Octet baryon electromagnetic form factor double ratios $(G_E^*/G_M^*)/(G_E/G_M)$ in a nuclear medium

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The ratios of the baryon electric (G_E) to magnetic (G_M) form factors, G_E/G_M, can provide us with important information on the structure of baryons in vacuum as demonstrated in recent studies for the proton and neutron systems. It has been argued that the corresponding in-medium ratios G_E^*/G_M^* can also provide useful information on the electromagnetic structure of baryons in a nuclear medium. Although the ratios may not be measured directly in experiments except for the proton at present, the double ratios $(G_E^*/G_M^*)/(G_E/G_M)$ can be directly related to the polarization-transfer measurements of bound baryons. In the present work we estimate, for the first time, the double ratios $(G_E^*/G_M^*)/(G_E/G_M)$ for all the members of octet baryons in symmetric nuclear matter in the range $Q^2 = 0$–$3\ \mathrm{GeV}^2$, where $Q^2 = -q^2$ with q being the four-momentum transfer.

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I. INTRODUCTION

The issue of whether or not hadrons change their properties in a nuclear medium has been one of the longstanding problems in nuclear physics [1–3]. Quantum chromodynamics (QCD) is established as the theory of strong interaction, and quarks and gluons are the degrees of freedom in the QCD Lagrangian. It is natural that in the strong mean fields which pervade nuclear matter, the motion of the quarks and gluons inside hadrons should be modified. Such changes are what is meant by the nuclear modification of hadron properties, and studying the effects is clearly central to the understanding of hadron properties in a nuclear medium within QCD.

In the last few decades new experimental methods have been proposed to probe the electromagnetic structure of baryons in a nuclear medium. The measurements made at JLab [4] and Mainz Microtron (MAMI) for polarization-transfer double ratio for a bound proton, which can be directly connected with the electromagnetic form factor (EMFF) double ratio $(G_E^*/G_M^*)/(G_E/G_M)$ [5–9], showed that the ratio is not unity and imply the medium modifications of the bound proton electromagnetic form factors.

The G_E^*/G_M^* ratio is measured based on the relation between the polarization-transfer coefficients in a quasielastic reaction where a proton is knocked out from a nucleus (quasifree proton). Although the final proton is in vacuum, one can still assume that the polarization-transfer coefficients carry the information of the bound proton, since the photon coupling with the proton occurs in a nuclear environment, and the polarization is conserved. This interpretation has been used by many different groups in the studies of the electromagnetic structure of the nucleon in a nuclear medium [10–16]. Although there are still debates in the interpretation of the measured polarization-transfer coefficients as representing the proton electromagnetic properties in a nuclear medium, since the knocked out proton is detected as a quasifree particle in a vacuum, this is presently considered to be the best and cleanest method to extract the effect of the nuclear medium for the bound proton electromagnetic form factors.

The results from JLab and MAMI motivated further theoretical studies of the double ratios, not only for the proton but also the neutron, using different frameworks [10–15,17–21]. Although model dependent, these studies predict a surprising fact that, while the double ratio for the proton is quenched, that for the neutron is enhanced in a nuclear medium [12–14]. Thus, one can suspect that the in-medium modifications of the electromagnetic form factors may be different for the charged and charge neutral particles. One of the important and interesting motivations of this study is to explore systematically the medium modifications of the octet baryon electromagnetic form factors focusing on the differences between the charged and charge neutral baryons.

In high-energy heavy-ion collisions and in the core of a neutron star and a compact star, the formation of baryon-nucleus bound states are less expected due to the disappearance of nuclei into nuclear, hadron, or quark matter. However, for matter produced in heavy-ion collisions and expected to exist in the core of neutron and compact stars, the medium effects that are similar to the system of the baryon-nucleus bound states may be produced. Thus, as a first step, we consider the situation that the octet baryons are immersed in symmetric nuclear matter and study the medium effects on the electromagnetic form factors of them. Such a study may provide us with very important information for understanding better heavy-ion collisions, neutron stars and compact stars.

In the present work we estimate the EMFF double ratios of octet baryons in symmetric nuclear matter. In addition to the nucleons, we predict the double ratios of the charged baryons Σ^+, Σ^-, and Ξ^-, as well as the charge neutral baryons Λ, Σ^0, and Ξ^0. We consider, in particular, nuclear matter densities $\rho = 0$ and $0.5\rho_0$, and ρ_0, with $\rho_0 = 0.15$ fm^{-3} being the normal nuclear matter density. The present estimates are based on the model results in Ref. [13]. In this work, however, we estimate directly the double ratios $(G_E^*/G_M^*)/(G_E/G_M)$ of octet baryons, since these quantities can possibly be compared with the polarization-transfer double ratio measurements, when the experimental technique advances in the future.

Our estimates are based on the combination of the covariant spectator quark model and the quark-meson coupling (QMC) model, extended in symmetric nuclear matter [13]. The free parameters of the covariant spectator quark model in vacuum are calibrated using lattice QCD EMFF data for the octet baryons avoiding the contamination from the meson cloud effects. The meson cloud contributions for the EMFFs are estimated based on an $SU(6)$ symmetry for the meson-baryon couplings. Since the meson cloud is expected to be dominated by the lightest meson, the pion, we reduce the calculations to the pion cloud contributions. The parameters associated with the pion cloud are fitted to the available physical data for the octet baryons including the nucleon EMFFs, octet baryon magnetic moments, and the electric and magnetic radii. The extension for symmetric nuclear matter was already made in Ref. [13] taking into account the medium modifications of relevant hadron masses and pion-baryon coupling constants combined with the QMC model [2,11]. The study of the properties of the nuclear matter is performed in the nuclear matter rest frame. Then, the nuclear matter densities considered in the present study refer to the nuclear matter rest frame. Note that, although the in-medium quantities used in this study are calculated in the rest frame of nuclear matter in the QMC model, they are all Lorentz-scalar quantities. The parameters of the QMC model relevant for describing nuclear matter, the coupling constants between the light quarks and the Lorentz-scalar as well as the Lorentz-vector mean fields, are fixed so as to reproduce the empirically extracted nuclear matter saturation properties, namely, the binding energy and the asymmetry energy coefficient at the saturation density point.

This article is organized as follows. In Sec. II, we introduce nomenclature associated with the octet baryon electromagnetic form factors in vacuum and in medium. In Sec. III, we discuss briefly the properties of the theoretical model used and explain the model extension for symmetric nuclear matter. The results for the octet baryon EMFF double ratios are presented and analyzed in Sec. IV. Finally, we give outlook and conclusions in Sec. V.

II. ELECTROMAGNETIC FORM FACTORS IN VACUUM AND IN MEDIUM

The electromagnetic current for the interaction between an octet baryon B and a virtual photon with momentum q in vacuum can be written as

$$J_B^\mu = F_{1B}(Q^2)\gamma^\mu + F_{2B}(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M_B}, \qquad (2.1)$$

where M_B is the baryon mass, $Q^2 = -q^2$ is the photon virtuality, and F_{1B} and F_{2B} are the Dirac and Pauli form factors, respectively. In Eq. (2.1) we represent the current in units $e = \sqrt{4\pi\alpha}$, where $\alpha \simeq 1/137$ is the electromagnetic fine structure constant. For simplicity, we omit the projection with the initial u_B and final $\bar{u}_B$ Dirac spinors.

At $Q^2 = 0$ the baryon form factors are normalized as $F_{1B}(0) = e_B$ and $F_{2B}(0) = \kappa_B$, where e_B is the baryon charge in units of e and κ_B is the baryon anomalous magnetic moment in natural units $\frac{e}{2M_B}$.

An alternative representation of EMFFs for a spin-$1/2$ particle is the Sachs representation: the electric G_{EB} and magnetic G_{MB} form factors. These form factors are defined by [13]

$$G_{EB}(Q^2) \equiv F_{1B}(Q^2) - \frac{Q^2}{4M_B^2}F_{2B}(Q^2), \qquad (2.2)$$

$$G_{MB}(Q^2) \equiv [F_{1B}(Q^2) + F_{2B}(Q^2)]\frac{M_N}{M_B}. \qquad (2.3)$$

Note that the expression for G_{MB} includes the factor $\frac{M_N}{M_B}$, where M_N is the free nucleon mass, in order to convert to the nucleon natural units ($\frac{e}{2M_N}$).

If we omit the conversion factor $\frac{M_N}{M_B}$, we have the usual relation $G_{MB} = F_{1B} + F_{2B}$ in the baryon B natural units ($\frac{e}{2M_B}$), and in the limit $Q^2 = 0$ we obtain the baryon magnetic moment $\mu_B = G_{MB}(0)\frac{e}{2M_B}$. To summarize, the inclusion of the extra factor $\frac{M_N}{M_B}$ in Eq. (2.3) is to convert G_{MB} to the nucleon natural units, allowing a simpler comparison among the magnetic form factors of different baryons.

We consider now the situation that baryons are immersed in symmetric nuclear matter. By assuming a spin-$1/2$ baryon B in symmetric nuclear matter as a quasiparticle which has an effective mass (Lorentz-scalar) and an effective vector potential or four-momentum (Lorentz-vector) due to the medium effects satisfying the quasiparticle Dirac equation, we can also define the bound baryon B EMFFs in a nuclear medium [22] as F^*_{1B} and F^*_{2B}, respectively, corresponding to those in vacuum without the asterisk *. The Sachs form factors G^*_{EB} and G^*_{MB} in a medium can also be obtained using the corresponding relations of Eqs. (2.2) and (2.3), with the baryon B effective mass M^*_B. The effective mass M^*_B appears also in the baryon B spinor in medium.

For convenience we use also $e^*_B = F^*_{1B}(0)$ and $\kappa^*_B = F^*_{2B}(0)$ to represent the charge and anomalous magnetic moments in medium, where $e^*_B = e_B$.

Note in particular that, according to our convention, the in-medium magnetic form factor takes the form

$$G^*_{MB}(Q^2) \equiv [F^*_{1B}(Q^2) + F^*_{2B}(Q^2)]\frac{M_N}{M^*_B}. \tag{2.4}$$

In this way, we represent the octet baryon in-medium magnetic form factors in natural units of the nucleon in a vacuum, where M_N is the free nucleon mass. For the magnetic moment of an octet baryon B in medium we use $\mu^*_B = G^*_{MB}(0)\frac{e}{2M_N}$.

Since the effective baryon mass is expected to be smaller than that in vacuum ($M^*_B < M_B$), $G^*_{MB}(Q^2)$ is expected to be enhanced in medium relative to that in vacuum. In particular, one has $|\mu^*_B| > |\mu_B|$.

As for the electric form factor, since there is no additional factor, the charges of the baryons are not modified by the medium effects (charge conservation).

III. DESCRIPTION OF THE MODEL

We explain in this section briefly the properties of the model used in the present work. We start with the description of the model in vacuum and, in the end, discuss the extension to the nuclear medium.

The electromagnetic interaction of a baryon B with a photon can be decomposed into the photon interaction with the valence quarks and the photon interaction with the (bare baryon)-meson system. Since the pion is the lightest meson, we consider only the pion cloud effects as a first, reasonable approximation. Then, we decompose the electromagnetic current into

$$J^\mu_B = Z_B[J^\mu_{0B} + J^\mu_\pi + J^\mu_{\gamma B}], \tag{3.1}$$

where J^μ_{0B} stands for the electromagnetic interaction with the baryon core (valence quarks without the pion cloud) and the remaining terms represent the interaction with the intermediate pion-baryon (πB) states, displayed in Fig. 1. In particular, J^μ_π represents the direct interaction with the pion [Fig. 1(a)] and the $J^\mu_{\gamma B}$ the interaction with the baryon while one pion is in the air [Fig. 1(b)]. Z_B is a normalization constant associated with the baryon B bare wave function. For more details, see Refs. [13,23–25].

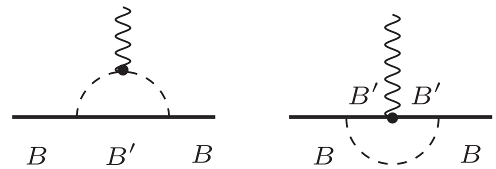

FIG. 1. Electromagnetic interaction of a baryon B with a photon within the one-pion loop level through the intermediate baryon states B'.

A. Valence quark contribution

The valence quark contributions for the octet baryon electromagnetic form factors are determined by the current J^μ_{0B} based on the covariant spectator quark model in Refs. [13,23].

In the covariant spectator quark model the structure of the baryon wave functions is based on an $SU(3)$ symmetry, which is broken by the quark masses, namely, by the heavier strange quark mass. In the relativistic impulse approximation we can separate the degrees of freedom of the interaction-acting quark from the spectator quark pair (diquark) and integrate into the internal degrees of freedom of the quark pair to obtain an effective radial wave function [26–31]. Then, the $SU(3)$ symmetry breaking in the model is included at the level of the octet baryon radial wave functions approximated by an S-wave state of the quark-diquark system [13,23]. The octet baryon radial wave functions are characterized by the Hulthen form associated with the two momentum-range scales and depend also on the baryon mass (M_B) [26]. We consider a common long-range scale for all octet baryons and a shorter-range scale associated with the number of strange quarks, namely, zero, one, or two [23,32]. Since we use different momentum-range scales for the different octet baryon families, we break the $SU(3)$ symmetry at the level of the radial wave functions. Overall, one has four momentum-range scales for the octet baryon radial wave functions. The $SU(3)$ symmetry breaking effect has a strong impact on the baryon systems with two strange quarks.

The electromagnetic structure of the valence constituent quarks is described by effective quark form factors, which can be parametrized in terms of the vector meson dominance (VMD) mechanism.

For the flavor $SU(2)$ sector, we represent the quark current based on the isoscalar (isovector) decomposition into an ω-meson (ρ-meson) associated term which parametrizes the long-range structure and a term associated with a heavy effective meson (mass M_h) to simulate the short-range structure. In numerical calculation we approximate $m_\omega \simeq m_\rho$ and fix M_h as $M_h = 2M_N$ (twice the nucleon mass).

For the strange quark, we replace the ρ term ($\pi\pi$ channel) by a term associated with the ϕ meson ($K\bar{K}$ channel). One obtains in the end a quark current with five adjustable parameters, combined with three bare-quark anomalous magnetic moments κ_u, κ_d and κ_s. The five parameters were determined in the study of the nucleon physical EMFFs [26] and in the study of decuplet baryon EMFFs in lattice QCD [27]. The quark bare-anomalous magnetic moments are readjusted in the study of octet baryon EMFFs in Ref. [13].

The covariant spectator quark model can be extended to the lattice QCD regime. In this extension, we generalize the quark currents and the radial wave functions to the lattice QCD conditions, in general, characterized by the value of the pion mass. The extension to lattice QCD regime assumes that the effect of the meson cloud is suppressed for simulations with large pion masses and that only the effect of the valence quark is relevant.

In the extension to the lattice QCD regime, the radial wave function is determined using the baryon mass M_B associated with the lattice QCD regime and the quark currents are determined using a VMD parametrization where the physical masses (M_N, m_ρ, m_ϕ and $M_h = 2M_N$) are replaced by the corresponding masses in the lattice QCD regime [33,34]. The parameters of the wave function and the parameters of the quark current are the same in the physical and in the lattice QCD regimes. The extension to lattice QCD regime was proved to be successful for the nucleon and $\Delta(1232)$ systems [33,34]. The method has been used in several studies of the octet and decuplet baryons in order to estimate the valence quark contributions for the EMFFs [13,23,27,35].

An important characteristic of the covariant spectator quark model is that it breaks $SU(2)$ symmetry in the u and d light-quark sector at the level of quark current. In the present model the strength of the isoscalar current is larger than that of the isovector compared with the other models [18,36–38]. This isovector-isoscalar asymmetry has an impact on the results of the neutron electric form factor and is responsible for the dominance of the valence quark component in the neutron electric form factor [13,26].

B. Pion cloud contribution

The pion cloud contributions for the octet baryon EMFFs are estimated using an $SU(3)$ symmetry-based model for the pion-baryon interaction parametrized by the constant $\alpha \equiv D/(D+F) = 0.6$ [$SU(6)$ symmetry-limit value]. We use the framework of Ref. [24], where all the couplings (πB, eB and κB) are determined by the $SU(6)$ symmetry model, apart the six independent functions of Q^2 [13]. Those functions are parametrized in terms of five independent coefficients and two cutoffs which regulate the falloff of the pion cloud contributions [13,23]. In the parametrizations we include a dependence on the pion mass in order to reproduce the leading order chiral behavior of the nucleon electromagnetic isovector square charge radii, corresponding to the Dirac and Pauli EMFFs [13].

C. Calibration of the model in vacuum

The parameters of the present model in vacuum are calibrated using lattice QCD EMFF data for the baryons p, n, Σ^+, Σ^0, Σ^- and Ξ^- [39]. Only for Λ and Ξ^0 are there no representative lattice QCD data.[1] In addition to the lattice QCD data, we use also physical data to constrain the model in vacuum, namely, EMFF data for p and n, the magnetic moments of octet baryons except for Σ^0, which is unknown, and the electric square charge radii of p, n and Σ^- as well as the magnetic square radii of p and n. The physical data are used to fix the parameters associated with the pion cloud, while the lattice data are used to fix the parameters associated with the valence quarks. The normalization constant Z_B is determined by the combination of the bare and pion cloud components. Details of the model calibration can be found in Ref. [13].

D. Extension to the nuclear medium

As discussed already, for the extension of the model in a vacuum for the nuclear medium, we assume that the properties of the octet baryons are modified in medium, and they are described by the modified Dirac equations for quasiparticles with the modified masses (M_B^*) and the modified pion-baryon coupling constants ($g_{\pi BB'}^*$). The medium modifications of baryon masses (Lorentz-scalar) and coupling constants $g_{\pi BB'}^*$ are calculated based on the QMC model [2] in the rest frame of a nuclear medium. Since the effective baryon masses are Lorentz-scalar and $g_{\pi BB'}^*$ are estimated using the corresponding effective baryon and pion masses, the Lorentz covariance is preserved.

The medium modifications of the valence quark contributions for the EMFFs are estimated as follows: the effects of masses M_B^* are included in the octet baryon radial wave functions Ψ_B and in the quark electromagnetic (EM) currents. In the extension of the quark EM currents to the nuclear medium, we take the advantage of the VMD parametrization in vacuum with the free masses $m_\rho(\simeq m_\omega)$, m_ϕ and $M_h = 2M_N$, and in medium we replace these masses by the respective values in medium, $m_\rho^*(\simeq m_\omega^*)$, m_ϕ^* and $M_h^* = 2M_N^*$.

[1] The lattice QCD simulations are more difficult to perform for charge neutral baryons and thus have poorer statistics. For these reasons the neutral baryons are in general excluded from the simulations. In the present case there are results for n and Ξ^0, although errors are large [39].

Concerning the medium modification of the pion cloud contribution, it is performed taking into account the medium modification of the pion-baryon coupling constants $g^*_{\pi BB'}$. For estimating the in-medium pion-baryon coupling constants $g^*_{\pi BB'}$, we use the Goldberger-Treiman relation and an estimate of the in-medium pion decay constant f^*_π [13].

The numerical values for the in-medium baryon (M^*_B) and meson masses, and coupling constants ($g^*_{\pi BB'}$) used in the present work, can be found in Tables 3 and 4 in Ref. [13], respectively.

E. Properties of the model in medium

Before presenting the results for the G_E/G_M double ratios of octet baryons, it is important to review briefly the properties of the model derived and calibrated in Ref. [13]. This will help better to understand the results of double ratios presented in this article.

From the calibration of the model in vacuum [13], it is clear that the fit to the lattice data provides a good description for the N and Σ systems but not so good for the Ξ systems. This may be the consequence of the $SU(3)$ symmetry breaking, which has a stronger impact on the systems with two strange quarks.

It was also concluded that the calibration provided only a crude estimate of the quark core effect for the charge neutral baryons, since these systems are not well constrained by lattice QCD data, except for the neutron and the Ξ^0. For the neutron, fortunately, there are physical data to be used. Nevertheless, the separation between the quark core and pion cloud contributions should be taken with care, particularly for G_{EB}, whose magnitudes are in general small for charge neutral baryons, where the small pion cloud effects and residual quark core effects cannot be separated with a good precision. The estimates for the charge neutral baryons can in principle be improved with the explicit inclusion of the kaon cloud as well. Although the effect of the kaon cloud is expected to be smaller than that of the pion due to the large kaon mass, it may still be relevant in the cases when both the bare quark core and the pion cloud contributions are small.

Based on the $SU(3)$ scheme applied in the present model, we conclude, in general, that the valence quark contribution is more than 80% of the total contribution for each octet baryon EMFF in vacuum, as well as in medium [13]. The exceptions are the charge neutral baryons which deserve specific discussions.

In general we notice that the ratio G^*_E/G_E decreases from unity as Q^2 increases except for the charge neutral baryons, which means that the magnitudes of the electric form factors decrease in symmetric nuclear matter faster than those in vacuum in the low-Q^2 region. [See Figs. 4, 7, 9 and 11 in Ref. [13] (but the effect is less significant for Ξ^-).] Such a feature can be interpreted as the enhancement of the in-medium charge square radii compared with those in vacuum. The falloff of the ratio G^*_E/G_E increases when the density increases. As for the charge neutral baryons n, Λ, Σ^0 and Ξ^0, the medium modifications show different features though. The Λ and Σ^0 systems are dominated by the pion cloud contributions and the medium modifications are small, while the n and Ξ^0 systems are dominated by the valence quark contributions, and the medium modifications can be significant. Those medium modifications can be understood by the result of the in-medium enhancement of the charge square radii.

Concerning the results of G^*_M/G_M, they are very similar for all the octet baryons. (See Figs. 4–11 in Ref. [13].) In a vacuum as well as in-medium cases we have falloffs with Q^2. The falloffs are faster for larger nuclear densities. In the region near $Q^2=0$, however, the magnitude of G^*_M/G_M is, in general, determined by the valence quark contributions, which are dominated by the in-medium masses of the baryons. The expressions for the valence quark contributions for the in-medium magnetic form factor G^*_M at $Q^2=0$ are presented in Table I. We can notice in Table I that $G^*_{MB}(0)$ can be decomposed into two main terms: a term in $\frac{M_N}{M^*_B}$ and a term expressed only in terms of the nucleon masses ($\frac{M_N}{M^*_N}$). These results are the consequence of our extension of the covariant spectator quark model to symmetric nuclear matter, based on a quark current which has the dependence of the quark anomalous magnetic moment (κ_q) in the form $\kappa_q/(2M^*_N)$. Note that in the case of the nucleon we need to take into account only one term. One has then $\frac{G^*_M}{G_M}\propto\frac{M_N}{M^*_N}$, and this means that the G_M is expected to be enhanced in medium. As for the other baryons it is expected that the term in $\frac{M_N}{M^*_N}$ has a strong impact on the medium modifications, since the factor $\frac{M_N}{M^*_B}$ is less sensitive to the mass variations in medium. The magnetic form factors in larger densities (or smaller

TABLE I. Explicit expressions for $G^*_{MB}(0)$ in terms of the octet baryon mass in medium (M^*_B) and the free nucleon mass to convert the nucleon natural units. The quark anomalous magnetic moment values are $\kappa_u=1.711$, $\kappa_d=1.987$ and $\kappa_s=1.462$ [13].

B	$G^*_{MB}(0)$
p	$[1+(\frac{8}{9}\kappa_u+\frac{1}{9}\kappa_d)]\frac{M_N}{M^*_N}$
n	$-[\frac{2}{3}+(\frac{2}{9}\kappa_u+\frac{4}{9}\kappa_d)]\frac{M_N}{M^*_N}$
Λ	$-\frac{1}{3}\frac{M_N}{M^*_\Lambda}-\frac{1}{3}\kappa_s\frac{M_N}{M^*_N}$
Σ^+	$\frac{M_N}{M^*_\Sigma}+(\frac{8}{9}\kappa_u+\frac{1}{9}\kappa_s)\frac{M_N}{M^*_N}$
Σ^0	$\frac{1}{3}\frac{M_N}{M^*_\Sigma}+(\frac{4}{9}\kappa_u-\frac{2}{9}\kappa_d+\frac{1}{9}\kappa_s)\frac{M_N}{M^*_N}$
Σ^-	$-\frac{1}{3}\frac{M_N}{M^*_\Sigma}-(\frac{4}{9}\kappa_d-\frac{1}{9}\kappa_s)\frac{M_N}{M^*_N}$
Ξ^0	$-\frac{2}{3}\frac{M_N}{M^*_\Xi}-(\frac{2}{9}\kappa_u+\frac{4}{9}\kappa_s)\frac{M_N}{M^*_N}$
Ξ^-	$-\frac{1}{3}\frac{M_N}{M^*_\Xi}+(\frac{1}{9}\kappa_d-\frac{4}{9}\kappa_s)\frac{M_N}{M^*_N}$

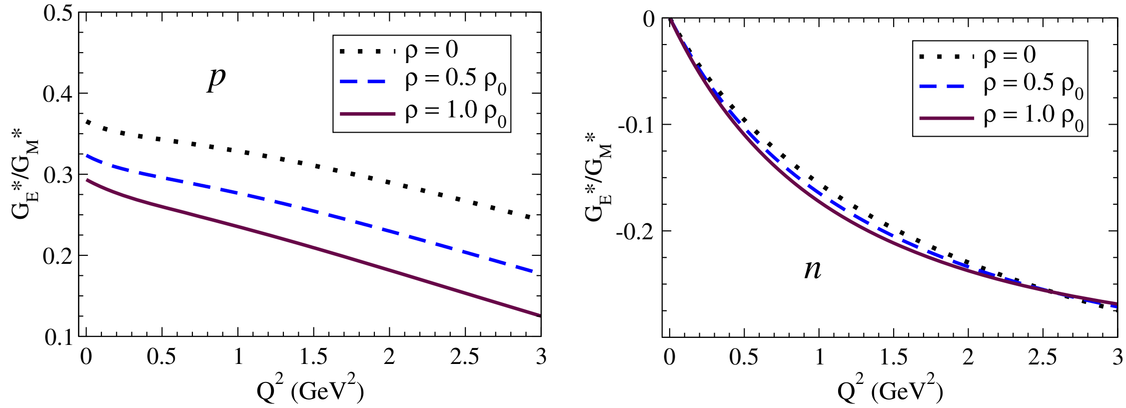

FIG. 2. EMFF single ratios G_E^*/G_M^* in symmetric nuclear matter for the proton (left) and the neutron (right), where $\rho_0 = 0.15\ \mathrm{fm}^{-3}$ [13].

effective masses) are more enhanced than those in smaller densities.

Once we have discussed the properties of the model in a medium, we can present our estimates for the octet baryon G_E/G_M double ratios for the nuclear matter densities $0.5\rho_0$ and ρ_0 with $\rho_0 = 0.15\ \mathrm{fm}^{-3}$.

IV. OCTET BARYON EMFF DOUBLE RATIOS $(G_E^*/G_M^*)/(G_E/G_M)$ IN SYMMETRIC NUCLEAR MATTER

We present here our estimates of the octet baryon electromagnetic form factor double ratios $\frac{G_E^*/G_M^*}{G_E/G_M}$ in symmetric nuclear matter.

In the cases of the proton and the neutron, we have experimental data for the ratio G_E/G_M in vacuum [13]. The estimates for G_E/G_M in vacuum and in symmetric nuclear matter for these cases are presented in Fig. 2. The results for the vacuum (dotted line) are in agreement with the corresponding experimental data (see Ref. [13]). The results of the double ratios for the proton and neutron can be better understood when we know the results for the proton and neutron single ratios G_E/G_M in vacuum and in symmetric nuclear matter.

For the discussion in symmetric nuclear matter, it is important to recall that the magnetic form factors are converted into the units of the nuclear magneton in vacuum, $\hat{\mu}_N = \frac{e}{2M_N}$. In the case of the nucleon the magnetic form factor near $Q^2 = 0$ can be expressed in the form $G_M^* \propto 1/M_N^*$ and thus the ratio $\frac{G_M^*}{G_M}$ can be expressed at low Q^2 by $\frac{G_M^*}{G_M^*} \approx \frac{M_N}{M_N^*}$. As for the other baryons, in the case when the valence quark contributions are dominant, we need to take into account the impact of the terms in $\frac{M_N}{M_B^*}$ and $\frac{M_N}{M_N^*}$ (see Table I). It is worth mentioning that the medium effects on $G_M(Q^2)$ are not restricted to the factors $\frac{M_N}{M_B^*}$ and $\frac{M_N}{M_N^*}$, since the form factors $F_1(Q^2)$ and $F_2(Q^2)$ are also modified in the medium. The effect of the mass factors is, however, in general the dominant effect.

For the EMFFs of charge neutral baryons, it is also important to notice that, in the low-Q^2 region, the electric form factor of a baryon B can be expressed as $G_{EB} \simeq -\frac{1}{6} r_{EB}^2 Q^2$, where r_{EB}^2 is the electric charge square radius of the baryon B. Then, we can write in the low-Q^2 region $\frac{G_{EB}^*}{G_{EB}} \simeq \frac{r_{EB}^{*2}}{r_{EB}^2}$, where r_{EB}^{*2} is the in-medium electric charge square radius.

Our results for the octet baryon double ratios $\frac{G_E^*/G_M^*}{G_E/G_M}$ are displayed in Figs. 3–6. We discuss separately each case in the following for proton, neutron, $\Sigma^\pm$ and Ξ^-, and the other charge neutral baryons (Λ, Σ^0 and Ξ^0).

A. Proton double ratios

We start our discussion with the proton double ratios displayed in the left panel of Fig. 3.

From the results of the single ratio G_E^*/G_M^*, the left panel of Fig. 2, one can see that the single ratio in vacuum ($\rho = 0$) decreases almost linearly with Q^2 (dotted line). A simple analysis of the result in vacuum for G_E/G_M can be made using the low-Q^2 expansion discussed in Ref. [12]: $\frac{G_E}{G_M} \simeq \frac{1}{\mu_p}[1 - \frac{Q^2}{6}(r_{Ep}^2 - r_{Mp}^2)]$, where r_{Ep}^2 and r_{Mp}^2 are, respectively, the proton electric charge and magnetic dipole square radii in vacuum. This estimate also holds in symmetric nuclear matter by replacing the in-vacuum quantities by the corresponding in-medium quantities: μ_p^*, r_{Ep}^{*2} and r_{Mp}^{*2}. The nearly linear falloff with Q^2 in vacuum is the consequence of the small magnitude of the factor $(r_{Ep}^2 - r_{Mp}^2)$, which means that the values of r_{Ep}^2 and r_{Mp}^2 are close. Although this analysis is only valid in the low-Q^2 region (say $Q^2 < 1\ \mathrm{GeV}^2$) [12], it is interesting to notice that the approximated linear dependence is observed also for large Q^2 in the present model. From Fig. 2, one can also conclude that the falloff increases with the nuclear matter density ρ, suggesting that $(r_{Ep}^{*2} - r_{Mp}^{*2})$ increases also with the nuclear matter density ρ.

Looking now at the double ratios, in the left panel of Fig. 3, we can see that the magnitudes at $Q^2 = 0$ are less

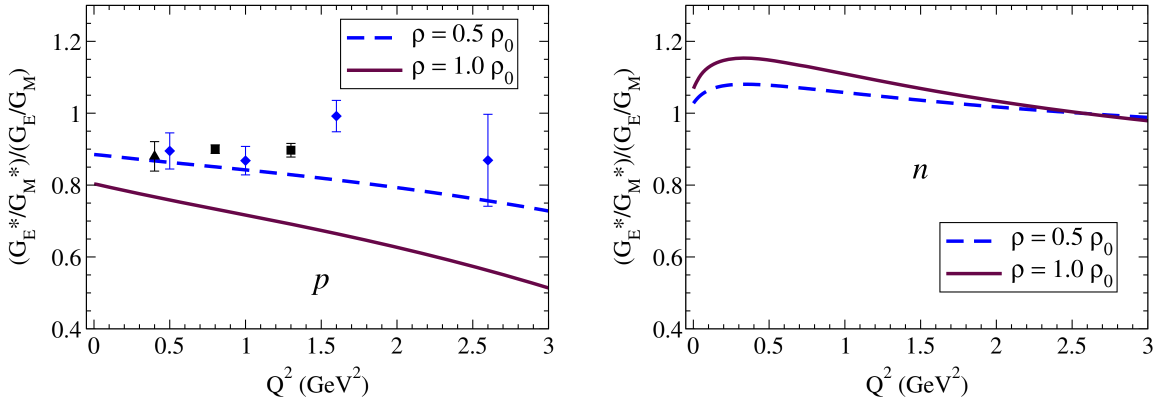

FIG. 3. EMFF double ratios in symmetric nuclear matter calculated for the proton (left) and the neutron (right). Experimental data for the proton are from Refs. [5–7].

than unity for both $0.5\rho_0$ and ρ_0. These results are essentially a consequence of the enhancement of the in-medium magnetic form factor, since $\frac{G_E^*}{G_E} \simeq 1$ and $\frac{G_M^*}{G_M} \approx \frac{M_N}{M_N^*}$, near $Q^2 = 0$. The Q^2 dependence of the double ratios can now be understood based on the results for the ratios G_E/G_M and the nearly linear falloff with increasing Q^2. The results for the proton double ratios indicate that the factor $(r_{Ep}^{*2} - r_{Mp}^{*2})$ is enhanced in symmetric nuclear matter. As a consequence, the falloff of G_E^*/G_M^* increases in symmetric nuclear matter.

Our results for $0.5\rho_0$ are similar to those in Ref. [14], for $\rho = 0.4\rho_0$. However, our estimates for both $0.5\rho_0$ and ρ_0 give slower falloff rates with Q^2 compared with the estimates based on the Nambu-Jona-Lasinio model in Ref. [12]. A relativistic light-front constituent quark model [36] predicts an almost constant double ratio. Other studies predict different Q^2 dependence [11,18,36].

B. Neutron double ratios

We now discuss the results for the neutron double ratios presented in the right panel of Fig. 3. It is worth to recall that, although there is no experimental extraction for the neutron double ratio at the moment, the double ratio results are expected from the $^2\mathrm{H}(\vec{e}, e'\vec{n})p$ and $^4\mathrm{He}(\vec{e}, e'\vec{n})^3\mathrm{He}$ reactions in the near future [9,12,16].

The results for the neutron double ratios are also very interesting. Contrary to the case of the proton, the neutron double ratios increase in the region $Q^2 < 2.8\ \mathrm{GeV}^2$. The medium effects enhance the neutron double ratios up to 7% for $0.5\rho_0$ and 15% for ρ_0 near $Q^2 = 0.3\ \mathrm{GeV}^2$, before starting to fall down slowly up to $Q^2 \simeq 2.8\ \mathrm{GeV}^2$, where the double ratios become closer to the unity. Beyond the point $Q^2 = 3\ \mathrm{GeV}^2$, the double ratios become smaller than unity (not shown in this article).

It is worth noticing that, in general, the amounts of increase in the neutron double ratios are smaller than those of the reduction in the proton double ratios for the corresponding nuclear densities. The important conclusion is that, contrary to the proton, the neutron double ratio $\frac{G_E^*/G_M^*}{G_E/G_M}$ increases in symmetric nuclear matter. The present result for $\frac{G_E^*/G_M^*}{G_E/G_M} > 1$ is consistent with the estimates made in Refs. [12,14], although the shapes are different. The magnitude of the enhancement for the neutron double ratios is, in general, smaller than the estimates made in other studies with similar nuclear densities [12,14].

The significant difference between the double ratios of the proton and the neutron for a given density ($0.5\rho_0$ and ρ_0) can be understood by the different behaviors of G_E/G_M in vacuum as well as in medium. In the right panel of Fig. 2, we can see that the neutron single ratio G_E^*/G_M^* decreases monotonically with Q^2 starting from $Q^2 = 0$, the point $G_E^*/G_M^* = 0$, for the three densities. The differences among the different densities are the slopes. The derivatives increase in absolute values with the density ρ. The common starting point ($G_E^*/G_M^* = 0$) is the consequence of $G_{En} \propto r_{En}^2 Q^2$, near $Q^2 = 0$. The slopes of G_{En} and G_{En}^* can be estimated by r_{En}^2 and r_{En}^{*2}, respectively. Those slopes have an important role in the double ratios.[2]

Since the neutron is a charge neutral baryon, the medium modification of G_E can be estimated using the low-Q^2 relation $\frac{G_{En}^*}{G_{En}} \approx \frac{r_{En}^{*2}}{r_{En}^2}$. One can then reduce the analysis of the effects in medium to the study of the variation of the neutron electric charge square radius in symmetric nuclear matter. The enhancement (faster falloff) of the electric form factors in medium at low Q^2 can be understood by the increasing of the neutron electric charge square radius in symmetric nuclear matter. In this aspect the medium effect in the electric form factor of the neutron differs significantly from that of the proton ($G_{Ep}^*/G_{Ep} \approx 1$). Looking at the numerical results, we obtain at the photon point

[2]The neutron electric charge square radius is negative in vacuum and also in symmetric nuclear matter. Thus, the signs are irrelevant for the discussion, since they cancel in G_E^*/G_E.

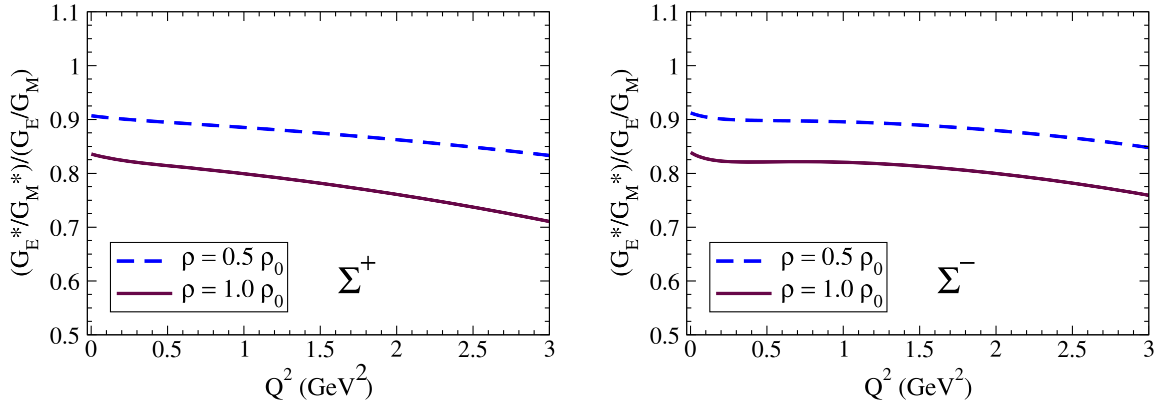

FIG. 4. EMFF double ratios in symmetric nuclear matter calculated for Σ^+ (left) and Σ^- (right).

$(Q^2 = 0)\ \frac{G^*_{En}}{G_{En}} \simeq 1.16$ for $0.5\rho_0$ and $\frac{G^*_{En}}{G_{En}} \simeq 1.33$ for ρ_0. These results show that the electric charge square radius increases 16% for $0.5\rho_0$ and 33% for ρ_0 relative to that in vacuum. In both cases, we conclude that the neutron electric charge square radius is enhanced in symmetric nuclear matter. The enhancement increases as the nuclear density increases.

Different from the proton case, one can conclude that to estimate the medium modifications of the neutron EMFF double ratio, it is not sufficient to take into account only the medium modification of the magnetic form factor. The medium modifications of G_M are given by the factor $(\propto \frac{M_N}{M_N^*} < 1)$. The impact of the medium modifications of G_E are given by the factor $\frac{r^{*2}_{En}}{r^2_{En}} > 1$, compensate the effect of G_M, and enhance the double ratio to values larger than unity. The combination of the two effects is that the neutron double ratio can be approximated by $\frac{G_E^*/G_M^*}{G_E/G_M} \approx \frac{r^{*2}_{En}}{r^2_{En}} \frac{M_N^*}{M_N}$ in the low-Q^2 region.

The particular Q^2 dependence obtained for the neutron EMFF double ratio (nonlinear shape) is a consequence of the properties of the model. Contrary to the analysis made in Ref. [12], which is restricted to the low-Q^2 region, our analysis cannot be exclusively restricted to the variables μ_B^*, r^{*2}_{EB} and r^{*2}_{MB}, since we take into account the medium modifications in a wider range of Q^2.

As mentioned in Sec. III, the covariant spectator quark model breaks the $SU(2)$ symmetry at the level of the quark EM currents. The main consequence is that the neutron form factors in vacuum are dominated by the valence quark contributions. In particular the valence quark contribution to G_{En} is about 86% near $Q^2 = 0$. In the extension for the symmetric nuclear matter we still have a dominance of the valence quark contributions. Since the impact of the in-medium effects is different for the valence and for the pion cloud components, we obtain nonlinear results for the neutron EMFF double ratios. In this aspect the present model differs from the usual other models in the literature, where the models for nucleons are based on an almost exact $SU(2)$ symmetry [18,36–38].

C. $\Sigma^\pm$ and Ξ^- double ratios

We now focus on the $\Sigma^\pm$ and Ξ^- EMFF double ratios displayed in Figs. 4 and 5.

The results for the Σ^+ and Σ^- EMFF double ratios shown in Fig. 4 are very similar, which may reflect the $SU(3)$ symmetry. Our first note goes to the reduction in the magnitude of the single ratio G_E^*/G_M^* in symmetric nuclear matter compared to that in vacuum. The reduction increases with the nuclear density ρ.

The Q^2 dependence of the $\Sigma^\pm$ EMFF double ratios is very similar to the case of the proton except for the slower falloff. The reduction of the falloff for $\Sigma^\pm$ with Q^2, in comparison with the proton case, reflects the presence of strange quarks. It is expected that baryons containing more strange quarks have smaller medium modifications due to the small modifications in the baryon masses. The small modification in the baryon mass has a small impact on the baryon wave function as well as on the pion cloud contributions. The final effect is the reduction in the falloff of the double ratio with Q^2. As in the proton case, the falloff increases with the nuclear density.

The results of the Ξ^- EMFF double ratios are presented in the left panel of Fig. 5. One can immediately notice that, contrary to the charged baryons discussed so far p, Σ^+ and Σ^-, the double ratio increases with Q^2, starting from values smaller than unity. The magnitude of the double ratio can be explained by observing that $\frac{G_E^*}{G_E} \approx 1$ in all ranges of Q^2, as a consequence of the very small contribution from the pion cloud contribution for G_E. As for the magnetic form factor which is dominated by the valence quark contribution, we can consider the approximation $\frac{G_M^*}{G_M} \approx 0.36 + 0.64 \frac{M_N}{M_N^*}$ (see Table I), where the first term includes the smooth dependence on M_Ξ^* and the last term the dependence on $\frac{M_N}{M_N^*}$, which is enhanced in medium. For a rough estimate at low Q^2, we can use $\frac{G_E^*/G_M^*}{G_E/G_M} \approx \frac{G_M}{G_M^*} \propto \frac{M_N^*}{M_N}$. For larger Q^2, one can estimate

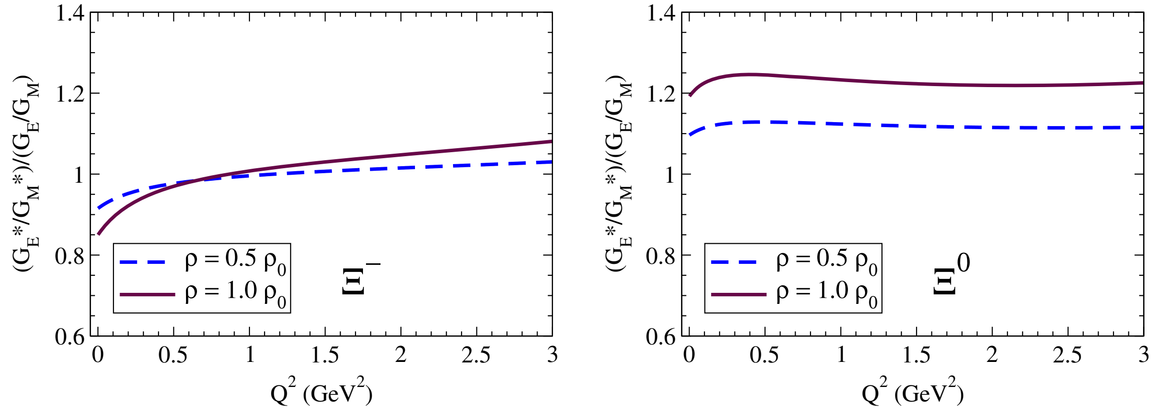

FIG. 5. EMFF double ratios in symmetric nuclear matter calculated for Ξ^- (left) and Ξ^0 (right).

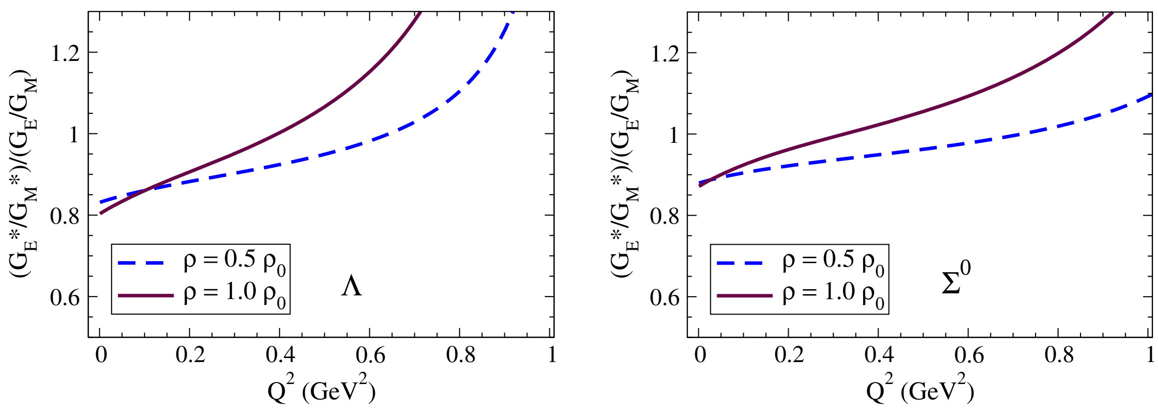

FIG. 6. EMFF double ratios in symmetric nuclear matter calculated for Λ (left) and Σ^0 (right).

as $\frac{G_E^*/G_M^*}{G_E/G_M} \approx 1$, reflecting less sensitivity of the strange quarks to the medium effects.

D. Charge neutral Λ, Σ^0 and Ξ^0 baryon double ratios

The EMFF double ratio results for the charge neutral baryons Λ, Σ^0 and Ξ^0, are presented in the right panel of Fig. 5 and in Fig. 6.

In the Λ and Σ^0 double ratios the electric form factors in vacuum G_E vanish at some finite points above $Q^2 = 1\ \mathrm{GeV}^2$ (see Figs. 6 and 8 in Ref. [13]). For this reason, there are singularities in both double ratios in the region $Q^2 > 1\ \mathrm{GeV}^2$. Thus, we restrict to present the results for the region $Q^2 \leq 1\ \mathrm{GeV}^2$.

Figure 6 shows that the Q^2 dependences of the Λ and Σ^0 double ratios are very similar. The double ratio increases faster in the Λ double ratio because of the singularity due to the $G_E = 0$, which happens at lower values of Q^2 than that for Σ^0.

As in the case of the neutron, we need to look at the factor G_E^*/G_E to understand better the results. For both the Λ and Σ^0 cases, we have $G_E^*/G_E \approx 1$, within a 5% uncertainty for $0.5\rho_0$ and ρ_0, meaning that $\frac{G_E^*/G_M^*}{G_E/G_M} \approx \frac{G_M}{G_M^*}$. The results in Fig. 6 reflect then the impact of medium modification on G_M. The medium effects are stronger for larger densities. From the analysis of the valence quark and pion cloud effects discussed in Ref. [13], we conclude that the magnetic form factors are largely dominated by the valence quark effects. The small medium modifications for the electric form factors, $G_E^*/G_E \approx 1$, on the other hand, are the consequence of the pion cloud dominance. The pion cloud contributions are only slightly modified in medium.

The results for the Ξ^0 EMFF double ratios (right panel of Fig. 5) show a significant enhancement in comparison with that in vacuum and an almost independence of Q^2 (nearly constant double ratios). In symmetric nuclear matter G_E^*/G_M^* is enhanced by about 10% for $0.5\rho_0$ and by about 20% for ρ_0. The nearly constant double ratios for both densities are the consequence of an almost cancellation of the Q^2 dependence in the division of $\frac{G_E^*}{G_E}$ by $\frac{G_M^*}{G_M}$, which means that $\frac{G_E^*}{G_E} \propto \frac{G_M^*}{G_M}$.

Since the double ratios are nearly constant, we can estimate their magnitudes by looking at the low-Q^2 region, where the Q^2 dependence is simplified. We then obtain $\frac{G_E^*}{G_E} \simeq \frac{r_{EB}^{*2}}{r_{EB}^2}$, and $\frac{G_M^*}{G_M} \simeq -(0.32 + 0.68\frac{M_N}{M_N^*})$, according to Table I.

As in the case of the neutron there is a significant enhancement for the electric charge radius in symmetric nuclear matter. The enhancement is of about 20% for $0.5\rho_0$ and 40% for ρ_0. These effects are partially canceled by the effects of the in-medium nucleon mass $\propto \frac{M_N}{M_N^*}$ for the two densities, which correspond to a 10% correction for $0.5\rho_0$ and 20% for ρ_0. Then the Ξ^0 double ratio can be estimated in a simple form: $\frac{G_E^*/G_M^*}{G_E/G_M} \propto \frac{r_{EB}^{*2}}{r_{EB}^2}\frac{M_N}{M_N^*}$. In the present model, these results are the consequence of the dominance of the valence quark contribution, since the pion cloud contribution is negligible for Ξ^0 (see Fig. 10 of Ref. [13]).

To summarize, our estimates of the Ξ^0 EMFF double ratios are based on the dominance of the valence quark contribution in both the electric and magnetic form factors. We then predict a significant enhancement of the double ratios in the nuclear medium and a weak Q^2 dependence.

V. OUTLOOK AND CONCLUSIONS

We have estimated, for the first time, the electromagnetic form factor double ratios $\frac{G_E^*/G_M^*}{G_E/G_M}$, namely, the in-medium electric to magnetic form factor ratio to that ratio in a vacuum, for all the octet baryon members. The estimates are based on the covariant spectator quark model plus an $SU(3)$ symmetry-based parametrization for the pion cloud contributions. The extension of the model for symmetric nuclear matter is performed using the in-medium inputs for the octet baryons and ρ- and ω-meson effective masses calculated in the quark-meson coupling model, as well as the in-medium pion-baryon coupling constants obtained using the Goldberger-Treiman relation. The model parameters in a vacuum are calibrated by the lattice QCD simulation data with large pion masses for the valence quark part and by physical nucleon electromagnetic form factor data and octet baryon magnetic moments for the pion cloud part. Our estimates are covariant and are performed for a wide range of Q^2 and not restricted to the low-Q^2 region.

One of the main interests of our work was originated from the different behaviors of the electromagnetic form factor double ratios for the proton and neutron, for which, so far, only a few studies have been made. While the double ratio of the proton decreases with Q^2 as was observed in the JLab experiments, that for the neutron is predicted to increase in medium in the low- and intermediate-Q^2 region ($Q^2 < 2.8$ GeV^2). This prediction can be tested in the near future with the advanced polarization-transfer experiment or the development of other new experimental methods.

The first estimates for the neutron electromagnetic form factor double ratio has motivated us to study the double ratios of the charge neutral particles in the octet baryon members. Our present estimates suggest that, contrary to the case of the neutron double ratio, those of the Λ and Σ^0 are suppressed in medium in the low-Q^2 region. On the other hand, for the large-Q^2 region, we expect the enhancement of the double ratios for Λ and Σ^0.

For the electromagnetic form factor double ratios of the charged particles Σ^- and Σ^+, we predict that they are quenched and become less than unity as in the case of the proton. The Q^2 dependence is weaker than the case of the proton, mainly because the systems with strange quarks are less affected by medium modifications.

As for the Ξ^- and Ξ^0 baryons containing two strange quarks, we conclude that the electromagnetic form factor double ratios have a weaker Q^2 dependence compared with those of the baryons containing a smaller number of strange quarks, although we admit that our estimates for the Ξ^- and Ξ^0 are less precise.

The medium effects as the ones estimated by the present study have an important impact on the baryons immersed in a nuclear medium, in such matter formed in heavy-ion collisions, and the one expected to exist in the core of a neutron star and a compact star. In order to better understand the medium effects, the measurement of octet baryon electromagnetic form factor double ratios is very important for testing and improving our theoretical estimates. At the moment, the measurement of the electromagnetic form factor double ratios is restricted to the case of the proton based on the polarization-transfer method, although the results for the neutron are expected in the near future. There is then some hope that the polarization-transfer method or new methods can be used to directly extract the in-medium modifications of octet baryon electromagnetic form factors.

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