Quasi-two-body decays $B_{(s)} \to PD_0^*(2400) \to PD\pi$ in the perturbative QCD approach

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We study the quasi-two-body decays $B \to PD_0^*(2400) \to PD\pi$ with $P = (\pi, K, \eta, \eta')$ in the perturbative QCD factorization approach. The predicted branching fractions for the considered decays are in the range of 10^{-9}–10^{-4}. The strong Cabibbo-Kobayashi-Maskawa (CKM) suppression factor $R_{\rm CKM} \approx \lambda^4(\bar\rho^2 + \bar\eta^2) \approx 3 \times 10^{-4}$ results in the great difference of the branching ratios for the decays with D_0^* and $\bar D_0^*$ as the intermediate states. The ratio $R_{\bar D_0^{*0}}$ between the decays $B^0 \to \bar D_0^{*0} K^0 \to D^-\pi^+ K^0$ and $B^0 \to \bar D_0^{*0}\pi^0 \to D^-\pi^+\pi^0$ is about $0.091^{+0.003}_{-0.005}$, consistent with the flavor-$SU(3)$ symmetry result. The ratio for the branching fractions is found to be $1.10^{+0.05}_{-0.02}$ between $\mathcal{B}(B_s^0 \to D_0^{*+}K^- \to D^0\pi^+K^-)$ and $\mathcal{B}(B^0 \to D_0^{*+}\pi^- \to D^0\pi^+\pi^-)$ and to be $1.03^{+0.06}_{-0.07}$ between $\mathcal{B}(B_s^0 \to \bar D_0^{*0}\bar K^0 \to D^-\pi^+\bar K^0)$ and $2\mathcal{B}(B^0 \to \bar D_0^{*0}\pi^0 \to D^-\pi^+\pi^0)$. The predictions in this work can be tested by the future experiments.

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I. INTRODUCTION

The strong dynamics contained in the three-body hadronic B meson decays is much more complicated than that in the two-body cases. There are resonant and nonresonant contributions, final-state interactions [1,2], and complex interplay between the weak processes and the low-energy strong interactions [3] in the three-body B meson decays. The traditional approaches for the two-body decays are no longer satisfactory in the three-body processes [4]. In order to extract the most information from the experimental data of those three-body processes, different methods have been adopted abundantly in theoretical works [5]. Three-body hadronic B decays are known, in most cases, to be dominated by the low-energy scalar, vector, and tensor resonant states. In this situation, for the numerous three-body B meson processes, it is urgent to study the resonance contributions, which could be handled in the quasi-two-body framework where the factorization procedure can be applied [4,6].

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The p-wave orbitally excited state D_0^*,[1] with its $j_q = 1/2$ [7–9] and $J^P = 0^+$ [10], decays rapidly through S-wave pion emission. It was thought to be the $c\bar q$ state in the traditional quark model [11–13], but the mass observed in experiments [14,15] is lower than the quark model predictions. One possible explanation is that the self-energy hadronic loop could pull down the mass of the heavy scalar [16] supported by [17] within the framework of heavy meson chiral perturbation theory. The tetraquark structure for D_0^* was investigated in [18] with the help of the QCD sum rule, and the authors of [18] suggested that the charmed scalar meson $D_0^{*0}(2308)$ observed by the Belle Collaboration [14] and $D_0^{*0(+)}(2405)$ observed by the FOCUS Collaboration [19] are different resonances. It was claimed that two poles exist in the D_0^* energy region [20], which has been supported by the lattice QCD analysis [21]. The resonant state D_0^* has also been explained as a mixture of $c\bar q$ and tetraquarks [22] or a meson-meson bound state [23]. Since the Belle Collaboration's announcement [14], much work [24–28] has emerged for the two-body hadronic B decays involving D_0^*.

By studying the three-body hadronic B meson decays involving D_0^*, one could provide the constraint on the unitary triangle [29–32] and probe the inner structure of the intermediate resonances. In Ref. [33], four quasi-two-body

[1]For the sake of convenience, we employ D_0^* to denote $D_0^*(2400)$ in this work.

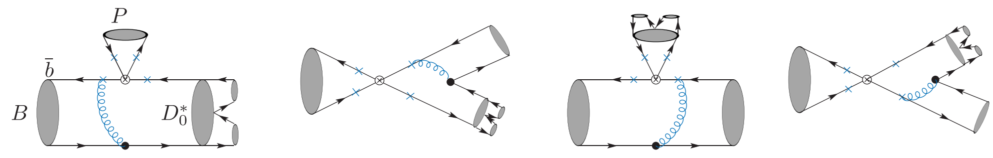

FIG. 1. Typical diagrams for the quasi-two-body decays $B_{(s)} \to PD_0^* \to PD\pi$. The diagram (a) for the $B \to D_0^*$ transition, and diagram (c) for the $B \to P$ transition, as well as the diagrams (b) and (d) for for annihilation contributions. The symbol $\otimes$ stands for the weak vertex and $\times$ denotes possible attachments of hard gluons.

decay processes involving D_0^* have been studied in the perturbative QCD (PQCD) approach [34–37]. In this work, we extend the study to the quasi-two-body decays $B_{(s)} \to PD_0^* \to PD\pi$, with the bachelor particle P which denotes the light pseudoscalar π, K, η, or η'. Typical diagrams for the $B_{(s)} \to PD_0^* \to PD\pi$ decays' processes are shown in Fig. 1. Inspired by the generalized parton distribution in hard exclusive two pion production [38–41], the two-meson distribution amplitude was introduced in three-body hadronic B decays in [42,43] as the universal nonperturbative input within the PQCD approach. The PQCD approach has been employed in [42–46] for the three-body and in [47–52] for the quasi-two-body B meson decays. The decay amplitude for a three-body or quasi-two-body B decay can be expressed as the convolution of the nonperturbative wave function and hard kernel [42,43,47]. Taking $B \to PD_0^* \to PD\pi$ as an example, we have the decay amplitude

$$\mathcal{A} = \phi_B \otimes H \otimes \phi_P \otimes \phi_{D\pi}^{\text{S-wave}}, \tag{1}$$

where hard kernel H is calculated at leading order which contains one hard gluon, and the distribution amplitudes ϕ_B, ϕ_P, and $\phi_{D\pi}^{\text{S-wave}}$ absorb the nonperturbative dynamics in the decay processes.

The layout of this paper is as follows. We give a brief introduction of the theoretical framework in Sec. II. Then the numerical results, a discussion, and conclusions are given in Secs. III and IV. The relevant factorization formulas for the decay amplitudes are collected in the Appendix.

II. FRAMEWORK

The definitions of the momenta for the $B_{(s)}$ meson, S-wave $D\pi$ system, and the bachelor meson are the same as those in Ref. [33]. The distribution amplitude and the parameters for the S-wave $D\pi$ system employed in this work are the same as those in [33]. The wave functions for $B_{(s)}$ and the relevant parameters can be found in [53]. The decay constants $f_{B^{0,\pm}} = 0.190$ GeV for $B^{0,\pm}$ and $f_{B_s^0} = 0.230$ GeV for B_s^0 were adopted from recent lattice QCD updated results with $N_f = 2+1+1$ [54]. The physical states η and η' are related to the flavor states η_q and η_s via [55–57]

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} |\eta_q\rangle \\ |\eta_s\rangle \end{pmatrix}, \tag{2}$$

with the decay constants $f_q = (1.07 \pm 0.02) f_\pi$ and $f_s = (1.34 \pm 0.06) f_\pi$ for η_q and η_s, respectively, and the mixing angle $\phi = 39.3° \pm 1.0°$, which is close to the recent measurement $\phi = (40.1 \pm 1.4_{\text{stat}} \pm 0.5_{\text{syst}})^\circ$ by the BESIII Collaboration [58]. The wave functions for the states π, K, η_q, and η_s in this work are written as

$$\Phi_P(p,z) = \frac{i}{\sqrt{2N_c}} \gamma_5 [\not{p} \phi^A(z) + m_0 \phi^P(z) + m_0(\not{v}\not{n} - 1)\phi^T(z)], \tag{3}$$

where m_0 is the chiral mass, $n = (1, 0, \mathbf{0}_T)$ and $v = (0, 1, \mathbf{0}_T)$ are the dimensionless lightlike unit vectors, p and z are, respectively, the momentum and corresponding momentum fraction of states π, K, η_q, and η_s. The distribution amplitudes $\phi^A(z), \phi^P(z), \phi^T(z)$ can be written as [59–62]

$$\begin{aligned}
\phi^A(z) &= \frac{f_P}{2\sqrt{2N_c}} 6z(1-z)[1 + a_1^P C_1^{3/2}(2z-1) + a_2^P C_2^{3/2}(2z-1) + a_4^P C_4^{3/2}(2z-1)], \\
\phi^P(z) &= \frac{f_P}{2\sqrt{2N_c}} \left[1 + \left(30\eta_3 - \frac{5}{2}\rho_P^2\right) C_2^{1/2}(2z-1) - 3\left[\eta_3\omega_3 + \frac{9}{20}\rho_P^2(1 + 6a_2^P)\right] C_4^{1/2}(2z-1)\right], \\
\phi^T(z) &= \frac{f_P}{2\sqrt{2N_c}} (1-2z)\left[1 + 6\left(5\eta_3 - \frac{1}{2}\eta_3\omega_3 - \frac{7}{20}\rho_P^2 - \frac{3}{5}\rho_P^2 a_2^P\right)(1 - 10z + 10z^2)\right],
\end{aligned} \tag{4}$$

where the Gegenbauer moments are $a_1^{\pi,\eta_{q,s}}=0$, $a_1^K=0.06$, $a_2^{\pi,K}=0.25$, $a_2^{\eta_{q,s}}=0.115$, $a_4^{\pi,\eta_{q,s}}=-0.015$, and the parameters are $\rho_\pi=m_\pi/m_0^\pi$, $\rho_K=m_K/m_0^K$, $\rho_{\eta_q}=2m_q/m_{qq}$, $\rho_{\eta_s}=2m_s/m_{ss}$, $\eta_3=0.015$, $\omega_3=-3$. Where m_q is the mass of the up or down quark, m_s is the mass of the strange quark, $m_{qq,ss}$ are related to $m_0^{\eta_q,\eta_s}$ by $m_0^{\eta_q}=m_{qq}^2/(m_u+m_d)$ and $m_0^{\eta_s}=m_{ss}^2/2m_s$, respectively. We adopt $m_0^\pi=(1.4\pm0.1)$ GeV, $m_0^K=(1.6\pm0.1)$ GeV, $m_0^{\eta_q}=1.07$ GeV, and $m_0^{\eta_s}=1.92$ GeV in the numerical calculation. The Gegenbauer polynomials are defined as

$$C_1^{\frac{3}{2}}(t)=3t,\quad C_2^{\frac{1}{2}}(t)=\frac{1}{2}(3t^2-1),\quad C_2^{\frac{3}{2}}(t)=\frac{3}{2}(5t^2-1),$$
$$C_4^{\frac{1}{2}}(t)=\frac{1}{8}(3-30t^2+35t^4),\qquad C_4^{\frac{3}{2}}(t)=\frac{15}{8}(1-14t^2+21t^4),\tag{5}$$

where the variable $t=2z-1$.

III. RESULTS

For the numerical calculations, we adopt from [10] the masses and mean lifetimes for the $B^{0,\pm}$ and B_s^0 mesons, the pole masses and width for $D_0^{*0,\pm}$, the masses and decay constants for the light pseudoscalar mesons pion and kaon, and the Wolfenstein parameters as

$$\begin{aligned}
&m_{B^{\pm,0}}=5.279,\quad m_{B_s^0}=5.367,\quad \tau_{B^0}=1.520,\quad \tau_{B^\pm}=1.638,\quad \tau_{B_s^0}=1.509,\\
&m_{D_0^{*0}}=2.318,\quad m_{D_0^{*\pm}}=2.351,\quad \Gamma_{D_0^{*0}}=0.267,\quad \Gamma_{D_0^{*\pm}}=0.230,\quad m_{\pi^0}=0.135,\\
&m_{\pi^\pm}=0.140,\quad m_K=0.496,\quad m_\eta=0.548,\quad m_{\eta'}=0.958,\quad f_K=0.156,\\
&f_\pi=0.130,\quad A=0.836,\quad \lambda=0.22453,\quad \bar\eta=0.355,\quad \bar\rho=0.122,
\end{aligned}\tag{6}$$

where the masses, decay constants, and widths are in units of GeV and lifetimes in units of ps.

By using the decay amplitudes for the decays $B_{(s)}\to PD_0^*\to PD\pi$ in the Appendix and the differential branching fraction ($\mathcal{B}$), Eq. (13) in [33], we obtain the branching fractions for the decays involving B^+ in Table I, the results for the processes including B^0 in Table II, and the values for the B_s^0 decay modes in Table III with the existing data from [14,15,63–67]. The first error of these results in Tables I–III comes from the shape parameters $\omega_{B^{0,\pm}}=0.40\pm0.04$ GeV for $B^{0,\pm}$ and $\omega_{B_s^0}=0.5\pm0.05$ GeV for B_s^0 [53]. The second error comes from the shape parameter $\omega_{D\pi}=0.40\pm0.10$ GeV for the $D\pi$ system, and the Gegenbauer moment $a_{D\pi}=0.40\pm0.10$ produces the third one [33]. The last one comes from the uncertainty of decay width $\Gamma_{D_0^{*0}}=267\pm40$ MeV or $\Gamma_{D_0^{*+}}=230\pm17$ MeV [10]. We have neglected the errors induced by the uncertainties of the parameters in the distribution amplitudes of the light pseudoscalar mesons and the Wolfenstein parameters since they are very small.

TABLE I. PQCD predictions for branching fractions of the quasi-two-body decays $B^+\to D_0^*P\to D\pi P$ together with the available experimental data.

Mode	Unit	$\mathcal{B}$	Data
$B^+\to D_0^{*0}\pi^+\to D^+\pi^-\pi^+$	(10^{-8})	$1.13^{+0.36}_{-0.26}(\omega_B)^{+0.13}_{-0.14}(\omega_{D\pi})^{+0.03}_{-0.05}(a_{D\pi})^{+0.06}_{-0.05}(\Gamma_{D_0^{*0}})$	⋯
$B^+\to \bar D_0^{*0}\pi^+\to D^-\pi^+\pi^+$	(10^{-4})	$5.95^{+2.37}_{-1.64}(\omega_B)^{+1.97}_{-1.55}(\omega_{D\pi})^{+0.54}_{-0.49}(a_{D\pi})^{+0.29}_{-0.21}(\Gamma_{D_0^{*0}})$	RPP [10]: 6.4 ± 1.4
			Belle [14]: $6.1\pm0.6\pm0.9\pm1.6$
			BABAR [15]: $6.8\pm0.3\pm0.4\pm2.0$
			LHCb [63]: $5.78\pm0.08\pm0.06\pm0.09\pm0.39$
$B^+\to D_0^{*0}K^+\to D^+\pi^-K^+$	(10^{-7})	$3.56^{+1.02}_{-0.78}(\omega_B)^{+0.46}_{-0.52}(\omega_{D\pi})^{+0.09}_{-0.15}(a_{D\pi})^{+0.16}_{-0.12}(\Gamma_{D_0^{*0}})$	⋯
$B^+\to \bar D_0^{*0}K^+\to D^-\pi^+K^+$	(10^{-5})	$4.65^{+1.89}_{-1.30}(\omega_B)^{+1.51}_{-1.24}(\omega_{D\pi})^{+0.40}_{-0.38}(a_{D\pi})^{+0.22}_{-0.18}(\Gamma_{D_0^{*0}})$	LHCb [64]: $0.61\pm0.19\pm0.05\pm0.14\pm0.04$
$B^+\to D_0^{*+}\pi^0\to D^0\pi^+\pi^0$	(10^{-7})	$1.40^{+0.48}_{-0.34}(\omega_B)^{+0.02}_{-0.01}(\omega_{D\pi})^{+0.01}_{-0.00}(a_{D\pi})^{+0.03}_{-0.02}(\Gamma_{D_0^{*+}})$	⋯
$B^+\to D_0^{*+}K^0\to D^0\pi^+K^0$	(10^{-9})	$5.52^{+0.15}_{-0.21}(\omega_B)^{+1.73}_{-1.42}(\omega_{D\pi})^{+0.41}_{-0.36}(a_{D\pi})^{+0.13}_{-0.12}(\Gamma_{D_0^{*+}})$	⋯
$B^+\to D_0^{*+}\eta\to D^0\pi^+\eta$	(10^{-8})	$6.26^{+2.11}_{-1.49}(\omega_B)^{+0.04}_{-0.03}(\omega_{D\pi})^{+0.03}_{-0.02}(a_{D\pi})^{+0.14}_{-0.10}(\Gamma_{D_0^{*+}})$	⋯
$B^+\to D_0^{*+}\eta'\to D^0\pi^+\eta'$	(10^{-8})	$4.01^{+1.34}_{-0.96}(\omega_B)^{+0.02}_{-0.03}(\omega_{D\pi})^{+0.02}_{-0.01}(a_{D\pi})^{+0.07}_{-0.06}(\Gamma_{D_0^{*+}})$	⋯

TABLE II. PQCD prediction of branching fraction for the quasi-two-body decays $B^0 \to D_0^* P \to D\pi P$ together with the available experimental data.

Mode	Unit	$\mathcal{B}$	Data
$B^0 \to D_0^{*-}\pi^+ \to \bar{D}^0\pi^-\pi^+$	(10^{-4})	$2.85^{+1.23}_{-0.80}(\omega_B)^{+1.05}_{-0.81}(\omega_{D\pi})^{+0.33}_{-0.31}(a_{D\pi})^{+0.06}_{-0.05}(\Gamma_{D_0^{*+}})$	RPP [10]: 0.76 ± 0.08 Belle [65]: $0.60 \pm 0.13 \pm 0.15 \pm 0.22$ LHCb [66]: $0.77 \pm 0.05 \pm 0.03 \pm 0.03 \pm 0.04$[a] LHCb [66]: $0.80 \pm 0.05 \pm 0.08 \pm 0.04 \pm 0.04$[b]
$B^0 \to D_0^{*+}\pi^- \to D^0\pi^+\pi^-$	(10^{-7})	$2.56^{+0.85}_{-0.65}(\omega_B)^{+0.01}_{-0.02}(\omega_{D\pi})^{+0.02}_{-0.03}(a_{D\pi})^{+0.03}_{-0.06}(\Gamma_{D_0^{*+}})$	...
$B^0 \to D_0^{*-}K^+ \to \bar{D}^0\pi^-K^+$	(10^{-5})	$2.38^{+0.95}_{-0.65}(\omega_B)^{+0.85}_{-0.68}(\omega_{D\pi})^{+0.30}_{-0.28}(a_{D\pi})^{+0.04}_{-0.03}(\Gamma_{D_0^{*+}})$	LHCb [67]: $1.77 \pm 0.26 \pm 0.19 \pm 0.67 \pm 0.20$
$B^0 \to D_0^{*0}\pi^0 \to D^+\pi^-\pi^0$	(10^{-9})	$4.20^{+1.62}_{-1.07}(\omega_B)^{+0.44}_{-0.48}(\omega_{D\pi})^{+0.09}_{-0.07}(a_{D\pi})^{+0.07}_{-0.12}(\Gamma_{D_0^{*0}})$	...
$B^0 \to \bar{D}_0^{*0}\pi^0 \to D^-\pi^+\pi^0$	(10^{-5})	$2.29^{+0.87}_{-0.61}(\omega_B)^{+0.51}_{-0.43}(\omega_{D\pi})^{+0.09}_{-0.06}(a_{D\pi})^{+0.12}_{-0.04}(\Gamma_{D_0^{*0}})$	...
$B^0 \to D_0^{*0}K^0 \to D^+\pi^-K^0$	(10^{-7})	$2.69^{+0.91}_{-0.66}(\omega_B)^{+0.30}_{-0.32}(\omega_{D\pi})^{+0.09}_{-0.08}(a_{D\pi})^{+0.12}_{-0.11}(\Gamma_{D_0^{*0}})$	...
$B^0 \to \bar{D}_0^{*0}K^0 \to D^-\pi^+K^0$	(10^{-6})	$4.15^{+1.54}_{-1.09}(\omega_B)^{+0.74}_{-0.72}(\omega_{D\pi})^{+0.03}_{-0.03}(a_{D\pi})^{+0.19}_{-0.14}(\Gamma_{D_0^{*0}})$	...
$B^0 \to D_0^{*0}\eta \to D^+\pi^-\eta$	(10^{-9})	$2.81^{+0.78}_{-0.58}(\omega_B)^{+0.30}_{-0.33}(\omega_{D\pi})^{+0.11}_{-0.14}(a_{D\pi})^{+0.13}_{-0.09}(\Gamma_{D_0^{*0}})$	...
$B^0 \to D_0^{*0}\eta' \to D^+\pi^-\eta'$	(10^{-9})	$1.80^{+0.49}_{-0.37}(\omega_B)^{+0.19}_{-0.21}(\omega_{D\pi})^{+0.07}_{-0.09}(a_{D\pi})^{+0.08}_{-0.06}(\Gamma_{D_0^{*0}})$	...
$B^0 \to \bar{D}_0^{*0}\eta \to D^-\pi^+\eta$	(10^{-5})	$1.79^{+0.60}_{-0.41}(\omega_B)^{+0.30}_{-0.28}(\omega_{D\pi})^{+0.07}_{-0.03}(a_{D\pi})^{+0.09}_{-0.06}(\Gamma_{D_0^{*0}})$	...
$B^0 \to \bar{D}_0^{*0}\eta' \to D^-\pi^+\eta'$	(10^{-5})	$1.15^{+0.38}_{-0.27}(\omega_B)^{+0.19}_{-0.18}(\omega_{D\pi})^{+0.04}_{-0.02}(a_{D\pi})^{+0.06}_{-0.04}(\Gamma_{D_0^{*0}})$	...

[a]Isobar model.
[b]K-matrix model.

The four quasi-two-body decays $B^+ \to \bar{D}_0^{*0}\pi^+ \to D^-\pi^+\pi^+$, $B^+ \to \bar{D}_0^{*0}K^+ \to D^-\pi^+K^+$, $B^0 \to D_0^{*-}\pi^+ \to \bar{D}^0\pi^-\pi^+$, and $B^0 \to D_0^{*-}K^+ \to \bar{D}^0\pi^-K^+$ have been discussed in Ref. [33]. For completeness, we keep their branching ratios in Tables I and II. In Fig. 2, we show the $D\pi$ invariant mass-dependent differential branching fraction for the quasi-two-body decay $B^0 \to \bar{D}_0^{*0}\pi^0 \to D^-\pi^+\pi^0$. One can find that the main portion of branching fraction for $B^0 \to \bar{D}_0^{*0}\pi^0 \to D^-\pi^+\pi^0$ comes from the region around the pole mass of the resonant state D_0^*. The contributions from the $m_{D\pi}$ mass region larger than 3 GeV can be neglected safely as argued in Ref. [33].

For the Cabibbo-Kobayashi-Maskawa (CKM) suppressed decay modes $B \to D_0^*\pi \to D\pi\pi$ and $B_s \to D_0^*\bar{K} \to D\pi\bar{K}$, their branching ratios are much smaller than the corresponding results of $B \to \bar{D}_0^*\pi \to D\pi\pi$ and $B_s \to \bar{D}_0^*\bar{K} \to D\pi\bar{K}$ decays as predicted by PQCD in this work. The major reason comes from the strong CKM suppression factor [51]

$$R_{\mathrm{CKM}} = \left|\frac{V_{ub}^* V_{cd}}{V_{cb}^* V_{ud}}\right|^2 \approx \lambda^4(\bar{\rho}^2 + \bar{\eta}^2) \approx 3 \times 10^{-4}. \quad (7)$$

For the CKM suppressed and CKM favored decay modes concerned in this work, we define the following ratios of the branching fractions for the corresponding decays as

TABLE III. PQCD prediction of branching fraction for the quasi-two-body decays $B_s^0 \to D_0^* P \to D\pi P$.

Mode	Unit	$\mathcal{B}$
$B_s^0 \to D_0^{*-}\pi^+ \to \bar{D}^0\pi^-\pi^+$	(10^{-7})	$2.70^{+0.29}_{-0.36}(\omega_B)^{+0.60}_{-0.58}(\omega_{D\pi})^{+0.43}_{-0.31}(a_{D\pi})^{+0.06}_{-0.01}(\Gamma_{D_0^{*+}})$
$B_s^0 \to D_0^{*+}\pi^- \to D^0\pi^+\pi^-$	(10^{-9})	$2.90^{+0.08}_{-0.15}(\omega_B)^{+0.95}_{-0.83}(\omega_{D\pi})^{+0.26}_{-0.23}(a_{D\pi})^{+0.07}_{-0.06}(\Gamma_{D_0^{*+}})$
$B_s^0 \to D_0^{*+}K^- \to D^0\pi^+K^-$	(10^{-7})	$2.82^{+1.09}_{-0.74}(\omega_B)^{+0.02}_{-0.01}(\omega_{D\pi})^{+0.01}_{-0.00}(a_{D\pi})^{+0.06}_{-0.04}(\Gamma_{D_0^{*+}})$
$B_s^0 \to D_0^{*0}\pi^0 \to D^+\pi^-\pi^0$	(10^{-9})	$1.48^{+0.03}_{-0.04}(\omega_B)^{+0.46}_{-0.42}(\omega_{D\pi})^{+0.12}_{-0.13}(a_{D\pi})^{+0.08}_{-0.07}(\Gamma_{D_0^{*0}})$
$B_s^0 \to \bar{D}_0^{*0}\pi^0 \to D^-\pi^+\pi^0$	(10^{-7})	$1.38^{+0.24}_{-0.19}(\omega_B)^{+0.48}_{-0.33}(\omega_{D\pi})^{+0.22}_{-0.16}(a_{D\pi})^{+0.07}_{-0.04}(\Gamma_{D_0^{*0}})$
$B_s^0 \to D_0^{*0}\bar{K}^0 \to D^+\pi^-\bar{K}^0$	(10^{-9})	$9.09^{+3.65}_{-2.38}(\omega_B)^{+0.84}_{-0.95}(\omega_{D\pi})^{+0.38}_{-0.23}(a_{D\pi})^{+0.41}_{-0.31}(\Gamma_{D_0^{*0}})$
$B_s^0 \to \bar{D}_0^{*0}\bar{K}^0 \to D^-\pi^+\bar{K}^0$	(10^{-5})	$4.70^{+2.05}_{-1.39}(\omega_B)^{+0.76}_{-0.75}(\omega_{D\pi})^{+0.04}_{-0.05}(a_{D\pi})^{+0.21}_{-0.16}(\Gamma_{D_0^{*0}})$
$B_s^0 \to D_0^{*0}\eta \to D^+\pi^-\eta$	(10^{-8})	$9.37^{+4.31}_{-2.70}(\omega_B)^{+0.67}_{-0.77}(\omega_{D\pi})^{+0.21}_{-0.15}(a_{D\pi})^{+0.43}_{-0.30}(\Gamma_{D_0^{*0}})$
$B_s^0 \to D_0^{*0}\eta' \to D^+\pi^-\eta'$	(10^{-7})	$1.62^{+0.65}_{-0.43}(\omega_B)^{+0.16}_{-0.15}(\omega_{D\pi})^{+0.06}_{-0.05}(a_{D\pi})^{+0.09}_{-0.05}(\Gamma_{D_0^{*0}})$
$B_s^0 \to \bar{D}_0^{*0}\eta \to D^-\pi^+\eta$	(10^{-6})	$1.27^{+0.55}_{-0.39}(\omega_B)^{+0.18}_{-0.20}(\omega_{D\pi})^{+0.04}_{-0.03}(a_{D\pi})^{+0.05}_{-0.04}(\Gamma_{D_0^{*0}})$
$B_s^0 \to \bar{D}_0^{*0}\eta' \to D^-\pi^+\eta'$	(10^{-6})	$2.24^{+0.93}_{-0.64}(\omega_B)^{+0.29}_{-0.30}(\omega_{D\pi})^{+0.02}_{-0.04}(a_{D\pi})^{+0.11}_{-0.08}(\Gamma_{D_0^{*0}})$

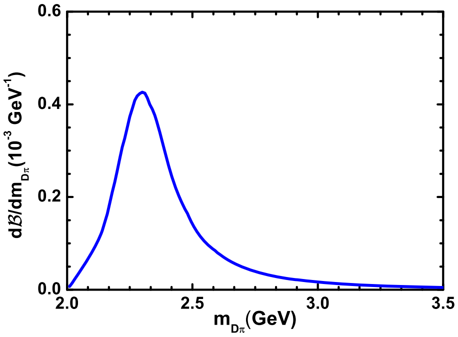

FIG. 2. The $D\pi$ invariant mass-dependent differential branching fraction for $B^0 \to \bar{D}_0^{*0}\pi^0 \to D^-\pi^+\pi^0$.

$$R_1 = \frac{B^0 \to D_0^{*0}\pi^0 \to D^+\pi^-\pi^0}{B^0 \to \bar{D}_0^{*0}\pi^0 \to D^-\pi^+\pi^0} \approx 1.83 \times 10^{-4},$$
$$R_2 = \frac{B^0 \to D_0^{*0}\eta \to D^+\pi^-\eta}{B^0 \to \bar{D}_0^{*0}\eta \to D^-\pi^+\eta} \approx 1.57 \times 10^{-4},$$
$$R_3 = \frac{B^0 \to D_0^{*0}\eta' \to D^+\pi^-\eta'}{B^0 \to \bar{D}_0^{*0}\eta' \to D^-\pi^+\eta'} \approx 1.57 \times 10^{-4},$$
$$R_4 = \frac{B_s \to D_0^{*0}\bar{K}^0 \to D^+\pi^-\bar{K}^0}{B_s \to \bar{D}_0^{*0}\bar{K}^0 \to D^-\pi^+\bar{K}^0} \approx 1.93 \times 10^{-4},$$
$$R_5 = \frac{B^+ \to D_0^{*0}\pi^+ \to D^+\pi^-\pi^+}{B^+ \to \bar{D}_0^{*0}\pi^+ \to D^-\pi^+\pi^+} \approx 1.91 \times 10^{-5}. \quad (8)$$

The ratios R_1, R_2, R_3, and R_4 are close to each other because all four decay pairs in these four ratios decay through the same color suppressed emission topologies, and the nonfactorizable diagrams in Fig. 1 play the dominant role. The nonvanishing charm quark mass in the fermion propagator generates the main differences between the $R_{\rm CKM}$ and $R_{1,2,3,4}$. For the decay process $B^+ \to \bar{D}_0^{*0}\pi^+ \to D^-\pi^+\pi^+$, one has the contributions from both the $B \to \bar{D}_0^{*0}$ transition and the $B \to \pi$ transition, while for $B^+ \to D_0^{*0}\pi^+ \to D^+\pi^-\pi^+$, one has only the color suppressed transition $B \to \pi$. So it is not surprising to have a quite small value for R_5.

Assuming factorization and flavor-$SU(3)$ symmetry, the ratio between the two decays $B^0 \to D_0^{*-}K^+ \to \bar{D}^0\pi^-K^+$ and $B^0 \to D_0^{*-}\pi^+ \to \bar{D}^0\pi^-\pi^+$ will not very far from 0.076, as discussed in Ref. [33]. The same situation should happen to the decays $B^0 \to \bar{D}_0^{*0}K^0 \to D^-\pi^+K^0$ and $B^0 \to \bar{D}_0^{*0}\pi^0 \to D^-\pi^+\pi^0$. With the PQCD predictions in Table II, we have

$$R_{\bar{D}_0^{*0}} = \frac{\mathcal{B}(B^0 \to \bar{D}_0^{*0}K^0 \to D^-\pi^+K^0)}{2\mathcal{B}(B^0 \to \bar{D}_0^{*0}\pi^0 \to D^-\pi^+\pi^0)} = 0.091^{+0.003}_{-0.005}. \quad (9)$$

The deviation between the $R_{\bar{D}_0^{*0}}$ and

$$\left|\frac{V_{us}}{V_{ud}}\right|^2 \cdot \frac{f_K^2}{f_\pi^2} = 0.076 \quad (10)$$

could be due to the violation of the flavor-$SU(3)$ symmetry and the contributions from annihilation diagrams in the $B^0 \to \bar{D}_0^{*0}\pi^0 \to D^-\pi^+\pi^0$ process.

The ratio of branching fractions with topologically similar decay processes $B_s^0 \to D_0^{*+}K^- \to D^0\pi^+K^-$ and $B^0 \to D_0^{*+}\pi^- \to D^0\pi^+\pi^-$ is expected to be close to 1 in the naive factorization because of the close values for the $B_s^0 \to K^-$ and $B^0 \to \pi^-$ transition form factors [53]. With the predictions in Tables II and III, we have

$$\frac{\mathcal{B}(B_s^0 \to D_0^{*+}K^- \to D^0\pi^+K^-)}{\mathcal{B}(B^0 \to D_0^{*+}\pi^- \to D^0\pi^+\pi^-)} = 1.10^{+0.05}_{-0.02}. \quad (11)$$

A similar relation for $B_s^0 \to \bar{D}_0^{*0}\bar{K}^0 \to D^-\pi^+\bar{K}^0$ and $B^0 \to \bar{D}_0^{*0}\pi^0 \to D^-\pi^+\pi^0$ is

$$\frac{\mathcal{B}(B_s^0 \to \bar{D}_0^{*0}\bar{K}^0 \to D^-\pi^+\bar{K}^0)}{2\mathcal{B}(B^0 \to \bar{D}_0^{*0}\pi^0 \to D^-\pi^+\pi^0)} = 1.03^{+0.06}_{-0.07} \quad (12)$$

induced from Tables II and III.

IV. CONCLUSION

We have studied the quasi-two-body decays $B_{(s)} \to PD_0^* \to PD\pi$, where the bachelor particle P denotes π, K, η, or η' in the PQCD approach. The predicted branching fractions for the considered decays are in the range of 10^{-9}–10^{-4}. For the decays $B \to D_0^*\pi \to D\pi\pi$ and $B \to \bar{D}_0^*\pi \to D\pi\pi$ as well as $B_s \to D_0^* \to D\pi\bar{K}$ and $B_s \to \bar{D}_0^*\bar{K} \to D\pi\bar{K}$, the great difference in their corresponding branching fractions can be understood by a strong CKM suppression factor $R_{\rm CKM} \approx \lambda^4(\bar{\rho}^2 + \bar{\eta}^2) \approx 3 \times 10^{-4}$. The flavor-$SU(3)$ symmetry can be employed to analyze the quasi-two-body decays with the same topologies, such as $B^0 \to \bar{D}_0^{*0}K^0 \to D^-\pi^+K^0$ and $B^0 \to \bar{D}_0^{*0}\pi^0 \to D^-\pi^+\pi^0$, while $R_{\bar{D}_0^{*0}}$ was predicted to be $0.091^{+0.003}_{-0.005}$ for their branching ratios. The ratio for the branching fractions was found to be $1.10^{+0.05}_{-0.02}$ between $\mathcal{B}(B_s^0 \to D_0^{*+}K^- \to D^0\pi^+K^-)$ and $\mathcal{B}(B^0 \to D_0^{*+}\pi^- \to D^0\pi^+\pi^-)$ and to be $1.03^{+0.06}_{-0.07}$ between $\mathcal{B}(B_s^0 \to \bar{D}_0^{*0}\bar{K}^0 \to D^-\pi^+\bar{K}^0)$ and $2\mathcal{B}(B^0 \to \bar{D}_0^{*0}\pi^0 \to D^-\pi^+\pi^0)$, which can be tested by the precise data from the future experiments.

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APPENDIX: DECAY AMPLITUDES

The amplitudes from Fig. 1 are written as

$$\mathcal{A}(B^+ \to \pi^+[D_0^{*0} \to]D^+\pi^-) = \frac{G_F}{\sqrt{2}} V_{ub}^* V_{cd}\{a_2 F_{TP} + C_2 M'_{TP} + a_1 F_{AD} + C_1 M_{AD}\},$$
$$\mathcal{A}(B^+ \to \pi^+[\bar{D}_0^{*0} \to]D^-\pi^+) = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud}\{a_2 F_{TP} + C_2 M_{TP} + a_1 F_{TD} + C_1 M_{TD}\},$$
$$\mathcal{A}(B^+ \to K^+[D_0^{*0} \to]D^+\pi^-) = \frac{G_F}{\sqrt{2}} V_{ub}^* V_{cs}\{a_2 F_{TP} + C_2 M'_{TP} + a_1 F_{AD} + C_1 M_{AD}\},$$
$$\mathcal{A}(B^+ \to K^+[\bar{D}_0^{*0} \to]D^-\pi^+) = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{us}\{a_2 F_{TP} + C_2 M_{TP} + a_1 F_{TD} + C_1 M_{TD}\},$$
$$\mathcal{A}(B^+ \to \pi^0[D_0^{*+} \to]D^0\pi^+) = \frac{G_F}{2} V_{ub}^* V_{cd}\{a_1(F_{TP} - F_{AD}) + C_1(M'_{TP} - M_{AD})\},$$
$$\mathcal{A}(B^+ \to K^0[D_0^{*+} \to]D^0\pi^+) = \frac{G_F}{\sqrt{2}} V_{ub}^* V_{cs}\{a_1 F_{AD} + C_1 M_{AD}\},$$
$$\mathcal{A}(B^+ \to \eta_q[D_0^{*+} \to]D^0\pi^+) = \frac{G_F}{2} V_{ub}^* V_{cd}\{a_1(F_{TP} + F_{AD}) + C_1(M'_{TP} + M_{AD})\},$$
$$\mathcal{A}(B^+ \to \eta[D_0^{*+} \to]D^0\pi^+) = \mathcal{A}(B^+ \to \eta_q[D_0^{*+} \to]D^0\pi^+)\cos\phi,$$
$$\mathcal{A}(B^+ \to \eta'[D_0^{*+} \to]D^0\pi^+) = \mathcal{A}(B^+ \to \eta_q[D_0^{*+} \to]D^0\pi^+)\sin\phi,$$
$$\mathcal{A}(B^0 \to \pi^+[D_0^{*-} \to]\bar{D}^0\pi^-) = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud}\{a_2 F_{AP} + C_2 M_{AP} + a_1 F_{TD} + C_1 M_{TD}\},$$
$$\mathcal{A}(B^0 \to \pi^-[D_0^{*+} \to]D^0\pi^+) = \frac{G_F}{\sqrt{2}} V_{ub}^* V_{cd}\{a_2 F_{AD} + C_2 M_{AD} + a_1 F_{TP} + C_1 M'_{TP}\},$$
$$\mathcal{A}(B^0 \to K^+[D_0^{*-} \to]\bar{D}^0\pi^-) = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{us}\{a_1 F_{TD} + C_1 M_{TD}\},$$
$$\mathcal{A}(B^0 \to \pi^0[D_0^{*0} \to]D^+\pi^-) = \frac{G_F}{2} V_{ub}^* V_{cd}\{a_2(F_{AD} - F_{TP}) + C_2(M_{AD} - M'_{TP})\},$$
$$\mathcal{A}(B^0 \to \pi^0[\bar{D}_0^{*0} \to]D^-\pi^+) = \frac{G_F}{2} V_{cb}^* V_{ud}\{a_2(F_{AP} - F_{TP}) + C_2(M_{AP} - M_{TP})\},$$
$$\mathcal{A}(B^0 \to K^0[D_0^{*0} \to]D^+\pi^-) = \frac{G_F}{\sqrt{2}} V_{ub}^* V_{cs}\{a_2 F_{TP} + C_2 M'_{TP}\},$$
$$\mathcal{A}(B^0 \to K^0[\bar{D}_0^{*0} \to]D^-\pi^+) = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{us}\{a_2 F_{TP} + C_2 M_{TP}\},$$
$$\mathcal{A}(B^0 \to \eta_q[D_0^{*0} \to]D^+\pi^-) = \frac{G_F}{2} V_{ub}^* V_{cd}\{a_2(F_{TP} + F_{AD}) + C_2(M'_{TP} + M_{AD})\},$$
$$\mathcal{A}(B^0 \to \eta[D_0^{*0} \to]D^+\pi^-) = \mathcal{A}(B^0 \to \eta_q[D_0^{*0} \to]D^+\pi^-)\cos\phi,$$
$$\mathcal{A}(B^0 \to \eta'[D_0^{*0} \to]D^+\pi^-) = \mathcal{A}(B^0 \to \eta_q[D_0^{*0} \to]D^+\pi^-)\sin\phi,$$
$$\mathcal{A}(B^0 \to \eta_q[\bar{D}_0^{*0} \to]D^-\pi^+) = \frac{G_F}{2} V_{cb}^* V_{ud}\{a_2(F_{TP} + F_{AP}) + C_2(M_{TP} + M_{AP})\},$$
$$\mathcal{A}(B^0 \to \eta[\bar{D}_0^{*0} \to]D^-\pi^+) = \mathcal{A}(B^0 \to \eta_q[\bar{D}_0^{*0} \to]D^-\pi^+)\cos\phi,$$
$$\mathcal{A}(B^0 \to \eta'[\bar{D}_0^{*0} \to]D^-\pi^+) = \mathcal{A}(B^0 \to \eta_q[\bar{D}_0^{*0} \to]D^-\pi^+)\sin\phi,$$
$$\mathcal{A}(B_s^0 \to \pi^+[D_0^{*-} \to]\bar{D}^0\pi^-) = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{us}\{a_2 F_{AP} + C_2 M_{AP}\},$$
$$\mathcal{A}(B_s^0 \to \pi^-[D_0^{*+} \to]D^0\pi^+) = \frac{G_F}{\sqrt{2}} V_{ub}^* V_{cs}\{a_2 F_{AD} + C_2 M_{AD}\},$$
$$\mathcal{A}(B_s^0 \to K^-[D_0^{*+} \to]D^0\pi^+) = \frac{G_F}{\sqrt{2}} V_{ub}^* V_{cd}\{a_1 F_{TP} + C_1 M'_{TP}\},$$

$$\mathcal{A}(B_s^0\to\pi^0[D_0^{*0}\to]D^+\pi^-)=\frac{G_F}{2}V_{ub}^*V_{cs}\{a_2F_{AD}+C_2M_{AD}\},$$
$$\mathcal{A}(B_s^0\to\pi^0[\bar{D}_0^{*0}\to]D^-\pi^+)=\frac{G_F}{2}V_{cb}^*V_{us}\{a_2F_{AP}+C_2M_{AP}\},$$
$$\mathcal{A}(B_s^0\to\bar{K}^0[D_0^{*0}\to]D^+\pi^-)=\frac{G_F}{\sqrt{2}}V_{ub}^*V_{cd}\{a_2F_{TP}+C_2M'_{TP}\},$$
$$\mathcal{A}(B_s^0\to\bar{K}^0[\bar{D}_0^{*0}\to]D^-\pi^+)=\frac{G_F}{\sqrt{2}}V_{cb}^*V_{ud}\{a_2F_{TP}+C_2M_{TP}\},$$
$$\mathcal{A}(B_s^0\to\eta_q[D_0^{*0}\to]D^+\pi^-)=\frac{G_F}{2}V_{ub}^*V_{cs}\{a_2F_{AD}+C_2M_{AD}\},$$
$$\mathcal{A}(B_s^0\to\eta_s[D_0^{*0}\to]D^+\pi^-)=\frac{G_F}{\sqrt{2}}V_{ub}^*V_{cs}\{a_2F_{TP}+C_2M'_{TP}\},$$
$$\mathcal{A}(B_s^0\to\eta[D_0^{*0}\to]D^+\pi^-)=\mathcal{A}(B_s^0\to\eta_q[D_0^{*0}\to]D^+\pi^-)\cos\phi-\mathcal{A}(B_s^0\to\eta_s[D_0^{*0}\to]D^+\pi^-)\sin\phi,$$
$$\mathcal{A}(B_s^0\to\eta'[D_0^{*0}\to]D^+\pi^-)=\mathcal{A}(B_s^0\to\eta_q[D_0^{*0}\to]D^+\pi^-)\sin\phi+\mathcal{A}(B_s^0\to\eta_s[D_0^{*0}\to]D^+\pi^-)\cos\phi,$$
$$\mathcal{A}(B_s^0\to\eta_q[\bar{D}_0^{*0}\to]D^-\pi^+)=\frac{G_F}{2}V_{cb}^*V_{us}\{a_2F_{AP}+C_2M_{AP}\},$$
$$\mathcal{A}(B_s^0\to\eta_s[\bar{D}_0^{*0}\to]D^-\pi^+)=\frac{G_F}{\sqrt{2}}V_{cb}^*V_{us}\{a_2F_{TP}+C_2M_{TP}\},$$
$$\mathcal{A}(B_s^0\to\eta[\bar{D}_0^{*0}\to]D^-\pi^+)=\mathcal{A}(B_s^0\to\eta_q[\bar{D}_0^{*0}\to]D^-\pi^+)\cos\phi-\mathcal{A}(B_s^0\to\eta_s[\bar{D}_0^{*0}\to]D^-\pi^+)\sin\phi,$$
$$\mathcal{A}(B_s^0\to\eta'[\bar{D}_0^{*0}\to]D^-\pi^+)=\mathcal{A}(B_s^0\to\eta_q[\bar{D}_0^{*0}\to]D^-\pi^+)\sin\phi+\mathcal{A}(B_s^0\to\eta_s[\bar{D}_0^{*0}\to]D^-\pi^+)\cos\phi,$$

where G_F is the Fermi constant, V's are the CKM matrix elements, C_1 and C_2 are Wilson coefficients, and $a_1=C_1/3+C_2$ and $a_2=C_2/3+C_1$. The factorization formulas for decay amplitudes from Fig. 1 are collected below:

$$F_{TD}=8\pi C_Fm_B^4f_P\int dx_Bdx_3\int b_Bdb_Bb_3db_3\phi_B(x_B,b_B)\phi_{D\pi}(x_3,b_3,s)(\eta-1)\{[\sqrt{\eta}(2x_3-1)-x_3-1]$$
$$\times E_{1ab}(t_{1a})h_{1a}(x_B,x_3,b_B,b_3)+(\eta+2\sqrt{\eta}(r_c-1)-r_c)E_{1ab}(t_{1b})h_{1b}(x_B,x_3,b_B,b_3)\},$$
$$M_{TD}=32\pi C_Fm_B^4/\sqrt{2N_c}\int dx_Bdzdx_3\int b_Bdb_Bbdb\phi_B(x_B,b_B)\phi_{D\pi}(x_3,b_3,s)\phi^A(\eta-1)$$
$$\times\{[\eta(1-x_3-z)+z+x_B+x_3\sqrt{\eta}-1]E_{1cd}(t_{1c})h_{1c}(x_B,z,x_3,b_B,b)$$
$$+[z(1-\eta)-x_B+x_3(1-\sqrt{\eta})]E_{1cd}(t_{1d})h_{1d}(x_B,z,x_3,b_B,b)\},$$
$$F_{AD}=8\pi C_Fm_B^4f_B\int dzdx_3\int bdbb_3db_3\phi_{D\pi}(x_3,b_3,s)\{[\phi^A(\eta-1)(1-x_3)-2\phi^P(x_3-2)\sqrt{\eta}r_0]E_{1ef}(t_{1e})$$
$$\times h_{1e}(z,x_3,b,b_3)+[(\eta-1)[2r_c\sqrt{\eta}+z(\eta-1)-\eta]\phi^A+2r_0\sqrt{\eta}(\eta-1)[z(\phi^P+\phi^T)-\phi^T]$$
$$+r_0(\eta+1)(-2\sqrt{\eta}+r_c)\phi^P+r_0r_c(\eta-1)\phi^T]\times E_{1ef}(t_{1f})h_{1f}(z,x_3,b,b_3)\},$$
$$M_{AD}=32\pi C_Fm_B^4/\sqrt{2N_c}\int dx_Bdzdx_3\int b_Bdb_Bbdb\phi_B(x_B,b_B)\phi_{D\pi}(x_3,b_3,s)\{[(1-\eta)z[(\eta-1)\phi^A$$
$$+\sqrt{\eta}r_0(\phi^P+\phi^T)]+x_B[(\eta-1)\phi^A+\sqrt{\eta}r_0(\phi^P+\phi^T)]+\sqrt{\eta}[\eta r_0(\phi^P+\phi^T)-r_0(x_3-3)\phi^P$$
$$+r_0(x_3-1)\phi^T+\sqrt{\eta}(\eta-1)(1-x_3)\phi^A]]\times E_{1gh}(t_{1g})h_{1g}(x_B,z,x_3,b_B,b)$$
$$+[\sqrt{\eta}r_0[(\phi^P-\phi^T)(\eta z-\eta-z+x_B)+(\phi^P+\phi^T)(x_3-1)]$$
$$+\phi^A(\eta^2-1)(x_3-1)]E_{1gh}(t_{1h})h_{1h}(x_B,z,x_3,b_B,b)\},$$
$$F_{TP}=8\pi C_Fm_B^4F_{D\pi}(s)\int dx_Bdz\int b_Bdb_Bbdb\phi_B(x_B,b_B)\{[\phi^A(1-\eta)(z(\eta-1)-1)$$
$$-r_0[\phi^P(\eta+2(\eta-1)z+1)+\phi^T(\eta-1)(2z-1)]]E_{2ab}(t_{2a})h_{2a}(x_B,z,b_B,b)$$
$$+[2r_0\phi^P(\eta+\eta x_B-1)+(\eta-1)\eta x_B\phi^A]E_{2ab}(t_{2b})\times h_{2b}(x_B,z,b_B,b)\},$$

$$
\begin{aligned}
M_{TP} = {} & 32\pi C_F m_B^4/\sqrt{2N_c}\int dx_B dz dx_3 \int b_B db_B b_3 db_3 \phi_B(x_B,b_B)\phi_{D\pi}(x_3,b_3,s)\{[(\eta-1)((\eta+1)(x_B+x_3-1) \\
& - r_c\sqrt{\eta})\phi^A + r_0[z(1-\eta)(\phi^T-\phi^P)+(x_B+x_3)\eta(\phi^T+\phi^P)-(2\eta+4r_c\sqrt{\eta})\phi^P]] \\
& \times E_{2cd}(t_{2c})h_{2c}(x_B,z,x_3,b_B,b_3) - [(\eta-1)z[(\eta-1)\phi^A + r_0(\phi^P+\phi^T)] \\
& + (x_B-x_3)[\eta r_0(\phi^P-\phi^T)+(\eta-1)\phi^A]]E_{2cd}(t_{2d})h_{2d}(x_B,z,x_3,b_B,b_3)\}, \\
M'_{TP} = {} & 32\pi C_F m_B^4/\sqrt{2N_c}\int dx_B dz dx_3 \int b_B db_B b_3 db_3 \phi_B(x_B,b_B)\phi_{D\pi}(x_3,b_3,s)\{[(1-x_B-x_3)(1-\eta^2)\phi^A \\
& + r_0[z(1-\eta)(\phi^T-\phi^P)+(x_B+x_3)\eta(\phi^T+\phi^P)-2\eta\phi^P]]E_{2cd}(t'_{2c})h'_{2c}(x_B,z,x_3,b_B,b_3) \\
& + [(\eta-1)[(1-\eta)z - x_B - r_c\sqrt{\eta} + x_3]\phi^A - r_0 z(\eta-1)(\phi^P+\phi^T) \\
& + r_0\eta(x_B-x_3)(\phi^T-\phi^P) - 4r_0 r_c\sqrt{\eta}\phi^P] \times E_{2cd}(t'_{2d})h'_{2d}(x_B,z,x_3,b_B,b_3)\}, \\
F_{AP} = {} & 8\pi C_F m_B^4 f_B \int dz dx_3 \int b db b_3 db_3 \phi_{D\pi}(x_3,b_3,s)\{[(\eta-1)[2\sqrt{\eta}r_c + (\eta-1)z+1]\phi^A \\
& - r_0[(\eta+1)r_c + 2\sqrt{\eta}(z(\eta-1)+2)]\phi^P + r_0(\eta-1)(r_c+2\sqrt{\eta}z)\phi^T]E_{2ef}(t_{2e})h_{2e}(z,x_3,b,b_3) \\
& + [2\sqrt{\eta}r_0\phi^P(-\eta+x_3+1) - \phi^A(\eta-1)x_3]E_{2ef}(t_{2f})h_{2f}(z,x_3,b,b_3)\}, \\
M_{AP} = {} & 32\pi C_F m_B^4/\sqrt{2N_c}\int dx_B dz dx_3 \int b_B db_B b db \phi_B(x_B,b_B)\phi_{D\pi}(x_3,b_3,s)\{[\eta\phi^A(1-\eta) \\
& + \sqrt{\eta}[-\eta r_0(z-1)(\phi^P+\phi^T) + r_0(z-3)\phi^P + r_0(z-1)\phi^T] + (x_B+x_3)[(\eta^2-1)\phi^A \\
& + r_0\sqrt{\eta}(\phi^T-\phi^P)]]E_{2gh}(t_{2g})h_{2g}(x_B,z,x_3,b_B,b) + [(1-\eta)\phi^A[\eta(x_3-x_B+z-1)-z+1] \\
& + r_0\sqrt{\eta}[(\eta-1)(z-1)(\phi^P-\phi^T) + (x_3-x_B)(\phi^P+\phi^T)]]E_{2gh}(t_{2h})h_{2h}(x_B,z,x_3,b_B,b)\},
\end{aligned}
$$

where x_B, x_3, and z are momentum fractions of the corresponding spectator quarks, as defined in Ref. [33]. b_B, b_3, and b are the conjugate variables of transverse momenta P_B, P_3, and P, respectively. Variable η is defined as $\eta = m_{D\pi}^2/m_B^2$. The ratio $r_0 = m_0/m_B$, where m_0 is the chiral mass of light pseudoscalars. $r_c = m_c/m_B$ is the ratio of the charm quark mass to the B meson mass. The functions E_{1mn} and $E_{2mn}(m = a, c, e, g$ and $n = b, d, f, h)$ are the evolution factors, which are given by

$$
\begin{aligned}
& E_{1ab}(t) = \alpha(t)\exp[-S_B(t)-S_D(t)], \qquad E_{2ab}(t) = \alpha(t)\exp[-S_B(t)-S_P(t)], \\
& E_{1cd}(t) = \alpha(t)\exp[-S_B(t)-S_D(t)-S_P(t)]_{b_3=b_B}, \qquad E_{2cd}(t) = \alpha(t)\exp[-S_B(t)-S_D(t)-S_P(t)]_{b=b_B}, \\
& E_{1ef}(t) = \alpha(t)\exp[-S_P(t)-S_D(t)], \qquad E_{2ef}(t) = E_{1ef}(t), \\
& E_{1gh}(t) = \alpha(t)\exp[-S_B(t)-S_D(t)-S_P(t)]_{b=b_3}, \qquad E_{2gh}(t) = E_{1gh}(t),
\end{aligned}
$$

in which Sudakov exponents $S_{(B,D,P)}(t)$ are defined as

$$
\begin{aligned}
S_B(t) &= s\left(\frac{x_B m_B}{\sqrt{2}}, b_B\right) + 2\int_{1/b_B}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu})), \\
S_D(t) &= s\left(\frac{x_3 m_B}{\sqrt{2}}, b_3\right) + 2\int_{1/b_3}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu})), \\
S_P(t) &= s\left(\frac{z m_B}{\sqrt{2}}, b\right) + s\left(\frac{(1-z)m_B}{\sqrt{2}}, b\right) + 2\int_{1/b}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu})),
\end{aligned}
$$

where the quark anomalous dimension $\gamma_q = -\alpha_s/\pi$. The explicit form for $s(Q,b)$ at one loop can be found in [68]. t_{1x} and $t_{2x}(x = a, b\cdots h)$ are hard scales which are chosen to be the maximum of the virtuality of the internal momentum transition in the hard amplitudes as

$$
\begin{aligned}
t_{1a} &= \mathrm{Max}\{m_B\sqrt{x_3}, 1/b_B, 1/b_3\},\\
t_{1b} &= \mathrm{Max}\{m_B\sqrt{x_3 x_B}, m_B\sqrt{|x_B-\eta+r_c^2|}, 1/b_B, 1/b_3\},\\
t_{1c} &= \mathrm{Max}\{m_B\sqrt{x_3 x_B}, m_B\sqrt{|x_3[x_B-(1-\eta)(1-z)]|}, 1/b_B, 1/b\},\\
t_{1d} &= \mathrm{Max}\{m_B\sqrt{x_3 x_B}, m_B\sqrt{|x_3[x_B-(1-\eta)z]|}, 1/b_B, 1/b\},\\
t_{1e} &= \mathrm{Max}\{m_B\sqrt{(1-x_3)[(1-\eta)z+\eta]}, m_B\sqrt{1-x_3}, 1/b_3, 1/b\},\\
t_{1f} &= \mathrm{Max}\{m_B\sqrt{(1-x_3)[(1-\eta)z+\eta]}, m_B\sqrt{|\eta+(1-\eta)z-r_c^2|}, 1/b_3, 1/b\},\\
t_{1g} &= \mathrm{Max}\{m_B\sqrt{(1-x_3)[(1-\eta)z+\eta]}, m_B\sqrt{1-x_3[(1-\eta)(1-z)-x_B]}, 1/b_B, 1/b\},\\
t_{1h} &= \mathrm{Max}\{m_B\sqrt{(1-x_3)[(1-\eta)z+\eta]}, m_B\sqrt{|(1-x_3)[x_B-\eta-(1-\eta)z]|}, 1/b_B, 1/b\},\\
t_{2a} &= \mathrm{Max}\{m_B\sqrt{(1-\eta)z}, 1/b_B, 1/b\},\\
t_{2b} &= \mathrm{Max}\{m_B\sqrt{(1-\eta)x_B}, 1/b_B, 1/b\},\\
t_{2c} &= \mathrm{Max}\{m_B\sqrt{(1-\eta)z x_B}, m_B\sqrt{|r_c^2-(1-x_3-x_B)[(1-\eta)z+\eta]|}, 1/b_B, 1/b_3\},\\
t_{2d} &= \mathrm{Max}\{m_B\sqrt{(1-\eta)z x_B}, m_B\sqrt{(1-\eta)z|x_B-x_3|}, 1/b_B, 1/b_3\},\\
t'_{2c} &= \mathrm{Max}\{m_B\sqrt{(1-\eta)z x_B}, m_B\sqrt{|1-x_3-x_B|[(1-\eta)z+\eta]}, 1/b_B, 1/b_3\},\\
t'_{2d} &= \mathrm{Max}\{m_B\sqrt{(1-\eta)z x_B}, m_B\sqrt{|r_c^2+(1-\eta)z(x_B-x_3)|}, 1/b_B, 1/b_3\},\\
t_{2e} &= \mathrm{Max}\{m_B\sqrt{(1-\eta)(1-z)x_3}, m_B\sqrt{|1-r_c^2-(1-\eta)z|}, 1/b_3, 1/b\},\\
t_{2f} &= \mathrm{Max}\{m_B\sqrt{(1-\eta)x_3}, 1/b_3, 1/b\},\\
t_{2g} &= \mathrm{Max}\{m_B\sqrt{(1-\eta)(1-z)x_3}, m_B\sqrt{|[\eta+(1-\eta)z](1-x_3-x_B)-1|}, 1/b_B, 1/b\},\\
t_{2h} &= \mathrm{Max}\{m_B\sqrt{(1-\eta)(1-z)x_3}, m_B\sqrt{|x_3-x_B|(1-\eta)(1-z)}, 1/b_B, 1/b\}.
\end{aligned}
$$

The hard functions can be written as

$$
\begin{aligned}
h_{1a}(x_B,x_3,b_B,b_3) &= K_0(m_B\sqrt{x_3x_B}b_B)[K_0(m_B\sqrt{x_3}b_B)I_0(m_B\sqrt{x_3}b_3)\theta(b_B-b_3)+(b_B\longleftrightarrow b_3)]S_t(x_3),\\
h_{1b}(x_B,x_3,b_B,b_3) &= K_0(m_B\sqrt{x_3x_B}b_3)S_t(x_B)\\
&\quad\times\begin{cases}[\theta(b_3-b_B)K_0(m_B\sqrt{r_c^2+x_B-\eta}b_3)\\ \times I_0(m_B\sqrt{r_c^2+x_B-\eta}b_B)+(b_3\longleftrightarrow b_B)], & r_c^2+x_B\ge\eta,\\ \frac{i\pi}{2}[\theta(b_3-b_B)H_0^{(1)}(m_B\sqrt{\eta-x_B-r_c^2}b_3)\\ \times J_0(m_B\sqrt{\eta-x_B-r_c^2}b_B)+(b_3\longleftrightarrow b_B)], & r_c^2+x_B<\eta,\end{cases}\\
h_{1c}(x_B,z,x_3,b_B,b) &= [K_0(m_B\sqrt{x_3x_B}b_B)I_0(m_B\sqrt{x_3x_B}b)\theta(b_B-b)+(b\longleftrightarrow b_B)]\\
&\quad\times\begin{cases}K_0(m_B\sqrt{x_3[x_B-(1-\eta)(1-z)]}b), & x_B\ge(1-\eta)(1-z),\\ \frac{i\pi}{2}H_0^{(1)}(m_B\sqrt{x_3[(1-\eta)(1-z)-x_B]}b), & x_B<(1-\eta)(1-z),\end{cases}\\
h_{1d}(x_B,z,x_3,b_B,b) &= [K_0(m_B\sqrt{x_3x_B}b_B)I_0(m_B\sqrt{x_3x_B}b)\theta(b_B-b)+(b\longleftrightarrow b_B)]\\
&\quad\times\begin{cases}K_0(m_B\sqrt{x_3[x_B-(1-\eta)z]}b), & x_B\ge(1-\eta)z,\\ \frac{i\pi}{2}H_0^{(1)}(m_B\sqrt{x_3[(1-\eta)z-x_B]}b), & x_B<(1-\eta)z,\end{cases}\\
h_{1e}(z,x_3,b,b_3) &= \left(\frac{i\pi}{2}\right)^2 H_0^{(1)}(m_B\sqrt{(1-x_3)[\eta+z(1-\eta)]}b)[H_0^{(1)}(m_B\sqrt{1-x_3}b)\\
&\quad\times J_0(m_B\sqrt{1-x_3}b_3)\theta(b-b_3)+(b_3\longleftrightarrow b)]S_t(x_3),
\end{aligned}
$$

$$
\begin{aligned}
h_{1f}(z,x_3,b,b_3) &= \frac{i\pi}{2}H_0^{(1)}(m_B\sqrt{(1-x_3)[\eta+z(1-\eta)]}b_3)S_t(z)\\
&\times\begin{cases}
[\theta(b_3-b)K_0(m_B\sqrt{r_c^2-[\eta+(1-\eta)z]}b_3)\\
\times I_0(m_B\sqrt{r_c^2-[\eta+(1-\eta)z]}b)+(b\longleftrightarrow b_3)] & r_c^2\geq\eta+(1-\eta)z,\\
\frac{i\pi}{2}[\theta(b_3-b)H_0^{(1)}(m_B\sqrt{[\eta+(1-\eta)z]-r_c^2}b_3)\\
\times J_0(m_B\sqrt{[\eta+(1-\eta)z]-r_c^2}b)+(b\longleftrightarrow b_3)] & r_c^2<\eta+(1-\eta)z,
\end{cases}\\
h_{1g}(x_B,z,x_3,b_B,b) &= \frac{i\pi}{2}K_0(m_B\sqrt{1-x_3[(1-z)(1-\eta)-x_B]}b_B)[H_0^{(1)}(m_B\sqrt{(1-x_3)[\eta+z(1-\eta)]}b_B)\\
&\times J_0(m_B\sqrt{(1-x_3)[\eta+z(1-\eta)]}b)\theta(b_B-b)+(b\longleftrightarrow b_B)],\\
h_{1h}(x_B,z,x_3,b_B,b) &= \left[\frac{i\pi}{2}H_0^{(1)}(m_B\sqrt{(1-x_3)[\eta+z(1-\eta)]}b_B)\right.\\
&\left.\times J_0(m_B\sqrt{(1-x_3)[\eta+z(1-\eta)]}b)\theta(b_B-b)+(b\longleftrightarrow b_B)\right]\\
&\times\begin{cases}
K_0(m_B\sqrt{(1-x_3)[x_B-\eta-z(1-\eta)]}b_B), & x_B\geq\eta+z(1-\eta),\\
\frac{i\pi}{2}H_0^{(1)}(m_B\sqrt{(1-x_3)[-x_B+\eta+z(1-\eta)]}b_B), & x_B<\eta+z(1-\eta),
\end{cases}\\
h_{2a}(x_B,z,b_B,b) &= K_0(m_B\sqrt{(1-\eta)zx_B}b_B)[K_0(m_B\sqrt{(1-\eta)z}b_B)\\
&\times I_0(m_B\sqrt{(1-\eta)z}b)\theta(b_B-b)+(b\longleftrightarrow b_B)]S_t(z),\\
h_{2b}(x_B,z,b_B,b) &= K_0(m_B\sqrt{(1-\eta)zx_B}b)[K_0(m_B\sqrt{(1-\eta)x_B}b)\\
&\times I_0(m_B\sqrt{(1-\eta)x_B}b_B)\theta(b-b_B)+(b\longleftrightarrow b_B)]S_t(x_B),\\
h_{2c}(x_B,z,x_3,b_B,b_3) &= [K_0(m_B\sqrt{(1-\eta)zx_B}b_B)I_0(m_B\sqrt{(1-\eta)zx_B}b_3)\theta(b_B-b_3)+(b_3\longleftrightarrow b_B)]\\
&\times\begin{cases}
K_0(m_B\sqrt{r_c^2-[\eta+(1-\eta)z](1-x_B-x_3)}b_3),\\
r_c^2\geq[\eta+(1-\eta)z](1-x_B-x_3),\\
\frac{i\pi}{2}H_0^{(1)}(m_B\sqrt{[\eta+(1-\eta)z](1-x_B-x_3)-r_c^2}b_3),\\
r_c^2<[\eta+(1-\eta)z](1-x_B-x_3),
\end{cases}\\
h_{2d}(x_B,z,x_3,b_B,b_3) &= [K_0(m_B\sqrt{(1-\eta)zx_B}b_B)I_0(m_B\sqrt{(1-\eta)zx_B}b_3)\theta(b_B-b_3)+(b_3\longleftrightarrow b_B)]\\
&\times\begin{cases}
K_0(m_B\sqrt{(1-\eta)(x_B-x_3)z}b_3), & x_B\geq x_3,\\
\frac{i\pi}{2}H_0^{(1)}(m_B\sqrt{(1-\eta)(x_3-x_B)z}b_3), & x_B<x_3,
\end{cases}\\
h'_{2c}(x_B,z,x_3,b_B,b_3) &= [K_0(m_B\sqrt{(1-\eta)zx_B}b_B)I_0(m_B\sqrt{(1-\eta)zx_B}b)\theta(b_B-b_3)+(b_3\longleftrightarrow b_B)]\\
&\times\begin{cases}
K_0(m_B\sqrt{[\eta+(1-\eta)z](x_B+x_3-1)}b_3), & x_B+x_3\geq 1,\\
\frac{i\pi}{2}H_0^{(1)}(m_B\sqrt{[\eta+(1-\eta)z](1-x_B-x_3)}b_3), & x_B+x_3<1,
\end{cases}\\
h'_{2d}(x_B,z,x_3,b_B,b_3) &= [K_0(m_B\sqrt{(1-\eta)zx_B}b_B)I_0(m_B\sqrt{(1-\eta)zx_B}b_3)\theta(b_B-b_3)+(b_3\longleftrightarrow b_B)]\\
&\times\begin{cases}
K_0(m_B\sqrt{r_c^2+(1-\eta)(x_B-x_3)z}b_3), & r_c^2\geq(1-\eta)(x_3-x_B)z,\\
\frac{i\pi}{2}H_0^{(1)}(m_B\sqrt{(1-\eta)(x_3-x_B)z-r_c^2}b_3), & r_c^2<(1-\eta)(x_3-x_B)z,
\end{cases}\\
h_{2e}(z,x_3,b,b_3) &= \left(\frac{i\pi}{2}\right)^2H_0^{(1)}(m_B\sqrt{(1-\eta)(1-z)x_3}b_3)[\theta(b_3-b)H_0^{(1)}\left(m_B\sqrt{1-(1-\eta)z-r_c^2}b_3\right)\\
&\times J_0\left(m_B\sqrt{1-(1-\eta)z-r_c^2}b\right)+(b_3\longleftrightarrow b)]S_t(z),
\end{aligned}
$$

$$h_{2f}(z,x_3,b,b_3) = \left(\frac{i\pi}{2}\right)^2 H_0^{(1)}(m_B\sqrt{(1-\eta)(1-z)x_3}b)[H_0^{(1)}(m_B\sqrt{x_3(1-\eta)}b)$$
$$\times J_0(m_B\sqrt{x_3(1-\eta)}b_3)\theta(b-b_3) + (b\longleftrightarrow b_3)]S_t(x_3),$$
$$h_{2g}(x_B,z,x_3,b_B,b) = \frac{i\pi}{2}K_0(m_B\sqrt{1-(1-x_B-x_3)[\eta+z(1-\eta)]}b_B)[H_0^{(1)}(m_B\sqrt{(1-\eta)(1-z)x_3}b_B)$$
$$\times J_0(m_B\sqrt{(1-\eta)(1-z)x_3}b)\theta(b_B-b) + (b\longleftrightarrow b_B)],$$
$$h_{2h}(x_B,z,x_3,b_B,b) = \left[\frac{i\pi}{2}H_0^{(1)}(m_B\sqrt{(1-\eta)(1-z)x_3}b_B)J_0(m_B\sqrt{(1-\eta)(1-z)x_3}b)\theta(b_B-b) + (b\longleftrightarrow b_B)\right]$$
$$\times\begin{cases} K_0(m_B\sqrt{(1-\eta)(1-z)(x_B-x_3)}b_B), & x_B\geq x_3,\\ \frac{i\pi}{2}H_0^{(1)}(m_B\sqrt{(1-\eta)(1-z)(x_3-x_B)}b_B), & x_B<x_3,\end{cases}$$

where K_0, I_0, and $H_0 = J_0 + iY_0$ are Bessel functions. The function $S_t(x)$ can be parametrized as

$$S_t(x) = \frac{2^{1+2c}\Gamma(3/2+c)}{\sqrt{\pi}\Gamma(1+c)}[x(1-x)]^c,$$

with $c = 0.4$ for numerical calculation [69,70].

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