for Λ_b and $\Lambda_c^{(*,**)}$, $m^S_{[us]}$ and $m^S_{[ds]}$ for Ξ_b and $\Xi_c^{(**)}$, and $m^A_{[ss]}$ for Ω_b and $\Omega_c^{(*)}$. The βs for states only differ in their radial quantum numbers should be identical. For example, the βs of Λ_c and of the radial excited state $\Lambda_c(2765)$ are identical, and the βs of the low-lying $3/2^-$ state $\Lambda_c(2625)$ and of the radial excited $3/2^-$ state $\Lambda_c(2765)$ are identical. These input parameters are chosen to satisfy the consistency constraints [see discussions after Eqs. (12) and (21)] and to reproduce the $\mathrm{Br}(\Lambda_b \to \Lambda_c P)$ data. In practice it is more convenient to

TABLE V. The transition form factors for various $\mathcal{B}_b(\bar{\mathbf{3}}_\mathbf{f}, 1/2^+) \to \mathcal{B}_c(\bar{\mathbf{3}}_\mathbf{f}, 1/2^+)$ transitions [types (i) and (i)*]. We employ a three parameter form for these form factors, see Eq. (39).

$\mathcal{B}_b \to \mathcal{B}_c$	F	$F(0)$	$F(q^2_{\max})$	a	b	F	$F(0)$	$F(q^2_{\max})$	a	b
$\Lambda_b \to \Lambda_c$	f_1^V	$0.474^{+0.069}_{-0.072}$	$0.764^{+0.111}_{-0.116}$	1.426	0.994	g_1^A	$0.468^{+0.067}_{-0.07}$	$0.743^{+0.106}_{-0.111}$	1.394	0.966
	f_2^V	$-0.153^{+0.027}_{-0.029}$	$-0.262^{+0.046}_{-0.050}$	1.753	1.623	g_2^A	$0.030^{+0.005}_{-0.007}$	$0.053^{+0.009}_{-0.012}$	1.921	1.963
	f_3^V	$0.069^{+0.021}_{-0.022}$	$0.130^{+0.039}_{-0.041}$	2.068	2.100	g_3^A	$-0.070^{+0.010}_{-0.009}$	$-0.114^{+0.016}_{-0.015}$	1.65	1.587
$\Xi_b \to \Xi_c^0$	f_1^V	$0.437^{+0.070}_{-0.072}$	$0.714^{+0.114}_{-0.118}$	1.676	1.504	g_1^A	$0.429^{+0.068}_{-0.071}$	$0.693^{+0.110}_{-0.115}$	1.635	1.452
	f_2^V	$-0.175^{+0.033}_{-0.035}$	$-0.294^{+0.056}_{-0.059}$	1.968	2.233	g_2^A	$0.034^{+0.006}_{-0.007}$	$0.057^{+0.010}_{-0.012}$	2.067	2.503
	f_3^V	$0.081^{+0.025}_{-0.025}$	$0.146^{+0.045}_{-0.045}$	2.257	2.760	g_3^A	$-0.078^{+0.011}_{-0.010}$	$-0.123^{+0.017}_{-0.016}$	1.825	2.119
$\Lambda_b \to \Lambda_c(2765)$	f_1^V	$-0.354^{+0.027}_{-0.028}$	$-0.494^{+0.038}_{-0.039}$	1.079	−0.063	g_1^A	$-0.341^{+0.024}_{-0.026}$	$-0.460^{+0.032}_{-0.035}$	0.985	−0.047
	f_2^V	$0.135^{+0.026}_{-0.018}$	$0.246^{+0.047}_{-0.034}$	1.844	0.346	g_2^A	$0.012^{+0.013}_{-0.010}$	$0.014^{+0.014}_{-0.012}$	1.874	5.804
	f_3^V	$0.047^{+0.039}_{-0.037}$	$0.057^{+0.048}_{-0.045}$	1.838	4.433	g_3^A	$0.062^{+0.006}_{-0.007}$	$0.120^{+0.012}_{-0.014}$	2.014	0.551

enforce the consistency constraints by using floating M and M', and the input parameters are determined by requiring M and M' to reproduce the physical masses of $\mathcal{B}_b$ and $\mathcal{B}_c$, respectively, within 10%. In fact, in most cases the agreements are better than 10%, but in the case of $\Lambda_c(2940)$ as a radial excited $3/2^-$ state, the corresponding M' is larger than $m_{\Lambda_c(2940)}$ by 25%.

Using the results in the previous section the form factors of various $\mathcal{B}_b \to \mathcal{B}_c$ transitions can be obtained. The form factors are calculated in spacelike region, as we are using the $q^+ = 0$ frame, we shall follow [16,42,50–54] to analytically continue them to the timelike region. We follow [42,50,51,53,54] and parametrize the form factors in the three-parameter form:

$$F(q^2) = \frac{F(0)}{1 - a(q^2/M^2) + b(q^2/M^2)^2} \qquad (39)$$

for $\mathcal{B}_b \to \mathcal{B}_c$ transitions with the parameters a, b expected to be of order $\mathcal{O}(1)$, while for some cases, where the corresponding a and b are much larger than 1, we shall use the following form [16,42,44,54]

$$F(q^2) = \frac{F(0)}{(1 - q^2/M^2)[1 - a(q^2/M^2) + b(q^2/M^2)^2]}, \qquad (40)$$

to reduce the size of a and b and gives better fits. As we shall see that there are cases where some of the parameters a, b are still larger than $\mathcal{O}(1)$, but usually the corresponding form factors are small and, consequently, they do not have much impact on the corresponding $\mathcal{B}_b \to \mathcal{B}_c M$ decay rates.

The $\mathcal{B}_b(\bar{\mathbf{3}}_\mathbf{f}, 1/2^+) \to \mathcal{B}_c(\bar{\mathbf{3}}_\mathbf{f}, 1/2^+)$ transition form factors $f^V_{1,2,3}(q^2)$ and $g^A_{1,2,3}(q^2)$ for $\Lambda_b \to \Lambda_c, \Lambda_c(2765)$ and $\Xi_b \to \Xi_c$ transitions are given in Table V and they are plotted in Fig. 2. The uncertainties in form factors $F(0)$ are obtained by varying m_b, m_c, $m_{[qq]}$, $\beta(\mathcal{B}_b)$, and $\beta(\mathcal{B}_c)$ by 10% separately and combine the uncertainties quadratically. Note that Λ_c and Ξ_c are low lying states, while $\Lambda_c(2765)$ is a radial excited state. From the table and the figures, we see that $f_1^V \simeq g_1^A$ and they dominate over $f^V_{2,3}$ and $g^A_{2,3}$. These are close to the predicted relations of form factors in the heavy quark limit, see Eq. (25). The values of f_1^V and g_1^A at $q^2_{\max}$ in $\Lambda_b \to \Lambda_c$ and $\Xi_b \to \Xi_c$ transitions are smaller than the ones predicted in heavy quark limit with $\zeta(1) = 1$, Eq. (33), by roughly 25%. The reduction can be more or less traced to the mismatch of the overlapping of the wave functions of $\mathcal{B}_b$ and $\mathcal{B}_c$, as $\beta(\mathcal{B}_b) \neq \beta(\mathcal{B}_c)$. Indeed, it is easy to see that, using $\beta(\Lambda_b)$ and $\beta(\Lambda_c)$ given in Table IV, the overlapping integral of wave functions of Λ_b and Λ_c is 0.66, which is smaller than the one in the ideal case with $\beta(\Lambda_b) = \beta(\Lambda_c)$. Things are similar in the $\Xi_b \to \Xi_c$ transition. In the $\Lambda_b \to \Lambda_c(2765)$ transition the sizes of the form factors are smaller than those in $\Lambda_b \to \Lambda_c$ and $\Xi_b \to \Xi_c$ transitions. and the signs of form factors are flipped. These changes are closely related to the fact that $\Lambda_c(2765)$ is a radial excited state. In fact, in the heavy quark limit, one expects $f_1^V = g_1^A = 0$ at $q^2 = q^2_{\max}$, as the wave functions of the low-lying $\mathcal{B}_b$ state and the radial excited $\mathcal{B}_c$ state are orthogonal [38]. This is not borne out as $\beta(\mathcal{B}_b) \neq \beta(\mathcal{B}_c)$, and instead the overlapping integral is −0.53. As the overlapping integral has smaller size and opposite sign compared to those in the low-lying $\mathcal{B}_b$ and $\mathcal{B}_c$ states, the reduction in sizes and the flipping of signs in f_1^V and g_1^A in this transition are expected.

The $\mathcal{B}_b(\mathbf{6}_\mathbf{f}, 1/2^+) \to \mathcal{B}_c(\mathbf{6}_\mathbf{f}, 1/2^+)$ transition form factors $f^V_{1,2,3}(q^2)$ and $g^A_{1,2,3}(q^2)$ for $\Omega_b \to \Omega_c$ and $\Omega_b \to \Omega_c(3090)$ transitions are given in Table VI and are plotted in Fig. 3. Note that Ω_c is a low lying state, while $\Omega_c(3090)$ is a radial excite state. Heavy quark symmetry and large N_c QCD predict that in these transitions, we have $\xi_1(\omega) = (1+\omega)\xi_2(\omega)$, see Eq. (32), which implies $g_2^A(q^2) = g_3^A(q^2) = 0$. We see from the table and the plots that we indeed have $g^A_{2,3}(q^2) \simeq 0$. In $\Omega_b \to \Omega_c$ transition, Eq. (31) gives $\xi_1(1) = 2\xi_2(1) = 1$, and, consequently,

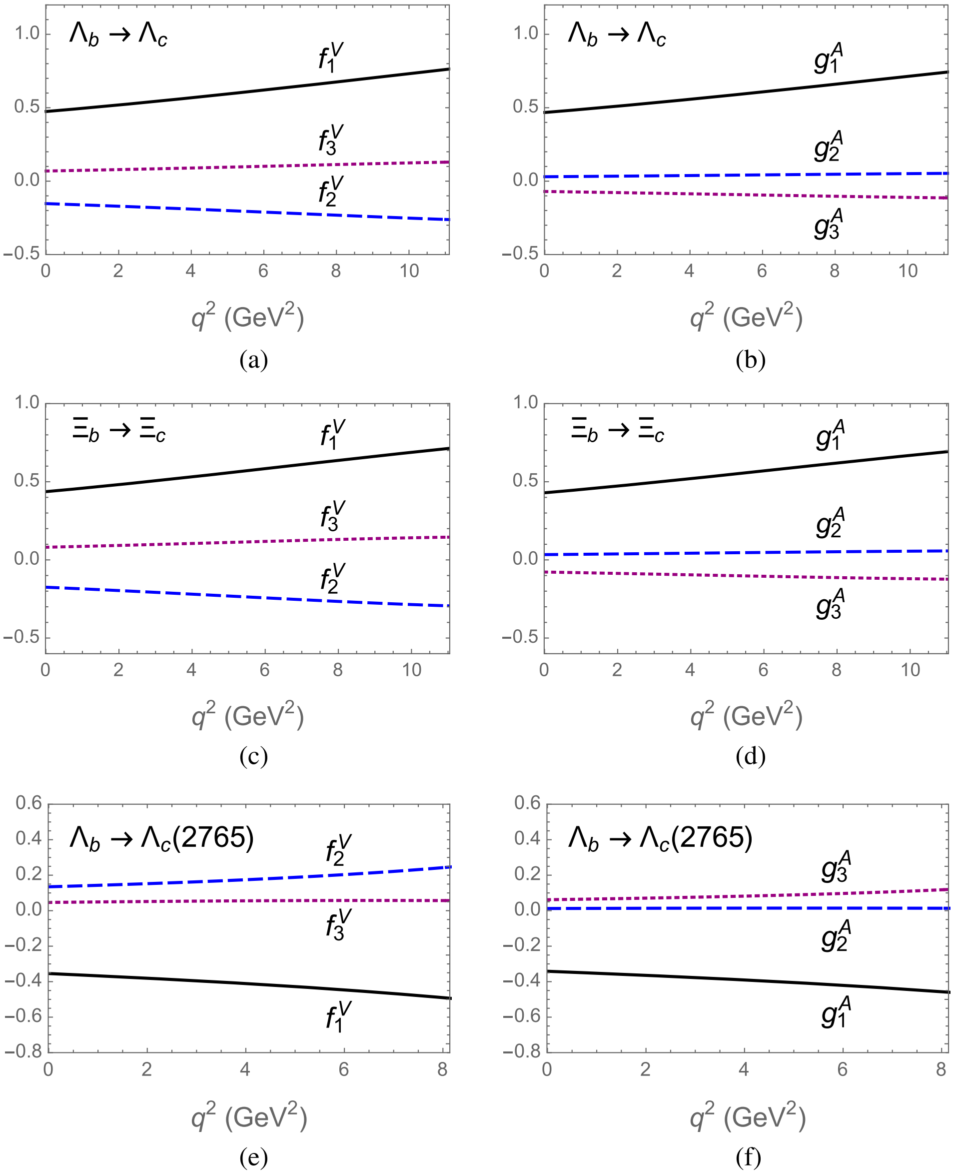

FIG. 2. Form factors $f_{1,2,3}(q^2)$ and $g_{1,2,3}(q^2)$ for $\Lambda_b \to \Lambda_c, \Lambda_c(2765)$ and $\Xi_b \to \Xi_c$ transitions. The transitions are $\mathcal{B}_b(\bar{\mathbf{3}}_{\mathbf{f}}, 1/2^+) \to \mathcal{B}_c(\bar{\mathbf{3}}_{\mathbf{f}}, 1/2^+)$ transitions [types (i) and (i)*].

TABLE VI. The transition form factors for various $\mathcal{B}_b(\mathbf{6}_{\mathbf{f}}, 1/2^+) \to \mathcal{B}_c(\mathbf{6}_{\mathbf{f}}, 1/2^+)$ transitions [types (ii) and (ii)*]. We employ a three parameter form for these form factors, Eq. (39), while for those with asterisks we employ Eq. (40).

$\mathcal{B}_b \to \mathcal{B}_c$	F	$F(0)$	$F(q^2_{\max})$	a	b	F	$F(0)$	$F(q^2_{\max})$	a	b
$\Omega_b \to \Omega_c$	f_1^{V*}	$0.292^{+0.062}_{-0.062}$	$0.605^{+0.128}_{-0.128}$	2.229	4.051	g_1^{A*}	$-0.097^{+0.021}_{-0.021}$	$-0.194^{+0.042}_{-0.042}$	1.421	1.723
	f_2^{V*}	$0.440^{+0.094}_{-0.093}$	$0.951^{+0.203}_{-0.201}$	2.027	3.082	g_2^{A*}	$-0.009^{+0.002}_{-0.002}$	$-0.018^{+0.004}_{-0.004}$	1.501	2.114
	f_3^{V*}	$-0.125^{+0.028}_{-0.026}$	$-0.246^{+0.055}_{-0.051}$	1.594	2.387	g_3^{A*}	$0.015^{+0.002}_{-0.003}$	$0.027^{+0.004}_{-0.005}$	1.230	1.895
$\Omega_b \to \Omega_c(3090)$	f_1^V	$-0.291^{+0.043}_{-0.043}$	$-0.483^{+0.071}_{-0.071}$	2.573	3.826	g_1^A	$0.097^{+0.013}_{-0.014}$	$0.151^{+0.02}_{-0.022}$	1.811	1.319
	f_2^V	$-0.433^{+0.065}_{-0.066}$	$-0.757^{+0.114}_{-0.115}$	2.467	2.836	g_2^A	$0.00^{+0.002}_{-0.002}$	$0.00^{+0.002}_{-0.002}$	1.044	1.006
	f_3^V	$0.159^{+0.033}_{-0.033}$	$0.259^{+0.054}_{-0.054}$	2.278	2.763	g_3^A	$-0.010^{+0.003}_{-0.003}$	$-0.017^{+0.005}_{-0.005}$	1.977	0.932

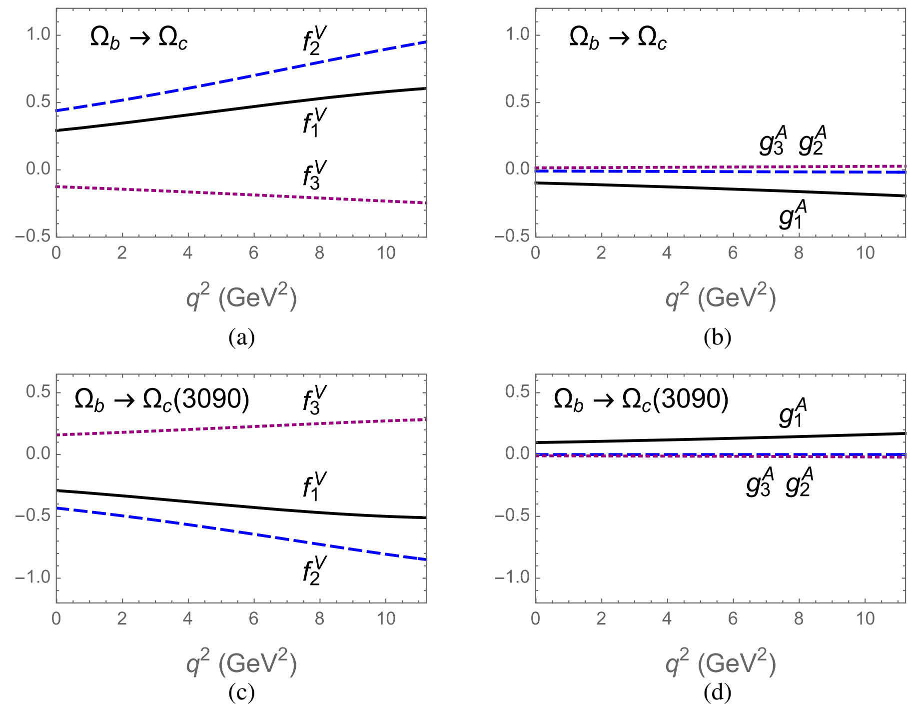

FIG. 3. Form factors $f_{1,2,3}(q^2)$ and $g_{1,2,3}(q^2)$ for $\Omega_b \to \Omega_c$ and $\Omega_c(3090)$ transitions. The transitions are $\mathcal{B}_b(\mathbf{6_f}, 1/2^+) \to \mathcal{B}_c(\mathbf{6_f}, 1/2^+)$ transitions [types (ii) and (ii)*].

form factors at $q^2_{\rm max}$ take the following values: $(f_1^V, f_2^V, f_3^V) = (1.23, 1.56, -0.60)$ and $(g_1^A, g_2^A, g_3^A) = (-0.33, 0, 0)$, see Eq. (34). From Table VI and Fig. 3(a), (b) we see that the form factors basically exhibit a similar pattern in sizes and signs, but the values of $|f_i^V|$ and $|g_1^A|$ are smaller by 40% to 60%, while the values of $g_{2,3}^A$ are closer to the HQS predicted values. The reductions can again be more or less traced to the smaller wave function overlapping integral, which is 0.46 in this case. In the $\Omega_b \to \Omega_c(3090)$ transition, we obtain form factors with similar sizes and signs flipped compared to the previous ones. These can also be more or less traced to the overlapping integral of the low lying Ω_b state and the radial excited $\Omega_c(3090)$ state. Explicitly, the corresponding overlapping integral is −0.46, which is nonvanishing (unlike the one in the ideal case), but has an opposite sign and a similar size compare to the one in the previous case.

The $\mathcal{B}_b(\mathbf{6_f}, 1/2^+) \to \mathcal{B}_c(\mathbf{6_f}, 3/2^+)$ transition form factors are related to the $\mathcal{B}_b(\mathbf{6_f}, 1/2^+) \to \mathcal{B}_c(\mathbf{6_f}, 1/2^+)$ ones in the heavy quark limit, see Eqs. (26) and (27). They are

TABLE VII. The transition form factors for various $\mathcal{B}_b(\mathbf{6_f}, 1/2^+) \to \mathcal{B}_c(\mathbf{6_f}, 3/2^\pm)$ transitions [types (iii), (iii)*, and (iv)]. We employ a three parameter form for these form factors, Eq. (39), while for those with asterisks we employ Eq. (40).

$\mathcal{B}_b \to \mathcal{B}_c$	F	$F(0)$	$F(q^2_{\rm max})$	a	b	F	$F(0)$	$F(q^2_{\rm max})$	a	b
$\Omega_b \to \Omega_c(2770)$	$\bar f_1^{V*}$	$-0.734^{+0.126}_{-0.127}$	$-1.270^{+0.218}_{-0.220}$	1.138	1.780	$\bar g_1^{A*}$	$0.526^{+0.079}_{-0.083}$	$1.023^{+0.153}_{-0.161}$	1.217	1.010
	$\bar f_2^{V*}$	$-0.300^{+0.063}_{-0.066}$	$-0.570^{+0.120}_{-0.125}$	1.633	2.614	$\bar g_2^{A*}$	$0.088^{+0.020}_{-0.019}$	$0.230^{+0.052}_{-0.049}$	2.137	1.960
	$\bar f_3^{V*}$	$0.340^{+0.069}_{-0.067}$	$0.573^{+0.117}_{-0.113}$	1.503	3.256	$\bar g_3^{A*}$	$-0.361^{+0.069}_{-0.072}$	$-0.838^{+0.160}_{-0.167}$	1.983	2.245
	$\bar f_4^{V*}$	$0.033^{+0.016}_{-0.012}$	$0.037^{+0.018}_{-0.014}$	0.108	3.272	$\bar g_4^{A*}$	$0.021^{+0.014}_{-0.013}$	$0.031^{+0.021}_{-0.019}$	1.233	3.716
$\Omega_b \to \Omega_c(3120)$	$\bar f_1^{V}$	$0.572^{+0.062}_{-0.053}$	$0.741^{+0.080}_{-0.068}$	1.250	1.184	$\bar g_1^{A}$	$-0.369^{+0.028}_{-0.032}$	$-0.487^{+0.037}_{-0.042}$	1.162	0.547
	$\bar f_2^{V}$	$0.270^{+0.039}_{-0.036}$	$0.403^{+0.058}_{-0.053}$	1.851	1.884	$\bar g_2^{A}$	$0.038^{+0.029}_{-0.028}$	$0.017^{+0.013}_{-0.012}$	4.054	40.596
	$\bar f_3^{V}$	$-0.263^{+0.028}_{-0.033}$	$-0.330^{+0.035}_{-0.041}$	1.445	2.477	$\bar g_3^{A}$	$0.163^{+0.021}_{-0.022}$	$0.174^{+0.022}_{-0.023}$	0.639	1.531
	$\bar f_4^{V}$	$0.010^{+0.016}_{-0.015}$	$0.013^{+0.02}_{-0.018}$	1.011	1.000	$\bar g_4^{A}$	$-0.106^{+0.034}_{-0.035}$	$-0.151^{+0.048}_{-0.05}$	2.565	5.564
$\Omega_b \to \Omega_c(3050)$	$\bar f_1^{A}$	$-0.667^{+0.115}_{-0.096}$	$-1.164^{+0.200}_{-0.167}$	2.007	1.089	$\bar g_1^{V}$	$0.163^{+0.024}_{-0.026}$	$0.127^{+0.019}_{-0.020}$	−0.463	2.900
	$\bar f_2^{A}$	$-0.376^{+0.077}_{-0.075}$	$-0.574^{+0.118}_{-0.115}$	2.122	2.913	$\bar g_2^{V*}$	$-0.147^{+0.048}_{-0.055}$	$-0.541^{+0.176}_{-0.202}$	3.160	2.247
	$\bar f_3^{A}$	$0.300^{+0.042}_{-0.049}$	$0.405^{+0.057}_{-0.066}$	2.034	3.976	$\bar g_3^{V}$	$0.335^{+0.120}_{-0.109}$	$0.255^{+0.092}_{-0.083}$	−1.021	0.997
	$\bar f_4^{A*}$	$0.047^{+0.018}_{-0.013}$	$0.157^{+0.060}_{-0.045}$	2.785	1.298	$\bar g_4^{V*}$	$0.426^{+0.115}_{-0.110}$	$0.774^{+0.208}_{-0.200}$	1.813	2.902

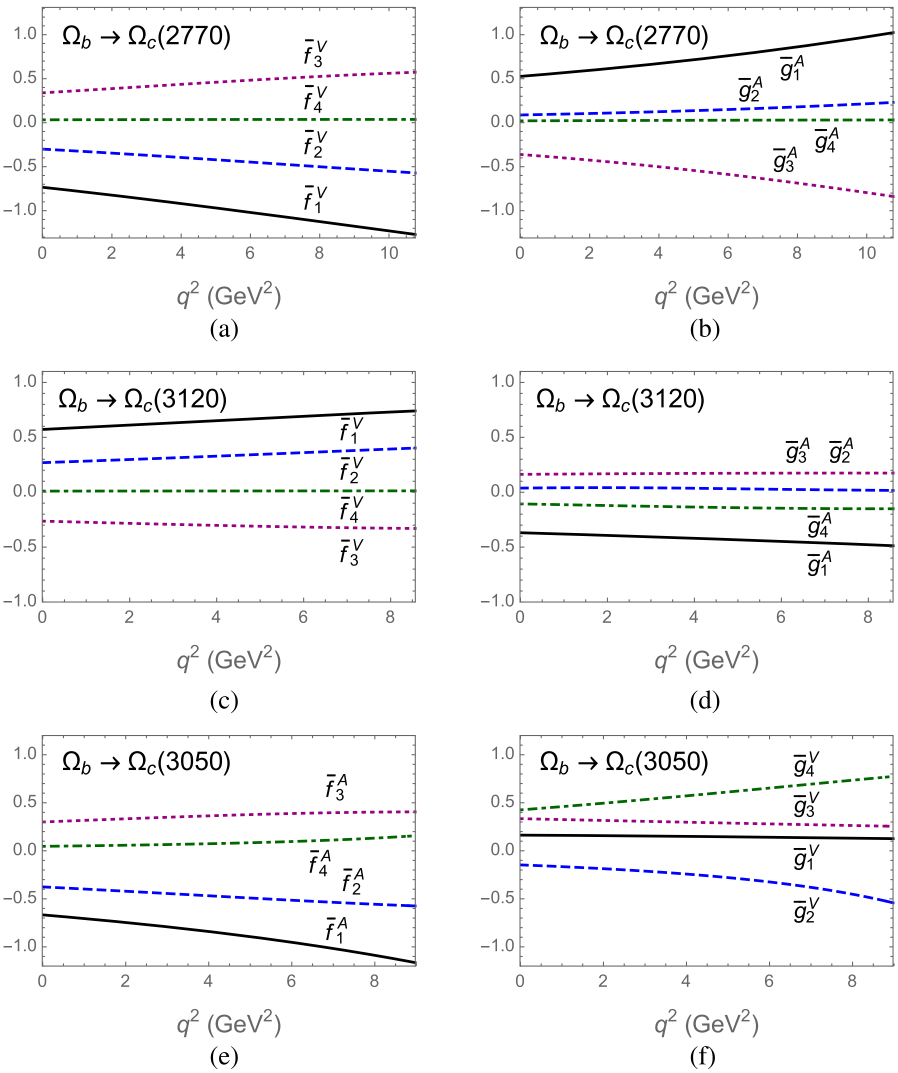

FIG. 4. Form factors $\bar f_i^{V,A}(q^2)$ and $\bar g_i^{A,V}(q^2)$ for $\Omega_b \to \Omega_c(2770)$, $\Omega_c(3120)$, and $\Omega_c(3050)$ transitions. The transitions are $\mathcal{B}_b(\mathbf{6_f}, 1/2^+) \to \mathcal{B}_c(\mathbf{6_f}, 3/2^+)$ transitions [types (iii) and (iii)*] and $\mathcal{B}_b(\mathbf{6_f}, 1/2^+) \to \mathcal{B}_c(\mathbf{6_f}, 3/2^-)$ transition [type (iv)].

governed by the same set of Isgur-Wise functions, namely $\xi_{1,2}$. In Table VII and Fig. 4, we show the form factors of $\mathcal{B}_b(\mathbf{6_f}, 1/2^+) \to \mathcal{B}_c(\mathbf{6_f}, 3/2^+)$ transitions. These involve $\Omega_b \to \Omega_c(2770)$ and $\Omega_c(3120)$ transitions, where $\Omega_c(2770)$ is a low lying state, while $\Omega_c(3120)$ a radial excited state. In the $\Omega_b \to \Omega_c(2770)$ and $\Omega_b \to \Omega_c(3120)$ transitions, as shown in the table and the figure, we have $\bar f_1^V(q^2) \simeq -\bar g_1^A(q^2)$, $\bar f_3^V(q^2) \simeq -\bar g_3^A(q^2) \simeq -\bar f_2^V(q^2)$, and $\bar f_4(q^2), \bar g_{2,4}^A(q^2)$ much smaller than other form factors, in accordance with the HQS relations using large N_c QCD, see Eqs. (27) and (32). The agreement is better in the $\Omega_b \to \Omega_c(2770)$ transition than in the $\Omega_b \to \Omega_c(3120)$ transition. Heavy quark symmetry and large N_c QCD predict the form factors in the $\Omega_b \to \Omega_c(2770)$ transition have the following values at $q^2_{\rm max}$: $(\bar f_1^V, \bar f_2^V, \bar f_3^V, \bar f_4^V) = (-1.15, -0.58, 0.58, 0)$ and $(\bar g_1^A, \bar g_2^A, \bar g_3^A, \bar g_4^A) = (1.15, 0, -0.58, 0)$, see Eq. (35). From Table VII, we see that $\bar f_{1,2,3}^V$ agree with the above predictions within 10%, $\bar g_{1,3}^A$ within 20% and 45% and $\bar f_4^V, \bar g_4^A$ are close to zero, the predicted values, while $\bar g_2^A$ is much smaller than $|\bar g_{1,3}^A|$ and closer to $\bar g_4^A$. As the overlapping integral is 0.56, the agreements in $\bar f_i^V$ and $\bar g_i^A$ are better than expected. In the $\Omega_b \to \Omega_c(3120)$ transition, most of the form factors are smaller in sizes and opposite in signs compared to the corresponding form factors in the previous transition. This is understandable as $\Omega_c(3120)$ is a radial excited state, the corresponding overlapping integral is -0.51, which has opposite sign compare to the previous one.

In Table VII and Fig. 4 we also show the form factors of the $\mathcal{B}_b(\mathbf{6_f}, 1/2^+) \to \mathcal{B}_c(\mathbf{6_f}, 3/2^-)$ transition, namely the $\Omega_b \to \Omega_c(3050)$ transition, where $\Omega_c(3050)$ is a p-wave state. The signs of $\bar f_{1,2}^V$ and $\bar g_{2,3,4}^A$ at $q^2_{\rm max}$ are in agreement with the HQS predictions, Eq. (36). If we naïvely make use of Eq. (36), we obtain $\xi_5(1) \simeq 1.4^{+0.2}_{-0.7}$ and $\xi_6(1) \simeq 0.7$ from these form factors, except from $\bar g_2^A$ we have $\xi_5(1) \simeq 3.0$.

TABLE VIII. The transition form factors for various $\mathcal{B}_b(\mathbf{3_f}, 1/2^+) \to \mathcal{B}_c(\bar{\mathbf{3}}_\mathbf{f}, 1/2^-)$ transitions [type (v) and (v)*].

$\mathcal{B}_b \to \mathcal{B}_c$	F	$F(0)$	$F(q^2_{\max})$	a	b	F	$F(0)$	$F(q^2_{\max})$	a	b
$\Lambda_b \to \Lambda_c(2595)$	f_1^A	$0.286^{+0.050}_{-0.051}$	$0.338^{+0.059}_{-0.060}$	0.667	0.483	g_1^V	$0.238^{+0.048}_{-0.049}$	$0.232^{+0.047}_{-0.048}$	0.120	0.722
	f_2^A	$-0.313^{+0.079}_{-0.081}$	$-0.439^{+0.111}_{-0.114}$	1.312	1.105	g_2^V	$-0.080^{+0.020}_{-0.022}$	$-0.112^{+0.028}_{-0.031}$	1.416	1.510
	f_3^A	$-0.299^{+0.080}_{-0.083}$	$-0.459^{+0.123}_{-0.128}$	1.722	1.791	g_3^V	$-0.167^{+0.039}_{-0.038}$	$-0.228^{+0.053}_{-0.052}$	1.286	1.234
$\Xi_b \to \Xi_c^0(2790)$	f_1^A	$0.269^{+0.056}_{-0.046}$	$0.330^{+0.068}_{-0.056}$	0.895	0.755	g_1^V	$0.221^{+0.046}_{-0.046}$	$0.221^{+0.046}_{-0.046}$	0.254	0.959
	f_2^A	$-0.319^{+0.083}_{-0.086}$	$-0.447^{+0.116}_{-0.121}$	1.487	1.544	g_2^V	$-0.071^{+0.020}_{-0.021}$	$-0.099^{+0.028}_{-0.029}$	1.619	2.187
	f_3^A	$-0.289^{+0.081}_{-0.084}$	$-0.436^{+0.122}_{-0.127}$	1.944	2.580	g_3^V	$-0.169^{+0.042}_{-0.040}$	$-0.228^{+0.057}_{-0.054}$	1.404	1.663
$\Lambda_b \to \Lambda_c(2940)$	f_1^A	$-0.255^{+0.019}_{-0.010}$	$-0.333^{+0.025}_{-0.013}$	0.996	−0.156	g_1^V	$-0.212^{+0.017}_{-0.009}$	$-0.241^{+0.019}_{-0.010}$	0.533	0.034
	f_2^A	$0.338^{+0.064}_{-0.066}$	$0.584^{+0.111}_{-0.114}$	1.997	0.642	g_2^V	$0.081^{+0.010}_{-0.013}$	$0.138^{+0.017}_{-0.022}$	2.029	0.839
	f_3^A	$0.331^{+0.071}_{-0.071}$	$0.592^{+0.127}_{-0.127}$	2.176	1.031	g_3^V	$0.157^{+0.009}_{-0.015}$	$0.301^{+0.017}_{-0.029}$	2.324	0.954

The $\mathcal{B}_b(\bar{\mathbf{3}}_\mathbf{f}, 1/2^+) \to \mathcal{B}_c(\bar{\mathbf{3}}_\mathbf{f}, 1/2^-)$ transition form factors $f^A_{1,2,3}(q^2)$ and $g^V_{1,2,3}(q^2)$ for $\Lambda_b \to \Lambda_c(2595)$, $\Lambda_c(2940)$ and $\Xi_b \to \Xi_c(2790)$ transitions are given in Table VIII and they are plotted in Fig. 5. We consider $\Lambda_c(2940)$ as a spin-1/2 p-wave radial excited state [6] in this case. HQS requires $f_1^A(q^2) = g_1^V(q^2)$, $f_2^A(q^2) = f_3^A(q^2)$ and

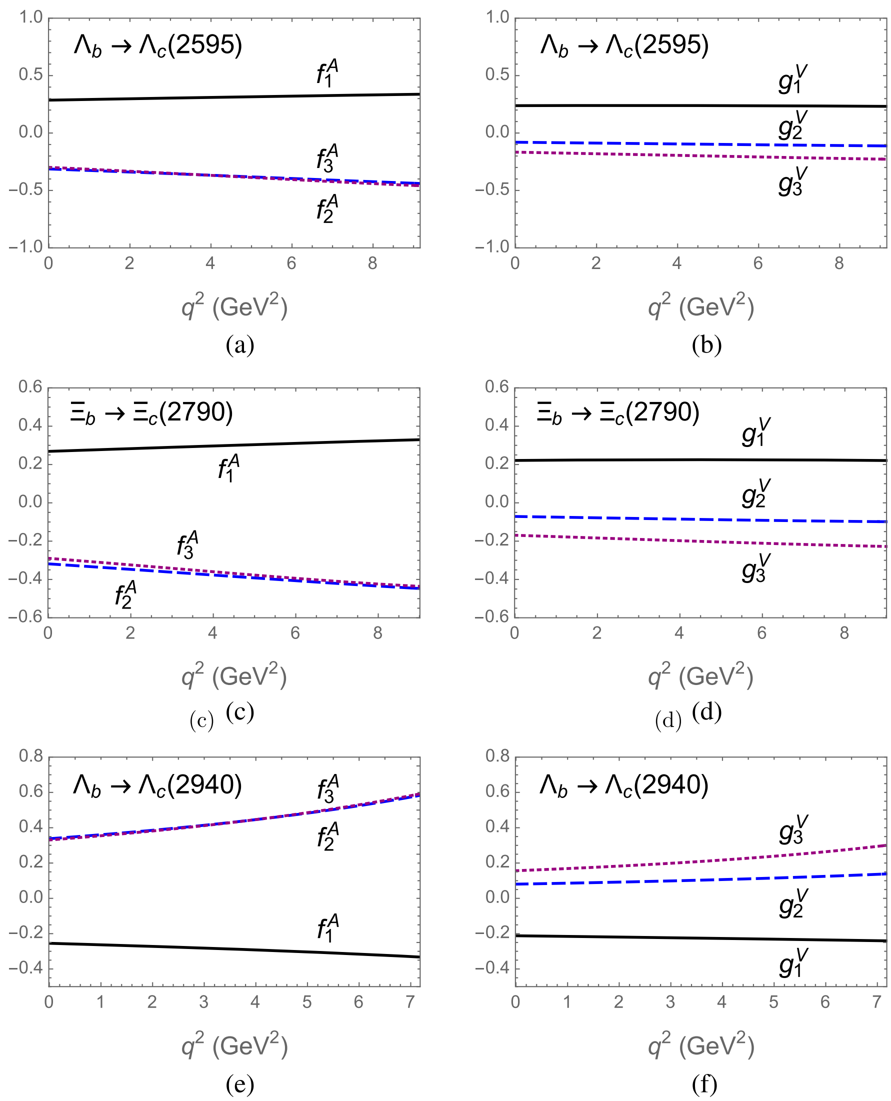

FIG. 5. Form factors $f_{1,2,3}(q^2)$ and $g_{1,2,3}(q^2)$ for $\Lambda_b \to \Lambda_c(2595)$, $\Lambda_c(2940)$, and $\Xi_b \to \Xi_c(2790)$ transitions. The transitions are $\mathcal{B}_b(\bar{\mathbf{3}}_\mathbf{f}, 1/2^+) \to \mathcal{B}_c(\bar{\mathbf{3}}_\mathbf{f}, 1/2^-)$ transitions [types (v) and (v)*].

TABLE IX. The transition form factors for various $\mathcal{B}_b(\bar{\mathbf{3}}_\mathbf{f}, 1/2^+) \to \mathcal{B}_c(\bar{\mathbf{3}}_\mathbf{f}, 3/2^-)$ transitions [type (vi)]. Note that the a and b for form factors denoted with asterisks are assumed, the corresponding form factors are small.

$\mathcal{B}_b \to \mathcal{B}_c$	F	$F(0)$	$F(q^2_{\rm max})$	a	b	F	$F(0)$	$F(q^2_{\rm max})$	a	b
$\Lambda_b \to \Lambda_c(2625)$	$\bar f_1^A$	$0.028^{+0.065}_{-0.032}$	$0.035^{+0.082}_{-0.040}$	1*	1*	$\bar g_1^V$	$-0.007^{+0.037}_{-0.026}$	$-0.009^{+0.046}_{-0.033}$	1*	1*
	$\bar f_2^A$	$0.545^{+0.111}_{-0.104}$	$0.756^{+0.154}_{-0.144}$	1.310	1.154	$\bar g_2^V$	$0.509^{+0.184}_{-0.173}$	$0.737^{+0.267}_{-0.251}$	1.388	1.043
	$\bar f_3^A$	$0.022^{+0.033}_{-0.091}$	$0.027^{+0.041}_{-0.114}$	1*	1*	$\bar g_3^V$	$0.088^{+0.039}_{-0.043}$	$0.115^{+0.051}_{-0.056}$	2.022	4.221
	$\bar f_4^A$	$-0.005^{+0.104}_{-0.068}$	$-0.006^{+0.131}_{-0.086}$	1*	1*	$\bar g_4^V$	$0.004^{+0.058}_{-0.053}$	$0.005^{+0.072}_{-0.066}$	1*	1*
$\Xi_b \to \Xi_c(2815)$	$\bar f_1^A$	$0.049^{+0.094}_{-0.048}$	$0.033^{+0.064}_{-0.033}$	−1.327	1.830	$\bar g_1^{V*}$	$0.015^{+0.046}_{-0.029}$	$0.018^{+0.057}_{-0.036}$	1*	1*
	$\bar f_2^A$	$0.675^{+0.128}_{-0.122}$	$0.987^{+0.187}_{-0.178}$	1.526	1.245	$\bar g_2^V$	$0.693^{+0.247}_{-0.216}$	$1.024^{+0.365}_{-0.320}$	1.557	1.249
	$\bar f_3^A$	$0.020^{+0.045}_{-0.127}$	$0.024^{+0.056}_{-0.158}$	1*	1*	$\bar g_3^V$	$0.055^{+0.050}_{-0.075}$	$0.068^{+0.062}_{-0.093}$	1*	1*
	$\bar f_4^A$	$-0.013^{+0.173}_{-0.138}$	$-0.017^{+0.215}_{-0.171}$	1*	1*	$\bar g_4^V$	$-0.020^{+0.083}_{-0.083}$	$-0.024^{+0.103}_{-0.103}$	1*	1*
$\Lambda_b \to \Lambda_c(2940)$	$\bar f_1^A$	$-0.036^{+0.053}_{-0.047}$	$-0.043^{+0.065}_{-0.057}$	1*	1*	$\bar g_1^{V*}$	$-0.061^{+0.013}_{-0.006}$	$-0.074^{+0.016}_{-0.007}$	1*	1*
	$\bar f_2^A$	$-0.459^{+0.067}_{-0.067}$	$-1.049^{+0.153}_{-0.153}$	2.400	−0.326	$\bar g_2^V$	$-0.820^{+0.192}_{-0.155}$	$-1.375^{+0.321}_{-0.260}$	1.957	0.793
	$\bar f_3^{A*}$	$-0.012^{+0.148}_{-0.098}$	$-0.004^{+0.050}_{-0.033}$	−8.054	18.712	$\bar g_3^V$	$0.198^{+0.080}_{-0.084}$	$0.358^{+0.145}_{-0.152}$	2.487	2.307
	$\bar f_4^{A*}$	$0.192^{+0.133}_{-0.120}$	$0.139^{+0.096}_{-0.087}$	−1.791	7.419	$\bar g_4^V$	$0.170^{+0.102}_{-0.095}$	$0.183^{+0.110}_{-0.102}$	0.924	2.713

$g_2^V(q^2) = g_3^V(q^2)$, see Eq. (29). From the table and the figures, we see that the form factors roughly exhibit this pattern. By naïvely compared to the HQ relations at $q^2_{\rm max}$, Eq. (37), we obtain $\sigma(1) \simeq 0.7^{+0.5}_{-0.3}$ in the $\Lambda_b \to \Lambda_c(2595)$ transition and $\sigma(1) \simeq 0.7^{+0.4}_{-0.3}$ in the $\Xi_b \to \Xi_c(2790)$ transition, while for the transition involving radial excite state, $\Lambda_b \to \Lambda_c(2940)$ transition, we have $\sigma^{(*)}(1) \simeq -0.8^{+0.3}_{-0.4}$. The sign flip in $\Lambda_b \to \Lambda_c(2940)$ transition can again be traced to the overlapping integral, whose value is −0.53, while the one in the $\Lambda_b \to \Lambda_c(2595)$ transition is 0.67. Roughly speaking these form factors exhibit the pattern predicted by HQS.

We now turn to the last case, the $\mathcal{B}_b(\bar{\mathbf{3}}_f, 1/2^+) \to \mathcal{B}_c(\bar{\mathbf{3}}_f, 3/2^-)$ transition, which includes $\Lambda_b \to \Lambda_c(2625)$, $\Xi_b \to \Xi_c(2815)$ and $\Lambda_b \to \Lambda_c(2940)$ transitions. We consider $\Lambda_c(2940)$ as a spin-3/2 p-wave radial excited state [1] in this case. The corresponding form factors for these transitions are shown in Table IX and are plotted in Fig. 6. Note that for the form factors denoted with asterisks, the parameters a and b are assumed to be one, while the values of the form factors at $q^2 = 0$ are obtained by using Eq. (23). These form factors are small and the assumptions on a and b should not have much impact on decay rates. HQS requires $\bar f_2^A = \bar g_2^V = \sigma(\omega)$, while all other form factors are vanishing, see Eq. (30). We see from the table and the plots that these form factors roughly exhibit the pattern predicted by HQS. By naïvely compared to the HQ relations at $q^2_{\rm max}$, Eq. (38), we obtain $\sigma(1) \simeq 0.7$, 0.8 in the $\Lambda_b \to \Lambda_c(2625)$ transition and $\sigma(1) \simeq 1$ in the $\Xi_b \to \Xi_c(2815)$ transition, while for the transition involving radial excite state, $\Lambda_b \to \Lambda_c(2940)$ transition, we have $\sigma^{(*)}(1) \simeq -1.0, -1.4$. These $\sigma(1)$ and $\sigma^{(*)}(1)$ are similar to those obtained in $\mathcal{B}_b(\bar{\mathbf{3}}_\mathbf{f}, 1/2^+) \to \mathcal{B}_c(\bar{\mathbf{3}}_\mathbf{f}, 1/2^-)$ transitions.

B. $\mathcal{B}_b \to \mathcal{B}_c M$ decay rates and up-down asymmetries

We will present the results of $\mathcal{B}_b \to \mathcal{B}_c M$ decay rates and up-down asymmetries including $1/2 \to 1/2$ and $1/2 \to 3/2$ transitions in this subsection. The decay rates and asymmetries for $1/2 \to 1/2$ transitions are updated from those in [16], as we are using different input parameters and more form factors.[4] Under the factorization approximation, the decay amplitudes for color-allowed $\mathcal{B}_b \to \mathcal{B}_c M^-$ decays are given by

$$\mathcal{A}(\mathcal{B}_b \to \mathcal{B}_c M^-) = \frac{G_F}{\sqrt{2}} V_{cb} V^*_{ij} a_1 \langle M^-(\bar q_i q_j)|V^\mu - A^\mu|0\rangle \langle \mathcal{B}_c|V_\mu - A_\mu|\mathcal{B}_b\rangle, \tag{41}$$

where V_{cb} and V_{ij} are the Cabibbo-Kobayashi-Maskawa matrix elements and a_1 is the color-allowed effective Wilson coefficient. The effective Wilson coefficient a_1 in the naïve factorization approximation is given by $c_1 + c_2/N_c$ with $N_c = 3$, $c_1 = 1.081$ and $c_2 = -0.190$ at the scale of $\mu = 4.2$ GeV [55]. The matrix element $\langle \mathcal{B}_c|V_\mu - A_\mu|\mathcal{B}_b\rangle$ is given by Eqs. (1)–(4), while $\langle M^-(\bar q_i q_j)|V^\mu - A^\mu|0\rangle$ are given by

$$\langle P|V^\mu - A^\mu|0\rangle = i q^\mu f_P, \qquad \langle V|V^\mu - A^\mu|0\rangle = m_V f_V \varepsilon^*_V,$$
$$\langle A|V^\mu - A^\mu|0\rangle = -m_A f_A \varepsilon^*_A, \tag{42}$$

with $f_{P,V,A}$ the corresponding meson decay constants.

In type (i) and (ii) transitions [$\mathcal{B}_b(\bar{\mathbf{3}}_\mathbf{f}, 1/2^+) \to \mathcal{B}_c(\bar{\mathbf{3}}_\mathbf{f}, 1/2^+)$ and $\mathcal{B}_b(\mathbf{6}_\mathbf{f}, 1/2^+) \to \mathcal{B}_c(\mathbf{6}_\mathbf{f}, 1/2^+)$ transitions], the decay amplitudes are given by [22]

[4] Note that errors in the code in [16] are also corrected.

$$\begin{aligned}
&\mathcal{A}[\mathcal{B}_b\to\mathcal{B}_c(1/2)P]=i\bar{u}'(A+B\gamma_5)u,\\
&\mathcal{A}[\mathcal{B}_b\to\mathcal{B}_c(1/2)V]\\
&\quad=\bar{u}'\varepsilon^{*\mu}(A_1\gamma_\mu\gamma_5+A_2P'_\mu\gamma_5+B_1\gamma_\mu+B_2P'_\mu)u,\\
&\mathcal{A}[\mathcal{B}_b\to\mathcal{B}_cA]\\
&\quad=\bar{u}'\varepsilon^{*\mu}(A'_1\gamma_\mu\gamma_5+A'_2P'_\mu\gamma_5+B'_1\gamma_\mu+B'_2P'_\mu)u, \qquad (43)
\end{aligned}$$

with

$$\begin{aligned}
A&=\frac{G_f}{\sqrt{2}}V_{cb}V^*_{q_1q_2}a_1f_P(M-M')\\
&\quad\times\left(f_1^V(m_P^2)+\frac{m_P^2}{M^2-M'^2}f_3^V(m_P^2)\right),\\
B&=\frac{G_f}{\sqrt{2}}V_{cb}V^*_{q_1q_2}a_1f_P(M+M')\\
&\quad\times\left(g_1^A(m_P^2)+\frac{m_P^2}{M^2-M'^2}g_3^A(m_P^2)\right),\\
A_1&=-\frac{G_f}{\sqrt{2}}V_{cb}V^*_{q_1q_2}a_1f_Vm_V[g_1^A(m_V^2)+g_2^A(m_V^2)],\\
A_2&=-2\frac{G_f}{\sqrt{2}}V_{cb}V^*_{q_1q_2}a_1f_Vm_V\frac{g_2^A(m_V^2)}{M-M'},\\
B_1&=\frac{G_f}{\sqrt{2}}V_{cb}V^*_{q_1q_2}a_1f_Vm_V[f_1^V(m_V^2)-f_2^V(m_V^2)],\\
B_2&=2\frac{G_f}{\sqrt{2}}V_{cb}V^*_{q_1q_2}a_1f_Vm_V\frac{f_2^V(m_V^2)}{M+M'},\\
A'_1&=\frac{G_f}{\sqrt{2}}V_{cb}V^*_{q_1q_2}a_1f_Am_A[g_1^A(m_A^2)+g_2^A(m_A^2)],\\
A'_2&=2\frac{G_f}{\sqrt{2}}V_{cb}V^*_{q_1q_2}a_1f_Am_A\frac{g_2^A(m_V^2)}{M-M'},\\
B'_1&=-\frac{G_f}{\sqrt{2}}V_{cb}V^*_{q_1q_2}a_1f_Am_A[f_1^V(m_A^2)-f_2^V(m_A^2)],\\
B'_2&=-2\frac{G_f}{\sqrt{2}}V_{cb}V^*_{q_1q_2}a_1f_Am_A\frac{f_2^V(m_A^2)}{M+M'}. \qquad (44)
\end{aligned}$$

For the type (v) transition $[\mathcal{B}_b(\bar{\mathbf{3}}_\mathbf{f},1/2^+)\to\mathcal{B}_c(\bar{\mathbf{3}}_\mathbf{f},1/2^-)$ transition], one simply replaces f_i^V and g_i^A in the above equations by $-f_i^A$ and $-g_i^V$, respectively.

In $\mathcal{B}_b(\mathbf{6}_\mathbf{f},1/2^+)\to\mathcal{B}_c(\mathbf{6}_\mathbf{f},3/2^+)$ transitions [type (iii) transitions], the $\mathcal{B}_b\to\mathcal{B}_cP$ and $\mathcal{B}_b\to\mathcal{B}_cV(A)$ decay amplitudes are given by [22]

$$\begin{aligned}
&\mathcal{A}[\mathcal{B}_b\to\mathcal{B}_c(3/2)P]=iq_\mu\bar{u}'^\mu(P')(C+D\gamma_5)u(P),\\
&\mathcal{A}[\mathcal{B}_b\to\mathcal{B}_c(3/2)V]\\
&\quad=\varepsilon^{*\mu}\bar{u}'^\nu(P')[g_{\nu\mu}(C_1+C_2\gamma_5)+q_\nu\gamma_\mu(C_2+D_2\gamma_5)\\
&\qquad+q_\nu P_\mu(C_3+D_3\gamma_5)]u(P),\\
&\mathcal{A}[\mathcal{B}_b\to\mathcal{B}_c(3/2)A]=\varepsilon^{*\mu}\bar{u}'^\nu(P')[g_{\nu\mu}(C'_1+C'_2\gamma_5)\\
&\qquad+q_\nu\gamma_\mu(C'_2+D'_2\gamma_5)+q_\nu P_\mu(C'_3+D'_3\gamma_5)]u(P), \qquad (45)
\end{aligned}$$

with

$$\begin{aligned}
C&=-\frac{G_f}{\sqrt{2}}V_{cb}V^*_{q_1q_2}a_1f_P\bigg[g_1^A(m_P^2)+(M-M')\frac{g_2^A(m_P^2)}{M}\\
&\quad+\frac{1}{2}(M^2-M'^2-m_P^2)\left(\frac{g_3^A(m_P^2)}{MM'}+\frac{g_4^A(m_P^2)}{M^2}\right)\\
&\quad-m_P^2\frac{g_3^A(m_P^2)}{MM'}\bigg],\\
D&=\frac{G_f}{\sqrt{2}}V_{cb}V^*_{q_1q_2}a_1f_P\bigg[f_1^V(m_P^2)-(M+M')\frac{f_2^V(m_P^2)}{M}\\
&\quad+\frac{1}{2}(M^2-M'^2-m_P^2)\left(\frac{f_3^V(m_P^2)}{MM'}+\frac{f_4^V(m_P^2)}{M^2}\right)\\
&\quad-m_P^2\frac{f_3^V(m_P^2)}{MM'}\bigg],\\
C_1^{(\prime)}&=\mp\frac{G_f}{\sqrt{2}}V_{cb}V^*_{q_1q_2}a_1m_{V(A)}f_{V(A)}\bar{g}_1^A(m^2_{V(A)}),\\
D_1^{(\prime)}&=\pm\frac{G_f}{\sqrt{2}}V_{cb}V^*_{q_1q_2}a_1m_{V(A)}f_{V(A)}\bar{f}_1^V(m^2_{V(A)}),\\
C_2^{(\prime)}&=\mp\frac{G_f}{\sqrt{2}}V_{cb}V^*_{q_1q_2}a_1m_{V(A)}f_{V(A)}\frac{\bar{g}_2^A(m^2_{V(A)})}{M},\\
D_2^{(\prime)}&=\pm\frac{G_f}{\sqrt{2}}V_{cb}V^*_{q_1q_2}a_1m_{V(A)}f_{V(A)}\frac{\bar{f}_2^V(m^2_{V(A)})}{M},\\
C_3^{(\prime)}&=\mp\frac{G_f}{\sqrt{2}}V_{cb}V^*_{q_1q_2}a_1m_{V(A)}f_{V(A)}\\
&\quad\times\left(\frac{\bar{g}_3^A(m^2_{V(A)})}{MM'}+\frac{\bar{g}_4^A(m^2_{V(A)})}{M^2}\right),\\
D_3^{(\prime)}&=\pm\frac{G_f}{\sqrt{2}}V_{cb}V^*_{q_1q_2}a_1m_{V(A)}f_{V(A)}\\
&\quad\times\left(\frac{\bar{f}_3^V(m^2_{V(A)})}{MM'}+\frac{\bar{f}_4^V(m^2_{V(A)})}{M^2}\right). \qquad (46)
\end{aligned}$$

For the $\mathcal{B}_b(\mathbf{6}_\mathbf{f},1/2^+)\to\mathcal{B}_c(\mathbf{6}_\mathbf{f},3/2^-)$ and $\mathcal{B}_b(\bar{\mathbf{3}}_\mathbf{f},1/2^+)\to\mathcal{B}_c(\bar{\mathbf{3}}_\mathbf{f},3/2^-)$ transitions [type (iv) and (vi) transitions], one simply replaces $\bar{f}_i^V$ and $\bar{g}_i^A$ in the above equations by $-\bar{f}_i^A$ and $-\bar{g}_i^V$, respectively. The formulas of decay rates and the up-down asymmetries are collected in Appendix C.

In our numerical study masses and life-times of all hadron are taken from PDG [4], while the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements are taken from the latest fitting results of the CKM fitter group [56]. We use [57]

$$f_\pi=130.2,\quad f_K=155.6,\quad f_D=211.9,\quad f_{D_s}=249.0, \qquad (47)$$

and [42]

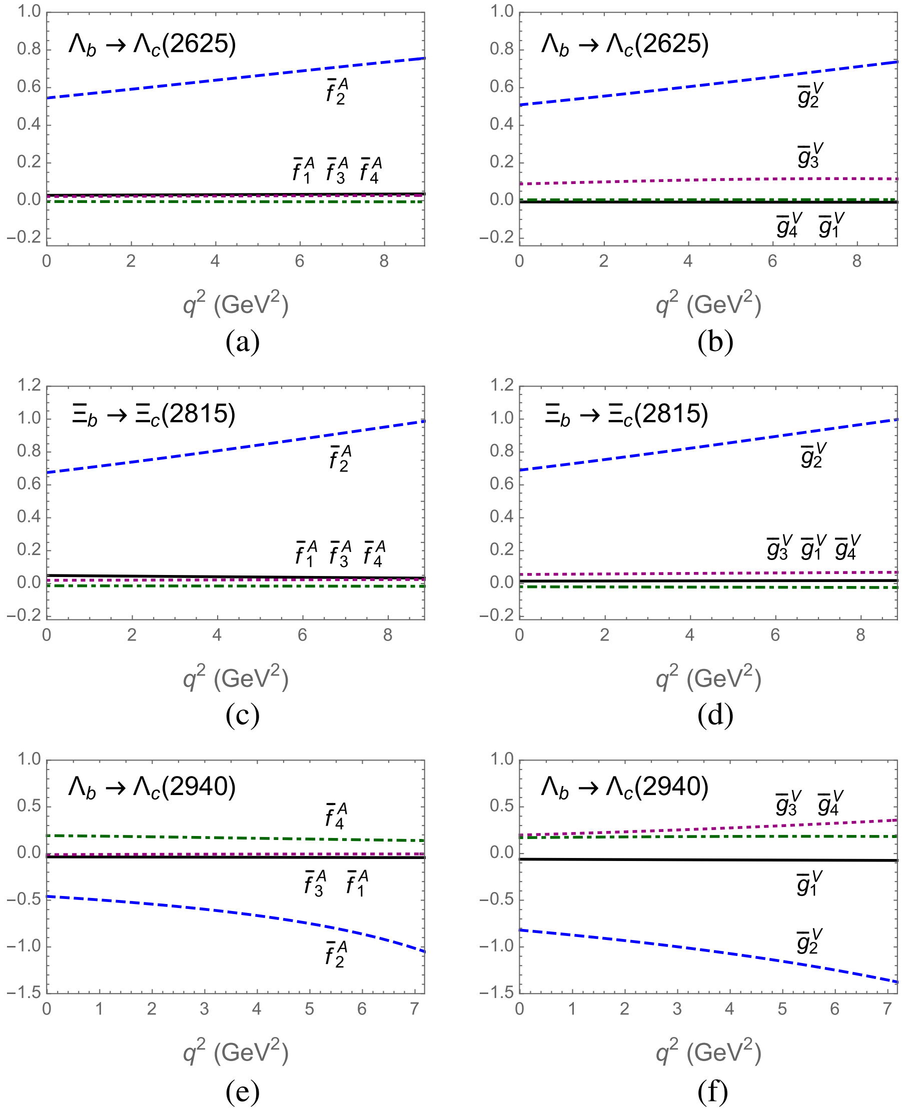

FIG. 6. Form factors $f_i^A(q^2)$ and $g_i^A(q^2)$ for $\Lambda_b \to \Lambda_c(2625)$, $\Lambda_c(2940)$, and $\Xi_b \to \Xi_c(2815)$ transitions. The transitions are $\mathcal{B}_b(\bar{\mathbf{3}}_\mathbf{f}, 1/2^+) \to \mathcal{B}_c(\bar{\mathbf{3}}_\mathbf{f}, 3/2^-)$ transitions [type (vi)].

$$f_\rho = 216, \qquad f_{K^*} = 210, \qquad f_{D^*} = 220,$$
$$f_{D_s^*} = 230, \qquad f_{K^*} = -203, \tag{48}$$

for the values (in unit of MeV) of decay constants of pseudoscalars, vectors, and the axial-vector mesons.

The decay rates are calculated using the naïve factorization approach. We assign 10% uncertainty in the effective Wilson coefficient a_1 for estimations and uncertainties in form factors shown in the previous subsection will be used. Some studies using QCD factorization on B meson and bottom baryon decays indicated that the effective Wilson coefficients a_1 in those decays are close to the one in naïve factorization [33,58] The authors in Ref. [58] obtained the $|a_1(DP)|$ agrees with the naïve factorization value within few % indicating that for color allowed modes naïve factorization is a good approximation.[5] A recent study of applying QCD factorization to Λ_b decays also shows a similar conclusion [33]. However, it has been shown that nonfactorizable contributions to $\mathcal{B}_b \to \mathcal{B}_c P$ nonleptonic decay amplitudes can be as large as 30% of the factorized ones [23,24]. Since a precise estimation of nonfactorization contributions is beyond the scope of the present work, we should stick to the naïve factorization approximation.

Note that in the cases of $\mathcal{B}_b \to D_{(s)}^{(*)} \mathcal{B}_c$ decays, penguin terms from $b \to d(s)\bar{c}c$ decays can contribute to the amplitudes. The most dominant penguin contributions to rates are the so-called strong penguin contributions, with

[5] They obtained $|a_1(\bar{B} \to DP)| = 1.055^{+0.019}_{-0.017} - (0.013^{+0.011}_{-0.006})\alpha_1^P$ with $\alpha_1^\pi = 0$ and $|\alpha_1^K| < 1$. This is to be compared with the naïve factorization value, $a_1^{\rm LO} = 1.025$.

TABLE X. Branching ratios (in the unit of 10^{-3}) of $\mathcal{B}_b \to \mathcal{B}_c P$ decays. Note that the asterisks denote the transitions where the final state charmed baryons are radial excited.

Transition type	Mode	$P=\pi^-$	$P=K^-$	$P=D^-$	$P=D_s^-$
(i) $(\frac{1}{2}^+ \to \frac{1}{2}^+)$	$\mathrm{Br}(\Lambda_b \to \Lambda_c P)$	$4.16^{+2.43}_{-1.73}$	$0.31^{+0.18}_{-0.13}$	$0.47^{+0.30}_{-0.21}$	$11.92^{+7.69}_{-5.28}$
(v) $(\frac{1}{2}^+ \to \frac{1}{2}^-)$	$\mathrm{Br}[\Lambda_b \to \Lambda_c(2595)P]$	$1.09^{+0.76}_{-0.51}$	$0.08^{+0.06}_{-0.04}$	$0.07^{+0.07}_{-0.04}$	$1.72^{+1.71}_{-1.01}$
(vi) $(\frac{1}{2}^+ \to \frac{3}{2}^-)$	$\mathrm{Br}[\Lambda_b \to \Lambda_c(2625)P]$	$2.40^{+4.09}_{-1.82}$	$0.17^{+0.30}_{-0.13}$	$0.13^{+0.22}_{-0.10}$	$2.88^{+4.92}_{-2.16}$
(i)* $(\frac{1}{2}^+ \to \frac{1}{2}^+)$	$\mathrm{Br}[\Lambda_b \to \Lambda_c(2765)P]$	$1.70^{+0.69}_{-0.52}$	$0.13^{+0.05}_{-0.04}$	$0.15^{+0.07}_{-0.05}$	$3.54^{+1.73}_{-1.24}$
(v)* $(\frac{1}{2}^+ \to \frac{1}{2}^-)$	$\mathrm{Br}[\Lambda_b \to \Lambda_c(2940)P]$	$0.68^{+0.21}_{-0.21}$	$0.05^{+0.02}_{-0.02}$	$0.04^{+0.02}_{-0.02}$	$0.87^{+0.46}_{-0.38}$
(vi)* $(\frac{1}{2}^+ \to \frac{3}{2}^-)$	$\mathrm{Br}[\Lambda_b \to \Lambda_c(2940)P]$	$1.00^{+2.00}_{-0.83}$	$0.07^{+0.14}_{-0.06}$	$0.07^{+0.10}_{-0.05}$	$1.69^{+2.30}_{-1.23}$
(i) $(\frac{1}{2}^+ \to \frac{1}{2}^+)$	$\mathrm{Br}(\Xi_b^- \to \Xi_c^0 P)$	$3.88^{+2.43}_{-1.69}$	$0.29^{+0.18}_{-0.13}$	$0.45^{+0.31}_{-0.21}$	$11.54^{+7.98}_{-5.34}$
(i) $(\frac{1}{2}^+ \to \frac{1}{2}^+)$	$\mathrm{Br}(\Xi_b^0 \to \Xi_c^+ P)$	$3.66^{+2.29}_{-1.59}$	$0.28^{+0.17}_{-0.12}$	$0.43^{+0.29}_{-0.20}$	$10.87^{+7.51}_{-5.03}$
(v) $(\frac{1}{2}^+ \to \frac{1}{2}^-)$	$\mathrm{Br}[\Xi_b^- \to \Xi_c^0(2790)P]$	$1.03^{+0.79}_{-0.48}$	$0.08^{+0.06}_{-0.04}$	$0.07^{+0.08}_{-0.04}$	$1.70^{+1.88}_{-0.99}$
(v) $(\frac{1}{2}^+ \to \frac{1}{2}^-)$	$\mathrm{Br}[\Xi_b^0 \to \Xi_c^-(2790)P]$	$0.97^{+0.74}_{-0.45}$	$0.07^{+0.06}_{-0.03}$	$0.07^{+0.07}_{-0.04}$	$1.60^{+1.76}_{-0.93}$
(vi) $(\frac{1}{2}^+ \to \frac{3}{2}^-)$	$\mathrm{Br}[\Xi_b^- \to \Xi_c^0(2815)P]$	$3.53^{+6.46}_{-2.80}$	$0.26^{+0.47}_{-0.20}$	$0.20^{+0.35}_{-0.15}$	$4.65^{+8.08}_{-3.48}$
(vi) $(\frac{1}{2}^+ \to \frac{3}{2}^-)$	$\mathrm{Br}[\Xi_b^0 \to \Xi_c^+(2815)P]$	$3.32^{+6.08}_{-2.63}$	$0.24^{+0.44}_{-0.19}$	$0.19^{+0.33}_{-0.14}$	$4.34^{+7.54}_{-3.25}$
(ii) $(\frac{1}{2}^+ \to \frac{1}{2}^+)$	$\mathrm{Br}(\Omega_b \to \Omega_c P)$	$1.10^{+0.85}_{-0.55}$	$0.08^{+0.07}_{-0.04}$	$0.15^{+0.14}_{-0.08}$	$4.03^{+3.72}_{-2.21}$
(iii) $(\frac{1}{2}^+ \to \frac{3}{2}^+)$	$\mathrm{Br}[\Omega_b \to \Omega_c(2770)P]$	$1.37^{+3.01}_{-1.19}$	$0.11^{+0.23}_{-0.09}$	$0.28^{+0.38}_{-0.20}$	$7.46^{+9.63}_{-5.04}$
(iv) $(\frac{1}{2}^+ \to \frac{3}{2}^-)$	$\mathrm{Br}[\Omega_b \to \Omega_c(3050)P]$	$3.40^{+4.45}_{-2.25}$	$0.24^{+0.31}_{-0.16}$	$0.09^{+0.15}_{-0.07}$	$1.78^{+3.16}_{-1.38}$
(ii)* $(\frac{1}{2}^+ \to \frac{1}{2}^+)$	$\mathrm{Br}[\Omega_b \to \Omega_c(3090)P]$	$0.85^{+0.50}_{-0.35}$	$0.06^{+0.04}_{-0.03}$	$0.10^{+0.07}_{-0.05}$	$2.43^{+1.79}_{-1.15}$
(iii)* $(\frac{1}{2}^+ \to \frac{3}{2}^+)$	$\mathrm{Br}[\Omega_b \to \Omega_c(3120)P]$	$0.96^{+0.95}_{-0.52}$	$0.07^{+0.07}_{-0.04}$	$0.10^{+0.08}_{-0.05}$	$2.37^{+1.81}_{-1.10}$

effective Wilson coefficients $a_4 = c_4 + c_3/3 = -0.03$ and $a_6 = c_6 + c_5/3 = -0.04$ at $\mu = 4.2\,\mathrm{GeV}$ [55]. The $a_1\langle M|(V-A)|0\rangle \times \langle\mathcal{B}_c|(V-A)|\mathcal{B}_b\rangle$ term in Eq. (41) needs the following additional terms: $a_4\langle M|(V-A)|0\rangle \times \langle\mathcal{B}_c|(V-A)|\mathcal{B}_b\rangle$ and $a_6(-2)\langle M|(S+P)|0\rangle \times \langle\mathcal{B}_c|(S-P)|\mathcal{B}_b\rangle$.[6] Note that we have neglected the subleading $V_{ub}V^*_{ud(s)}$ terms in the above expression. For $M = D^*_{(s)}$, we have $\langle M|(S+P)|0\rangle = 0$ and the resulting a_6 contributions are vanishing, and the $\mathcal{B}_b \to \mathcal{B}_c D^*_{(s)}$ decay amplitudes can be obtained with the following replacement

$$a_1 \to a_1 + a_4, \tag{49}$$

in the corresponding amplitudes in Eqs. (44) and (46). The penguin contributions are destructively interfere with the tree contributions in these modes. For $\mathcal{B}_b \to D_{(s)}\mathcal{B}_c$ decays, through equations of motion, we have

$$\begin{aligned}
&-2\langle D_{(s)}(q)|(S+P)|0\rangle\langle|\mathcal{B}_c|(S-P)|\mathcal{B}_b\rangle \\
&\quad= \frac{2m^2_{D_{(s)}}}{(m_c+m_{q(s)})(m_b-m_c)}\langle D_{(s)}(q)|V-A|0\rangle\langle\mathcal{B}_c|V|\mathcal{B}_b\rangle \\
&\qquad- \frac{2m^2_{D_{(s)}}}{(m_c+m_{q(s)})(m_b+m_c)}\langle D_{(s)}(q)|V-A|0\rangle \\
&\qquad\times\langle\mathcal{B}_c|(-A)|\mathcal{B}_b\rangle,
\end{aligned} \tag{50}$$

[6]See e.g., Eq. (4) in [55] for a similar expression.

where the quark masses are current quark masses with $m_b = 4.2\,\mathrm{GeV}$, $m_c = 1.3\,\mathrm{GeV}$, $m_s = 0.08\,\mathrm{GeV}$ and $m_q = 0.003$ at $\mu = 4.2\,\mathrm{GeV}$. Therefore, the $\mathcal{B}_b(1/2^+) \to \mathcal{B}_c(1/2^+, 3/2^+)D_{(s)}$ decay amplitudes can be obtained with the following replacements:

$$\begin{aligned}
a_1 &\to a_1 + a_4 + \frac{2m^2_{D_{(s)}}}{(m_c+m_{q(s)})(m_b-m_c)}a_6, \\
a_1 &\to a_1 + a_4 - \frac{2m^2_{D_{(s)}}}{(m_c+m_{q(s)})(m_b+m_c)}a_6,
\end{aligned} \tag{51}$$

where the first (second) one is applicable for the A (B) term in Eq. (44) and the D (C) term in Eq. (46), while for the $\mathcal{B}_b(1/2^+) \to \mathcal{B}_c(1/2^-, 3/2^-)D_{(s)}$ amplitudes, one needs to switch the above replacements. We also apply 10% uncertainties on a_4 and a_6.

In this work we do not consider the effects of final state interactions (FSI). In fact, from the studies of final state interactions in B decays, one will expect the effects of FSI be more prominent in color suppressed modes, where various subleading contributions including FSI can compete with each other and be surfaced, see example [59]. Whereas, for color allowed modes, the leading contributions dominate over subleading contributions, and, consequently, we expect that FSI has little effect on the decay rates and asymmetries of color allowed modes.

TABLE XI. Same as Table X but for $\mathcal{B}_b \to \mathcal{B}_c V$ and $B_b \to \mathcal{B}_c A$ decays.

Type	Mode	$M = \rho^-$	$M = K^{*-}$	$M = D^{*-}$	$M = D_s^{*-}$	$M = a_1^-$
(i)	$\mathrm{Br}(\Lambda_b \to \Lambda_c M)$	$12.28^{+7.19}_{-5.11}$	$0.63^{+0.37}_{-0.26}$	$0.84^{+0.51}_{-0.36}$	$17.49^{+10.60}_{-7.48}$	$11.91^{+6.98}_{-4.97}$
(v)	$\mathrm{Br}[\Lambda_b \to \Lambda_c(2595) M]$	$2.99^{+2.20}_{-1.44}$	$0.15^{+0.11}_{-0.07}$	$0.12^{+0.11}_{-0.07}$	$2.28^{+2.21}_{-1.29}$	$2.57^{+2.01}_{-1.29}$
(vi)	$\mathrm{Br}[\Lambda_b \to \Lambda_c(2625) M]$	$4.38^{+6.78}_{-3.17}$	$0.22^{+0.33}_{-0.16}$	$0.13^{+0.17}_{-0.08}$	$2.41^{+2.98}_{-1.52}$	$3.50^{+5.11}_{-2.45}$
(i)*	$\mathrm{Br}[\Lambda_b \to \Lambda_c(2765) M]$	$4.84^{+2.01}_{-1.50}$	$0.25^{+0.10}_{-0.08}$	$0.26^{+0.12}_{-0.09}$	$5.29^{+2.54}_{-1.84}$	$4.45^{+1.91}_{-1.42}$
(v)*	$\mathrm{Br}[\Lambda_b \to \Lambda_c(2940) M]$	$1.85^{+0.63}_{-0.60}$	$0.09^{+0.03}_{-0.03}$	$0.06^{+0.03}_{-0.03}$	$1.16^{+0.62}_{-0.48}$	$1.57^{+0.59}_{-0.54}$
(vi)*	$\mathrm{Br}[\Lambda_b \to \Lambda_c(2940) M]$	$1.93^{+3.19}_{-1.43}$	$0.10^{+0.15}_{-0.07}$	$0.06^{+0.06}_{-0.03}$	$1.11^{+1.07}_{-0.62}$	$1.58^{+2.26}_{-1.08}$
(i)	$\mathrm{Br}(\Xi_b^- \to \Xi_c^0 M)$	$11.56^{+7.25}_{-5.04}$	$0.60^{+0.37}_{-0.26}$	$0.82^{+0.53}_{-0.37}$	$17.26^{+11.2}_{-7.70}$	$11.37^{+7.14}_{-4.97}$
(i)	$\mathrm{Br}(\Xi_b^0 \to \Xi_c^+ M)$	$10.88^{+6.83}_{-4.74}$	$0.56^{+0.35}_{-0.24}$	$0.77^{+0.50}_{-0.35}$	$16.24^{+10.54}_{-7.25}$	$10.70^{+6.72}_{-4.67}$
(v)	$\mathrm{Br}[\Xi_b^- \to \Xi_c^0(2790) M]$	$2.86^{+2.28}_{-1.36}$	$0.14^{+0.12}_{-0.07}$	$0.12^{+0.12}_{-0.06}$	$2.25^{+2.33}_{-1.26}$	$2.48^{+2.10}_{-1.23}$
(v)	$\mathrm{Br}[\Xi_b^0 \to \Xi_c^+(2790) M]$	$2.69^{+2.15}_{-1.28}$	$0.13^{+0.11}_{-0.06}$	$0.11^{+0.11}_{-0.06}$	$2.11^{+2.19}_{-1.19}$	$2.33^{+1.98}_{-1.16}$
(vi)	$\mathrm{Br}[\Xi_b^- \to \Xi_c^0(2815) M]$	$6.49^{+10.58}_{-4.84}$	$0.32^{+0.51}_{-0.24}$	$0.20^{+0.26}_{-0.13}$	$3.74^{+4.58}_{-2.32}$	$5.24^{+7.92}_{-3.72}$
(vi)	$\mathrm{Br}[\Xi_b^0 \to \Xi_c^+(2815) M]$	$6.10^{+9.95}_{-4.55}$	$0.30^{+0.48}_{-0.22}$	$0.19^{+0.24}_{-0.12}$	$3.51^{+4.30}_{-2.18}$	$4.92^{+7.45}_{-3.50}$
(ii)	$\mathrm{Br}(\Omega_b \to \Omega_c M)$	$3.07^{+2.41}_{-1.53}$	$0.16^{+0.12}_{-0.08}$	$0.16^{+0.13}_{-0.08}$	$3.18^{+2.69}_{-1.61}$	$2.76^{+2.20}_{-1.37}$
(iii)	$\mathrm{Br}[\Omega_b \to \Omega_c(2770) M]$	$2.37^{+4.68}_{-1.85}$	$0.13^{+0.24}_{-0.10}$	$0.28^{+0.30}_{-0.16}$	$6.20^{+6.19}_{-3.49}$	$2.78^{+4.35}_{-1.93}$
(iv)	$\mathrm{Br}[\Omega_b \to \Omega_c(3050) M]$	$4.09^{+5.62}_{-2.71}$	$0.20^{+0.27}_{-0.13}$	$0.08^{+0.10}_{-0.05}$	$1.43^{+1.78}_{-0.88}$	$2.84^{+3.88}_{-1.86}$
(ii)*	$\mathrm{Br}[\Omega_b \to \Omega_c(3090) M]$	$2.29^{+1.36}_{-0.94}$	$0.11^{+0.07}_{-0.05}$	$0.09^{+0.06}_{-0.04}$	$1.69^{+1.06}_{-0.71}$	$1.92^{+1.15}_{-0.79}$
(iii)*	$\mathrm{Br}[\Omega_b \to \Omega_c(3120) M]$	$1.50^{+1.37}_{-0.76}$	$0.08^{+0.07}_{-0.04}$	$0.11^{+0.07}_{-0.04}$	$2.37^{+1.33}_{-0.88}$	$1.55^{+1.21}_{-0.71}$

TABLE XII. Comparisons of data and theoretical results on the branching ratios (in the unit of 10^{-3}) of $\Lambda_b \to \Lambda_c M$, $\Xi_b \to \Xi_c M$, $\Omega_b \to \Omega_c M$, and $\Omega_b \to \Omega_c(2770) M$ decays.

Mode	Data [4]	This work	[21]	[22]	[23,24]	[25]	[26]	[27]	[33]	[35]	[36]
$\Lambda_b \to \Lambda_c \pi^-$	4.9 ± 0.4	$4.16^{+2.43}_{-1.73}$	$4.6^{+2.0}_{-3.1}$	4.6	5.62	3.91	…	1.75	4.96	…	5.67
$\Lambda_b \to \Lambda_c K^-$	0.359 ± 0.030	$0.31^{+0.18}_{-0.13}$	…	…	…	…	…	0.13	0.393	…	0.46
$\Lambda_b \to \Lambda_c D^-$	0.46 ± 0.06	$0.47^{+0.30}_{-0.21}$	…	…	…	…	…	0.30	0.522	…	0.76
$\Lambda_b \to \Lambda_c D_s^-$	11.0 ± 1.0	$11.92^{+7.69}_{-5.28}$	23^{+3}_{-4}	13.7	…	12.91	22.3	7.70	12.4	14.78	19.94
$\Lambda_b \to \Lambda_c \rho^-$	…	$12.28^{+7.19}_{-5.11}$	$6.6^{+2.4}_{-4.0}$	12.9	…	10.82	…	4.91	8.65	…	16.71
$\Lambda_b \to \Lambda_c K^{*-}$	…	$0.63^{+0.37}_{-0.26}$	…	…	…	…	…	0.27	0.441	…	0.87
$\Lambda_b \to \Lambda_c D^{*-}$	…	$0.84^{+0.51}_{-0.36}$	…	…	…	…	…	0.49	0.520	…	1.38
$\Lambda_b \to \Lambda_c D_s^{*-}$	…	$17.49^{+10.60}_{-7.48}$	$17.3^{+2.0}_{-3.0}$	21.8	…	19.83	32.6	14.14	10.5	25.16	30.86
$\Lambda_b \to \Lambda_c a_1^-$	…	$11.91^{+6.98}_{-4.97}$	…	…	…	…	…	5.32	…	…	16.53
$\Xi_b^0 \to \Xi_c^+ \pi^-$	…	$3.66^{+2.29}_{-1.59}$	…	4.9	7.08	…	…	…	…	…	…
$\Xi_b^- \to \Xi_c^0 \pi^-$	…	$3.88^{+2.43}_{-1.69}$	…	5.2	10.13	…	…	…	…	…	…
$\Xi_b^0 \to \Xi_c^+ D^-$	…	$0.43^{+0.29}_{-0.20}$	…	…	…	…	…	…	…	0.45	…
$\Xi_b^0 \to \Xi_c^+ D_s^-$	…	$10.87^{+7.51}_{-5.03}$	…	14.6	…	…	…	…	…	…	…
$\Xi_b^0 \to \Xi_c^+ D^{*-}$	…	$0.77^{+0.50}_{-0.35}$	…	…	…	…	…	…	…	0.95	…
$\Xi_b^0 \to \Xi_c^+ D_s^{*-}$	…	$16.24^{+10.54}_{-7.25}$	…	23.1	…	…	…	…	…	…	…
$\Omega_b \to \Omega_c \pi^-$	…	$1.10^{+0.85}_{-0.55}$	…	4.92	5.81	…	…	…	…	1.88	…
$\Omega_b \to \Omega_c D_s^-$	…	$4.03^{+3.72}_{-2.21}$	…	17.9	…	…	…	…	…	…	…
$\Omega_b \to \Omega_c \rho^-$	…	$3.07^{+2.41}_{-1.53}$	…	12.8	…	…	…	…	…	5.43	…
$\Omega_b \to \Omega_c D_s^{*-}$	…	$3.18^{+2.69}_{-1.61}$	…	11.5	…	…	…	…	…	…	…
$\Omega_b \to \Omega_c^* \pi^-$	…	$1.37^{+3.01}_{-1.19}$	…	2.69	…	…	…	…	…	1.70	…
$\Omega_b \to \Omega_c^* D^-$	…	$0.28^{+0.38}_{-0.20}$	…	…	…	…	…	…	…	0.16	…
$\Omega_b \to \Omega_c^* D_s^-$	…	$7.46^{+9.63}_{-5.04}$	…	3.53	…	…	…	…	…	…	…
$\Omega_b \to \Omega_c^* \rho^-$	…	$2.37^{+4.68}_{-1.85}$	…	3.81	…	…	…	…	…	5.58	…
$\Omega_b \to \Omega_c^* D^{*-}$	…	$0.28^{+0.30}_{-0.16}$	…	…	…	…	…	…	…	0.58	…
$\Omega_b \to \Omega_c^* D_s^{*-}$	…	$6.20^{+6.19}_{-3.49}$	…	3.93	…	…	…	…	…	…	…

TABLE XIII. The predicted up-down asymmetries (in the unit of %) of $\mathcal{B}_b \to \mathcal{B}_c P$ decays. Note that the asterisks denote the transitions where the final state charmed baryons are radial excited.

Type	Mode	$P=\pi^-$	$P=K^-$	$P=D^-$	$P=D_s^-$
(i) $(\frac{1}{2}^+\to\frac{1}{2}^+)$	$\alpha(\Lambda_b\to\Lambda_c P)$	$-99.99^{+4.70}_{-0.00}$	$-99.97^{+5.02}_{-0.01}$	$-99.45^{+7.94}_{-0.55}$	$-99.19^{+8.59}_{-0.81}$
(v) $(\frac{1}{2}^+\to\frac{1}{2}^-)$	$\alpha[\Lambda_b\to\Lambda_c(2595)P]$	$-98.33^{+12.89}_{-1.67}$	$-98.12^{+13.51}_{-1.88}$	$-88.05^{+26.57}_{-11.95}$	$-86.49^{+27.83}_{-13.51}$
(vi) $(\frac{1}{2}^+\to\frac{3}{2}^-)$	$\alpha[\Lambda_b\to\Lambda_c(2625)P]$	$-97.76^{+39.39}_{-2.24}$	$-97.64^{+39.37}_{-2.36}$	$-97.44^{+37.86}_{-2.56}$	$-97.07^{+38.04}_{-2.93}$
(i)* $(\frac{1}{2}^+\to\frac{1}{2}^+)$	$\alpha[\Lambda_b\to\Lambda_c(2765)P]$	$-99.93^{+1.65}_{-0.02}$	$-99.87^{+1.87}_{-0.11}$	$-98.03^{+5.04}_{-1.97}$	$-97.23^{+5.70}_{-2.70}$
(v)* $(\frac{1}{2}^+\to\frac{1}{2}^-)$	$\alpha[\Lambda_b\to\Lambda_c(2940)P]$	$-98.32^{+2.86}_{-1.47}$	$-98.10^{+3.11}_{-1.64}$	$-86.24^{+11.11}_{-9.26}$	$-84.04^{+12.19}_{-10.66}$
(vi)* $(\frac{1}{2}^+\to\frac{3}{2}^-)$	$\alpha[\Lambda_b\to\Lambda_c(2940)P]$	$-99.41^{+65.88}_{-0.59}$	$-99.06^{+61.14}_{-0.94}$	$-89.25^{+31.59}_{-10.75}$	$-86.81^{+30.72}_{-13.19}$
(i) $(\frac{1}{2}^+\to\frac{1}{2}^+)$	$\alpha(\Xi_b^-\to\Xi_c^0 P)$	$-99.98^{+5.73}_{-0.00}$	$-99.96^{+6.10}_{-0.00}$	$-99.29^{+9.61}_{-0.71}$	$-98.99^{+10.34}_{-1.01}$
(i) $(\frac{1}{2}^+\to\frac{1}{2}^+)$	$\alpha(\Xi_b^0\to\Xi_c^+ P)$	$-99.98^{+5.73}_{-0.00}$	$-99.96^{+6.10}_{-0.00}$	$-99.29^{+9.61}_{-0.71}$	$-98.99^{+10.34}_{-1.01}$
(v) $(\frac{1}{2}^+\to\frac{1}{2}^-)$	$\alpha[\Xi_b^-\to\Xi_c^0(2790)P]$	$-98.13^{+14.56}_{-1.87}$	$-97.88^{+15.26}_{-2.11}$	$-86.62^{+28.66}_{-13.38}$	$-84.85^{+29.84}_{-15.15}$
(v) $(\frac{1}{2}^+\to\frac{1}{2}^-)$	$\alpha[\Xi_b^0\to\Xi_c^-(2790)P]$	$-98.13^{+14.56}_{-1.87}$	$-97.88^{+15.26}_{-2.11}$	$-86.60^{+28.67}_{-13.40}$	$-84.83^{+29.85}_{-15.16}$
(vi) $(\frac{1}{2}^+\to\frac{3}{2}^-)$	$\alpha[\Xi_b^-\to\Xi_c^0(2815)P]$	$-97.63^{+42.32}_{-2.37}$	$-97.48^{+42.09}_{-2.52}$	$-96.48^{+38.46}_{-3.52}$	$-95.89^{+38.40}_{-4.11}$
(vi) $(\frac{1}{2}^+\to\frac{3}{2}^-)$	$\alpha[\Xi_b^0\to\Xi_c^+(2815)P]$	$-97.70^{+42.27}_{-2.30}$	$-97.56^{+42.03}_{-2.44}$	$-96.71^{+38.31}_{-3.29}$	$-96.16^{+38.26}_{-3.84}$
(ii) $(\frac{1}{2}^+\to\frac{1}{2}^+)$	$\alpha(\Omega_b\to\Omega_c P)$	$59.94^{+21.34}_{-18.76}$	$59.39^{+21.45}_{-18.70}$	$56.04^{+23.79}_{-19.29}$	$55.16^{+23.98}_{-19.18}$
(iii) $(\frac{1}{2}^+\to\frac{3}{2}^+)$	$\alpha[\Omega_b\to\Omega_c(2770)P]$	$2.60^{+97.40}_{-102.23}$	$1.17^{+98.43}_{-100.15}$	$-11.02^{+55.88}_{-59.25}$	$-11.70^{+50.63}_{-55.10}$
(iv) $(\frac{1}{2}^+\to\frac{3}{2}^-)$	$\alpha[\Omega_b\to\Omega_c(3050)P]$	$18.07^{+52.52}_{-41.45}$	$17.73^{+53.31}_{-42.50}$	$9.16^{+75.06}_{-71.89}$	$7.09^{+78.45}_{-76.03}$
(ii)* $(\frac{1}{2}^+\to\frac{1}{2}^+)$	$\alpha[\Omega_b\to\Omega_c(3090)P]$	$59.75^{+14.13}_{-13.17}$	$59.15^{+14.21}_{-13.16}$	$54.01^{+16.49}_{-13.95}$	$52.73^{+16.61}_{-13.86}$
(iii)* $(\frac{1}{2}^+\to\frac{3}{2}^+)$	$\alpha[\Omega_b\to\Omega_c(3120)P]$	$4.58^{+42.35}_{-41.22}$	$3.81^{+41.17}_{-40.26}$	$-3.74^{+22.18}_{-24.05}$	$-4.20^{+20.50}_{-22.52}$

The predicted branching ratios for $\mathcal{B}_b\to\mathcal{B}_c P$ and $\mathcal{B}_b\to\mathcal{B}_c V, \mathcal{B}_c A$ decays, are summarized in Tables X and XI, respectively In Tables XII, we compare our results on $\Lambda_b\to\Lambda_c M$, $\Xi_b\to\Xi_c M$, $\Omega_b\to\Omega_c M$, and $\Omega_b\to\Omega_c(2770)M$ decay rates to data [4] and the results of other theoretical studies. Overall speaking, our results agree reasonably well with data and with most of the results obtained in other works [21–27,33,35,36]. It is interesting that the penguin contributions slightly reduce the $\Lambda_b\to\Lambda_c D$ and ΛD_s rates resulting a better agreement with data. Indeed, when the penguin contributions are included, the central values of the $\Lambda_b\to\Lambda_c D$ and ΛD_s branching ratios (in the unit of 10^{-3}) are reduced from 0.53 and 13.50 to 0.47 and 11.92, respectively, which are closer to the experimental values, 0.46 ± 0.06 and 11.0 ± 1.0, respectively.

In Tables XIII and XIV, we show the prediction of up-down asymmetries for $\mathcal{B}_b\to\mathcal{B}_c M$ decays with $M=P$, V, A. From the tables we see that most of the signs of α are negative, except those in $\Omega_b\to\Omega_c M$, $\Omega_b\to\Omega_c(3050)M$ and $\Omega_b\to\Omega_c(3090)M$ decays. In Tables XV, we compare our results on the up-down asymmetries of $\Lambda_b\to\Lambda_c M$, $\Xi_b\to\Xi_c M$, $\Omega_b\to\Omega_c M$ and $\Omega_b\to\Omega_c(2770)M$ decays to other results [21–24,26,27,33,35,36]. Our results agree well within errors with almost all of the results obtained in other works.

It will be interesting to comparing $\Lambda_b\to\Lambda_c(2940)M$ decays with two different assignments of the configurations of $\Lambda_c(2940)$, a radial excite p-wave spin-1/2 or spin-3/2 particle. From Tables X and XI, we have $\mathrm{Br}[\Lambda_b\to\Lambda_c(2940,3/2^-)P]\simeq(1.5\sim2)\times\mathrm{Br}[\Lambda_b\to\Lambda_c(2940,1/2^-)P]$, $\mathrm{Br}[\Lambda_b\to\Lambda_c(2940,3/2^-)V]\simeq\mathrm{Br}[\Lambda_b\to\Lambda_c(2940,1/2^-)V]$ and $\mathrm{Br}[\Lambda_b\to\Lambda_c(2940,3/2^-)A]\simeq\mathrm{Br}[\Lambda_b\to\Lambda_c(2940,1/2^-)A]$. The asymmetries in $\Lambda_b\to\Lambda_c(2940,1/2^-)M$ and $\Lambda_b\to\Lambda_c(2940,3/2^-)M$ decays are similar in most cases, but have larger deviations in the cases of heavy vector mesons. In $\Lambda_b\to\Lambda_c(2940)D^{*-}$ and $\Lambda_c(2940)D_s^{*-}$ decays, the predictions based on the spin-1/2 configuration, give −25% and −19%, respectively, while the ones based on the spin-3/2 configurations are −48% and −45%, respectively.

It will be useful to understand why the $\Lambda_b\to\Lambda_c(2940,3/2^-)P$ is greater than the $\Lambda_b\to\Lambda_c(2940,1/2^-)P$ rate. Using Eqs. (C1) and (C4), we have

$$\frac{\Gamma[\Lambda_b\to\Lambda_c(2940,\frac{3}{2})P]}{\Gamma[\Lambda_b\to\Lambda_c(2940,\frac{1}{2})P]}=\frac{2m_{\Lambda_b}^2}{3m_{\Lambda_c(2940)}^2}\times\frac{[(m_{\Lambda_b}+m_{\Lambda_c(2940)})^2-m_P^2]|p_c C|^2+[(m_{\Lambda_b}-m_{\Lambda_c(2940)})^2-m_P^2]|p_c D|^2}{[(m_{\Lambda_b}+m_{\Lambda_c(2940)})^2-m_P^2]|A|^2+[(m_{\Lambda_b}-m_{\Lambda_c(2940)})^2-m_P^2]M^2|B|^2}. \tag{52}$$

TABLE XIV. Same as Table XIII but for $B_b \to \mathcal{B}_c V$ and $B_b \to \mathcal{B}_c A$ decays.

Type	Mode	$M = \rho^-$	$M = K^{*-}$	$M = D^{*-}$	$M = D_s^{*-}$	$M = a_1^-$
(i)	$\alpha(\Lambda_b \to \Lambda_c M)$	$-86.96^{+5.60}_{-0.87}$	$-82.96^{+6.02}_{-1.11}$	$-36.85^{+7.08}_{-4.88}$	$-32.69^{+6.82}_{-5.05}$	$-70.00^{+7.00}_{-2.30}$
(v)	$\alpha[\Lambda_b \to \Lambda_c(2595)M]$	$-85.6^{+12.41}_{-3.85}$	$-81.65^{+12.21}_{-4.46}$	$-33.47^{+15.76}_{-11.40}$	$-28.68^{+15.97}_{-11.88}$	$-68.66^{+11.32}_{-6.19}$
(vi)	$\alpha[\Lambda_b \to \Lambda_c(2625)M]$	$-91.48^{+42.04}_{-3.89}$	$-89.01^{+41.55}_{-4.91}$	$-53.92^{+34.18}_{-18.71}$	$-49.80^{+33.31}_{-20.08}$	$-80.52^{+39.81}_{-8.58}$
(i)*	$\alpha[\Lambda_b \to \Lambda_c(2765)M]$	$-86.15^{+2.71}_{-1.04}$	$-81.89^{+3.24}_{-1.47}$	$-31.54^{+5.31}_{-4.91}$	$-26.87^{+5.07}_{-4.82}$	$-67.99^{+4.60}_{-2.94}$
(v)*	$\alpha[\Lambda_b \to \Lambda_c(2940)M]$	$-84.01^{+4.44}_{-3.07}$	$-79.56^{+4.84}_{-3.51}$	$-24.77^{+10.24}_{-9.27}$	$-19.04^{+10.34}_{-9.71}$	$-64.97^{+6.29}_{-4.97}$
(vi)*	$\alpha[\Lambda_b \to \Lambda_c(2940)M]$	$-87.25^{+66.15}_{-7.21}$	$-84.14^{+63.92}_{-8.45}$	$-48.02^{+36.43}_{-18.07}$	$-45.28^{+33.21}_{-18.13}$	$-73.90^{+56.57}_{-12.00}$
(i)	$\alpha(\Xi_b^- \to \Xi_c^0 M)$	$-86.61^{+6.57}_{-1.09}$	$-82.52^{+7.04}_{-1.38}$	$-36.00^{+8.09}_{-5.51}$	$-31.85^{+7.78}_{-5.73}$	$-69.34^{+8.13}_{-2.69}$
(i)	$\alpha(\Xi_b^0 \to \Xi_c^+ M)$	$-86.61^{+6.57}_{-1.09}$	$-82.52^{+7.04}_{-1.38}$	$-35.98^{+8.08}_{-5.51}$	$-31.84^{+7.78}_{-5.72}$	$-69.33^{+8.13}_{-2.69}$
(v)	$\alpha[\Xi_b^- \to \Xi_c^0(2790)M]$	$-85.14^{+13.71}_{-4.14}$	$-81.11^{+13.40}_{-4.77}$	$-32.40^{+16.14}_{-11.43}$	$-27.57^{+16.30}_{-11.88}$	$-67.92^{+12.18}_{-6.55}$
(v)	$\alpha[\Xi_b^0 \to \Xi_c^+(2790)M]$	$-85.13^{+13.71}_{-4.15}$	$-81.09^{+13.40}_{-4.78}$	$-32.35^{+16.14}_{-11.43}$	$-27.52^{+16.30}_{-11.88}$	$-67.90^{+12.17}_{-6.55}$
(vi)	$\alpha[\Xi_b^- \to \Xi_c^0(2815)M]$	$-91.55^{+46.65}_{-4.08}$	$-89.08^{+45.9}_{-5.17}$	$-55.07^{+35.28}_{-18.5}$	$-51.32^{+34.12}_{-19.77}$	$-80.63^{+43.30}_{-8.94}$
(vi)	$\alpha[\Xi_b^0 \to \Xi_c^+(2815)M]$	$-91.48^{+46.84}_{-4.15}$	$-89.00^{+46.09}_{-5.27}$	$-54.83^{+35.37}_{-18.69}$	$-51.06^{+34.18}_{-20.08}$	$-80.52^{+43.48}_{-9.07}$
(ii)	$\alpha(\Omega_b \to \Omega_c M)$	$61.63^{+22.03}_{-19.83}$	$62.20^{+22.22}_{-20.19}$	$72.15^{+22.11}_{-25.51}$	$73.53^{+21.54}_{-26.05}$	$64.27^{+22.74}_{-21.45}$
(iii)	$\alpha[\Omega_b \to \Omega_c(2770)M]$	$-5.22^{+82.03}_{-88.99}$	$-7.51^{+78.52}_{-85.12}$	$-22.24^{+45.94}_{-51.42}$	$-22.62^{+42.82}_{-48.63}$	$-13.64^{+68.97}_{-74.18}$
(iv)	$\alpha[\Omega_b \to \Omega_c(3050)M]$	$24.95^{+60.39}_{-59.79}$	$24.72^{+59.86}_{-60.40}$	$13.22^{+49.96}_{-62.37}$	$10.39^{+49.66}_{-61.11}$	$23.53^{+57.68}_{-62.28}$
(ii)*	$\alpha[\Omega_b \to \Omega_c(3090)M]$	$61.51^{+14.81}_{-13.97}$	$62.10^{+15.02}_{-14.24}$	$73.1^{+15.51}_{-17.68}$	$74.63^{+14.95}_{-17.81}$	$64.31^{+15.63}_{-15.17}$
(iii)*	$\alpha[\Omega_b \to \Omega_c(3120)M]$	$1.15^{+44.49}_{-48.55}$	$-0.31^{+41.58}_{-46.29}$	$-11.06^{+21.35}_{-24.74}$	$-11.43^{+19.79}_{-22.92}$	$-4.44^{+33.80}_{-39.57}$

TABLE XV. Various theoretical results on the up-down asymmetries (α in the unit of %) of $\Lambda_b \to \Lambda_c M$, $\Xi_b \to \Xi_c M$, $\Omega_b \to \Omega_c M$, and $\Omega_b \to \Omega_c(2770)M$ decays are compared.

Mode	This work	[21]	[22]	[23,24]	[26]	[27]	[33]	[35]	[36]
$\Lambda_b \to \Lambda_c \pi^-$	$-99.99^{+4.70}_{-0.00}$	−100	−99	−99	…	−99.9	−99.8	…	−100
$\Lambda_b \to \Lambda_c K^-$	$-99.97^{+5.02}_{-0.01}$	…	…	…	…	−100	−100	…	−100
$\Lambda_b \to \Lambda_c D^-$	$-99.45^{+7.94}_{-0.55}$	…	…	…	…	−98.7	−99.9	−98.9	−98.3
$\Lambda_b \to \Lambda_c D_s^-$	$-99.19^{+8.59}_{-0.81}$	−99.1	−99	…	−98	−98.4	−100	−98.6	−97.8
$\Lambda_b \to \Lambda_c \rho^-$	$-86.96^{+5.60}_{-0.87}$	−90.3	−88	…	…	−89.8	−88.8	…	−87.5
$\Lambda_b \to \Lambda_c K^{*-}$	$-82.96^{+6.02}_{-1.11}$	…	…	…	…	−86.5	−85.9	…	−83.6
$\Lambda_b \to \Lambda_c D^{*-}$	$-36.85^{+7.08}_{-4.88}$	…	…	…	…	−45.9	−47.8	…	−37.1
$\Lambda_b \to \Lambda_c D_s^{*-}$	$-32.69^{+6.82}_{-5.05}$	−43.7	−36	…	−40	−41.9	−43.9	−36.4	−32.7
$\Lambda_b \to \Lambda_c a_1^-$	$-70.00^{+7.00}_{-2.30}$	…	…	…	…	−75.8	…	…	−70.9
$\Xi_b^0 \to \Xi_c^+ \pi^-$	$-99.98^{+5.73}_{-0.00}$	…	−100	−100	…	…	…	…	…
$\Xi_b^- \to \Xi_c^0 \pi^-$	$-99.98^{+5.73}_{-0.00}$	…	−100	−97	…	…	…	…	…
$\Xi_b^0 \to \Xi_c^+ D_s^-$	$-98.99^{+10.34}_{-1.01}$	…	−99	…	…	…	…	…	…
$\Xi_b^0 \to \Xi_c^- D_s^{*-}$	$-31.84^{+7.78}_{-5.72}$	…	−36	…	…	…	…	…	…
$\Omega_b \to \Omega_c \pi^-$	$59.94^{+21.34}_{-18.76}$	…	51	60	…	…	…	…	…
$\Omega_b \to \Omega_c D_s^-$	$55.16^{+23.98}_{-19.18}$	…	42	…	…	…	…	…	…
$\Omega_b \to \Omega_c \rho^-$	$61.63^{+22.03}_{-19.83}$	…	53	…	…	…	…	…	…
$\Omega_b \to \Omega_c D_s^{*-}$	$73.53^{+21.54}_{-26.05}$	…	64	…	…	…	…	…	…
$\Omega_b \to \Omega_c(2770)\pi^-$	$2.60^{+97.40}_{-102.23}$	…	−38	…	…	…	…	…	…
$\Omega_b \to \Omega_c(2770)D_s^-$	$-11.70^{+50.63}_{-55.10}$	…	−22	…	…	…	…	…	…
$\Omega_b \to \Omega_c(2770)\rho^-$	$-5.22^{+82.03}_{-88.99}$	…	−75	…	…	…	…	…	…
$\Omega_b \to \Omega_c(2770)D_s^{*-}$	$-22.62^{+42.82}_{-48.63}$	…	−31	…	…	…	…	…	…

The first factor in the right-hand side (r.h.s.) of the above equation is an enhancement factor, while the second factor is expected to be close to unity as the form factors shown in Tables VII and IX for $\Lambda_b \to \Lambda_c(2940, 1/2^-)$ and $\Lambda_b \to \Lambda_c(2940, 3/2^-)$ transitions are of similar sizes. For example, in $\Lambda_b \to \Lambda_c(2940, 3/2^-)\pi^-$ and $\Lambda_b \to \Lambda_c(2940, 1/2^-)\pi^-$ decays, we have $(A, B) = (-3.09, -8.20) \times 10^{-8}$ and $p_c(C, D) = (-2.33, -6.69) \times 10^{-8}$, and the ratio of the decay rates are given by

$$\frac{\Gamma[\Lambda_b \to \Lambda_c(2940, \frac{3}{2}^-)\pi^-]}{\Gamma[\Lambda_b \to \Lambda_c(2940, \frac{1}{2}^-)\pi^-]} = 2.44 \times 0.61 = 1.49, \quad (53)$$

and, hence, the $\Lambda_b \to \Lambda_c(2940, 3/2^-)\pi^-$ rate is greater than the $\Lambda_b \to \Lambda_c(2940, 1/2^-)\pi^-$ rate by about 50%. The enhancements in other $\Lambda_b \to \Lambda_c(2940, 3/2^-)P$ decay rates can be understood similarly.

The predictions on rates and asymmetries presented in Tables X, XI, XIII, and XIV can be verified experimentally. These information may shed light on the quantum numbers of $\Lambda_c(2765)$, $\Lambda_c(2940)\Omega_c(3050)$, $\Omega_c(3090)$, and $\Omega_c(3120)$.

IV. CONCLUSIONS

In this work, we study color allowed $\mathcal{B}_b \to \mathcal{B}_c M$ decays with $\mathcal{B}_b = \Lambda_b, \Xi_b, \Omega_b$, $M = \pi, K, \rho, K^*, a_1, D, D_s, D^*, D_s^*$ and s-wave and p-wave charmed baryons, $\mathcal{B}_c$, including $\Lambda_c^{(*,**)} = \Lambda_c, \Lambda_c(2595), \Lambda_c(2625), \Lambda_c(2765), \Lambda_c(2940)$, $\Xi_c^{(**)} = \Xi_c, \Xi_c(2815), \Xi_c(2790)$, and $\Omega_c^{(*,**)} = \Omega_c, \Omega_c(2770)$, $\Omega_c(3050), \Omega_c(3090), \Omega_c(3120)$. There are six types of transitions, namely (i) $\mathcal{B}_b(\bar{\mathbf{3}}_\mathbf{f}, 1/2^+)$ to $\mathcal{B}_c(\bar{\mathbf{3}}_\mathbf{f}, 1/2^+)$ transition, (ii) $\mathcal{B}_b(\mathbf{6}_\mathbf{f}, 1/2^+)$ to $\mathcal{B}_c(\mathbf{6}_\mathbf{f}, 1/2^+)$ transition, (iii) $\mathcal{B}_b(\mathbf{6}_\mathbf{f}, 1/2^+)$ to $\mathcal{B}_c(\mathbf{6}_\mathbf{f}, 3/2^+)$ transition, (iv) $\mathcal{B}_b(\mathbf{6}_\mathbf{f}, 1/2^+)$ to $\mathcal{B}_c(\mathbf{6}_\mathbf{f}, 3/2^-)$ transition, (v) $\mathcal{B}_b(\bar{\mathbf{3}}_\mathbf{f}, 1/2^+)$ to $\mathcal{B}_c(\bar{\mathbf{3}}_\mathbf{f}, 1/2^-)$ transition, and (vi) $\mathcal{B}_b(\bar{\mathbf{3}}_\mathbf{f}, 1/2^+)$ to $\mathcal{B}_c(\bar{\mathbf{3}}_\mathbf{f}, 3/2^-)$ transition. Types (i) to (iii) involve spin 1/2 and 3/2 s-wave charmed baryons, while types (iv) to (vi) involve spin 1/2 and 3/2 p-wave charmed baryons. We have scalar or axial vector light diquarks in the baryons. The light diquarks are spectating in these transitions. The bottom baryon to s-wave and p-wave charmed baryon form factors are calculated in the light-front quark model approach. The analysis and the scope of this work is improved and enlarged compared to a previous study [16] in several aspects. All of the form factors in the $1/2 \to 1/2$ and $1/2 \to 3/2$ transitions are extracted, while we only have $f_{1,2}$ and $g_{1,2}$ for $1/2 \to 1/2$ transition in [16]. Some consistency constraints are found and are imposed. We find that the form factors can reasonably satisfy the relations obtained in the heavy quark limit. In fact, we do not expect them to satisfy the relations exactly as we are using heavy but finite m_b and m_c.

Using naïve factorization decay rates and up-down asymmetries for various $\Lambda_b \to \Lambda_c^{(*,**)} M^-$, $\Xi_b \to \Xi_c^{(**)} M^-$ and $\Omega_b \to \Omega_c^{(*,**)} M^-$ decays are predicted and can be checked experimentally. We find that most of the signs of α are negative, except those in $\Omega_b \to \Omega_c M$, $\Omega_b \to \Omega_c(3050)M$ and $\Omega_b \to \Omega_c(3090)M$ decays. We compare our results of rates and up-down asymmetries of $\Lambda_b \to \Lambda_c M$, $\Xi_b \to \Xi_c M$, $\Omega_b \to \Omega_c M$ and $\Omega_b \to \Omega_c(2770)M$ decays to existing data and other theoretical results. Our predictions agree well with data and with most of the results of other works.

The study on these decay modes may shed light on the quantum numbers of $\Lambda_c(2765)$, $\Lambda_c(2940)\Omega_c(3050)$, $\Omega_c(3090)$, and $\Omega_c(3120)$, as the decays depend on bottom baryon to charmed baryon form factors, which are sensitive to the configurations of the final state charmed baryons. For example, there are two possible quantum numbers for $\Lambda_c(2940)$, it can either be a $\mathcal{B}_c(\bar{\mathbf{3}}_\mathbf{f}, 1/2^-)$ state [6] or a $\mathcal{B}_c(\bar{\mathbf{3}}_\mathbf{f}, 3/2^-)$ one [1]. Both possibilities are considered. Comparing predictions on $\Lambda_b \to \Lambda_c(2940)M$ rates, we have $\mathrm{Br}[\Lambda_b \to \Lambda_c(2940, 3/2^-)P] \simeq (1.5 \sim 2) \times \mathrm{Br}[\Lambda_b \to \Lambda_c(2940, 1/2^-)P]$, $\mathrm{Br}[\Lambda_b \to \Lambda_c(2940, 3/2^-)V] \simeq \mathrm{Br}[\Lambda_b \to \Lambda_c(2940, 1/2^-)V]$ and $\mathrm{Br}[\Lambda_b \to \Lambda_c(2940, 3/2^-)A] \simeq \mathrm{Br}[\Lambda_b \to \Lambda_c(2940, 1/2^-)A]$. The asymmetries in $\Lambda_b \to \Lambda_c(2940, 1/2^-)M$ and $\Lambda_b \to \Lambda_c(2940, 3/2^-)M$ decays have larger deviations in the cases of heavy vector mesons.

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APPENDIX A: WAVE FUNCTIONS

In the light-front quark model the baryon state, which consists of a heavy quark $Q = b$, c and a scalar diquark $[qq]$ or an axial-vector diquark $[qq]$, can be expressed as (see, [16,42,43])

$$\begin{aligned} |\mathcal{B}_Q(P, J, J_z)\rangle = & \int \{d^3 p_1\}\{d^3 p_2\} 2(2\pi)^3 \delta^3(\tilde{P} - \tilde{p}_1 - \tilde{p}_2) \\ & \times \sum_{\lambda_1, \lambda_2, \alpha, \beta, \gamma, b, c} \Psi^{JJ_z}_{nL_K S_{[qq]} J_l}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \lambda_2) C_{\alpha\beta\gamma} F^{bc} \\ & \times |Q^\alpha(p_1, \lambda_1)[q_b^\beta q_c^\gamma](p_2, \lambda_2)\rangle, \end{aligned} \quad (A1)$$

where $S_{[qq]}$, L_K, and J_l denote the spin of the diquark, the orbital angular momentum of the $Q - [qq]$ system, and the total angular momentum of the light degree of freedom, respectively. In the above equation, n, (α, β, γ), $(b, c)\lambda_i$, and $p_{1,2}$ are the quantum number of the wave function, color indices, flavor indices, helicity and the on-mass-shell light-front momenta, respectively. The following notations are used

$$\tilde{p} = (p^+, \vec{p}_\perp), \quad \vec{p}_\perp = (p^1, p^2), \quad p^- = \frac{m^2 + p_\perp^2}{p^+}, \quad \text{(A2)}$$

and

$$\begin{aligned} \{d^3p\} &\equiv \frac{dp^+ d^2p_\perp}{2(2\pi)^3}, \\ \delta^3(\tilde{p}) &= \delta(p^+)\delta^2(\vec{p}_\perp), \\ |Q(p_1,\lambda_1)[q_b q_c](p_2,\lambda_2)\rangle &= b^\dagger_{\lambda_1}(p_1)a^\dagger_{\lambda_2}(p_2)|0\rangle, \\ [a_{\lambda'}(p'), a^\dagger_\lambda(p)] &= 2(2\pi)^3\delta^3(\tilde{p}'-\tilde{p})\delta_{\lambda',\lambda}, \\ \{b_{\lambda'}(p'), b^\dagger_\lambda(p)\} &= 2(2\pi)^3\delta^3(\tilde{p}'-\tilde{p})\delta_{\lambda'\lambda}, \quad \text{(A3)} \end{aligned}$$

with $\lambda_2 = S_2 = 0$ for a scalar diquark and $\lambda_2 = 0, \pm 1$ and $S_2 = 1$ for an axial vector diquark. The coefficient $C_{\alpha\beta\gamma}$ is a normalized color factor and F^{bc} is a normalized flavor coefficient, obeying the relation

$$\begin{aligned} &C_{\alpha'\beta'\gamma'}F^{b'c'}C_{\alpha\beta\gamma}F^{bc}\langle Q^{\alpha'}(p_1',\lambda_1')[q^{\beta'}_{b'}q^{\gamma'}_{c'}](p_2',\lambda_2')|Q^\alpha(p_1,\lambda_1) \\ &\times [q^\beta_a q^\gamma_b](p_2,\lambda_2)\rangle \\ &\quad = 2^2(2\pi)^6\delta^3(\tilde{p}_1'-\tilde{p}_1)\delta^3(\tilde{p}_2'-\tilde{p}_2)\delta_{\lambda_1'\lambda_1}\delta_{\lambda_2'\lambda_2}. \quad \text{(A4)} \end{aligned}$$

The momenta can be defined in terms of the light-front internal momentum variables, $(x_i, \vec{k}_{i\perp})$ for $i = 1, 2$,

$$\begin{aligned} p_i^+ &= x_i P^+, & \sum_{i=1}^2 x_i &= 1, \\ \vec{p}_{i\perp} &= x_i \vec{P}_\perp + \vec{k}_{i\perp}, & \sum_{i=1}^2 \vec{k}_{i\perp} &= 0. \quad \text{(A5)} \end{aligned}$$

The momentum-space wave-function $\Psi^{JJ_z}_{nL_K S_{[qq]} J_l}$ can be expressed as

$$\begin{aligned} &\Psi^{JJ_z}_{nL_K S_{[qq]} J_l}(\tilde{p}_1,\tilde{p}_2,\lambda_1,\lambda_2) \\ &= \sum_{s_1,s_2,L_z,J_{lz}} \langle\lambda_1|\mathcal{R}^\dagger_M(p_1^+,\vec{p}_{1\perp},m_1)|s_1\rangle \\ &\quad\times\langle\lambda_2|\mathcal{R}^\dagger_M(p_2^+,\vec{p}_{2\perp},m_2)|s_2\rangle\langle S_1 J_l; s_1 J_{lz}|S_1 J_l; JJ_z\rangle \\ &\quad\times\langle L_K S_{[qq]}; L_z s_2|L_k S_{[qq]}; J_l J_{lz}\rangle\phi_{nL_K L_z}(x_1,x_2,k_{1\perp},k_{2\perp}), \end{aligned} \quad \text{(A6)}$$

where $\phi_{nL_K L_z}(x_1,x_2,k_{1\perp},k_{2\perp})$ is the momentum distribution of the constituents in the bound state, $\langle J'J''; m'm''|J'J''; Jm\rangle$ the Clebsch-Gordan coefficients and $\langle\lambda_i|\mathcal{R}^\dagger_M(p_1^+,\vec{p}_{1\perp},m_i)|s_i\rangle$ the Melosh transform matrix element.

We normalize the state as

$$\langle\mathcal{B}_Q(P',J',J_z')|\mathcal{B}_Q(P,J,J_z)\rangle = 2(2\pi)^3P^+\delta^3(\tilde{P}'-\tilde{P})\delta_{J'J}\delta_{J_z'J_z}, \quad \text{(A7)}$$

consequently, $\phi_{nLL_z}(x,p_\perp)$ satisfies the following orthonormal condition,

$$\int\frac{dx d^2p_\perp}{2(2\pi)^3}\phi'^*_{n'L'L_z'}(x,p_\perp)\phi_{nLL_z}(x,p_\perp) = \delta_{n',n}\delta_{L',L}\delta_{L_z',L_z}. \quad \text{(A8)}$$

The wave function is defined as

$$\phi_{nLm}(\{x\},\{k_\perp\}) = \sqrt{\frac{dk_{2z}}{dx_2}}\varphi_{nLm}\left(\frac{\vec{k}_1-\vec{k}_2}{2},\beta\right), \quad \text{(A9)}$$

with

$$\begin{aligned} \varphi_{n00}(\vec{k},\beta) &= \varphi_{ns}(\vec{k},\beta), \\ \varphi_{n1m}(\vec{k},\beta) &= k_m\varphi_{np}(\vec{k},\beta) = -\varepsilon(k_1+k_2,m)\cdot k\varphi_{np}(\vec{k},\beta), \\ \varphi_{n2m}(\vec{k},\beta) &= (kk)_m\varphi_{nd}(\vec{k},\beta) \\ &= \varepsilon_{\mu\nu}(k_1+k_2,m)k^\mu k^\nu\varphi_{nd}(\vec{k},\beta), \quad \text{(A10)} \end{aligned}$$

where $k_m \equiv \vec{\varepsilon}(m)\cdot\vec{k}$ (or, explicitly $k_{L_z=\pm1} \equiv \mp(k^x \pm ik^y)/\sqrt{2}$, $k_{L_z=0} \equiv k^z$), and φ_{ns} and φ_{np} are s-wave and p-wave wave functions, respectively. The kinematics are given by

$$\begin{aligned} M_0^{(\prime)2} &= \sum_{i=1}^2\frac{m_i^{(\prime)2}+k_{i\perp}^{(\prime)2}}{x_i}, \\ k_i^{(\prime)} &= \left(\frac{m_i^{(\prime)2}+k_{i\perp}^{(\prime)2}}{x_i^{(\prime)}M_0^{(\prime)}}, x_i^{(\prime)}M_0^{(\prime)}, \vec{k}_{i\perp}^{(\prime)}\right) \\ &= (e_i^{(\prime)}-k_{iz}^{(\prime)}, e_i^{(\prime)}+k_{iz}^{(\prime)}, \vec{k}_{i\perp}^{(\prime)}), \\ M_0^{(\prime)} &= e_1^{(\prime)}+e_2^{((\prime)}, \\ e_i^{(\prime)} &= \sqrt{m_i^{(\prime)2}+k_{i\perp}^{(\prime)2}+k_{iz}^{(\prime)2}} \\ &= \frac{x_i^{(\prime)}M_0^{(\prime)}}{2}+\frac{m_i^{(\prime)2}+k_{i\perp}^{(\prime)2}}{2x_i^{(\prime)}M_0^{(\prime)}}, \\ k_{iz}^{(\prime)} &= \frac{x_i^{(\prime)}M_0^{(\prime)}}{2}-\frac{m_i^{(\prime)2}+k_{i\perp}^{(\prime)2}}{2x_i^{(\prime)}M_0^{(\prime)}}, \\ 2M_0^{(\prime)}(e_{1(2)}^{(\prime)}+m_{1(2)}^{(\prime)}) &= (M_0^{(\prime)}+m_{1(2)}^{(\prime)})^2-m_{2(1)}^{(\prime)2}. \quad \text{(A11)} \end{aligned}$$

Under the constraint of $\sum_{i=1}^2 x_i = 1$ and $\sum_{i=1}^2 \vec{k}_i = 0$, one can easily obtain

$$\frac{dk_{2z}}{dx_2} = \frac{e_1 e_2}{x_1 x_2 M_0} = \frac{dk_{1z}}{dx_1}. \tag{A12}$$

For the heavy quark, the corresponding Melosh transform is [60,61],

$$\langle \lambda_1 | \mathcal{R}_M^\dagger (p_1^+, \vec{p}_{1\perp}, m_1) | s_1 \rangle = \frac{\bar{u}(p_1, \lambda_1) u_D(p_1, s_1)}{2m_1} \tag{A13}$$

with u and u_D the Dirac spinors in the light-front and instant forms, respectively. The Melosh transform for a scalar diquark is a trivial one, i.e.,

$$\langle \lambda_2 | \mathcal{R}_M^\dagger (p_2^+, \vec{p}_{2\perp}, m_2) | s_2 \rangle = 1, \tag{A14}$$

whereas the Melosh transform for an axial-vector diquark is more complicated

$$\langle \lambda_2 | \mathcal{R}_M^\dagger (p_2^+, \vec{p}_{2\perp}, m_2) | s_2 \rangle = -\varepsilon_{LF}^*(p_2, \lambda_2) \cdot \varepsilon_I(p_2, s_2), \tag{A15}$$

with ε_{LF} and ε_I the polarization vectors in light-front and instant forms, respectively.

It is convenient to use the covariant form for $\Psi^{JJ_z}_{nL_K S_{[qq]} J_l}$. We have

$$\begin{aligned}
\Psi^{1/2 J_z}_{nL_K S_{[qq]} J_l}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \lambda_2) &= \frac{1}{\sqrt{(M_0+m_1)^2 - m_2^2}} \bar{u}(p_1, \lambda_1) \Gamma_{L_K S_{[qq]} J_l} u(\bar{P}, J_z) \phi_{nL_K}(x_1, x_2, k_{1\perp}, k_{2\perp}), \\
\Psi^{3/2 J_z}_{nL_K S_{[qq]} J_l}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \lambda_2) &= \frac{1}{\sqrt{(M_0+m_1)^2 - m_2^2}} \bar{u}(p_1, \lambda_1) \Gamma^\mu_{L_K S_{[qq]} J_l} u_\mu(\bar{P}, J_z) \phi_{nL_K}(x_1, x_2, k_{1\perp}, k_{2\perp}),
\end{aligned} \tag{A16}$$

with $\Gamma^{(\mu)}_{L_K, n, S_{[qq]}}$ given in Eq. (7). Note that we have

$$\phi_{nL_K} \equiv \sqrt{\frac{dk_{2z}}{dx_2}} \varphi_{nL_K}, \tag{A17}$$

with φ_{nL_K} given in Eq. (9). The vertex functions Γ_{s00}, Γ_{s11}, and Γ_{p01} are taken from [16], while Γ^μ are new and the derivations can be found in Appendix B.

APPENDIX B: VERTEX FUNCTIONS

1. Some useful identities

We collect some useful identities for the derivation of vertex functions. Relations involving Melosh transform for spin-1/2 and spin-1 particles are given by

$$\langle \lambda_1 | \mathcal{R}_M^\dagger (x_1, k_{1\perp}, m_1) | s_1 \rangle \bar{u}_D(k_1, s_1) = \bar{u}(k_1, \lambda_1) \frac{u_D(k_1, s_1) \bar{u}_D(k_1, s_1)}{2m_1} = \bar{u}(k_1, \lambda_1), \tag{B1}$$

$$\langle \lambda_2 | \mathcal{R}_M^\dagger (x_2, k_{2\perp}, m_2) | s_2 \rangle \varepsilon_I^*(k_2, s_2) = -\varepsilon_{LF}^*(k_2, \lambda_2) \cdot \varepsilon_I(k_2, s_2) \varepsilon_I^*(k_2, s_2) = \varepsilon_{LF}^*(k_2, \lambda_2), \tag{B2}$$

where the polarization vector in the light-front form ε_{LF} has the following expression,

$$\begin{aligned}
\varepsilon^\mu_{LF}(\pm 1) &= \left[\frac{2}{P^+} \vec{\varepsilon}_\perp(\pm 1) \cdot \vec{P}_\perp, 0, \vec{\varepsilon}_\perp(\pm 1)\right], \qquad \vec{\varepsilon}_\perp(\pm 1) = \mp (1, \pm i)/\sqrt{2}, \\
\varepsilon^\mu_{LF}(0) &= \frac{1}{M_0}\left(\frac{-M_0^2 + P_\perp^2}{P^+}, P^+, P_\perp\right).
\end{aligned} \tag{B3}$$

Note that in the particle rest frame, the polarization vectors in the instant and light-front forms, ε_I and ε_{LF}, are identical, and likewise the spinors in the two different forms, u_D and u, are identical.

It is useful to expressed the relevant Clebsch-Gordan coefficients occurring in Eq. (A6) in compact forms:

$$\begin{aligned}
\left\langle \frac{1}{2} 1; s_1 s_2 \middle| \frac{1}{2} 1; \frac{1}{2} J_z \right\rangle &= \frac{1}{\sqrt{3}} \chi^\dagger_{s_1} \vec{\sigma} \cdot \vec{\varepsilon}^*(k_1 + k_2, s_2) \chi_{J_z} \\
&= \frac{1}{\sqrt{3[(M_0+m_1)^2 - m_2^2]}} \bar{u}_D(k_1, s_1) \gamma_5 \not{\varepsilon}^*(k_1 + k_2, s_2) u(k_1 + k_2, J_z),
\end{aligned} \tag{B4}$$

$$\left\langle \frac{1}{2}1; s_1 J_{lz} \middle| \frac{1}{2}1; \frac{3}{2}J_z \right\rangle = -\chi^\dagger_{s_1} \varepsilon^{*\mu}(k_1+k_2, J_{lz}) \chi_{\mu J_z} = -\frac{1}{\sqrt{(M_0+m_1)^2 - m_2^2}} \bar{u}_D(k_1, s_1) \varepsilon^{*\mu}(k_1+k_2, J_{lz}) u_\mu(k_1+k_2, J_z), \tag{B5}$$

$$\left\langle \frac{1}{2}2; s_1 J_{lz} \middle| \frac{1}{2}2; \frac{3}{2}J_z \right\rangle = -\sqrt{\frac{2}{5}} \frac{1}{\sqrt{(M_0+m_1)^2 - m_2^2)}} \varepsilon^*_{\mu\nu}(\bar{P}, J_{lz}) \bar{u}_D(k_1, s_1) \gamma_5 \gamma^\nu u^\mu(k_1+k_2, J_z), \tag{B6}$$

$$\langle 11; s_2 s_4 | 11; 2J_{lz} \rangle = \varepsilon^\nu(\bar{P}, s_2) \varepsilon^\mu(\bar{P}, s_4) \varepsilon^*_{\mu\nu}(\bar{P}, J_{lz}), \tag{B7}$$

where u^μ is the Rarita-Schwinger spinor in the light-front form with

$$u^\mu(k_1+k_2, J_z) = \left\langle \frac{1}{2}1; s_3 s_4 \middle| \frac{1}{2}1; \frac{3}{2}J_z \right\rangle \varepsilon^\mu_{LF}(k_1+k2, s_4) u(k_1+k_2, s_3), \tag{B8}$$

and $\varepsilon_{\rho\sigma}(\bar{P}, m)$ is the spin-2 polarization vector defined as

$$\varepsilon_{\mu\nu}(\bar{P}, s) \equiv \langle 11; m'm'' | 11; 2s \rangle \varepsilon_\mu(\bar{P}, m') \varepsilon_\nu(\bar{P}, m''). \tag{B9}$$

Note the Rarita-Schwinger spinor satisfies the following relations: (see, e.g., [46])

$$\bar{P}^\mu u_\mu(\bar{P}, J_z) = 0, \qquad \gamma^\mu u_\mu(\bar{P}, J_z) = 0, \qquad \bar{P\!\!\!/} u_\mu(\bar{P}) = M_0 u_\mu(\bar{P}, J_z). \tag{B10}$$

It is useful to note that we have

$$u^+(\bar{P}, J_z) = \sqrt{\frac{2}{3}} \delta_{J_z S_z} \varepsilon^+_{LF}(\bar{P}, 0) u(\bar{P}, S_z) = \sqrt{\frac{2}{3}} \delta_{J_z S_z} \frac{\bar{P}^+}{M_0} u(\bar{P}, S_z), \tag{B11}$$

which can be easily proved by using Eqs. (B3) and (B8), in particular, $u^+(\bar{P}, \pm 3/2) = 0$.

The following relations of the polarization vectors will be proved to be useful,

$$\varepsilon^\mu(k_2, s_2) = \varepsilon^\mu(\bar{P}, s_2) - \frac{M_0 k_2^\mu + m_2 \bar{P}^\mu}{m_2 M_0} \frac{\varepsilon(\bar{P}, s_2) \cdot k_2}{e_2 + m_2}, \tag{B12}$$

$$\varepsilon^*_\mu(\bar{P}, m) \varepsilon_\nu(\bar{P}, m) = -G_{\mu\nu}, \tag{B13}$$

$$\varepsilon^*_{\mu\nu}(\bar{P}, m) \varepsilon_{\rho\sigma}(\bar{P}, m) = \frac{1}{2} G_{\mu\rho} G_{\nu\sigma} + \frac{1}{2} G_{\mu\sigma} G_{\nu\rho} - \frac{1}{3} G_{\mu\nu} G_{\rho\sigma}, \tag{B14}$$

where $G_{\mu\nu}$ is defined as

$$G_{\mu\nu}(\bar{P}) \equiv g_{\mu\nu} - \frac{\bar{P}_\mu \bar{P}_\nu}{M_0^2}. \tag{B15}$$

The relations in Eqs. (B4), (B5), and (B7) can be easily proved by using Eq. (B9) and Eq. (A11). Explicitly, we use

$$\begin{aligned} u_D(k_1,s) &= \frac{k_1\cdot\gamma+m_1}{\sqrt{e_1+m_1}}\begin{pmatrix}\chi_s\\0\end{pmatrix} = \frac{1}{\sqrt{e_1+m_1}}\begin{pmatrix}(e_1+m_1)\chi_s\\ \vec\sigma\cdot\vec p\chi_s\end{pmatrix},\\ u(k_1+k_2,\lambda) &= \frac{(k_1+k_2)\cdot\gamma+M_0}{\sqrt{2M_0}}\gamma^+\gamma^0\begin{pmatrix}\chi_\lambda\\0\end{pmatrix} = \sqrt{2M_0}\begin{pmatrix}\chi_\lambda\\0\end{pmatrix},\\ u^\mu(k_1+k_2,\lambda) &= \frac{(k_1+k_2)\cdot\gamma+M_0}{\sqrt{2M_0}}\gamma^+\gamma^0\begin{pmatrix}\chi^\mu_\lambda\\0\end{pmatrix} = \sqrt{2M_0}\begin{pmatrix}\chi^\mu_\lambda\\0\end{pmatrix},\\ \chi^\mu_\lambda &\equiv \left\langle\frac{1}{2}1;m'm''\middle|\frac{1}{2}1;\frac{3}{2},\lambda\right\rangle\varepsilon^\mu(k_1+k_2,m'')\chi_{m'}, \end{aligned} \tag{B16}$$

the standard Dirac representation of γ^μ, γ_5, $\varepsilon(k_1+k_2,s)=(0,\vec\varepsilon(s))$ with $\vec\varepsilon(\pm1)=\mp(1,\pm i,0)/\sqrt{2}$, $\vec\varepsilon(0)=(0,0,1)$. One can see [16] for the derivation of Eqs. (B4) and (B12). Equation (B5) can be easily proved using the above equation and the usual normalization of the polarization vector: $\varepsilon^*_\mu(\bar P,m)\varepsilon^\mu(\bar P,m')=-\delta_{m,m'}$.

To prove Eq. (B6), we need to use the following identity on Clebsch-Gordan coefficient:

$$\left\langle\frac{1}{2}2;s_1J_{lz}\middle|\frac{1}{2}2;\frac{3}{2}J_z\right\rangle = \sqrt{\frac{6}{5}}\left\langle\frac{1}{2}1;s_1s_2\middle|\frac{1}{2}1;\frac{1}{2}s_3\right\rangle\langle 11;s_2s_4|11;2J_{lz}\rangle\left\langle\frac{1}{2}1;s_3s_4\middle|\frac{1}{2}1;\frac{3}{2}J_z\right\rangle, \tag{B17}$$

and use Eqs. (B4), (B7), (B8), and (B13),

$$\begin{aligned} \left\langle\frac{1}{2}2;s_1J_{lz}\middle|\frac{1}{2}2;\frac{3}{2}J_z\right\rangle &= \sqrt{\frac{6}{5}}\left\langle\frac{1}{2}1;s_1s_2\middle|\frac{1}{2}1;\frac{1}{2}s_3\right\rangle\langle 11;s_2s_4|11;2J_{lz}\rangle\left\langle\frac{1}{2}1;s_3s_4\middle|\frac{1}{2}1;\frac{3}{2}J_z\right\rangle\\ &= -\sqrt{\frac{2}{5}}\frac{1}{\sqrt{(M_0+m_1)^2-m_2^2}}\varepsilon^*_{\mu\nu}(J_{lz})\bar u_D(k_1,s_1)\gamma_5\gamma^\nu u^\mu(k_1+k_2,J_z). \end{aligned} \tag{B18}$$

Using Eq. (B9) and the following identity of the Clebsch-Gordan coefficients,

$$\langle 11;a,b|11;2m\rangle\langle 11;c,d|11;2m\rangle = \frac{1}{2}\delta_{a,c}\delta_{b,d}+\frac{1}{2}\delta_{a,d}\delta_{b,c}-\frac{1}{3}(-)^{a+c}\delta_{a,-b}\delta_{c,-d}, \tag{B19}$$

and noting that $\varepsilon^*(\bar P,m)=(-)^m\varepsilon(\bar P,-m)$, we have

$$\begin{aligned} \varepsilon^*_{\mu\nu}(\bar P,m)\varepsilon_{\rho\sigma}(\bar P,m) &= \frac{1}{2}\varepsilon^*_\mu(\bar P,m)\varepsilon_\rho(\bar P,m)\varepsilon^*_\nu(\bar P,m')\varepsilon_\sigma(\bar P,m')+\frac{1}{2}\varepsilon^*_\mu(\bar P,m)\varepsilon_\sigma(\bar P,m)\varepsilon^*_\nu(\bar P,m')\varepsilon_\rho(\bar P,m')\\ &\quad -\frac{1}{3}\varepsilon^*_\mu(\bar P,m)\varepsilon_\nu(\bar P,m)\varepsilon_\rho(\bar P,m')\varepsilon^*_\sigma(\bar P,m'), \end{aligned} \tag{B20}$$

which leads to Eq. (B14) after performing the polarization sum, Eq. (B13).

Note that using Eqs. (B12), (B2) and (B13), we have

$$\begin{aligned} \langle\lambda_2|\mathcal{R}^\dagger_M(x_2,k_{2\perp},m_2)|s_2\rangle\varepsilon^{*\nu}(\bar P,s_2) &= -\varepsilon^*_{LF}(k_2,\lambda_2)\cdot\varepsilon(k_2,s_2)\varepsilon^{*\nu}(\bar P,s_2)\\ &= \varepsilon^{*\nu}_{LF}(k_2,\lambda_2)-\frac{M_0k_2^\nu+m_2\bar P^\nu}{(\bar P\cdot k_2+m_2M_0)}\frac{\varepsilon^*_{LF}(k_2,\lambda_2)\cdot\bar P}{M_0}, \end{aligned} \tag{B21}$$

which will be useful in obtaining vertex functions.

2. Γ^μ for states with configuration $(L_K, S^P_{[qq]}, J^P_l, J^P) = (1, 0^+, 1^-, \frac{3}{2}^-)$

From Eq. (A6) the corresponding momentum-space wave-function $\Psi^{JJ_z}_{nL_K S_{[qq]} J_l}$ is given by

$$\Psi^{\frac{3}{2}J_z}_{np01}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \lambda_2) = \langle \lambda_1 | \mathcal{R}^\dagger_M(p_1^+, \vec{p}_{1\perp}, m_1) | s_1 \rangle \left\langle \frac{1}{2} 1; s_1 J_{lz} \middle| \frac{1}{2} 1; \frac{3}{2} J_z \right\rangle \langle 10; L_z 0 | 10; 1 J_{lz} \rangle \phi_{n1L_z}(x_1, x_2, k_{1\perp}, k_{2\perp}). \tag{B22}$$

It can be expressed as

$$\Psi^{\frac{3}{2}J_z}_{np01}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \lambda_2) = \frac{1}{\sqrt{(M_0 + m_1)^2 - m_2^2}} \bar{u}(p_1, \lambda_1) \Gamma^\mu_{p01} u_\mu(\bar{P}, J_z) \phi_{np}(x_1, x_2, k_{1\perp}, k_{2\perp}), \tag{B23}$$

with

$$\Gamma^\mu_{p01} = -\frac{1}{2}(p_1 - p_2)^\mu, \tag{B24}$$

where Eqs. (B1), (B5), (A10), (B13), and (B10) have been used.

3. Γ^μ for states with configuration $(L_K, S^P_{[qq]}, J^P_l, J^P) = (0, 1^+, 1^+, \frac{3}{2}^+)$

From Eq. (A6) the corresponding momentum-space wave-function $\Psi^{JJ_z}_{nL_K S_{[qq]} J_l}$ is given by

$$\Psi^{3/2 J_z}_{ns11}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \lambda_2) = \langle \lambda_1 | \mathcal{R}^\dagger_M(p_1^+, \vec{p}_{1\perp}, m_1) | s_1 \rangle \langle \lambda_2 | \mathcal{R}^\dagger_M(p_2^+, \vec{p}_{2\perp}, m_2) | s_2 \rangle \left\langle \frac{1}{2} 1; s_1 J_{lz} \middle| \frac{1}{2} 1; \frac{3}{2} J_z \right\rangle \langle 01; 0 s_2 | 01; 1 J_{lz} \rangle \times \phi_{n00}(x_1, x_2, k_{1\perp}, k_{2\perp}). \tag{B25}$$

It can be expressed as

$$\Psi^{3/2 J_z}_{ns11}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \lambda_2) = \frac{1}{\sqrt{(M_0 + m_1)^2 - m_2^2}} \bar{u}(p_1, \lambda_1) \Gamma^\mu_{s11} u_\mu(\bar{P}, J_z) \phi_{ns}(x_1, x_2, k_{1\perp}, k_{2\perp}), \tag{B26}$$

with

$$\Gamma^\mu_{s11} = -\left(\varepsilon^{*\mu}_{LF}(p_2, \lambda_2) - \frac{p_2^\mu}{\bar{P} \cdot p_2 + m_2 M_0} \varepsilon^*_{LF}(p_2, \lambda_2) \cdot \bar{P} \right), \tag{B27}$$

where Eqs. (B1), (B5), (B21), and (B10) have been used.

4. Γ^μ for states with configuration $(L_K, S^P_{[qq]}, J^P_l, J^P) = (1, 1^+, 2^-, \frac{3}{2}^-)$

From Eq. (A6) the corresponding momentum-space wave function $\Psi^{JJ_z}_{nL_K S_{[qq]} J_l}$ is given by

$$\Psi^{3/2 J_z}_{np12}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \lambda_2) = \langle \lambda_1 | \mathcal{R}^\dagger_M(p_1^+, \vec{p}_{1\perp}, m_1) | s_1 \rangle \langle \lambda_2 | \mathcal{R}^\dagger_M(p_2^+, \vec{p}_{2\perp}, m_2) | s_2 \rangle \left\langle \frac{1}{2} 2; s_1 J_{lz} \middle| \frac{1}{2} 2; \frac{3}{2} J_z \right\rangle \langle 11; L_z s_2 | 11; 2 J_{lz} \rangle \times \phi_{n1L_z}(x_1, x_2, k_{1\perp}, k_{2\perp}). \tag{B28}$$

It can be expressed as

$$\Psi^{3/2 J_z}_{np12}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \lambda_2) = \frac{1}{\sqrt{(M_0 + m_1)^2 - m_2^2}} \bar{u}(p_1, \lambda_1) \Gamma^\mu_{p12} u_\mu(\bar{P}, J_z) \phi_{np}(x_1, x_2, k_{1\perp}, k_{2\perp}), \tag{B29}$$

with

$$\Gamma^{\mu}_{p12} = -\frac{1}{2\sqrt{10}}\gamma_5\left[(\varepsilon^{*\mu}(p_2,\lambda_2) + \left(p_1 - p_2\right)^{\mu}\frac{\varepsilon^{*}_{LF}(p_2,\lambda_2)\cdot\bar{P}}{\bar{P}\cdot p_2 + M_0 m_2}\right)\left(\not{p}_1 - \not{p}_2 - \frac{m_1^2 - m_2^2}{M_0}\right) + (p_1 - p_2)^{\mu}\left(\not{\varepsilon}^{*}_{LF}(p_2,\lambda_2) - \frac{\varepsilon^{*}_{LF}(p_2,\lambda_2)\cdot\bar{P}}{M_0}\right)\right]. \tag{B30}$$

The above result can be obtained by using Eqs. (B1), (B2), (B6), (A10), (B12), (B9), and (B14). In fact, from Eqs. (B2), (B12), and (B9), we have

$$\begin{aligned}
&\langle 11; L_z s_2 | 11; 2J_{lz}\rangle \varepsilon_{\sigma}(\bar{P}, L_z)\langle \lambda_2 | \mathcal{R}^{\dagger}_{M}(x_2, k_{2\perp}, m_2) | s_2\rangle \\
&= -\langle 11; L_z s_2 | 11; 2J_{lz}\rangle \varepsilon^{*}_{LF}(k_2,\lambda_2)\cdot\varepsilon(k_2, s_2)\varepsilon_{\sigma}(\bar{P}, L_z) \\
&= -\left(\varepsilon^{*\rho}_{LF}(k_2,\lambda_2) + \frac{\varepsilon^{*}_{LF}(k_2,\lambda_2)\cdot\bar{P}}{M_0(e_2+m_2)}\frac{(k_1-k_2)^{\rho}}{2}\right)\varepsilon_{\rho\sigma}(\bar{P}, J_{lz})
\end{aligned} \tag{B31}$$

Putting them together we obtain

$$\begin{aligned}
\Gamma^{\mu}_{p12}u_{\mu} &= -\sqrt{\frac{2}{5}}\left(\frac{k_1-k_2}{2}\right)^{\sigma}\gamma_5\gamma^{\nu}\left(\varepsilon^{*\rho}_{LF}(k_2,\lambda_2) + \frac{\varepsilon^{*}_{LF}(k_2,\lambda_2)\cdot\bar{P}}{M_0(e_2+m_2)}\left(\frac{k_1-k_2}{2}\right)^{\rho}\right)\left(\frac{1}{2}G_{\mu\rho}G_{\nu\sigma} + \frac{1}{2}G_{\mu\sigma}G_{\nu\rho} - \frac{1}{3}G_{\mu\nu}G_{\rho\sigma}\right)u^{\mu} \\
&= -\frac{1}{\sqrt{10}}\gamma_5\left(\varepsilon^{*\rho}_{LF}(k_2,\lambda_2)\left(\frac{k_1-k_2}{2}\right)^{\sigma} + \frac{\varepsilon^{*}_{LF}(k_2,\lambda_2)\cdot\bar{P}}{M_0(e_2+m_2)}\left(\frac{k_1-k_2}{2}\right)^{\sigma}\left(\frac{k_1-k_2}{2}\right)^{\rho}\right) \\
&\quad\times\left[g_{\mu\rho}\left(\gamma_{\sigma} - \frac{\bar{P}_{\sigma}}{M_0}\right) + g_{\mu\sigma}\left(\gamma_{\rho} - \frac{\bar{P}_{\rho}}{M_0}\right)\right]u^{\mu}.
\end{aligned} \tag{B32}$$

It can be further simplified into the following expression:

$$\Gamma^{\mu}_{p12}u_{\mu} = -\frac{1}{2\sqrt{10}}\gamma_5\left[(\varepsilon^{*\mu}(k_2,\lambda_2) + (k_1-k_2)^{\mu}\frac{\varepsilon^{*}_{LF}(k_2,\lambda_2)\cdot\bar{P}}{M_0(e_2+m_2)}\right)\left(\not{k}_1 - \not{k}_2 - \frac{m_1^2-m_2^2}{M_0}\right) + (k_1-k_2)^{\mu}\left(\not{\varepsilon}^{*}_{LF}(k_2,\lambda_2) - \frac{\varepsilon^{*}_{LF}(k_2,\lambda_2)\cdot\bar{P}}{M_0}\right)\right]u_{\mu}. \tag{B33}$$

Boosting $k_i \to p_i$ in the LF boost we obtain Eqs. (B29) and (B30).

APPENDIX C: KINEMATICS

The decay rates and asymmetries read [22]

$$\begin{aligned}
\Gamma[\mathcal{B}_b \to \mathcal{B}_c(1/2)P] &= \frac{p_c}{8\pi}\left[\frac{(M+M')^2 - m_P^2}{M^2}|A|^2 + \frac{(M-M')^2 - m_P^2}{M^2}|B|^2\right], \\
\Gamma[\mathcal{B}_b \to \mathcal{B}_c(1/2)V(A)] &= \frac{p_c}{4\pi}\frac{E'+M'}{M}\left[2(|S^{(\prime)}|^2 + |P_2^{(\prime)}|^2) + \frac{E^2_{V(A)}}{m^2_{V(A)}}(|S^{(\prime)} + D^{(\prime)}|^2 + |P_1^{(\prime)}|^2)\right],
\end{aligned} \tag{C1}$$

$$\begin{aligned}
\alpha[\mathcal{B}_b \to \mathcal{B}_c(1/2)P] &= -\frac{2\kappa\mathrm{Re}(A^*B)}{|A|^2 + \kappa^2|B|^2}, \\
\alpha[\mathcal{B}_b \to \mathcal{B}_c(1/2)V(A)] &= \frac{4m^2_{V(A)}\mathrm{Re}(S^{(\prime)*}P_2) + 2E_V^2\mathrm{Re}(S^{(\prime)} + D^{(\prime)})^* P_1^{(\prime)}}{2m^2_{V(A)}(|S^{(\prime)}|^2 + |P_2^{(\prime)}|^2) + E^2_{V(A)}(|S^{(\prime)} + D^{(\prime)}|^2 + |P_1^{(\prime)}|^2)},
\end{aligned} \tag{C2}$$

with $\kappa \equiv p_c/(E'+M')$ and p_c is the momentum in the center of mass frame

$$S^{(\prime)} = -A_1^{(\prime)}, \qquad P_1^{(\prime)} = -\frac{p_c}{E_{V(A)}}\left(\frac{M+M'}{E'+M'}B_1^{(\prime)} + MB_2^{(\prime)}\right),$$
$$P_2^{(\prime)} = \frac{p_c}{E'+M'}B_1^{(\prime)}, \qquad D^{(\prime)} = -\frac{p_c^2}{E_{V(A)}(E'+M')}(A_1^{(\prime)} - MA_2^{(\prime)}). \tag{C3}$$

The $\mathcal{B}_b \to \mathcal{B}_c(3/2)P$ decay rates and asymmetries are given by

$$\Gamma[\mathcal{B}_b \to \mathcal{B}_c(3/2)P] = \frac{p_c^3}{12\pi}\left[\frac{(M+M')^2 - m_P^2}{M'^2}|C|^2 + \frac{(M-M')^2 - m_P^2}{M'^2}|D|^2\right] \tag{C4}$$

and

$$\alpha[\mathcal{B}_b \to \mathcal{B}_c(3/2)P] = -\frac{2\kappa \mathrm{Re}(A^* B)}{|A|^2 + \kappa^2|B|^2}, \tag{C5}$$

respectively. The decay rates and asymmetries of $\mathcal{B}_b \to \mathcal{B}_c(3/2)V$ decays read [20,22,62]

$$\Gamma = \frac{p_c}{16\pi M^2}\sum_{\lambda_V}(|h_{\lambda_V+1/2,\lambda_V;1/2}|^2 + |h_{-(\lambda_V+1/2),-\lambda_V;-1/2}|^2) = \frac{p_c}{32\pi M^2}\sum_{\lambda_V}(|h^{PV}_{\lambda_V+1/2,\lambda_V;1/2}|^2 + |h^{PC}_{\lambda_V+1/2,\lambda_V;1/2}|^2)$$
$$\alpha = \frac{\sum_{\lambda_V}|h_{\lambda_V+1/2,\lambda_V;1/2}|^2 - |h_{-(\lambda_V+1/2),-\lambda_V;-1/2}|^2}{\sum_{\lambda_V}|h_{\lambda_V+1/2,\lambda_V;1/2}|^2 + |h_{-(\lambda_V+1/2),-\lambda_V;-1/2}|^2} = \frac{\sum_{\lambda_V} 2h^{PV}_{\lambda_V+1/2,\lambda_V;1/2}h^{PC}_{(\lambda_V+1/2),\lambda_V;1/2}}{\sum_{\lambda_V}|h^{PV}_{\lambda_V+1/2,\lambda_V;1/2}|^2 + |h^{PC}_{\lambda_V+1/2,\lambda_V;1/2}|^2} \tag{C6}$$

where $h_{\lambda',\lambda_V;\lambda} \equiv \langle \mathcal{B}_c(\lambda')V(\lambda_V)|H_W|\mathcal{B}_b(\lambda)\rangle$ is the helicity amplitudes and

$$h^{PV}_{\lambda',\lambda_V;1/2} \equiv h_{\lambda',\lambda_V,1/2} \pm h_{-\lambda',-\lambda_V,-1/2}, \tag{C7}$$

with

$$h^{PV(PC)}_{3/2,1;1/2} = \mp\sqrt{2Q_\pm}E_1^{PV(PC)},$$
$$h^{PV(PC)}_{-1/2,-1;1/2} = \mp\sqrt{\frac{2}{3}}\sqrt{Q_\pm}\left[E_1^{PV(PC)} - \frac{Q_\mp}{M'}E_2^{PV(PC)}\right],$$
$$h^{PV(PC)}_{1/2,0;1/2} = \mp\frac{\sqrt{Q_\pm}}{2\sqrt{3}M'm_V}[2(M^2 - M'^2 - m_V^2)E_1^{PV(PC)} \pm 2Q_\mp(M \pm M')E_2^{PV(PC)} + Q_+Q_-E_3^{PV(PC)}], \tag{C8}$$

$E_i^{PV} \equiv C_i$, $E_i^{PC} \equiv D_i$ and $Q_\pm \equiv (M \pm M')^2 - m_V^2$. The formulas for the decay rates and asymmetries of $\mathcal{B}_b \to \mathcal{B}_c(3/2)V$ decays can be obtained by using the above formulas, but with λ_V, m_V, C_i and D_i and replaced by λ_A, m_A, C'_i and D'_i, respectively.

[1] R. Aaij et al. (LHCb Collaboration), Study of the $D^0 p$ amplitude in $\Lambda_b^0 \to D^0 p\pi^-$ decays, J. High Energy Phys. 05 (2017) 030.

[2] R. Aaij et al. (LHCb Collaboration), Observation of Five New Narrow Ω_c^0 States Decaying to $\Xi_c^+ K^-$, Phys. Rev. Lett. **118**, 182001 (2017).

[3] J. Yelton et al. (Belle Collaboration), Observation of excited Ω_c charmed Baryons in e^+e^- collisions, Phys. Rev. D **97**, 051102 (2018).

[4] M. Tanabashi et al. (Particle Data Group), Review of particle physics, Phys. Rev. D **98**, 030001 (2018).

[5] C. G. Wohl, reviews on charmed baryons in [4].

[6] H. Y. Cheng and C. W. Chiang, Quantum numbers of Ω_c states and other charmed baryons, Phys. Rev. D **95**, 094018 (2017).

[7] H. X. Chen, Q. Mao, W. Chen, A. Hosaka, X. Liu, and S. L. Zhu, Decay properties of P-wave charmed baryons from light-cone QCD sum rules, Phys. Rev. D **95**, 094008 (2017).

[8] M. Karliner and J. L. Rosner, Very narrow excited Ω_c baryons, Phys. Rev. D **95**, 114012 (2017).
[9] K. L. Wang, L. Y. Xiao, X. H. Zhong, and Q. Zhao, Understanding the newly observed Ω_c states through their decays, Phys. Rev. D **95**, 116010 (2017).
[10] M. Padmanath and N. Mathur, Quantum Numbers of Recently Discovered Ω_c^0 Baryons from Lattice QCD, Phys. Rev. Lett. **119**, 042001 (2017).
[11] Z. G. Wang, Analysis of $\Omega_c(3000)$, $\Omega_c(3050)$, $\Omega_c(3066)$, $\Omega_c(3090)$ and $\Omega_c(3119)$ with QCD sum rules, Eur. Phys. J. C **77**, 325 (2017).
[12] B. Chen and X. Liu, New Ω_c^0 baryons discovered by LHCb as the members of $1P$ and $2S$ states, Phys. Rev. D **96**, 094015 (2017).
[13] S. S. Agaev, K. Azizi, and H. Sundu, On the nature of the newly discovered Ω states, Europhys. Lett. **118**, 61001 (2017).
[14] H. Y. Cheng and C. K. Chua, Strong decays of charmed baryons in heavy hadron chiral perturbation theory, Phys. Rev. D **75**, 014006 (2007).
[15] H. Y. Cheng and C. K. Chua, Strong decays of charmed baryons in heavy hadron chiral perturbation theory: An update, Phys. Rev. D **92**, 074014 (2015).
[16] C. K. Chua, Color-allowed bottom baryon to charmed baryon nonleptonic decays, Phys. Rev. D **99**, 014023 (2019).
[17] R. Aaij *et al.* (LHCb Collaboration), Study of the kinematic dependences of Λ_b^0 production in pp collisions and a measurement of the $\Lambda_b^0 \to \Lambda_c^+ \pi^-$ branching fraction, J. High Energy Phys. 08 (2014) 143.
[18] R. Aaij *et al.* (LHCb Collaboration), Searches for Λ_b^0 and Ξ_b^0 decays to $K_S^0 p\pi^-$ and $K_S^0 pK^-$ final states with first observation of the $\Lambda_b^0 \to K_S^0 p\pi^-$ decay, J. High Energy Phys. 04 (2014) 087.
[19] R. Aaij *et al.* (LHCb Collaboration), Study of Beauty Hadron Decays into Pairs of Charm Hadrons, Phys. Rev. Lett. **112**, 202001 (2014).
[20] D. Ebert, R. N. Faustov, and V. O. Galkin, Semileptonic decays of heavy baryons in the relativistic quark model, Phys. Rev. D **73**, 094002 (2006).
[21] T. Mannel and W. Roberts, Nonleptonic Lambda(b) decays at colliders, Z. Phys. C **59**, 179 (1993).
[22] H. Y. Cheng, Nonleptonic weak decays of bottom baryons, Phys. Rev. D **56**, 2799 (1997); Erratum, Phys. Rev. D **99**, 079901(E) (2019).
[23] M. A. Ivanov, J. G. Korner, V. E. Lyubovitskij, and A. G. Rusetsky, Exclusive nonleptonic bottom to charm baryon decays including nonfactorizable contributions, Mod. Phys. Lett. A **13**, 181 (1998).
[24] M. A. Ivanov, J. G. Korner, V. E. Lyubovitskij, and A. G. Rusetsky, Exclusive nonleptonic decays of bottom and charm baryons in a relativistic three quark model: Evaluation of nonfactorizing diagrams, Phys. Rev. D **57**, 5632 (1998).
[25] A. K. Giri, L. Maharana, and R. Mohanta, Two-body nonleptonic Λ_b decays with $1/M_Q$ corrections, Mod. Phys. Lett. A **13**, 23 (1998).
[26] Fayyazuddin and Riazuddin, Two-body nonleptonic Λ_b decays in quark model with factorization ansatz, Phys. Rev. D **58**, 014016 (1998).
[27] R. Mohanta, A. K. Giri, M. P. Khanna, M. Ishida, S. Ishida, and M. Oda, Hadronic weak decays of Λ_b baryon in the covariant oscillator quark model, Prog. Theor. Phys. **101**, 959 (1999).
[28] H. H. Shih, S. C. Lee, and H. n. Li, Applicability of perturbative QCD to $\Lambda_b \to \Lambda_c$ decays, Phys. Rev. D **61**, 114002 (2000).
[29] C. Albertus, E. Hernandez, and J. Nieves, Nonrelativistic constituent quark model and HQET combined study of semileptonic decays of Lambda(b) and Xi(b) baryons, Phys. Rev. D **71**, 014012 (2005).
[30] H. W. Ke, X. Q. Li, and Z. T. Wei, Diquarks and Lambda(b) → Lambda(c) weak decays, Phys. Rev. D **77**, 014020 (2008).
[31] H. W. Ke, X. H. Yuan, X. Q. Li, Z. T. Wei, and Y. X. Zhang, $\Sigma_b \to \Sigma_c$ and $\Omega_b \to \Omega_c$ weak decays in the light-front quark model, Phys. Rev. D **86**, 114005 (2012).
[32] W. Detmold, C. Lehner, and S. Meinel, $\Lambda_b \to p\ell^-\bar{\nu}_\ell$ and $\Lambda_b \to \Lambda_c\ell^-\bar{\nu}_\ell$ form factors from lattice QCD with relativistic heavy quarks, Phys. Rev. D **92**, 034503 (2015).
[33] J. Zhu, Z. T. Wei, and H. W. Ke, Semileptonic and nonleptonic weak decays of Λ_b^0, Phys. Rev. D **99**, 054020 (2019).
[34] Z. X. Zhao, Weak decays of heavy baryons in the light-front approach, Chin. Phys. C **42**, 093101 (2018).
[35] T. Gutsche, M. A. Ivanov, J. G. Krner, and V. E. Lyubovitskij, Nonleptonic two-body decays of single heavy baryons Λ_Q, Ξ_Q, and Ω_Q ($Q = b, c$) induced by W emission in the covariant confined quark model, Phys. Rev. D **98**, 074011 (2018).
[36] H. W. Ke, N. Hao, and X. Q. Li, Revisiting $\Lambda_b \to \Lambda_c$ and $\Sigma_b \to \Sigma_c$ weak decays in the light-front quark model, Eur. Phys. J. C **79**, 540 (2019).
[37] N. Isgur and M. B. Wise, Heavy baryon weak form-factors, Nucl. Phys. **B348**, 276 (1991).
[38] N. Isgur, M. B. Wise, and M. Youssefmir, Excited charm baryons in semileptonic Lambda(b) decay and their contribution to a Bjorken sum rule, Phys. Lett. B **254**, 215 (1991).
[39] T. M. Yan, H. Y. Cheng, C. Y. Cheung, G. L. Lin, Y. C. Lin, and H. L. Yu, Heavy quark symmetry and chiral dynamics, Phys. Rev. D **46**, 1148 (1992); Erratum, Phys. Rev. D **55**, 5851(E) (1997).
[40] Q. P. Xu, A Bjorken sum rule for semileptonic Omega(b) decays to ground and excited charmed baryon states, Phys. Rev. D **48**, 5429 (1993).
[41] C. K. Chow, New universality of the baryon Isgur-Wise form-factor in the large N(c) limit, Phys. Rev. D **51**, 1224 (1995).
[42] H. Y. Cheng, C. K. Chua, and C. W. Hwang, Covariant light front approach for s wave and p wave mesons: Its application to decay constants and form-factors, Phys. Rev. D **69**, 074025 (2004).
[43] H. Y. Cheng, C. Y. Cheung, and C. W. Hwang, Mesonic form-factors and the Isgur-Wise function on the light front, Phys. Rev. D **55**, 1559 (1997).
[44] H. Y. Cheng, C. K. Chua, and C. W. Hwang, Light front approach for heavy pentaquark transitions, Phys. Rev. D **70**, 034007 (2004).

[45] F. Schlumpf, Relativistic constituent quark model of electroweak properties of baryons, Phys. Rev. D **47**, 4114 (1993); Erratum Phys. Rev. D **49**, 6246(E) (1994).

[46] T. Moroi, Effects of the gravitino on the inflationary universe, arXiv:hep-ph/9503210.

[47] R. Mertig, M. Bohm, and A. Denner, FEYN CALC: Computer algebraic calculation of Feynman amplitudes, Comput. Phys. Commun. **64**, 345 (1991).

[48] V. Shtabovenko, R. Mertig, and F. Orellana, New developments in FeynCalc 9.0, Comput. Phys. Commun. **207**, 432 (2016).

[49] D. Ebert, R. N. Faustov, and V. O. Galkin, Masses of tetraquarks with open charm and bottom, Phys. Lett. B **696**, 241 (2011).

[50] W. Jaus, Semileptonic decays of B and d mesons in the light front formalism, Phys. Rev. D **41**, 3394 (1990).

[51] W. Jaus and D. Wyler, The rare decays of $B \to K$ lepton anti-lepton and $B \to K^*$ lepton anti-lepton, Phys. Rev. D **41**, 3405 (1990).

[52] W. Jaus, Semileptonic, radiative, and pionic decays of B, B^* and D, D^* mesons, Phys. Rev. D **53**, 1349 (1996); Erratum, Phys. Rev. D **54**, 5904(E) (1996).

[53] W. Jaus, Covariant analysis of the light front quark model, Phys. Rev. D **60**, 054026 (1999).

[54] H. Y. Cheng and C. K. Chua, Covariant light front approach for $B \to K^*\gamma, K_1\gamma, K_2^*\gamma$ decays, Phys. Rev. D **69**, 094007 (2004); Erratum, Phys. Rev. D **81**, 059901(E) (2010).

[55] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, QCD factorization in $B \to \pi K, \pi\pi$ decays and extraction of Wolfenstein parameters, Nucl. Phys. **B606**, 245 (2001).

[56] J. Charles, A. Höcker, H. Lacker, S. Laplace, F. R. Le Diberder, J. Malclés, J. Ocariz, M. Pivk, and L. Roos, *CP* violation and the CKM matrix: Assessing the impact of the asymmetric B factories, Eur. Phys. J. C **41**, 1 (2005), updated results available at, http://ckmfitter.in2p3.fr.

[57] J. Rosner, S. Stone, and R. Van de Water, Review on leptonic decays of charged pseudoscalar mesons in [4].

[58] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, QCD factorization for exclusive, nonleptonic B meson decays: General arguments and the case of heavy light final states, Nucl. Phys. **B591**, 313 (2000).

[59] C. K. Chua, W. S. Hou, and K. C. Yang, Final state rescattering and color suppressed $\bar{B}^0 \to D^{(*)0}h^0$ decays, Phys. Rev. D **65**, 096007 (2002); H. Y. Cheng, C. K. Chua, and A. Soni, Final state interactions in hadronic B decays, Phys. Rev. D **71**, 014030 (2005) C. K. Chua, Rescattering effects in charmless $\bar{B}_{u,d,s} \to PP$ decays, Phys. Rev. D **78**, 076002 (2008); Revisiting final state interaction in charmless $B_q \to PP$ decays, Phys. Rev. D **97**, 093004 (2018).

[60] W. Jaus, Semileptonic decays of B and D mesons in the light front formalism, Phys. Rev. D **41**, 3394 (1990).

[61] W. R. B. de Araujo, M. Beyer, T. Frederico, and H. J. Weber, Feynman versus Bakamjian-Thomas in light front dynamics, J. Phys. G **25**, 1589 (1999).

[62] J. G. Korner and M. Kramer, Exclusive nonleptonic charm baryon decays, Z. Phys. C **55**, 659 (1992).