Effective repulsion in dense quark matter from nonperturbative gluon exchange

Yifan Song,^{1,2} Gordon Baym,^{1,2} Tetsuo Hatsuda,^{2,3} and Toru Kojo^{4,2}

¹Department of Physics, University of Illinois at Urbana-Champaign,

1110 West Green Street, Urbana, Illinois 61801, USA

²Interdisciplinary Theoretical and Mathematical Sciences (iTHEMS) Program,

RIKEN, Wako, Saitama 351-0198, Japan

³Quantum Hadron Physics Laboratory, RIKEN Nishina Center, Wako, Saitama 351-0198, Japan

⁴Key Laboratory of Quark and Lepton Physics (MOE) and Institute of Particle Physics,

Central China Normal University, Wuhan 430079, China

(Received 7 May 2019; published 20 August 2019)

A moderately strong vector repulsion between quarks in dense quark matter is needed to explain how a quark core can support neutron stars heavier than 2 solar masses. We study this repulsion, parametrized by a four-fermion interaction with coupling g_V , in terms of nonperturbative gluon exchange in QCD in the Landau gauge. Matching the energy of quark matter $g_V n_q^2$ (where n_q is the number density of quarks) with the quark exchange energy calculated in QCD with a gluon propagator parametrized by a finite gluon masse m_g and a frozen coupling α_s , at moderate quark densities, we find that gluon masses m_g in the range 200–600 MeV and $\alpha_s = 2-6$ lead to a g_V consistent with neutron star phenomenology. Estimating the effects of quark masses and a color-flavor-locked (CFL) pairing gap, we find that g_V can be well approximated by a flavor-symmetric, decreasing function of density. We briefly discuss similar matchings for the isovector repulsion and for the pairing attraction.

DOI: 10.1103/PhysRevD.100.034018

I. INTRODUCTION

Quarks are active degrees of freedom in the deep interior of massive neutron stars. For a comprehensive review of quark matter and the QCD phase diagram, see Refs. [1,2] and references therein. In Refs. [3–5], we constructed a family of quark-hadron equations of state in which matter is described at densities up to about twice the nuclear saturation density: $n_0 \approx 0.16$ baryons per fm³ by interacting nucleons, and at higher densities, $n_B \gtrsim 5-10 n_0$, by interacting quark matter with a highly constrained interpolation of the equation of state between the two regimes. This equation of state describes neutron star properties quite consistent with recent LIGO inferences from the binary neutron star merger GW170817 [6]. Version QHC18 of this equation of state at zero temperature is reviewed in Ref. [1], and the latest version, QHC19, was recently made available [7,8].

We describe quark matter in terms of a Nambu–Jona-Lasinio (NJL) model with point interactions in the scalar, diquark, and vector-isoscalar channels, with a Lagrangian schematically of the form [9,10]

$$\mathcal{L}_{\rm int} = G(\bar{q}q)^2 + H(\bar{q}\,\bar{q})(qq) - g_V(\bar{q}\gamma^\mu q)^2,\qquad(1)$$

where the vector repulsion in the isoscalar channel [11] is needed for quark matter to support heavy neutron stars.

The resultant energy density from the vector repulsion is $g_V n_q^2$, where $n_q = 3n_B$ is the quark number density.

While the scalar coupling G and the ultraviolet cutoff Λ_{NJL} of the NJL model can be directly related to physical observables such as the properties of pseudoscalar mesons, the vector repulsion at present is constrained only by comparing the equation of state of matter with observations of neutron stars. As we have found in our QHC19 equation of state, to support neutron stars of masses above 2 solar masses (including the recently measured neutron star mass, 2.17 ± 0.1 solar masses in the pulsar PSR J0740 + 6620 [12]) requires that g_V be well within the range 0.6–1.3 G_0 , and H within the range $1.35-1.65 G_0$ [7], where $G_0 =$ $1.835\Lambda_{\text{NIL}}^{-2}$, with $\Lambda_{\text{NIL}} = 631.4$ MeV, is the scalar coupling in the vacuum obtained by a fitting of pion observables [9,10]. Our aim in this paper is to explore a further understanding of the structure of Eq. (1) in terms of QCD and the strength of the vector repulsion in particular. A simple Fierz transformation of the color-current-colorcurrent interaction, $\sim (\bar{q}\gamma_{\mu}\lambda^{\alpha}q)^2$, leads to NJL couplings [Eq. (1)] with the ratios $g_{V0}/G_0 = 1/2$ and $H_0/G_0 = 3/4$ (see the Appendix A) [10], where the "0" continues to indicate vacuum values. But in the fully interacting system, these ratios need not hold; as in OHC18 and OHC19, we focus on more general in-medium values of g_V and H, studying here the density dependence of g_V in particular.

Since q_V has dimensions of mass⁻², at asymptotically large densities, where the only energy scale is the quark Fermi momentum p_F , the value g_V should behave as $\sim \alpha_s/p_F^2$, where α_s is the QCD running coupling constant. On the other hand, in the highly nonperturbative vacuum at zero baryon density, the relevant scale is Λ_{QCD} , and we expect $g_V \sim \alpha_s / \Lambda_{\rm OCD}^2$. Thus, the matter-density dependence of g_V can be ignored only when $p_F \ll \Lambda_{\text{OCD}}$, provided that α_s also freezes at low energy [13]. To smoothly connect g_V at low density with that at high density, we adopt a model of massive gluons [14,15] which includes the nonperturbative generation of the gluon mass m_a , as well as the freezing of α_s in the Landau gauge at low energies. This approach is a step towards conceptually connecting the NJL model and perturbative QCD [16–18]. As we estimate, a gluon mass $m_a \sim 0.4$ GeV and a moderately strong quark-gluon coupling $\alpha_s \sim 3$ at $5n_0$ (or similar values, shown in Fig. 3 below, with α_s/m_a^2 roughly constant) can produce a strong enough $g_V \sim G_0$ to allow quark matter to support neutron stars above 2 solar masses.

At high density, where the matter tends to have equal populations of up, down, and strange quarks, flavor singlet channels are much more important than nonsinglet flavor channels. This allows us to focus on the flavor singlet scalar and vector couplings as well as CFL-type diquark pairing [19], favored for equal flavor population. Flavor nonsinglet interactions are nonetheless important at low densities (see Appendix B).

This paper is organized as follows: In Sec. II, we present the single-gluon exchange energy calculation starting with free-quark and gluon Green's functions, at first using the two-loop running coupling constant in perturbative QCD. The Landau pole in the running coupling leads to a strongly divergent result at a density $\lesssim 5n_0$. To avoid such a divergence, we consider in Sec. III a range of α_s and gluon masses, m_a , as estimated nonperturbatively below the 1 GeV scale, and we comment on the connection of the QHC19 neutron star equation of state, constrained by neutron star observations, to sub-GeV theories of α_s and massive gluons. We also provide an approximate densitydependent parametrization of q_V , connecting the lowdensity and high-density limits. Next, in Sec. IV, we estimate the effects on g_V of a finite quark mass, M_q , arising from chiral condensation in the quark sector, and in Sec. V, we estimate the effects of diquark pairing. As we show, a quark mass term tends to enhance q_V , while diquark pairing decreases it; both effects are suppressed by a gluon mass, and as a result, a flavor-independent g_V is a good approximation in the NJL model. We summarize our discussion in Sec. VI. In Appendix A, we show how the color-current-current interactions can be rearranged via the Fierz transformation. In Appendix B, we consider effective vector-isovector couplings, possibly important at intermediate and low densities, and in Appendix C, we estimate the value of H from the $N - \Delta$ mass splitting.

Throughout, we work in natural units $\hbar = c = 1$ with the metric $g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$, and we focus on zero temperature with $N_f = N_c = 3$ and equal quark masses, unless stated otherwise. We use the notation $\int_p = \int d^4 p / (2\pi)^4$.

II. WEAK COUPLING LIMIT

The quark-gluon interaction to leading order in α_s leads to the energy-density shift of the quark matter

$$E_{\rm QCD} = -\frac{i\pi\alpha_s}{2} \int d^4x \langle J^{\alpha}_{\mu}(x) A^{\mu}_{\alpha}(x) J^{\beta}_{\nu}(0) A^{\nu}_{\beta}(0) \rangle, \quad (2)$$

where the expectation value is in a Fermi gas, $x = (t, \mathbf{x})$, and *t* is integrated from 0 to -i/T (with *T* being the temperature). The currents are $J^{\alpha}_{\mu}(x) \equiv \bar{q}(x)\gamma_{\mu}\lambda^{\alpha}q(x)$, where the λ_{α} 's are the color SU(3) Gell-Mann matrices normalized to $tr\lambda_{\alpha}\lambda_{\beta} = 2\delta_{\alpha\beta}$.

In the weak coupling limit, neglecting diquark pairing, Eq. (2) becomes the Fock term in terms of the two-quark interaction

$$E_{\text{QCD}} \approx \frac{\pi \alpha_s}{2} \int_{p,p'} \text{Tr}[S(p)\lambda_{\alpha}\gamma^{\mu}S(p')\lambda_{\beta}\gamma^{\nu}] D_{\mu\nu}^{\alpha\beta}(p-p').$$
(3)

Here the trace Tr runs over flavor, color, and Dirac indices, and the integrations over the frequencies p_0 and p'_0 are understood as the fermion Matsubara frequency summations, $\int dp_0 f(p_0) \rightarrow 2\pi i T \sum_n f(i\omega_n)$, where $\omega_n = 2\pi T n$, with $n = \pm 1/2, \pm 3/2, \ldots$ The time-ordered quark Green's function is

$$S_{ij}^{ab}(x-y) = -i\langle \mathcal{T}q_i^a(x)\bar{q}_j^b(y)\rangle \tag{4}$$

and is denoted by S(p) in momentum space; here *a*, *b* are color indices and *i*, *j* flavor indices. The gluon Green's function is

$$D^{\alpha\beta}_{\mu\nu}(x-y) = -i\langle \mathcal{T}A^{\alpha}_{\mu}(x)A^{\beta}_{\nu}(y)\rangle.$$
 (5)

With no medium modification of the gluons, D in the Landau gauge takes the form in the momentum space,

$$D^{\alpha\beta}_{\mu\nu}(q) = -\delta^{\alpha\beta} \left(g_{\mu\nu} - \frac{q_{\mu}q_{\nu}}{q^2} \right) D(q).$$
 (6)

The full calculation of the energy leads to divergent Dirac sea contributions involving antiparticles. Only the $g_{\mu\nu}$ term in $D^{\alpha\beta}_{\mu\nu}(q)$ contributes to the particle-particle exchange (Fock) energy, and we keep only this term.

The traces in Eq. (3) can be reorganized, via a Fierz transformation (see Appendix A), into traces over quark Green's functions in the quark-antiquark channels. The NJL model contains two such channels: the scalar $\bar{q}q$

channel, which is used to characterize the spontaneous chiral symmetry breaking, and the vector-isoscalar $\bar{q}\gamma^{\mu}q$ channel. The energies corresponding to the scalar and vector channels, after the Fierz expansion of Eq. (3), denoted by E_{OCD}^{s} and E_{OCD}^{v} , are

$$E_{\text{QCD}}^{\text{s}} = -\frac{8\pi\alpha_s}{27} \int_{p,p'} \text{Tr}S(p)\text{Tr}S(p')D(p-p'), \tag{7}$$

$$E_{\text{QCD}}^{\text{v}} = \frac{4\pi\alpha_s}{27} \int_{p,p'} \text{Tr}[S(p)\gamma^{\mu}] \text{Tr}[S(p')\gamma_{\mu}] D(p-p').$$
(8)

We first outline how these results are related to the effective G and g_V in the NJL model. Since the detailed relation depends on the gluon propagator, we first illustrate the results in the two limiting extremes, low and high density. Owing to the nonperturbative infrared cutoff of order Λ_{QCD} , the gluon propagator has a finite limit $D(q \rightarrow 0)$ at low energy; thus, at low densities we have

$$E_{\text{QCD}}^{\text{s,v}} = C_{\text{s,v}} \alpha_s D(0) \left(\int_p \text{Tr}[S(p)\Gamma_{\text{s,v}}] \right)^2, \qquad (9)$$

where $C_s = -8\pi/27 = -2C_v$ and $\Gamma_s = 1$, and provided that $\int_p \text{Tr}[S(p)\gamma_j] = 0$, $\Gamma_v = \gamma^0$. In this form, one can readily identify the NJL couplings as $G = 2g_V = C_s \alpha_s D(0)$.

At higher densities, we must keep the momentum dependence of the gluon propagators. For example, with massless free quark and gluon propagators,

$$S_{ij}^{0,ab}(p) = \delta_{ab} \delta_{ij} \frac{\gamma_{\mu} p^{\mu}}{(p_0 + \mu)^2 - p^2}, \qquad (10)$$

$$D^0(p) = \frac{1}{p^2},$$
 (11)

where μ is the quark chemical potential, and we find the perturbative result,¹

$$E_{\rm QCD}^{\rm v} = 24\pi\alpha_s \left(\int \frac{d^3p}{(2\pi)^3} \frac{f(|\pmb{p}| - \mu)}{|\pmb{p}|}\right)^2, \qquad (12)$$

where $f(z) = [\exp(z/T) + 1]^{-1}$ is the Fermi distribution function; at zero temperature, Eq. (12) reduces to

$$E_{\rm QCD}^{\rm v} = \frac{3\alpha_s p_F^4}{2\pi^3}.$$
 (13)

This result is identical to the exchange energy of a highly relativistic electron gas to within the flavor and color factors.^{2,3}

The vector repulsion contributes an energy density in the NJL model [7],

$$E_{\rm NJL}^{\rm v} = g_V n_q^2, \tag{15}$$

which we identify with $E_{\text{QCD}}^{\text{v}}$ in the matching density region ~5–20 n_0 corresponding to $p_F \sim 0.4$ –0.6 GeV, and one finds

$$g_V = \frac{\pi \alpha_s}{6p_F^2}.$$
 (16)

The dashed line in Fig. 1 shows g_V obtained using Eq. (16) and the two-loop running coupling constant $\alpha_s(\mu_q)$:

$$\alpha_s(\mu_q) = \frac{4\pi}{9\ln\tilde{\mu}^2} \left(1 - \frac{64\ln\ln\tilde{\mu}^2}{81\ln\tilde{\mu}^2}\right),$$
 (17)

with $\tilde{\mu} \equiv \mu_q / \Lambda_{\rm QCD}$ and $\Lambda_{\rm QCD} = 340$ MeV [13]. The shaded horizontal band indicates the range of (constant) g_V in QHC19 [7]. Although g_V in Fig. 1 approaches the needed range below $20n_0$, the factor p_F^{-2} and the running α_s near the Landau pole at $\Lambda_{\rm QCD}$ already causes strongly divergent behavior of g_V even at $5n_0$ (corresponding to $p_F \sim 400$ MeV), in contrast to the simple treatment in NJL of g_V as constant in this regime. However, extending the pQCD calculation down to $\Lambda_{\rm QCD}$ is not reliable. The solid line in Fig. 1 shows g_V for α_s frozen at 3.0 at low energies [13]. Although the divergence from the Landau pole is removed in this case, g_V still increases rapidly at low energy.

²Equation (12) includes the interactions between quarknumber densities $\bar{q}\gamma_0 q$, as well as those between spatial currents, $\bar{q}\gamma_j q$. These contributions yield the matrix element, for on-shell momenta,

$$\operatorname{Tr}[S(p)\gamma^{\mu}]\operatorname{Tr}[S(p')\gamma_{\mu}] \propto \frac{|\boldsymbol{p}||\boldsymbol{p}'| - \boldsymbol{p} \cdot \boldsymbol{p}'}{2|\boldsymbol{p}||\boldsymbol{p}'|}, \qquad (14)$$

whose numerator cancels the pole from the massless gluon propagator, giving Eq. (12).

³In deriving E_{QCD}^v in Eq. (12) from Eq. (9) with a momentumdependent gluon propagator, the correlation functions $\langle \bar{q} \, \vec{\gamma} \, q \rangle$ are as important as $\langle \bar{q} \gamma^0 q \rangle$; the former is not included in the NJL mean field description. Such a deficiency in the NJL model can be compensated by absorbing the contribution from $\langle \bar{q} \, \vec{\gamma} \, q \rangle$ into the density dependence of g_V itself; in this way, we can directly compare the NJL g_V with the current definition of g_V in terms of QCD parameters.

¹While the full trace in Eq. (3) contains contributions from both particles and antiparticles, we focus only on modifications due to nonzero particle densities here.



FIG. 1. The dashed line indicates the single-gluon exchange result for g_V in perturbative QCD as a function of the quark matter Fermi momentum, p_F . The horizontal shaded region shows the range of g_V in QHC19 [7], while the vertical shaded region shows the baryon density ~5–20 n_0 . The solid line indicates the result for α_s frozen at 3.0 at low energies [13].

III. NONPERTURBATIVE α_s AND MASSIVE GLUONS BELOW 1 GeV

We now examine the consequences of the nonperturbative behavior of the strong coupling constant α_s and the gluon propagator below the 1 GeV scale. For reviews, see Refs. [13,15] and references therein. In various nonperturbative approaches for the gluon sector (lattice gauge theory, Schwinger-Dyson equations, and gauge/gravity duality), under gauge fixing, α_s is of order unity below 1 GeV (with freezing or decoupling behaviors in the deep infrared limit, $q \rightarrow 0$). Here we focus on gluons dynamically acquiring a mass, favored by the lattice results (and corresponding to the decoupling solution of the gluon Schwinger-Dyson equations in the Landau gauge),

$$D(p) = \frac{1}{p^2 - m_g^2}.$$
 (18)

Estimates of m_g tend to lie in the range ~500 ± 200 MeV [14,15]. The present use of a massive gluon propagator is conceptually different from using in-medium generated masses of the gluon field, as in quasiparticle models, e.g., Ref. [20] and references therein; the massive gluon here originates from nonperturbative physics and exists even in the vacuum. Equation (18) regulates the divergent behavior of g_V as $p_F \rightarrow 0$ in Fig. 1 and leads to

$$E_{\text{QCD}}^{\text{v}}(m_g) = E_{\text{QCD}}^{\text{v}}(0) + \delta E_{\text{QCD}}^{\text{v}}(m_g).$$
(19)

As in the derivation of Eq. (12), $E_{\text{QCD}}^{\text{v}}(0)$ results from a cancellation between the massive gluon propagator and a part of the quark matrix elements, while the remaining terms are proportional to m_q^2

$$\delta E_{\text{QCD}}^{\text{v}}(m_g) = -\frac{3\alpha_s m_g^2}{2\pi^3} \int_0^{p_F} \int_0^{p_F} dp dp' \ln\left(1 + \frac{4p\,p'}{m_g^2}\right) \\ = \frac{3\alpha_s m_g^4}{8\pi^3} K(x), \tag{20}$$

where $z \equiv (2p_F/m_g)^2$ and $K(z) \equiv 2z - (1+z)\ln(1+z) +$ Li₂(-z), with Li₂(-z) $\equiv \sum_{\ell=1}^{\infty} (-z)^{\ell}/\ell^2$ being the polylogarithm function with n = 2. Thus, one finds

$$E_{\rm QCD}^{\rm v}(m_g) = \frac{3\alpha_s p_F^4}{2\pi^3} \left(1 + \frac{K(z)}{z^2}\right).$$
 (21)

Note that for positive z, $0 \le 1 + K(z)/z^2 < 1$, implying that the finite gluon mass softens the repulsion while keeping the total vector energy positive.

Matching Eq. (15) with Eqs. (16) and (21), one finds

$$g_V(p_F; z \gg 1) \to \frac{\pi \alpha_s}{6p_F^2},$$

$$g_V(p_F; z \ll 1) \to \frac{4\pi \alpha_s}{27m_q^2}.$$
 (22)

Figure 2 shows g_V for different gluon masses m_g , with a typical value of the frozen $\alpha_s = 3.0$ at low energies $\lesssim 1$ GeV [13]. In the infrared, g_V is regulated by the gluon mass, m_g , so that there is no divergent behavior at $p_F = 0$.

Figure 3 gives contour plots of the resulting vector coefficient g_V for given different α_s and gluon masses m_g , at $5n_0$ and $20n_0$. For the resulting g_V/G_0 to be in the interval 0.6–1.3 at $5n_0$ with $m_g = 400$ MeV, one needs a strong $\alpha_s \sim 2$ –4, within the range of possible quark-gluon coupling strengths at low energies [13]. Future theories of the quark-gluon vertex α_s together with detailed forms of gluon correlation functions below 1 GeV will be of interest, as they can be directly related to effective quark models constrained by neutron star observations.



FIG. 2. The vector coefficient g_V as a function of quark Fermi momentum generated by a frozen $\alpha_s = 3$ below 1 GeV and different gluon masses m_q .



FIG. 3. The vector coefficient g_V generated by different constant α_s and gluon masses m_g , at $p_F = 400$ MeV (~5 n_0). The central cross indicates $\alpha_s = 3$ and $m_g = 400$ MeV.

In the density range $\sim 5 n_0$ in a neutron star, where the quark Fermi momentum lies well below 1 GeV, it is reasonable to assume an approximately constant α_s and m_g . The two limiting results, Eq. (22), thus suggest an approximate density-dependent parametrization of g_V based on explicit single-gluon exchange:

$$g_V(p_F; m_g) \simeq \frac{4\pi \alpha_s/3}{9m_g^2 + 8p_F^2}.$$
 (23)

This parametrization is useful for including the density dependence of g_V in the quark-hadron crossover equations of state.

IV. EFFECT OF FINITE QUARK MASS

At high densities, quark matter contains a small chiral condensate $\sim \langle \bar{q}q \rangle$ as well as a diquark condensate $\sim \langle qq \rangle$, as a consequence of the six-quark Kobayashi-Maskawa-'t Hooft (KMT) effective interaction [21]. At densities $\gtrsim 5n_0$, the chiral condensate enhanced by the





FIG. 4. Vector repulsion coefficient g_V for different values of M_q with $m_q = 400$ MeV and $\alpha_s = 3$.

KMT interaction could result in an effective quark mass $M_q \sim 50-70$ MeV for the light quarks, and $\sim 250-300$ MeV for the *s* quark [1]. These masses are not small compared to the quark Fermi momentum at these densities, and must be taken into account in the exchange energy calculation.

Here we calculate the effects of M_q on g_V only by modifying the quark propagators in Eq. (9), and not further correcting the vertices. We recognize that this is not a selfconsistent calculation; rather, we aim here to get a sense of the effects of a finite quark mass on the vector channel of the matrix element (2), which is connected to perturbative QCD at asymptotic density. We take the quark Green's function to be

$$S_{ij}^{ab}(p) = \delta_{ab}\delta_{ij}\frac{\gamma_{\mu}p^{\mu} + M_q}{(p_0 + \mu)^2 - p^2 - M_q^2}$$
(24)

and assume the same effective mass M_q for all flavors.

With this S, we obtain after some algebra, with $\epsilon_p = (|\mathbf{p}|^2 + M_a^2)^{1/2}$,

$$E_{\rm QCD}^{\rm v} = 24\pi\alpha_s \left[\left(\int \frac{d^3p}{(2\pi)^3} \frac{f(\epsilon_p - \mu)}{\epsilon_p} \right)^2 - (2M_q^2 - m_g^2) \int \frac{d^3p d^3p'}{(2\pi)^6} \frac{1}{\epsilon_p \epsilon_{p'}} \cdot \frac{f(\epsilon_p - \mu_q) f(\epsilon_{p'} - \mu_q)}{(\epsilon_p - \epsilon_{p'})^2 - |\mathbf{p} - \mathbf{p'}|^2 - m_g^2} \right].$$
(25)

The asymptotic forms of Eq. (25) for $p_F \gg M_q$ and m_g , and for $p_F \ll M_q$ and m_g , can be readily found, with the result that $g_V(p_F; m_g, M_q)$ agrees in these limits with Eq. (22). In particular, g_V is independent of M_q at $p_F = 0$ as long as m_g is finite. The combined effects of M_q and m_g are shown in Fig. 4, which compares g_V at several different values of M_q and $m_g = 400$ MeV. We find that the effect of M_q on g_V is almost negligible. Thus, the assumption that g_V is flavor independent is reasonable, despite flavor symmetry being significantly broken by the strange quark mass; the parametrization (23) is approximately useful independent of flavor.

V. EFFECT OF DIQUARK PAIRING

We next consider the effects on E_{QCD}^{v} of scalar colorflavor-locked pairing among quarks through modification of the normal quark Green's function *S* in Eq. (9).⁴ In the CFL phase, it is convenient to expand the quark field [with *SU*(3) flavor and *SU*(3) color indices], as $q_{ia} = \sum_{A=0}^{8} \lambda_{ia}^{A} q_{A} / \sqrt{2}$, in term of the Gell-Mann matrices, λ^{A} (A = 1, 2, ..., 8), and $\lambda^{0} = \mathbf{1}\sqrt{2/3}$. In this basis, the normal quark propagator becomes diagonal:

$$S_{ij}^{ab}(x-y) = \sum_{A} \frac{1}{2} \lambda_{ia}^{A} \lambda_{bj}^{A} S_{A}(x-y).$$
(26)

With CFL pairing, the $S_{A=1,...,8}$ describe eight paired quark quasiparticles with the same gap, $\Delta_{A=1,...,8}(p) = \Delta(p)$, and one quasiparticle S_0 with double the gap, $\Delta_0(p) = 2\Delta(p)$.

For massless quarks ($\epsilon_p = |\mathbf{p}|$), one finds

$$E_{\text{QCD}}^{v} = \frac{4\pi\alpha_{s}}{27} \sum_{A,B} \int_{pp'} \text{tr}[S_{A}(p)\gamma^{\mu}] \text{tr}[S_{B}(p')\gamma_{\mu}] \frac{1}{(p-p')^{2} - m_{g}^{2}},$$
(27)

$$= \frac{\alpha_s}{54\pi^3} \sum_{A,B} \int_0^\infty dp dp' v_{Ap}^2 v_{Bp'}^2 \left[4p p' - J_{AB}(p, p', m_g) \right] \times \ln \left| 1 + \frac{4p p'}{J_{AB}(p, p', m_g)} \right|,$$
(28)

with $v_{Ap}^2 = \frac{1}{2}(1 - (\epsilon_p - \mu)/E_p^A)$, $E_p^A = [(\epsilon_p - \mu)^2 + \Delta_A^2]^{1/2}$, and $J_{AB}(p, p', m_g) = m_g^2 + (p - p')^2 - (E_p^A - E_{p'}^B)^2$. Generalization to the case with finite quark mass M_q is straightforward. Note that the total quark density is given by

$$n_q = 2\sum_A \int \frac{d^3p}{(2\pi)^3} v_{Ap}^2.$$
 (29)

The integral in Eq. (28) converges only with a momentum-dependent gap. Following the numerical study in Refs. [22,23], we approximate the spatial momentum dependence of Δ by

$$\Delta(p) = \frac{\Delta(\mu)}{(1 + b(p - \mu)^2 / \mu^2)^{\zeta}};$$
(30)

the constant b > 0 parametrizes how fast $\Delta(p)$ falls off away from the Fermi surface, and the exponent $\zeta > 0$ parametrizes the behavior of $\Delta(p)$ at high momenta (see Fig. 5). In the weak coupling limit, $\zeta = 1 + O(\alpha_s)$ and $\Delta \sim \mu g^{-5} e^{-3\pi^2/\sqrt{2}g}$ [24,25]. Here we simply vary the gap in the range, $\Delta(\mu) = 100\text{--}300$ MeV, consistent with the QHC19 equation of state. This range of CFL gaps results from the



FIG. 5. The parametrization [Eq. (30)] of the momentumdependence gap $\Delta(p)$ for $\mu = 500$ MeV, b = 1.0, $\Delta(\mu) = 50$ MeV, and $\zeta = 1.0$.

phenomenologically derived pairing strength [7] $H \sim 1.5G_0$ being notably larger than the conventional Fierz value 3/4 G in the NJL model; the latter yields the more commonly discussed range of gaps, ~10–100 MeV [19]. See particularly Fig. 33 of Ref. [1], and Appendix C, which argues that the level splitting between N and Δ in the NJL model favors $H \gtrsim 1.4 G$. Such larger pairing strengths are reasonable, since with decreasing density one expects two-(and three-) quark correlations to build up in quark matter, eventually leading to well-defined nucleons at lower densities. Indeed, before diquarks are confined into nucleons, the size of diquarks is expected to be ~1 fm, consistent with a gap ~200 MeV.

As we see in Fig. 6, a gap decreases g_V at all densities, and the dependence of the gap is significant for massless gluons. For gluon masses $m_g \sim 400$ MeV, however, even a large variation of Δ from 0 to 300 MeV does not change the qualitative behavior of g_V . In comparison with the effects of M_q , a large gap $\Delta(\mu) = 200$ MeV (as in QHC19) still has a sizable impact: at $5n_0$, a 200 MeV CFL gap reduces g_V from $\sim 0.9G_0$ to $\sim 0.55G_0$, even with $m_q = 400$ MeV.



FIG. 6. The vector repulsion coefficient g_V for different $\Delta(\mu)$ with $m_g = 400$ MeV and $\alpha_s = 3$. The curves show how inclusion of pairing in the presence of a massive gluon has only a small effect on g_V .

⁴The anomalous Green's function, $F_{ij}^{ab}(x - y) = -i\langle \mathcal{T}q_i^a(x) (q^T C)_b^j(y) \rangle$, leads as well to the familiar energy shift $E_{\text{QCD}}^{\text{pair}}$ proportional to the square of the pairing gap, an effect related to inferring the in-medium modification of H.

The gluon propagator is also modified in a dense quark medium by Landau damping [24–26], and the Debye screening mass in the longitudinal sector, and in the presence of diquark pairing by Meissner masses in the transverse sector [27,28], of order $\sqrt{\alpha_s}\mu$. The interplay of these modifications of the gluon propagator in the quark matter in neutron stars and their effects on neutron star properties are open questions worthy of future research.

VI. CONCLUSION

We have computed the vector repulsion coefficient q_V from the explicit gluon exchange energy in quark matter, modifying the quark and gluon Green's functions to account for a nonperturbative gluon mass m_a and for chiral condensate and diquark pairing, including as well a possible infrared-finite α_s . In the density range $\sim 5-20n_0$ with reasonable parameters for α_s , gluon mass, quark mass, and pairing gap, we can begin to understand the origin of a g_V of order ~0.6–1.3G. The parameters we have chosen, despite their uncertainties, lie within estimates from a variety of models and theoretical frameworks of sub-GeV QCD. Among the nonperturbative effects we have considered, the resulting g_V is most sensitive to α_s and m_q , while M_q and Δ induce only relatively small changes owing to suppression by a gluon mass. Thus, the parametrization [Eq. (23)] should be a good approximate description of the density dependence of q_V , to be included in the equation of state for neutron star matter with a strongly interacting quark phase.

Many open questions remain. The vector repulsion between quarks at densities $\gtrsim 5n_0$ may also come from nonperturbative QCD beyond the single gluon-exchange contribution treated in this paper; such uncertainty is not under control at present. As α_s could range anywhere from 0 to 10 (or even be divergent at low-momentum scales), the assumption that the vector repulsion is dominated by a single gluon exchange with a fixed α_s and m_q is overly simplified. Our treatment can be improved and extended in several directions. The first would be the inclusion of more realistic quark and gluon propagators, including possible momentum dependence of masses and differences between transverse and longitudinal gluons. The second would be to include the nonperturbative running of α_s . Including the density dependence of g_V , as in the parametrization (23), can have a significant effect on model studies of quark matter. In particular, corrections to the contributions from the light and heavy quarks could shift the phase boundaries and modify the equation of state. Including the density dependence of the diquark coupling, H, would have a similar effect.

We note that relating the effective QCD vector couplings g_V and g_τ^{α} (Appendix B) in the NJL model of dense matter (an effective field theory for quarks) to nucleon-meson models (effective field theories for hadrons) would provide

a further probe of quark-hadron continuity [21,29]. If the transition from nuclear to quark matter is essentially smooth, one expects the vector repulsion from hadronic to quark matter to be similarly smooth, since in the quark-hadron continuity picture, the spectrum of light gluonic excitations is tightly connected to that of hadronic vector mesons [30], while quarks are mapped to the baryons in nuclear matter. Low-energy quark-gluon matter treated in this way becomes an extension of the baryon-meson picture of nuclear matter, plausibly enabling a relatively smooth crossover and in turn mapping g_V and g_τ^{α} from the hadronic to the quark phases.⁵

ACKNOWLEDGMENTS

G. B. and Y. Song are grateful to the RIKEN iTHEMS program for hospitality during this work. Their research was supported in part by National Science Foundation Grant No. PHY1714042. T. H. was partially supported by the RIKEN iTHEMS program and by JSPS Grant-in-Aid for Scientific Research (S), No. 18H05236, and T. K. was supported by NSFC Grants No. 11650110435 and No. 11875144, and by the KMI for his long-term stay at the Nagoya University. The authors are grateful to the Aspen Center for Physics, supported by NSF Grant No. PHY1607611, where part of this research began, and to Hajime Togashi and Shun Furusawa for discussions there.

APPENDIX A: FIERZ TRANSFORMATION

The Fierz transformation is a rearrangement of fermion operator products in the Dirac, flavor, and color spaces using index-exchanging properties of the gamma and SU(N) generator matrices. In the quark-antiquark channel, the rearrangement of the Dirac indices reads

$$\begin{aligned} (\gamma^{\mu})_{mn}(\gamma_{\mu})_{m'n'} &= \mathbf{1}_{mn'}\mathbf{1}_{m'n} + (i\gamma_{5})_{mn'}(i\gamma_{5})_{mn'} \\ &- \frac{1}{2}(\gamma^{\mu})_{mn'}(\gamma_{\mu})_{m'n} \\ &- \frac{1}{2}(\gamma^{\mu}\gamma_{5})_{mn'}(\gamma_{\mu}\gamma_{5})_{m'n}, \end{aligned}$$
(A1)

and those of the flavor and color indices $(N_f = N_c = 3)$ read

⁵One may ask how vector repulsions in the nucleon-meson description of nuclear matter, a gauge-invariant theory, can be mapped onto vector repulsions in the gauge-dependent theory of quarks and gluons, despite the vector repulsions in both being effective fermion-fermion interactions mediated by massive boson exchange. In fact, including color charge screening by CFL diquark condensates [31,32] leads to a low-energy gauge-invariant description of quarks and gluons of the same form as a baryon-meson Lagrangian [31].

$$\mathbf{1}_{ij}\mathbf{1}_{kl} = \frac{1}{3}\mathbf{1}_{il}\mathbf{1}_{kj} + \frac{1}{2}(\tau_a)_{il}(\tau_a)_{kj},$$
$$\lambda_{\alpha}^{ab}\lambda_{\alpha}^{a'b'} = \frac{16}{9}\mathbf{1}_{ab'}\mathbf{1}_{a'b} - \frac{1}{3}\lambda_{\alpha}^{ab'}\lambda_{\alpha}^{a'b}.$$
(A2)

In the quark-quark channel,

$$\begin{aligned} (\gamma^{\mu})_{mn}(\gamma_{\mu})_{m'n'} &= (i\gamma^{5}C)_{mm'}(i\gamma^{5}C)_{nn'} + C_{mm'}C_{nn'} \\ &- \frac{1}{2}(\gamma^{\mu}\gamma^{5}C)_{mm'}(\gamma_{\mu}\gamma^{5}C)_{nn'} \\ &- \frac{1}{2}(\gamma^{\mu}C)_{mm'}(\gamma_{\mu}C)_{nn'}, \end{aligned}$$
(A3)

and

$$\mathbf{1}_{ij}\mathbf{1}_{kl} = \frac{1}{2}(\tau_S)_{ik}(\tau_S)_{lj} + \frac{1}{2}(\tau_A)_{ik}(\tau_A)_{lj},$$
$$\lambda_{\alpha}^{ab}\lambda_{\alpha}^{cd} = \frac{2}{3}\lambda_{ac}^S\lambda_{bd}^S - \frac{4}{3}\lambda_{ac}^A\lambda_{bd}^A, \tag{A4}$$

where *S* and *A* stand for symmetric and antisymmetric indices, and $\tau_{\alpha=1,...,8}$ are the eight Gell-Mann flavor matrices. Using these relations, one can transform a single trace into products of two traces, as is done in, e.g., Eq. (9):

$$\operatorname{Tr}[S(p)\Gamma^{I}S(p')\Gamma^{I}] = \sum_{M} g_{M}\operatorname{Tr}[S(p)\Gamma^{M}]\operatorname{Tr}[S(p')\Gamma^{M}],$$
(A5)

where Γ^{I} are Dirac, flavor, and color matrices.

APPENDIX B: THE VECTOR-ISOVECTOR INTERACTION

The discussion in the main text focuses on the flavorsymmetric case, where in the absence of pairing, the vector component of single-gluon exchange contributes only to the isoscalar channel. [In the CFL phase, one finds nonvanishing contributions in the flavor-color vector channel $(\bar{q}\gamma^{\mu}\tau_{a}\lambda_{A}q)^{2}$ as well.] For realistic constituent quark masses, however, the vector-isovector channel (denoted by τ), corresponding to the interaction $(\bar{q}\gamma^{\mu}\tau_{a}q)^{2}$, also contributes to the single-gluon exchange energy:

$$E_{\text{QCD}}^{\text{v},\tau} = \frac{2\pi\alpha_s}{9} \int_{p,p'} \text{Tr}[S(p)\gamma^{\mu}\tau_{\alpha}] \text{Tr}[S(p')\gamma_{\mu}\tau_{\alpha}] D(p-p').$$
(B1)

In particular, the $\alpha = 3$ and 8 terms yield an exchange energy at low density of the form

$$g_{\tau}^{(3)}(n_u - n_d)^2 + \frac{g_{\tau}^{(8)}}{3}(n_u + n_d - 2n_s)^2.$$
 (B2)

This vector-isovector energy is analogous to the neutronproton symmetry energy in nuclear matter. For single-gluon exchange, $g_{\tau}^{(3)} = g_{\tau}^{(8)} = \frac{3}{2}g_V$, indicating an vector-isovector energy comparable to the vector-isoscalar energy for significant differences in flavor densities. It is an interesting future problem to estimate the in-medium values of $g_{\tau}^{(3,8)}$ as well as g_V by matching with, e.g., the chiral nucleon-meson model [33].

APPENDIX C: ESTIMATING H FROM THE $N - \Delta$ MASS SPLITTING

One of the important ingredients in the QHC19 equation of state is the parameter H that quantifies the strength of attractive diquark correlations. At high density, diquark correlations are the driving force of color superconductivity, while at low density, the correlations appear in the context of hadron mass splittings, e.g., the $N - \Delta$ splitting, $m_{\Delta} - m_N \simeq 293$ MeV. The density $n_B \sim 5n_0 \simeq 0.8$ fm⁻³ is roughly that inside of baryons, and so it suggests the possibility of inferring the value of H at $n_B \sim 5n_0$ from the $N - \Delta$ splitting.

This splitting has been derived by Ishii *et al.* [34] by solving the Faddeev equations of three-quark systems within the NJL model. They included effective four-quark interactions in the isoscalar scalar and isovector axialvector diquark channels, which in our notation are

$$\mathcal{L}_{S} = H \sum_{A=2,5,7} (\bar{\psi} i \gamma_{5} \tau_{2} \lambda_{A} \psi_{C}) (\bar{\psi}_{C} i \gamma_{5} \tau_{2} \lambda_{A} \psi), \qquad (C1)$$

$$\mathcal{L}_A = H' \sum_{A=2,5,7} (\bar{\psi} \gamma_\mu \tau_2 \vec{\tau} \lambda_A \psi_C) (\bar{\psi}_C \gamma^\mu \tau_2 \vec{\tau} \lambda_A \psi).$$
(C2)

Reference [34] finds the approximate formulas:

$$M_N \simeq 1.70 - 0.21 r'_H - 0.33 r_H \,[\text{GeV}],$$
 (C3)

$$M_{\Delta} \simeq 1.52 - 0.22 r'_H \,[\text{GeV}].$$
 (C4)

where $r_H = H/G_0$ and $r'_H = H'/G_0$. The absolute values of these masses are not quite trustworthy, as they are sensitive to the physics beyond the NJL model, e.g., confinement. In the mass splitting, such uncertainties are largely canceled, and the physics of short-range correlations become dominant. Using the empirical $M_{\Delta} - M_N$, we find

$$-0.01r'_H + 0.33r_H \simeq 0.47$$
 [GeV]. (C5)

Provided $r'_H \ge 0$ as expected from typical models, we arrive at

$$H/G_0 \gtrsim 1.4,$$
 (C6)

consistent with the range in QHC19, $H/G_0 = 1.35-1.65$. More comprehensive studies will be given elsewhere [35].

- G. Baym, T. Hatsuda, T. Kojo, P. D. Powell, Y. Song, and T. Takatsuka, From hadrons to quarks in neutron stars: A review, Rep. Prog. Phys. 81, 056902 (2018).
- [2] K. Fukushima and T. Hatsuda, The phase diagram of dense QCD, Rep. Prog. Phys. 74, 014001 (2010).
- [3] K. Masuda, T. Hatsuda, and T. Takatsuka, Hadron-quark crossover and massive hybrid stars with strangeness, Astrophys. J. 764, 12 (2013).
- [4] K. Masuda, T. Hatsuda, and T. Takatsuka, Hadron-quark crossover and massive hybrid stars, Prog. Theor. Exp. Phys. 2013, 073D01 (2013).
- [5] T. Kojo, P. D. Powell, Y. Song, and G. Baym, Phenomenological QCD equation of state for massive neutron stars, Phys. Rev. D 91, 045003 (2015).
- [6] B. P. Abbott *et al.* (LIGO Scientific and Virgo Collaborations), Properties of the Binary Neutron Star Merger GW170817, Phys. Rev. X 9, 011001 (2019).
- [7] G. Baym, S. Furusawa, T. Hatsuda, T. Kojo, and H. Togashi, New neutron star equation of state with quark-Hadron crossover, arXiv:1903.08963.
- [8] Posted at https://compose.obspm.fr/eos/140/.
- [9] T. Hatsuda and T. Kunihiro, QCD phenomenology based on a chiral effective Lagrangian, Phys. Rep. **247**, 221 (1994).
- [10] M. Buballa, NJL-model analysis of dense quark matter, Phys. Rep. 407, 205 (2005).
- [11] T. Kunihiro, Quark-number susceptibility and fluctuations in the vector channel at high temperatures, Phys. Lett. B 271, 395 (1991).
- [12] H. T. Cromartie *et al.*, A very massive neutron star: Relativistic Shapiro delay measurements of PSR J0740 + 6620, arXiv:1904.06759.
- [13] A. Deur, S. J. Brodsky, and G. F. de Teramond, The QCD running coupling, Prog. Part. Nucl. Phys. 90, 1 (2016).
- [14] J. M. Cornwall, Dynamical mass generation in continuum quantum chromodynamics, Phys. Rev. D 26, 1453 (1982).
- [15] A. C. Aguilar, D. Binosi, and J. Papavassiliou, The gluon mass generation mechanism: A concise primer, Front. Phys. 11, 111203 (2016).
- [16] A. Kurkela, P. Romatschke, and A. Vuorinen, Cold quark matter, Phys. Rev. D 81, 105021 (2010).
- [17] B. A. Freedman and L. D. McLerran, Fermions and gauge vector mesons at finite temperature and density: III. The ground state energy of a relativistic quark gas, Phys. Rev. D 16, 1169 (1977).
- [18] T. Gorda, A. Kurkela, P. Romatschke, M. Säppi, and A. Vuorinen, NNNLO Pressure of Cold Quark Matter: Leading Logarithm, Phys. Rev. Lett. **121**, 202701 (2018).
- [19] M.G. Alford, A. Schmitt, K. Rajagopal, and T. Schäfer, Color superconductivity in dense quark matter, Rev. Mod. Phys. 80, 1455 (2008).

- [20] A. Peshier, B. Kämpfer, and G. Soff, The equation of state of deconfined matter at finite chemical potential in a quasiparticle description, Phys. Rev. C 61, 045203 (2000).
- [21] T. Hatsuda, M. Tachibana, N. Yamamoto, and G. Baym, New Critical Point Induced by the Axial Anomaly in Dense QCD, Phys. Rev. Lett. **97**, 122001 (2006); N. Yamamoto, M. Tachibana, T. Hatsuda, and G. Baym, Phase structure, collective modes, and the axial anomaly in dense QCD, Phys. Rev. D **76**, 074001 (2007); H. Abuki, G. Baym, T. Hatsuda, and N. Yamamoto, The NJL model of dense three flavor matter with axial anomaly: The low temperature critical point and BEC-BCS diquark crossover, Phys. Rev. D **81**, 125010 (2010).
- [22] M. Matsuzaki, Spatial structure of quark Cooper pairs in a color superconductor, Phys. Rev. D 62, 017501 (2000).
- [23] H. Abuki, T. Hatsuda, and K. Itakura, Structural change of Cooper pairs and momentum-dependent gap in color superconductivity, Phys. Rev. D 65, 074014 (2002).
- [24] D. T. Son, Superconductivity by long-range color magnetic interaction in high-density quark matter, Phys. Rev. D 59, 094019 (1999).
- [25] R. D. Pisarski and D. H. Rischke, Color superconductivity in weak coupling, Phys. Rev. D **61**, 074017 (2000).
- [26] G. Baym, C. J. Pethick, and H. Monien, Kinetics of quark-gluon plasmas, Nucl. Phys. A498, 313 (1989); G. Baym, H. Monien, C. J. Pethick, and D. G. Ravenhall, Transverse Interactions and Transport in Relativistic Quark-Gluon and Electromagnetic Plasmas, Phys. Rev. Lett. 64, 1867 (1990).
- [27] K. Fukushima, Analytical and numerical evaluation of the Debye and Meissner masses in dense neutral three-flavor quark matter, Phys. Rev. D 72, 074002 (2005).
- [28] D. H. Rischke, Debye screening and Meissner effect in a three-flavor color superconductor, Phys. Rev. D 62, 054017 (2000).
- [29] T. Schaefer and F. Wilczek, Continuity of Quark and Hadron Matter, Phys. Rev. Lett. 82, 3956 (1999).
- [30] T. Hatsuda, M. Tachibana, and N. Yamamoto, Spectral continuity in dense QCD, Phys. Rev. D 78, 011501(R) (2008).
- [31] A. Kryjevski and T. Schaefer, An effective theory for baryons in the CFL phase, Phys. Lett. B 606, 52 (2005).
- [32] Y. Song and G. Baym (to be published).
- [33] M. Drews and W. Weise, Functional renormalization group studies of nuclear and neutron matter, Prog. Part. Nucl. Phys. 93, 69 (2017).
- [34] N. Ishii, W. Bentz, and K. Yazaki, Baryons in the NJL model as solutions of the relativistic Faddeev equation, Nucl. Phys. A587, 617 (1995).
- [35] T. Kojo (to be published).