

# Overcharging a multi-black-hole system and cosmic censorship

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(Received 6 May 2019; published 15 July 2019)

We study the generalization of the gedanken experiment of overcharging an extremal black hole proposed by Wald in the context of a multi-black-hole solution. In particular, we attempt to overcharge a system of two extremal black holes via test particle absorption to produce a system involving a black hole and a naked singularity. If such a process is possible, then this would be a potential violation of the cosmic censorship hypothesis. However, we find that, analogous to Wald's result for a single charged black hole, such a test particle which can expose the singularity would not be able to enter the horizon. This provides an interesting and nontrivial example that supports the validity of the cosmic censorship hypothesis in four-dimensional general relativity.

DOI: [10.1103/PhysRevD.100.024030](https://doi.org/10.1103/PhysRevD.100.024030)

## I. INTRODUCTION

Gravity is best described in the framework of general relativity as the manifestation of the curvature of space and time. Even after more than 100 years, general relativity still stands as one of the most beautiful theories the human mind has ever produced and has often surprised us by its various bold yet interesting predictions like black holes, gravitational waves, expansion of the Universe, etc. General relativity has passed all the observational tests covering a wide range of scales starting from Solar System measurements like the perihelion precession of Mercury, up to tests at the cosmological level. Despite having such theoretical beauty and observational consistencies, general relativity certainly cannot be considered as the final theory of gravity because of several limitations. One among such limitations of general relativity is the inevitable occurrence of singularities in the solutions of Einstein's equation [1]. Gravitational singularities are well known to physicists ever since the discovery of the Schwarzschild black hole solution and have been one of the major themes of research. All the physically relevant quantities diverge at the singularity, and the underlying physical theory loses its predictive power. In fact, since the notion of space and time becomes ill defined at the singularity, the definition of singularities in general relativity is far from obvious. Gravitational singularities, as of now, are best understood in terms of the geodesic incompleteness of causal curves [1,2].

The issue of occurrence of singularities in generic solutions of Einstein's equation has been pursued by numerous researchers over many decades, and the most definitive work in this regard is the Hawking and Penrose

singularity theorems [1,2]. These theorems are a set of results which asserts that the solution of Einstein's equation with physically reasonable initial data would always evolve to a singularity via gravitational collapse. One of the major key insights in classical general relativity is the realization that such a singularity which results from the gravitational collapse of an object may always be hidden inside the event horizon of a black hole and not accessible to an outside observer. This is known as the weak cosmic censorship conjecture (WCCC), first stated by Penrose [2,3], and is essential in ensuring the predictability of physical laws outside the black hole. More precisely, the WCCC is the statement that regular initial data cannot be evolved by Einstein's equation to a naked singularity solution, a spacetime singularity without an event horizon. For some interesting reviews on the cosmic censorship conjecture and gravitational collapse, refer to Refs. [4–6], and references therein.

Considering the complexity involved in Einstein's equations, it is challenging and almost impossible to come up with a general proof of the statement, and therefore cosmic censorship as of now remains a major open problem in classical general relativity. However, attempts have been made by numerous researchers to approach the problem in the other way around, i.e., by looking for a possible counterexample of the hypothesis. The strategy is to start with a black hole solution as an initial configuration (extremal or nonextremal) and look for any possible physical process that would give rise to a naked singularity in the final state. Such an analysis was pioneered by Wald in his seminal paper [7], where he attempts to overcharge and overspun an extremal Kerr-Newman black hole via the process of charged test particle absorption to produce a naked singularity. Interestingly, the electromagnetic repulsion between the test particle and the black hole turns out to

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be enough to prevent the black hole from capturing such a particle.

The WCCC is supported as well as challenged by several such examples and counterexamples. Extending Wald's calculation further, an analogous result of overcharging a Kerr-Newman black hole in the presence of both electric and magnetic charges was studied in [8]. Overcharging via test field absorption instead of test particle was pursued in [9–11]. Various other attempts of overcharging have been carried out in the last few decades over a wide range of black hole solutions in general relativity in order to look for any possible violation of cosmic censorship [12–24].

In Ref. [25], Hubeny showed that, starting with a slightly nonextremal charged black hole, it is possible to create a naked singularity in the final configuration via charged particle absorption. Such a near-extremal black hole overcharging problem was further pursued in [26–28]. Further such counterexamples to the WCCC can be found in [29,30]. However, it is also argued that the violation of the WCCC can be avoided by taking into account the backreaction effect of the test particle. For follow-up works on the WCCC with the self-force effect refer to [31–35].

Recently, in [36], the authors have presented a general analysis of the overcharging problem for arbitrary matter falling into the black hole (extremal and nonextremal) by taking into account up to second-order variation of the parameters of the black hole. The main result of [36] is that overcharging of extremal as well as nonextremal black holes is not possible, as long as the matter stress-energy tensor satisfies the null energy condition.

Hence, considering the current status and in the absence of a general proof, it is very much desirable for one to look for various other possible tests of the cosmic censorship hypothesis. Such tests as of now available in the literature only involve the overcharging of a single black hole configuration, and till now, no attempts have been made to study the WCCC in the context of a system comprising more than a single black hole. This motivates us to test the cosmic censorship hypothesis in a multi-black-hole setting. In this article we attempt to study the problem of overcharging via test particle absorption in the context of the well-known Majumdar-Papapetrou black hole solution, which corresponds to a system of two extremal black holes in equilibrium [37–40]. The final configuration that we would be interested in is a dihole, which is a stable system of a nonextremal black hole and a naked singularity and exists as a nontrivial solution of the Einstein-Maxwell equations [41]. Essentially, we would attempt to address the following question: Is it possible to obtain a stable dihole configuration in the final state, starting from two extremal black holes in equilibrium? The question of stability is far richer in a dihole configuration than that of a single black hole and requires additional conditions on the parameters involved, i.e., the mass ( $m_i$ ), charge ( $q_i$ ) and distance between the two sources [41]. Also, the spacetime structure

in such a system is vastly different than that of a single black hole because of different symmetry of the spacetime [42]. *A priori* there is no reason to believe that the WCCC would hold for such cases and requires careful analysis.

To our surprise, even for such a nontrivial system, the overcharging is found to be impossible and does not lead to any violation of the cosmic censorship hypothesis. Specifically, we show that it is impossible to obtain an equilibrium dihole system in the final configuration via test particle absorption. We also show that one cannot even overcharge to obtain a naked singularity without using the equilibrium condition. The result seems to be a strong and nontrivial example in favor of the cosmic censorship hypothesis. In this analysis, we neglect the self-force effect.

## II. OVERCHARGING A MAJUMDAR-PAPAPETROU CONFIGURATION

The equilibrium configuration of multiple massive objects in Newtonian physics is vastly different than the same situation in general relativity [43–48]. The only equilibrium configuration in general relativity is known to be either a system of two extremal black holes or a combination of a nonextremal black hole and a naked singularity satisfying some stability conditions. The former one is the well-known Majumdar-Papapetrou solution and the latter we would refer as a dihole [41]. As of now, the dihole configuration represents the only solution of the Einstein-Maxwell equation known in the literature consisting of two nonextreme sources in equilibrium. We start this section by providing an overview of the dihole geometry and subsequently present an analysis of the motion of test particles on such a background. In terms of Weyl cylindrical coordinate, the line element can be written as [41]

$$ds^2 = -H(\rho, z)dt^2 + f(\rho, z)(d\rho^2 + dz^2) + \frac{\rho^2}{H}d\phi^2, \quad (1)$$

$$A_t = \Phi(\rho, z), \quad A_\rho = A_z = A_\phi = 0, \quad (2)$$

where  $A_\mu$  is the electromagnetic vector potential and  $z$  is the symmetry axis. Considering the symmetry of the problem, it is convenient to express the metric components in terms of bipolar coordinates  $(r_1, \theta_1)$  and  $(r_2, \theta_2)$ , centered at both the sources respectively and spanning the entire spacetime, which can be achieved by the following coordinate transformations:

$$\rho = \sqrt{(r_1 - m_1)^2 - \sigma_1^2} \sin \theta_1, \quad z = z_1 + (r_1 - m_1) \cos \theta_1, \quad (3)$$

$$\rho = \sqrt{(r_2 - m_2)^2 - \sigma_1^2} \sin \theta_2, \quad z = z_2 + (r_2 - m_2) \cos \theta_2. \quad (4)$$

The notations in the above expression are

$$\sigma_1^2 = m_1^2 - e_1^2 + 2e_1\gamma, \quad \sigma_2^2 = m_2^2 - e_2^2 - 2e_2\gamma, \quad (5)$$

$$\gamma = \frac{m_2 e_1 - m_1 e_2}{l + m_1 + m_2}, \quad l = z_2 - z_1. \quad (6)$$

Here  $m_1$ ,  $m_2$  and  $e_1$ ,  $e_2$  represent the physical mass and charge of the two nonextreme Reissner-Nordström sources located at points  $z_1$  and  $z_2$ , respectively, on the symmetry axis, with  $l$  being the distance between them. For this configuration, it turns out that the region between the two

sources contains conical singularities (struts) at each point on the symmetry axis. However, the condition

$$m_1 m_2 = (e_1 - \gamma)(e_2 + \gamma) \quad (7)$$

ensures the equilibrium between the two sources without the presence of any such struts on the symmetry axis. Note that, when one of the sources is absent, the equilibrium condition (7) is trivially satisfied. The components of metric ( $H$ ,  $f$ ) and electrostatic potential ( $\Phi$ ) in this bipolar coordinate system read [41]

$$H = \frac{[(r_1 - m_1)^2 - \sigma_1^2 + \gamma^2 \sin^2 \theta_2] \times [(r_2 - m_2)^2 - \sigma_2^2 + \gamma^2 \sin^2 \theta_1]}{[r_1 r_2 - (e_1 - \gamma - \gamma \sin \theta_1)(e_2 + \gamma - \gamma \sin \theta_2)]^2}, \quad (8)$$

$$\Phi = -\frac{[(e_1 - \gamma)(r_2 - m_2) + (e_2 + \gamma)(r_1 - m_1) + \gamma(m_1 \cos \theta_1 + m_2 \cos \theta_2)]}{[r_1 r_2 - (e_1 - \gamma - \gamma \sin \theta_1)(e_2 + \gamma - \gamma \sin \theta_2)]}, \quad (9)$$

$$f = \frac{[r_1 r_2 - (e_1 - \gamma - \gamma \sin \theta_1)(e_2 + \gamma - \gamma \sin \theta_2)]^2}{[(r_1 - m_1)^2 - \sigma_1^2 \cos^2 \theta_1] \times [(r_2 - m_2)^2 - \sigma_2^2 \cos^2 \theta_2]}. \quad (10)$$

Note that  $\sigma_k^2 = 0$  ( $k = 1, 2$  for both sources) represents the extremal limit and the above line element reduces to that of a Majumdar-Papapetrou solution consisting of two extremal black holes. However,  $\sigma_k^2 > 0$  represents a nonextreme black hole whereas  $\sigma_k^2 < 0$  corresponds to a naked singularity. The location of the horizon can be obtained by setting  $H = 0$ , which corresponds to  $r_1 = m_1$  and  $r_2 = m_2$  in the extremal limit or equivalently  $\rho = 0$ .

Having discussed the geometry of a dihole system, now we would like to understand the overcharging problem and the cosmic censorship in this context. To that end, let us start with a system of two extremal black holes ( $\sigma_{1(i)}^2 = \sigma_{2(i)}^2 = 0, \gamma = 0$ ) as our initial configuration. Let the first extremal black hole absorb a test particle of mass  $\delta m_1$  and charge  $\delta e_1$  and turn into a nonextreme black hole of final mass  $m_1 + \delta m_1$  and final charge  $e_1 + \delta e_1$ . The test particle assumption is ensured by the conditions  $\delta m_k \ll m_k$  and  $\delta e_k \ll e_k$ . For a black hole in the final state, we have  $\sigma_{1(f)}^2 > 0$ , which further reduces to

$$(\delta m_1 - \delta e_1) > -\delta\gamma. \quad (11)$$

And the second extremal black hole absorbs another test particle of mass  $\delta m_2$  and charge  $\delta e_2$  and turns into a naked singularity of final mass  $m_2 + \delta m_2$  and final charge  $e_2 + \delta e_2$ . For a naked singularity in the final state we have  $\sigma_{2(f)}^2 < 0$ , which leads to

$$(\delta m_2 - \delta e_2) < \delta\gamma. \quad (12)$$

Equations (11) and (12) represent the constraints on the mass and charge of both the test particles for which the final

state would correspond to a configuration of a nonextreme black hole and a naked singularity, which can be regarded as a potential violation of the cosmic censorship hypothesis. Now it is important to understand if the initial extremal black holes can capture such test particles satisfying the above two conditions. We carry out this analysis by studying the geodesic motion of test particles on the Majumdar-Papapetrou background. Let us start with the motion of first test particle with mass  $\delta m_1$  and charge  $\delta e_1$ . For a timelike geodesic, with metric signature  $\text{diag}(-, +, +, +)$  we have

$$-1 = -H\dot{t}^2 + f(\dot{\rho}^2 + \dot{z}^2) + \frac{\rho^2}{H}\dot{\phi}^2, \quad (13)$$

where the derivatives are with respect to the proper time  $\tau$  of the test particle. The energy of the particle is given by

$$-\delta m_1 = \delta e_1 \Phi - \dot{t} H. \quad (14)$$

Now we eliminate  $\dot{t}$  in (13), which reduces to

$$(\delta m_1 + \delta e_1 \Phi)^2 - H - \rho^2 \dot{\phi}^2 = H f (\dot{\rho}^2 + \dot{z}^2). \quad (15)$$

Since our initial configuration is a system of two extremal sources, we can use  $H F = 1$  to simplify the above expression further. Writing  $\dot{\rho}$  and  $\dot{z}$  in terms of  $\dot{r}_1$  and  $\dot{\theta}_1$ ,

$$\dot{\rho} = \sqrt{(r_1 - m_1)^2 - \sigma_1^2} \cos \theta_1 \dot{\theta}_1 + \frac{(r_1 - m_1) \dot{r}_1}{\sqrt{(r_1 - m_1)^2 - \sigma_1^2}} \sin \theta_1, \quad (16)$$

$$\dot{z} = \dot{r}_1 \cos \theta_1 - (r_1 - m_1) \sin \theta_1 \dot{\theta}_1. \quad (17)$$

At the extremal horizon ( $r_1 = m_1$ ) of the first black hole, one has  $\Phi = -1$  and  $H = 0$  and the geodesic equation further reduces to

$$(\delta m_1 - \delta e_1)^2 = \dot{r}_1^2. \quad (18)$$

Now, in order to cross the extremal horizon, the test particle must have  $\dot{r}_1 < 0$  at the horizon. Further note that the energy of the particle measured at infinity is positive, where the potential vanishes, and we have

$$\dot{r}_1 = -(\delta m_1 - \delta e_1). \quad (19)$$

Therefore the mass and charge of the test particle must satisfy  $\delta m_1 > \delta e_1$  in order to get captured by the black hole. Similarly, the second test particle must satisfy  $\delta m_2 > \delta e_2$  in order to cross the horizon of the second black hole. Also notice that so far we have not used the equilibrium condition to hold in the final configuration and we shall study both cases of presence and absence of struts on the symmetry axis separately. Let us start with the case where the equilibrium condition holds in the final configuration, i.e.,  $m_{1(f)}m_{2(f)} = (e_{1(f)} - \gamma_f)(e_{2(f)} + \gamma_f)$ , which ensures the absence of any struts on the symmetry axis between the two sources and using the fact that  $\gamma = 0$  for the initial configuration it further reduces to

$$\gamma_f = \delta\gamma = \frac{m_1(\delta m_2 - \delta e_2) + m_2(\delta m_1 - \delta e_1)}{(m_1 - m_2)}. \quad (20)$$

However, from the general expression in (6), by taking the first-order variation of  $\gamma$ , we obtain

$$\delta\gamma = \frac{m_1(\delta m_2 - \delta e_2) - m_2(\delta m_1 - \delta e_1)}{(l + m_1 + m_2)}. \quad (21)$$

Now, assuming the equilibrium condition to hold in the final configuration, we equate Eqs. (20) and (21) to solve for  $l$ , in terms of the variations, which gives rise to

$$\delta_1 + \delta_2 = -\frac{l(m_1\delta_2 + m_2\delta_1)}{2m_1m_2}, \quad (22)$$

where  $\delta_1 = \delta m_1 - \delta e_1$  and  $\delta_2 = \delta m_2 - \delta e_2$ . It is evident from the above expression that, as far as the test particles are entering the horizons, i.e.,  $\delta_1 > 0$  and  $\delta_2 > 0$ , (22) cannot be satisfied for the positive value of the length  $l$  and hence the equilibrium condition will not hold in the final configuration. Therefore, although we have started with a Majumdar-Papapetrou configuration initially, it would not be possible to maintain the equilibrium condition in the final configuration by test particle absorption. This rules out the possibilities of obtaining any equilibrium system (two extremal black holes, a black hole and a naked singularity or two naked singularities) by starting from a Majumdar-Papapetrou

system. Note that here the equilibrium condition ensures the absence of any struts on the symmetry axis.

Having illustrated that test particle absorption would not lead to any equilibrium configuration, now we would like to understand whether one can produce any naked singularity in the final configuration without demanding the equilibrium condition. In other words, we would like to see if we can produce a black hole and a naked singularity in the final configuration, with struts on the symmetry axis. To that end, we substitute the general first-order variation of  $\gamma$  (which does not use the equilibrium condition) from (21) in Eqs. (11) and (12) to obtain

$$(\delta m_1 - \delta e_1) > -\left(\frac{m_1}{l + m_1}\right)(\delta m_2 - \delta e_2), \quad (23)$$

$$(\delta m_2 - \delta e_2) < -\left(\frac{m_2}{l + m_2}\right)(\delta m_1 - \delta e_1). \quad (24)$$

Here, Eqs. (23) and (24) represent the conditions for obtaining a nonextremal black hole and a naked singularity in the final configuration, respectively. Note that the condition of source 1 becoming a black hole explicitly depends on the constraint of a test particle entering the second source and vice versa. It is not difficult to realize that the condition for obtaining a nonextremal black hole is consistent with the condition for the test particle entering the horizon. However, the condition for obtaining a naked singularity is not compatible with the test particle entering the horizon. Therefore, it is not possible to obtain any naked singularity, with or without using the equilibrium condition via test particle absorption by starting from two extremal sources. Hence, no violation of the WCCC can be achieved in the context of a Majumdar-Papapetrou configuration.

### III. DISCUSSIONS

Even after more than five decades since its proposal by Penrose, the cosmic censorship hypothesis still remains one unsolved puzzle in classical general relativity. One of the potential counterexamples to the hypothesis is the overcharging of a slightly nonextremal black hole to produce a naked singularity. However, it is widely believed that such processes would not occur when the backreaction effect of the test particle is taken into account. Although the conjecture lacks a general proof, there seems to be a growing amount of evidence in its favor. The general theme of approach in this regard has been to look for any possible counterexample to the hypothesis, which may result from regular initial data via some physical process. Following the gedanken experiment of Wald to overcharge an extremal Kerr-Newman black hole via test particle absorption, in this article, we attempted to generalize this result to a multi-black-hole solution of general relativity. In particular, we studied the overcharging problem of a system of two



extremal black holes represented by a Majumdar-Papapetrou solution in the initial configuration to produce a stable system of two nonextreme sources consisting of a black hole and a naked singularity in the final configuration. If such a physical process happens to exist, this will correspond to a potential violation of the cosmic censorship hypothesis. Because of distinct geodesic structure than that of a single source and an additional equilibrium condition, it certainly not evident whether the cosmic censorship hypothesis would still hold for such settings. Surprisingly, we found that such test particles which would expose the naked singularity upon absorption by the black hole would not be able to enter the black hole horizon and the cosmic censorship hypothesis even in such a nontrivial setting remains intact. We have also shown that test particle

absorption would not lead to any equilibrium final configuration (absence of struts). This is an important result supporting the validity of cosmic censorship since the dihole system is the only equilibrium solution of general relativity involving two nonextreme sources known in the literature. An interesting direction to pursue further would be to carry out this analysis for a system of  $N$ -extremal charged black holes.

## ACKNOWLEDGMENTS

We thank Avirup Ghosh for many helpful discussions. Research of S. S. is supported by the Department of Science and Technology, Government of India under the SERB Fast Track Scheme for Young Scientists (YSS/2015/001346).

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- [1] R. M. Wald, *General Relativity* (Chicago University Press, Chicago, 1984).
  - [2] S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-Time*, Cambridge Monographs on Mathematical Physics (Cambridge University Press, Cambridge, England, 1973).
  - [3] R. Penrose, Gravitational collapse: The role of general relativity, *Riv. Nuovo Cimento* **1**, 252 (1969); “Golden oldie”: Gravitational collapse: The role of general relativity, *Gen. Relativ. Gravit.* **34**, 1141 (2002).
  - [4] R. M. Wald, *Gravitational collapse and cosmic censorship*, Black Holes, Gravitational Radiation and the Universe: Essays in Honor of C.V. Vishveshwara (Springer, Netherlands, Dordrecht, 1999), [https://doi.org/10.1007/978-94-017-0934-7\\_5](https://doi.org/10.1007/978-94-017-0934-7_5).
  - [5] P. S. Joshi, Cosmic censorship: A current perspective, *Mod. Phys. Lett. A* **17**, 1067 (2002).
  - [6] C. J. S. Clarke, A title of cosmic censorship, *Classical Quantum Gravity* **11**, 1375 (1994).
  - [7] R. Wald, Gedanken experiments to destroy a black hole, *Ann. Phys. (N.Y.)* **82**, 548 (1974).
  - [8] I. Semiz, Dyon black holes do not violate cosmic censorship, *Classical Quantum Gravity* **7**, 353 (1990).
  - [9] I. Semiz, Dyonic Kerr-Newman black holes, complex scalar field and cosmic censorship, *Gen. Relativ. Gravit.* **43**, 833 (2011).
  - [10] G. Z. Toth, Test of the weak cosmic censorship conjecture with a charged scalar field and dyonic Kerr-Newman black holes, *Gen. Relativ. Gravit.* **44**, 2019 (2012).
  - [11] J. Natário, L. Queimada, and R. Vicente, Test fields cannot destroy extremal black holes, *Classical Quantum Gravity* **33**, 175002 (2016).
  - [12] S. Hod, Black hole polarization and cosmic censorship, *Phys. Rev. D* **60**, 104031 (1999).
  - [13] S. Hod, Weak Cosmic Censorship: As Strong as Ever, *Phys. Rev. Lett.* **100**, 121101 (2008).
  - [14] G. Chirco, S. Liberati, and T. P. Sotiriou, Gedanken experiments on nearly extremal black holes and the Third Law, *Phys. Rev. D* **82**, 104015 (2010).
  - [15] M. Bouhmadi-Lopez, V. Cardoso, A. Nerozzi, and J. V. Rocha, Black holes die hard: Can one spin-up a black hole past extremality?, *Phys. Rev. D* **81**, 084051 (2010).
  - [16] A. Saa and R. Santarelli, Destroying a near-extremal Kerr-Newman black hole, *Phys. Rev. D* **84**, 027501 (2011).
  - [17] J. V. Rocha and R. Santarelli, Flowing along the edge: Spinning up black holes in AdS spacetimes with test particles, *Phys. Rev. D* **89**, 064065 (2014).
  - [18] C. Fairoos, A. Ghosh, and S. Sarkar, Massless charged particles: Cosmic censorship, and the third law of black hole mechanics, *Phys. Rev. D* **96**, 084013 (2017).
  - [19] K. S. Revelar and I. Vega, Overcharging higher-dimensional black holes with point particles, *Phys. Rev. D* **96**, 064010 (2017).
  - [20] J. An, J. Shan, H. Zhang, and S. Zhao, Five-dimensional Myers-Perry black holes cannot be overspun in Gedanken experiments, *Phys. Rev. D* **97**, 104007 (2018).
  - [21] T.-Y. Yu and W.-Y. Wen, Cosmic censorship and weak gravity conjecture in the Einstein-Maxwell-dilaton theory, *Phys. Lett. B* **781**, 713 (2018).
  - [22] S. Jana, R. Shaikh, and S. Sarkar, Overcharging black holes and cosmic censorship in Born-Infeld gravity, *Phys. Rev. D* **98**, 124039 (2018).
  - [23] S. Shaymatov, N. Dadhich, and B. Ahmedov, The higher dimensional Myers-Perry black hole with single rotation always obeys the cosmic censorship conjecture, [arXiv:1809.10457](https://arxiv.org/abs/1809.10457).
  - [24] B. Ge, Y. Mo, S. Zhao, and J. Zheng, Higher-dimensional charged black holes cannot be over-charged by Gedanken experiments, *Phys. Lett. B* **783**, 440 (2018).
  - [25] V. E. Hubeny, Overcharging a black hole and cosmic censorship, *Phys. Rev. D* **59**, 064013 (1999).
  - [26] S. Hod, Cosmic censorship, area theorem, and self-energy of particles, *Phys. Rev. D* **66**, 024016 (2002).

- [27] G.E.A. Matsas and A.R.R. da Silva, Overspinning a Nearly Extreme Charged Black Hole via a Quantum Tunneling Process, *Phys. Rev. Lett.* **99**, 181301 (2007).
- [28] T. Jacobson and T.P. Sotiriou, Over-Spinning a Black Hole with a Test Body, *Phys. Rev. Lett.* **103** (2009) 141101; Erratum, *Phys. Rev. Lett.* **103**, 209903(E) (2009).
- [29] L. Lehner and F. Pretorius, Black Strings, Low Viscosity Fluids, and Violation of Cosmic Censorship, *Phys. Rev. Lett.* **105**, 101102 (2010).
- [30] S. Gao and Y. Zhang, Destroying extremal Kerr-Newman black holes with test particles, *Phys. Rev. D* **87**, 044028 (2013).
- [31] E. Barausse, V. Cardoso, and G. Khanna, Test Bodies and Naked Singularities: Is the Self-Force the Cosmic Censor?, *Phys. Rev. Lett.* **105**, 261102 (2010).
- [32] E. Barausse, V. Cardoso, and G. Khanna, Testing the cosmic censorship conjecture with point particles: The effect of radiation reaction and the self-force, *Phys. Rev. D* **84**, 104006 (2011).
- [33] S. Isoyama, N. Sago, and T. Tanaka, Cosmic censorship in overcharging a Reissner-Nordström black hole via charged particle absorption, *Phys. Rev. D* **84**, 124024 (2011).
- [34] P. Zimmerman, I. Vega, E. Poisson, and R. Haas, Self-force as a cosmic censor, *Phys. Rev. D* **87**, 041501 (2013).
- [35] M. Colleoni and L. Barack, Overspinning a Kerr black hole: The effect of self-force, *Phys. Rev. D* **91**, 104024 (2015).
- [36] J. Sorce and R.M. Wald, Gedanken experiments to destroy a black hole. II. Kerr-Newman black holes cannot be overcharged or overspun, *Phys. Rev. D* **96**, 104014 (2017).
- [37] S.D. Majumdar, A class of exact solutions of Einstein's field equations, *Phys. Rev.* **72**, 390 (1947).
- [38] A. Papapetrou, A static solution of the equations of the gravitational field for an arbitrary charge-distribution, *Proc. R. Irish Acad., Sect. A* **51**, 191 (1945).
- [39] J.B. Hartle and S.W. Hawking, Solutions of the Einstein-Maxwell equations with many black holes, *Commun. Math. Phys.* **26**, 87 (1972).
- [40] O. Semerák and M. Basovnick, Geometry of deformed black holes. I. Majumdar-Papapetrou binary, *Phys. Rev. D* **94**, 044006 (2016).
- [41] G.A. Alekseev and V.A. Belinski, Equilibrium configurations of two charged masses in general relativity, *Phys. Rev. D* **76**, 021501 (2007).
- [42] J. Ryzner and M. Žofka, Electrogeodesics in the di-hole Majumdar-Papapetrou spacetime, *Classical Quantum Gravity* **32**, 205010 (2015).
- [43] W.B. Bonnor, The equilibrium of two charged masses in general relativity, *Phys. Lett.* **83A**, 414 (1981).
- [44] T. Ohta and T. Kimura, Static balance of two charged particles in general relativity, *Prog. Theor. Phys.* **68**, 1175 (1982).
- [45] A. Tomimatsu, Condition for equilibrium of two Reissner-Nordström black holes, *Prog. Theor. Phys.* **71**, 409 (1984).
- [46] T. Azuma and T. Koikawa, Equilibrium condition in the axisymmetric N-Reissner-Nordstrom solution, *Prog. Theor. Phys.* **92**, 1095 (1994).
- [47] D. Bini, A. Geralico, and R. Ruffini, On the equilibrium of a charged massive particle in the field of a Reissner-Nordstrom black hole, *Phys. Lett. A* **360**, 515 (2007).
- [48] V.S. Manko, The Double-Reissner-Nordstrom solution and the interaction force between two spherically symmetric charged particles, *Phys. Rev. D* **76**, 124032 (2007).