Study of the electromagnetic Dalitz decays $\psi(\Upsilon) \to \eta_c(\eta_b) l^+ l^-$

Li-Min Gu,[1,*] Hai-Bo Li,[2,3,†] Xin-Xin Ma,[2,3,‡] and Mao-Zhi Yang[4,§]

[1]*Nanjing University, Nanjing 210093, People's Republic of China*
[2]*Institute of High Energy Physics, Beijing 100049, People's Republic of China*
[3]*University of Chinese Academy of Sciences, Beijing 100049, People's Republic of China*
[4]*School of Physics, Nankai University, Tianjin 300071, China*

(Received 16 April 2019; published 30 July 2019)

We study the electromagnetic Dalitz decays, $\psi \to \eta_c l^+ l^-$ and $\Upsilon \to \eta_b l^+ l^-$ ($l = e$ or μ), in which the lepton pair comes from the virtual photon emitted by the M1 transition from $c\bar{c}$ ($b\bar{b}$) spin triplet state to the spin singlet state. We estimate the partial width of $\psi(\Upsilon) \to \eta_c(\eta_b) l^+ l^-$, based on the simple pole approximation. Besides, based on different QCD models, the partial width of $\psi(\Upsilon) \to \eta_c(\eta_b)\gamma$ is determined.

DOI: 10.1103/PhysRevD.100.016018

I. INTRODUCTION

The electromagnetic (EM) Dalitz decays, $V \to P l^+ l^-$, where V and P are vectors and pseudoscalar mesons, and l denotes lepton (e, μ), provide an ideal opportunity to probe the structure of hadronic states and to investigate the fundamental mechanisms of the interactions between photons and hadrons [1,2]. The lepton pair $l^+ l^-$ comes from an off-shell photon, radiated from the transition between V and P. Assuming pointlike particles, the process can be exactly described by QED [3]. Otherwise, the structure-dependent partial width can be modified by transition form factor $f_{\rm VP}(q^2)$, which can be estimated based on QCD models [4–8] and provides information of the EM structure arising from the V-P transition. The M1 transition between $\psi(\Upsilon)$ and $\eta_c(\eta_b)$ is widely studied in theory [9–20], and the M1 transition between ψ and $\eta_{\rm c}$ has been observed with the average branching fraction, $\mathcal{B}(J/\psi \to \gamma\eta_{\rm c}(1S)) = (1.7 \pm 0.4)\%$ [21]. In the following ratio of branching fractions:

$$R \equiv \frac{B(\psi \to e^+ e^- \eta_c)}{B(\psi \to \gamma\eta_c)}, \quad (1)$$

many theoretical uncertainties can cancel; therefore, it can be used to test theoretical models. Experimentally, the EM Dalitz decays of light unflavored vector mesons (ρ^0, ω, ϕ) have been widely observed [21] and several decays of charmonium vector mesons ($J/\psi, \psi'$) to light pseudoscalar mesons, which are studied in Ref. [22], have been observed recently by BESIII experiment [21,23,24]. In Table I, we summarize the experimental results of the EM decays for the light unflavored vector mesons (ρ^0, ω, ϕ) and charmonium vector mesons ($J/\psi, \psi'$). The results indicate that the ratios of the EM Dalitz decay to the corresponding radiative decays are suppressed by 2 orders of magnitude.

However, in previous paper [22], which focus on the EM Dalitz decays of J/ψ, it is assumed that the $\psi(\Upsilon)$ is totally unpolarized. In this paper, we investigate the polarization of $\psi(\Upsilon)$ produced in e^+e^- collisions, then deduce the general form of the decay width, as a function of polarization vector of $\psi(\Upsilon)$, at last apply it to the $\psi(\Upsilon) \to l^+ l^- \eta_c(\eta_b)$ decay to predict its branching fraction.

TABLE I. The branching fractions of EM Dalitz decays $V \to P l^+ l^-$ and ratios of the EM Dalitz decays to the corresponding radiative decays of the vector mesons. These data are from PDG2018 [21].

Decay mode	Branching fraction	$\frac{\Gamma(V \to P l^+ l^-)}{\Gamma(V \to P\gamma)}$
$\rho^0 \to \pi^0 e^+ e^-$	$< 1.2 \times 10^{-5}$	$< 2.6 \times 10^{-2}$
$\omega \to \pi^0 e^+ e^-$	$(7.7 \pm 0.6) \times 10^{-4}$	$(0.91 \pm 0.08) \times 10^{-2}$
$\omega \to \pi^0 \mu^+ \mu^-$	$(1.34 \pm 0.18) \times 10^{-4}$	$(0.16 \pm 0.02) \times 10^{-2}$
$\phi \to \pi^0 e^+ e^-$	$(1.33^{+0.07}_{-0.10}) \times 10^{-5}$	$(1.02^{+0.07}_{-0.09}) \times 10^{-2}$
$\phi \to \eta e^+ e^-$	$(1.08 \pm 0.04) \times 10^{-4}$	$(0.83 \pm 0.03) \times 10^{-2}$
$\phi \to \eta \mu^+ \mu^-$	$< 9.4 \times 10^{-6}$	$< 0.07 \times 10^{-2}$
$J/\psi \to \pi^0 e^+ e^-$	$(7.6 \pm 1.4) \times 10^{-7}$	$(2.18^{+0.45}_{-0.44}) \times 10^{-2}$
$J/\psi \to \eta e^+ e^-$	$(1.16 \pm 0.09) \times 10^{-5}$	$(1.05 \pm 0.09) \times 10^{-2}$
$J/\psi \to \eta' e^+ e^-$	$(5.81 \pm 0.35) \times 10^{-5}$	$(1.13 \pm 0.08) \times 10^{-2}$
$\psi' \to \eta' e^+ e^-$	$(1.90 \pm 0.27) \times 10^{-6}$	$(1.53 \pm 0.22) \times 10^{-2}$

*gulm@ihep.ac.cn
†lihb@ihep.ac.cn
‡maxx@ihep.ac.cn
§yangmz@nankai.edu.cn

II. GENERAL FORMULA FOR THE EM DALITZ DECAY $V \to Pl^+l^-$

The amplitude of the EM Dalitz decay, $V \to Pl^+l^-$, has been obtained in the literature previously, which can be written in a Lorentz-invariant form [1,2,22]

$$T(V \to Pl^+l^-) = 4\pi\alpha f_{VP}\epsilon^{\mu\nu\rho\sigma}p_\mu q_\nu \epsilon_\rho \frac{1}{q^2}\bar{u}_1\gamma_\sigma\nu_2, \quad (2)$$

where α is the fine-structure constant, f_{VP} the transition form factor, $\epsilon^{\mu\nu\rho\sigma}$ the Levi-Civita tensor, p_μ the momentum of the pseudoscalar meson, and $q_\nu = k_1 + k_2$ with k_1 and k_2 the momenta of the l^+ and l^-. After summing over the spin of leptons, the amplitude squared is

$$|T(V \to Pl^+l^-)|^2 = 16\pi^2\alpha^2 \frac{|f_{VP}(q^2)|^2}{q^4} \cdot h, \quad (3)$$

where

$$\begin{aligned} h = {} & 8m_V^2 m_l^2(q^2\epsilon\cdot\epsilon^* - q\cdot\epsilon q\cdot\epsilon^*) - 2m_V^2 q^4(k_1-k_2) \\ & \cdot\epsilon(k_1-k_2)\cdot\epsilon^* + 8m_l^2 q\cdot p[q\cdot\epsilon p.\epsilon^* - \epsilon\cdot\epsilon^* q\cdot p] \\ & + 2m_l^2(k_1-k_2)\cdot p[\epsilon\cdot\epsilon^*(k_1-k_2)\cdot p + (k_1-k_2) \\ & \cdot\epsilon(p\cdot\epsilon^*)] + 8[(k_2\cdot p)(k_1\cdot\epsilon) - (k_1\cdot p)(k_2\cdot\epsilon)] \\ & \times[(k_2\cdot p)(k_1\cdot\epsilon^*) - (k_1\cdot p)(k_2\cdot\epsilon^*)], \end{aligned} \quad (4)$$

where m_V and m_l are masses of vector and pseudoscalar mesons. The differential decay width of $V \to Pl^+l^-$ is obtained as

$$\begin{aligned} d\Gamma(V \to Pl^+l^-) = {} & \frac{1}{(2\pi)^5}\frac{1}{16m_V^2}|T(\psi(\Upsilon) \to Pl^+l^-)|^2 \\ & \times |\mathbf{k}^*||\mathbf{p}_3| dm_{l^+l^-} d\Omega_3 d\Omega_1^*, \end{aligned} \quad (5)$$

where $|\mathbf{k}^*|$ is the momentum of l^+ or l^- in the rest frame of l^+l^- system, $|\mathbf{p}_3|$ momentum of the pseudoscalar meson P in the rest frame of V, $d\Omega_3 = d\phi_3 d(\cos\theta_3)$ is the solid angle of P in the rest frame of V, and $d\Omega_1^* = d\phi_1^* d(\cos\theta_1^*)$ the solid angle of l^+ or l^- in the rest frame of l^+l^- system (the z direction is defined as the momentum direction of l^+l^- system in the rest frame of V).

A. Differential partial widths for polarized ψ/Υ EM Dalitz decays

There are totally three polarization states for the massive vector mesons, which are defined as

$$\begin{aligned} \epsilon^\mu_{\rm LP} &= (0,0,0,1) \\ \epsilon^\mu_{\rm TL} &= \frac{1}{\sqrt{2}}(0,1,-i,0) \\ \epsilon^\mu_{\rm TR} &= \frac{1}{\sqrt{2}}(0,1,i,0), \end{aligned} \quad (6)$$

where $\epsilon^\mu_{\rm LP}$, $\epsilon^\mu_{\rm TL}$, and $\epsilon^\mu_{\rm TR}$ are the longitudinal polarization, the left-hand transverse polarization, and the right-hand transverse polarization component, respectively. Correspondingly, it is generally known that the angular distribution for each component that has the following form:

$$\begin{aligned} \frac{d\Gamma(\psi(\Upsilon) \to Pl^+l^-)_{\rm LP}}{d\cos\theta} &\sim 1-\cos^2\theta \\ \frac{d\Gamma(\psi(\Upsilon) \to Pl^+l^-)_{\rm TL}}{d\cos\theta} &\sim \frac{1+\cos^2\theta}{2} \\ \frac{d\Gamma(\psi(\Upsilon) \to Pl^+l^-)_{\rm TR}}{d\cos\theta} &\sim \frac{1+\cos^2\theta}{2}, \end{aligned} \quad (7)$$

where $\theta = \theta_3$ is the polar angle of the pseudoscalar P in the rest frame of $\psi(\Upsilon)$. Therefore, one can determine the polarization of $\psi(\Upsilon)$ by a likelihood fit to the $\cos\theta$ distribution

$$\mathcal{L}(\cos\theta) = 1 + \alpha_{\rm polar}\cos^2\theta, \quad (8)$$

where nonzero $\alpha_{\rm polar}$ means partial polarization of $\psi(\Upsilon)$, $\alpha_{\rm polar} = +1$ means only transverse polarization of $\psi(\Upsilon)$, and $\alpha_{\rm polar} = -1$ means only longitudinal polarization of $\psi(\Upsilon)$, respectively. We show the angular distributions of $\psi(\Upsilon) \to Pl^+l^-$ with different $\alpha_{\rm polar}$ values in Fig. 1. In $\psi(\Upsilon) \to Pl^+l^-$, an explicit value of $\alpha_{\rm polar}$ from experiment indicates the polarization information of $\psi(\Upsilon)$.

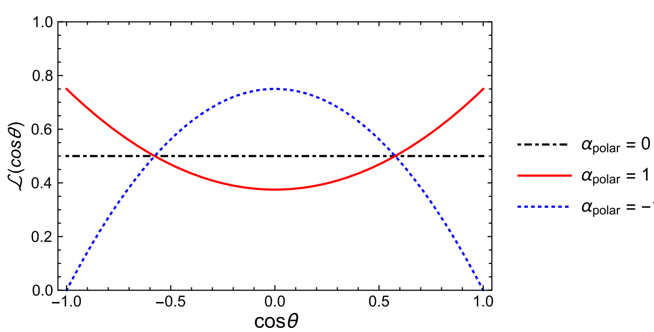

FIG. 1. The angular distributions of $\psi(\Upsilon) \to Pl^+l^-$ with different $\alpha_{\rm polar}$ values, where the black dot-dashed line refers to $\alpha_{\rm polar} = 0$ (nonpolarization); the red solid curve refers to $\alpha_{\rm polar} = 1$ (transverse polarization); and the blue dotted curve refers to $\alpha_{\rm polar} = -1$ (longitudinal polarization).

B. Polarization of $\psi(\Upsilon)$ in $e^+e^- \to \psi(\Upsilon)$

In this section, we study the production rate for each polarization state of the vector mesons ψ or Υ in electron-positron colliders. In electron-positron collisions, such as BESIII and Belle II experiments, the amplitude squared of $e^+e^- \to \psi$ or Υ can be written as

$$|T(\psi)|^2 = \frac{16\pi^2\alpha^2 e_c^2}{q^4}|f_\psi|^2 m_\psi^2 \epsilon_\mu^* \epsilon_\nu \times (k_1^\mu k_2^\nu + k_2^\mu k_1^\nu - g^{\mu\nu} k_1 \cdot k_2 + g^{\mu\nu} m_e^2), \quad (9)$$

where e_c is the electrical charge of the charm quark, f_ψ the form factor of $c\bar{c} \to \psi$, ϵ_μ the polarization four vector of ψ, m_ψ, and m_e the masses of ψ and electron, $q = k_1 + k_2$ with k_1 and k_2 the momenta of e^+ and e^-, respectively. Then, the relative probabilities for three polarization states can be obtained

$$|T(\psi)|^2_{\mathrm{LP}} : |T(\psi)|^2_{\mathrm{TL}} : |T(\psi)|^2_{\mathrm{TR}} = m_e^2 : m_\psi^2 : m_\psi^2 \approx 2.7 \times 10^{-8} : 1 : 1. \quad (10)$$

The above result shows that the longitudinal polarization can be neglected and ψ is totally polarized in transverse state in unpolarized e^+e^- collider, which is consistent with the conclusion in Ref. [25]. It is easy and intuitive to extend the conclusion to Υ case.

C. Decay rate of $\psi(\Upsilon)$ in $\psi(\Upsilon) \to \eta_c(\eta_b) l^+ l^-$

To remove most part of the uncertainty caused by the form factor $f_{VP}(q^2)$, one can consider the ratio of the decay widths of $\psi(\Upsilon) \to P l^+ l^-$ and $\psi(\Upsilon) \to P\gamma$. The q^2-dependent differential decay width of $\psi(\Upsilon) \to P l^+ l^-$ normalized to the width of the corresponding radiative decay $\psi(\Upsilon) \to P\gamma$ has been given in Ref. [22]; here for clearness we quote it in the following:

$$\frac{d\Gamma(\psi(\Upsilon) \to P l^+ l^-)}{dq^2 \Gamma(\psi(\Upsilon) \to P\gamma)} = |F_{VP}(q^2)|^2 \times [\mathrm{QED}(q^2)], \quad (11)$$

where the normalized transition form factor for the $\psi(\Upsilon) \to P$ transition is defined as $F_{VP}(q^2) \equiv f_{VP}(q^2)/f_{VP}(0)$, and $\mathrm{QED}(q^2)$ represents the QED calculations for pointlike particles,

$$\mathrm{QED}(q^2) = \frac{\alpha}{3\pi}\frac{1}{q^2}\left(1 - \frac{4m_l^2}{q^2}\right)^{\frac{1}{2}}\left(1 + \frac{2m_l^2}{q^2}\right) \times \left[\left(1 + \frac{q^2}{m_V^2 - m_P^2}\right)^2 - \frac{4m_V^2 q^2}{(m_V^2 - m_P^2)^2}\right]^{\frac{3}{2}}. \quad (12)$$

In experiment, by comparing the measured spectrum of the lepton pair in the EM Dalitz decay with the QED calculation for pointlike particle, one can determine the transition form factor in the timelike region of the momentum transfer [2]. Namely, the transition form factor can modify the lepton spectrum as compared with that obtained for pointlike particles.

To estimate the partial width of the $\psi(\Upsilon)$ EM Dalitz decay, the vector dominance model (VDM) is adopted, in which the hadronic EM current is proportional to vector meson fields [26,27]. Hence, the transition form factor can be parametrized in the simple pole approximation

$$F_{VP}(q^2) = \frac{1}{1 - \frac{q^2}{\Lambda^2}}, \quad (13)$$

where the pole mass Λ should be the mass of the vector resonance near the energy scale of the decaying particle according to the VDM model. In $\psi \to \eta_c l^+ l^-$ decays, the pole mass for J/ψ and ψ' could be the mass of ψ' and $\psi(3770)$, respectively. Similarly, in $\Upsilon \to \eta_b l^+ l^-$ decays, the pole mass for $\Upsilon(2\mathrm{S})$ ($\Upsilon(3\mathrm{S})$) decay could be the mass of $\Upsilon(3\mathrm{S})$ ($\Upsilon(4\mathrm{S})$).

By assuming the simple pole approximation, in which we take $\Lambda = m_{\psi'}, m_{\psi(3770)}, m_{\Upsilon(3\mathrm{S})}, m_{\Upsilon(4\mathrm{S})}$ for $J/\psi, \psi'$, $\Upsilon(2\mathrm{S})$ and $\Upsilon(3\mathrm{S})$ EM Dalitz decays, respectively; the partial decay widths of $\psi(\Upsilon) \to \eta_c(\eta_b) l^+ l^-$ are estimated and presented in Table II. It is shown that the decay rates for $\psi(\Upsilon) \to \eta_c(\eta_b) l^+ l^-$ when $l^+ l^-$ being $\mu^+\mu^-$ are approximately one-order smaller then the relevant case when the lepton pair is e^+e^-, which is due to the suppression of the phase space and the fast decrease of partial decay rate as the q^2 raises shown in Fig. 2.

To study the dependence of the decay rates on the value of the pole mass, we varied the pole mass. The numerical result shows both the differential and the total decay rates are not sensitive to the value of the pole mass. The reason can be well understood. The dominant contribution to the decay rate comes from the region of the small value of q^2 for the sake of phase space suppression in the region of large q^2. For the precesses considered in this work, the pole masses are larger then the maximum value of q^2; therefore, q^2/Λ^2 is small, thus this term cannot give large effect. Since the decay rates are not sensitive to the pole mass in

TABLE II. The estimated partial decay widths of $\psi \to \eta_c l^+ l^-$ and $\Upsilon \to \eta_b l^+ l^-$ based on Eq. (11) by assuming the simple pole approximation according to VDM model. Here we take $\Lambda = m_{\psi'}, m_{\psi(3770)}, m_{\Upsilon(3\mathrm{S})}$, and $m_{\Upsilon(4\mathrm{S})}$ for $J/\psi, \psi', \Upsilon(2\mathrm{S})$, and $\Upsilon(3\mathrm{S})$ EM Dalitz decays, respectively. The uncertainties are from the errors on measured $\Gamma(\psi \to \eta_c\gamma)$ and $\Gamma(\Upsilon \to \eta_b\gamma)$.

Decay mode	$\Gamma^{VDM}_{e^+e^-}$ (keV)	$\Gamma^{VDM}_{\mu^+\mu^-}$ (keV)
$J/\psi \to \eta_c l^+ l^-$	$(9.6 \pm 2.3) \times 10^{-3}$	…
$\psi' \to \eta_c l^+ l^-$	$(8.9 \pm 1.3) \times 10^{-3}$	$(8.2 \pm 1.2) \times 10^{-4}$
$\Upsilon(2\mathrm{S}) \to \eta_b l^+ l^-$	$(10.8 \pm 4.2) \times 10^{-5}$	$(8.2 \pm 3.2) \times 10^{-6}$
$\Upsilon(3\mathrm{S}) \to \eta_b l^+ l^-$	$(9.7 \pm 1.6) \times 10^{-5}$	$(12.6 \pm 2.1) \times 10^{-6}$

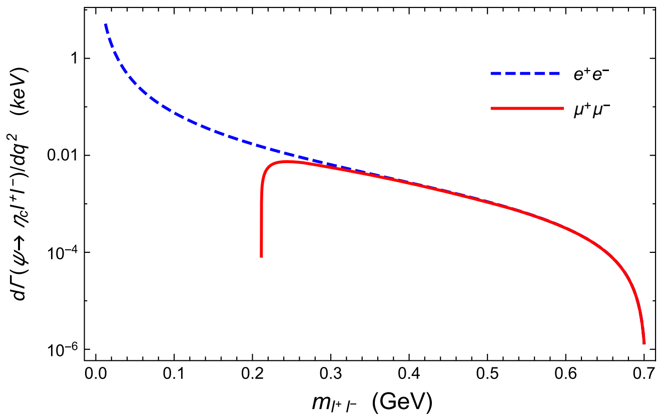

FIG. 2. The differential decay rates for $\psi' \to \eta_c l^+ l^-$, where the red solid curve is for $\psi' \to \mu^+\mu^-$, the blue dashed curve for $\psi' \to \eta_c e^+ e^-$.

the transition form factor, the estimated partial decay widths in Table II based on the VDM are reliable.

III. DISCUSSION

A. Model predictions of $\psi(\Upsilon) \to \eta_c(\eta_b)\gamma$

The EM Dalitz decays, $\psi(\Upsilon) \to \eta_c(\eta_b) l^+ l^-$, are related to the radiative decays, $\psi(\Upsilon) \to \eta_c(\eta_b)\gamma$, by the transition form factor $f_{VP}(q^2)$ and $f_{VP}(0)$. Models describing $\psi(\Upsilon) \to \eta_c(\eta_b)\gamma$ can provide information for $f_{VP}(q^2)$. In Ref. [28], using the theories of NRQCD and pNRQCD, one studied the M1 transitions between two heavy quarkonia and obtained

$$\Gamma(\psi(\Upsilon) \to \eta_c(\eta_b)\gamma) = \frac{16 e_Q^2}{3} \frac{\alpha (m^2_{\psi(\Upsilon)} - m^2_{\eta_c(\eta_b)})^3}{8 m^3_{\psi(\Upsilon)}} w(\alpha_s), \tag{14}$$

where e_Q is the electrical charge of the heavy quark ($e_c = 2/3$, $e_b = -1/3$), and $w(\alpha_s)$ the function of the strong coupling constant α_s. The predicted result for $J/\psi \to \eta_c\gamma$ is consistent with data [28]. However, the predicted results for $\psi' \to \eta_c\gamma$ and $\Upsilon(2S) \to \eta_b\gamma$ are larger than data by 2 and 1 order of magnitude, respectively. For $\Upsilon(3S) \to \eta_b\gamma$, the prediction of the branching ratio in the relativistic quark model is $\mathrm{Br}(\Upsilon(3S) \to \eta_b\gamma) = 4.0 \times 10^{-4}$ in Ref. [13], which is approximately consistent with experimental measurement $\mathrm{Br}(\Upsilon(3S) \to \eta_b\gamma)_{\mathrm{EXP}} = (5.1 \pm 0.7) \times 10^{-4}$ [21]. The prediction from the light-front quark model depends on the model parameters, which can be $(1.0 \sim 2.5) \times 10^{-4}$ [17]. This can also be consistent with experimental data after taking into account both experimental and theoretical errors. For $J/\psi \to \eta_c\gamma$, the $w(\alpha_s)$ has the following form [28]:

$$w(\alpha_s) = 1 + C_F \frac{\alpha_s(m_{J/\psi}/2)}{\pi} - \frac{2}{3}(C_F \alpha_s(p_{J/\psi}))^2, \tag{15}$$

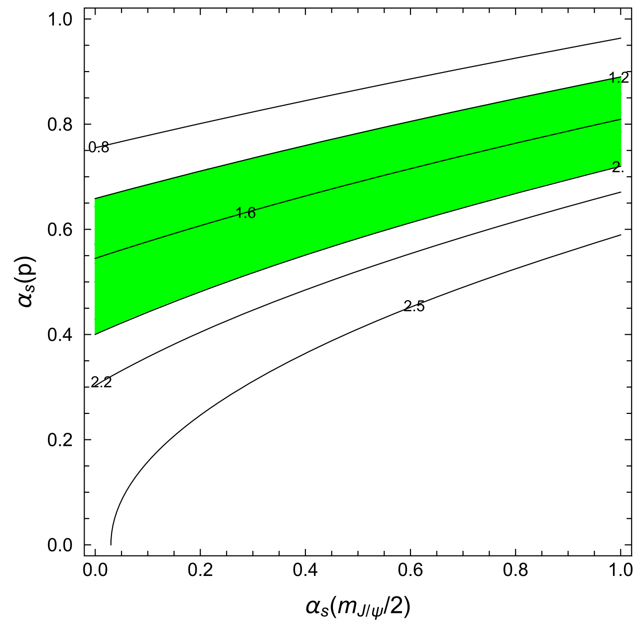

FIG. 3. The contour line of $\Gamma(J/\psi \to \eta_c\gamma)$ with $\alpha_s(m_{J/\psi}/2)$ and $\alpha_s(p_{J/\psi})$ from Eqs. (14) and (15). The green shadow represents the result in experiment, $\Gamma(J/\psi \to \eta_c\gamma)_{\mathrm{EXP}} = (1.6 \pm 0.4)$.

where $C_F = 4/3$ is the color coefficient, $\alpha_s(m_{J/\psi}/2)$ and $\alpha_s(p_{J/\psi})$ represent the values of α_s in corresponding energy scale, respectively. Typically, $\alpha_s(p_{J/\psi})$ satisfies $p_{J/\psi} \approx m_c C_F \alpha_s(p_{J/\psi})/2 \approx 0.8$ GeV [28]. To study the dependence of $\Gamma(J/\psi \to \eta_c\gamma)$ on the value of $\alpha_s(m_{J/\psi}/2)$ and $\alpha_s(p_{J/\psi})$, we varied the $\alpha_s(m_{J/\psi}/2)$ and $\alpha_s(p_{J/\psi})$. The results are shown in Fig. 3. In this way, a precision measurement of the partial width of $J/\psi \to \eta_c\gamma$ can be used to test QCD model and provide stringent restriction on α_s in charm energy scale.

B. Discussion on the absolute measurement of the partial widths of $\psi(\Upsilon) \to \eta_c(\eta_b) l^+ l^-$

With the explicit form factor $f_{VP}(q^2)$, one can obtain the partial decay widths of $\psi(\Upsilon) \to \eta_c(\eta_b)\gamma$ where $q^2 = 0$. Hence, it is important to measure the q^2-dependent form factor $f_{VP}(q^2)$, since it can be used to determine the partial decay widths of $\psi(\Upsilon) \to \eta_c(\eta_b)\gamma$ at $q^2 = 0$.

Actually, for the $\psi(\Upsilon) \to \eta_c(\eta_b) l^+ l^-$ decay in e^+e^- collisions at BESIII [29], Belle [30], and *BABAR* [31] experiments, several hundred million ψ (Υ) are collected. The lepton usually could be clearly distinguished from pion, kaon, and proton, so one needs only to reconstruct the lepton pair and then look at the recoiling mass of the lepton pair to obtain the signal. In this way, the results are irrelevant to the decay modes of $\eta_c(\eta_b)$. Hundreds of million of $\Upsilon(2S)$ and $\Upsilon(3S)$ events are collected by Belle and *BABAR* detector; we estimate that about 30 signal events could be observed for $\eta_b e^+ e^-$ mode with typical signal efficiency 4%. Since there are more than 10 billion and 800 million J/ψ and ψ' events collected by BESIII detector up to date, we expect 10^5 and 10^3 signal events from J/ψ and ψ', respectively, could be observed by BESIII with typical signal efficiency 10%. So it is possible to measure the branching fraction precisely and probe the transition form factor.

IV. SUMMARY

In summary, the EM Dalitz decays, $\psi(\Upsilon) \to \eta_c(\eta_b)l^+l^-$, are studied in this work. We investigate the effect of polarization of $\psi(\Upsilon)$ and estimate the partial decay widths of $\psi(\Upsilon) \to \eta_c(\eta_b)l^+l^-(l = e, \mu)$ by assuming the simple pole approximation. We obtain the polarization components for the vector mesons ψ and/or Υ produced in e^+e^- colliders. We find the transverse polarization states for ψ and/or Υ dominate in unpolarized e^+e^- colliders. We obtain the angular distribution of $\psi(\Upsilon) \to \eta_c(\eta_b)l^+l^-$ decays for each polarization state of ψ and/or Υ, which is helpful to determine the polarization of $\psi(\Upsilon)$ in the EM Dalitz decays $\psi(\Upsilon) \to \eta_c(\eta_b)l^+l^-$ in experiment. The decay widths of $\psi(\Upsilon) \to \eta_c(\eta_b)l^+l^-$ are also obtained. Besides, we discuss a QCD model where $\Gamma(J/\psi \to \eta_c\gamma)$ is related to α_s and suggest that absolute partial decay widths of EM Dalitz decays should be measured with reconstruction of only lepton pairs by taking advantage of the e^+e^- collision with known initial four momentum of the electron and positron beams.

ACKNOWLEDGMENTS

This work is supported in part by the National Natural Science Foundation of China under Contracts No. 11335009, No. 11875054, and No. 11875168; the Joint Large-Scale Scientific Facility Funds of the NSFC and CAS under Contract No. U1532257; CAS under Contract No. QYZDJ-SSW- SLH003; and the National Key Basic Research Program of China under Contract No. 2015CB856700.

[1] L. G. Landsberg, Electromagnetic leptonic decays and structure of light mesons, Sov. Phys. Usp. **28**, 435 (1985).
[2] L. G. Landsberg, Electromagnetic decays of light mesons, Phys. Rep. **128**, 301 (1985).
[3] N. M. Kroll and W. Wada, Internal pair production associated with the emission of high-energy gamma rays, Phys. Rev. **98**, 1355 (1955).
[4] N. N. Achasov and A. A. Kozhevnikov, Rare decays of the phi meson forbidden by the OZI rule and by G parity, Sov. J. Nucl. Phys. **55**, 449 (1992).
[5] F. Klingl, N. Kaiser, and W. Weise, Effective Lagrangian approach to vector mesons, their structure and decays, Z. Phys. A **356**, 193 (1996).
[6] A. Faessler, C. Fuchs, and M. I. Krivoruchenko, Dilepton spectra from decays of light unflavored mesons, Phys. Rev. C **61**, 035206 (2000).
[7] C. Terschlusen and S. Leupold, Electromagnetic transition form factors of light vector mesons, Phys. Lett. B **691**, 191 (2010).
[8] S. Ivashyn, Vector to pseudoscalar meson radiative transitions in chiral theory with resonances, Prob. Atomic Sci. Technol. **2012N1**, 179 (2012).
[9] M. A. Shifman, eta(c) as we know it, Z. Phys. C **4**, 345 (1980); Erratum, Z. Phys. C **6**, 282(E) (1980).
[10] A. Y. Khodjamirian, On the calculation of $J/\psi \to \eta_c\gamma$ width in QCD, Sov. J. Nucl. Phys. **39**, 614 (1984).
[11] V. A. Beilin and A. V. Radyushkin, Borelized sum rules for the radiative decays of charmonium in QCD, Sov. J. Nucl. Phys. **45**, 342 (1987).
[12] X. Zhang, K. J. Sebastian, and H. Grotch, M1 decay rates of heavy quarkonia with a nonsingular potential, Phys. Rev. D **44**, 1606 (1991).
[13] D. Ebert, R. N. Faustov, and V. O. Galkin, Properties of heavy quarkonia and B_c mesons in the relativistic quark model, Phys. Rev. D **67**, 014027 (2003).
[14] T. A. Lahde, Exchange current operators and electromagnetic dipole transitions in heavy quarkonia, Nucl. Phys. **A714**, 183 (2003).
[15] C. W. Hwang and Z. T. Wei, Covariant light-front approach for heavy quarkonium: Decay constants, $P \to \gamma\gamma$ and $V \to P\gamma$, J. Phys. G **34**, 687 (2007).
[16] J. J. Dudek, R. G. Edwards, and D. G. Richards, Radiative transitions in charmonium from lattice QCD, Phys. Rev. D **73**, 074507 (2006).
[17] H. W. Ke, X. Q. Li, Z. T. Wei, and X. Liu, Re-Study on the wave functions of (nS) states in LFQM and the radiative decays of (nS)b+, Phys. Rev. D **82**, 034023 (2010).
[18] G. C. Donald, C. T. H. Davies, R. J. Dowdall, E. Follana, K. Hornbostel, J. Koponen, G. P. Lepage, and C. McNeile, Precision tests of the J/ψ from full lattice QCD: Mass, leptonic width and radiative decay rate to η_c, Phys. Rev. D **86**, 094501 (2012).
[19] D. Becirevic and F. Sanfilippo, Lattice QCD study of the radiative decays $J/\psi \to \eta_c\gamma$ and $h_c \to \eta_c\gamma$, J. High Energy Phys. 01 (2013) 028.
[20] A. Pineda and J. Segovia, Improved determination of heavy quarkonium magnetic dipole transitions in potential nonrelativistic QCD, Phys. Rev. D **87**, 074024 (2013).
[21] M. Tanabashi *et al.* (Particle Data Group), Review of particle physics, Phys. Rev. D **98**, 030001 (2018).
[22] J. Fu, H. B. Li, X. Qin, and M. Z. Yang, Study of the electromagnetic transitions $J/\psi \to Pl^+l^-$ and probe dark photon, Mod. Phys. Lett. A **27**, 1250223 (2012).

[23] M. Ablikim *et al.* (BESIII Collaboration), Observation of electromagnetic Dalitz decays $J/\psi \to Pe^+e^-$, Phys. Rev. D **89**, 092008 (2014).
[24] M. Ablikim *et al.* (BESIII Collaboration), Observation of $\psi(3686) \to \eta' e^+e^-$, Phys. Lett. B **783**, 452 (2018).
[25] J. D. Richman, An experimenter's guide to the helicity formalism, Report No. CALT-68-1148.
[26] M. Gell-Mann and F. Zachariasen, Form-factors and vector mesons, Phys. Rev. **124**, 953 (1961).
[27] T. H. Bauer, R. D. Spital, D. R. Yennie, and F. M. Pipkin, The hadronic properties of the photon in high-energy interactions, Rev. Mod. Phys. **50**, 261 (1978); Erratum, Rev. Mod. Phys.**51**, 407(E) (1979).
[28] N. Brambilla, Y. Jia, and A. Vairo, Model-independent study of magnetic dipole transitions in quarkonium, Phys. Rev. D **73**, 054005 (2006).
[29] D. M. Asner *et al.*, Physics at BES-III, Int. J. Mod. Phys. A **24**, 499 (2009).
[30] E. Kou *et al.* (Belle-II Collaboration), The Belle II Physics Book, arXiv:1808.10567.
[31] J. P. Lees *et al.* (*BABAR* Collaboration), Study of radiative bottomonium transitions using converted photons, Phys. Rev. D **84**, 072002 (2011).