Strong decays of observed Λ_c baryons in the $^3P_0$ model

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All excited Λ_c baryon candidates are systematically studied in a $^3P_0$ strong decay model. Possible Okubo-Zweig-Iizuka-allowed strong decay channels of $\Lambda_c(2595)^+$, $\Lambda_c(2625)^+$, $\Lambda_c(2765)^+$ ($\Sigma_c(2765)^+$), $\Lambda_c(2860)^+$, $\Lambda_c(2880)^+$, and $\Lambda_c(2940)^+$ are given. The strong decay widths and some important branching ratios of these states are computed, and possible assignments of these Λ_c baryons are given. 1) $\Lambda_c(2595)^+$ and $\Lambda_c(2625)^+$ are possibly the $1P$-wave charmed baryons $\Lambda_{c1}(\frac{1}{2}^-)$ and $\Lambda_{c1}(\frac{3}{2}^-)$, respectively. 2) $\Lambda_c(2765)^+$ [$\Sigma_c(2765)^+$] is not the $1P$-wave Λ_c, but it could be the $2S$-wave or $1D$-wave charmed baryon. The available experimental data is not sufficient to identify this state. 3) $\Lambda_c(2860)^+$ is not the $2S$-wave charmed baryon, it may be the P-wave $\tilde{\Lambda}_{c2}(\frac{3}{2}^-)$ or $\tilde{\Lambda}_{c2}(\frac{5}{2}^-)$, it could also be the D-wave $\check{\Lambda}^2_{c1}(\frac{1}{2}^+)$ or $\check{\Lambda}^2_{c1}(\frac{3}{2}^+)$. If the hypothesis that $\Lambda_c(2860)^+$ has $J^P=\frac{3}{2}^+$ is true, $\Lambda_c(2860)^+$ is possibly the D-wave $\check{\Lambda}^2_{c1}(\frac{3}{2}^+)$ which has a predicted branching ratio $R=\Gamma(\Sigma_c(2520)\pi)/\Gamma(\Sigma_c(2455)\pi)=2.8-3.0$. 4) $\Lambda_c(2880)^+$ is not a $1P$-wave or $2S$-wave charmed baryon, it may be a D-wave $\check{\Lambda}^2_{c3}(\frac{5}{2}^+)$ with $\Gamma_{\mathrm{total}}=0.1$–$1.3$ MeV. The predicted branching ratio $R=\Gamma(\Sigma_c(2520)\pi)/\Gamma(\Sigma_c(2455)\pi)=0.35$–$0.36$, which is consistent with experimental data. 5) $\Lambda_c(2940)^+$ is the P-wave $\tilde{\Lambda}_{c2}(\frac{3}{2}^-)$ or $\tilde{\Lambda}_{c2}(\frac{5}{2}^-)$, it is also possibly the D-wave $\check{\Lambda}^2_{c3}(\frac{5}{2}^+)$ or $\check{\Lambda}^2_{c3}(\frac{7}{2}^+)$. It is possible to distinguish the two assignments in P-wave or D-wave excitations through the measurement of $R=\Gamma(\Sigma_c(2520)\pi)/\Gamma(\Sigma_c(2455)\pi)$.

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I. INTRODUCTION

In the past years, in addition to established ground states, more and more highly excited charmed baryons have been observed by the Belle, *BABAR*, CLEO, and LHCb collaborations [1]. Λ_c baryons have two light u and d quarks and one heavy c quark. The two light quarks couple with isospin zero. The heavy-quark symmetry works approximately in Λ_c baryons, and the light quarks in Λ_c baryons may correlate and make a diquark. The Λ_c states provide an excellent window to explore the baryon structure and quark dynamics in baryons.

So far, Λ_c^+, $\Lambda_c(2595)^+$, $\Lambda_c(2625)^+$, $\Lambda_c(2765)^+$ [or $\Sigma_c(2765)^+$], $\Lambda_c(2860)^+$, $\Lambda_c(2880)^+$, $\Lambda_c(2940)^+$ have been listed in the Review of Particle Physics [1]. The masses, total decay widths, and possible decay channels of these Λ_c states are presented in Table I. The spins and parities of these Λ_c states have not been measured by experiments. In order to identify these states, it is important to determine their J^P quantum numbers and to learn their internal dynamics in every model.

Heavy baryons have been studied in many models, which can be found in some reviews [2–7] and references therein. Many tentative J^P assignments to these Λ_c states have been made in many models [3,8–32]. In addition to normal charmed baryon interpretations [3,8–22], there are also coupled-channel effect interpretations [23,24] and molecular state interpretations of these Λ_c states [25–32]. In Table II, some possible J^P assignments of Λ_c states within the charmed baryon interpretations are presented.

For low-lying Λ_c states, $\Lambda_c(2286)^+$ is, without any doubt, the ground $1S$-wave charmed baryon with $J^P=\frac{1}{2}^+$. $\Lambda_c(2595)^+$ and $\Lambda_c(2625)^+$ are popularly believed to be the $1P$-wave charmed baryons with $J^P=\frac{1}{2}^-$ and $J^P=\frac{3}{2}^-$, respectively. However, the J^P assignments of the highly excited $\Lambda_c(2765)^+$ [$\Sigma_c(2765)^+$], $\Lambda_c(2860)^+$, $\Lambda_c(2880)^+$, and $\Lambda_c(2940)^+$ states differ throughout the literature, although the $J^P=\frac{3}{2}^+$ hypothesis is preferred for $\Lambda_c(2860)^+$, and $J^P=\frac{5}{2}^+$ was determined for the $\Lambda_c(2880)^+$ state by the LHCb Collaboration [33]. Furthermore, it is not yet clear whether $\Lambda_c(2765)^+$ [$\Sigma_c(2765)^+$] is an excited Λ_c or Σ_c.

As is known, a study of the strong decays of Λ_c baryons is an important way to determine their J^P quantum numbers. The $^3P_0$ model was proposed as a phenomenological method

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TABLE I. Masses, decay widths (MeV), and possible strong decay channels of Λ_c states [1].

States	J^P	Mass	Width	Decay channels (experiment)	Decay channels in $^3P_0$ model
Λ_c^+	$\frac{1}{2}^+$	2286.46 ± 0.14	/	Weak	/
$\Lambda_c(2595)^+$	$\frac{1}{2}^-$	2592.25 ± 0.28	$2.59 \pm 0.30 \pm 0.47$	$\Sigma_c^{++,0}\pi^{-,+}$	$\Sigma_c^{++,0}\pi^{-,+}, \Sigma_c^+\pi^0$
$\Lambda_c(2625)^+$	$\frac{3}{2}^-$	2628.11 ± 0.19	<0.97	$\Sigma_c^{++,0}\pi^{-,+}$	$\Sigma_c^{++,0}\pi^{-,+}, \Sigma_c^+\pi^0$
$\Lambda_c(2765)^+$	??	2766.6 ± 2.4	50	/	$\Sigma_c^{(*)++,0,+}\pi^{-,+,0}$
$\Lambda_c(2860)^+$	$\frac{3}{2}^+$	$2856.1^{+2.0}_{-1.7} \pm 0.5^{+1.1}_{-5.6}$	$67.6^{+10.1}_{-8.1} \pm 1.4^{+5.9}_{-20.0}$	D^0p	$\Sigma_c^{(*)++,0,+}\pi^{-,+,0}, D^0p, D^+N$
$\Lambda_c(2880)^+$	$\frac{5}{2}^+$	2881.63 ± 0.24	$5.6^{+0.8}_{-0.6}$	$\Sigma_c^{(*)++,0}\pi^{-,+}, D^0p$	$\Sigma_c^{(*)++,0}\pi^{-,+}, D^0p, \Sigma_c^{(*)+}\pi^0, D^+N$
$\Lambda_c(2940)^+$	??	$2939.6^{+1.3}_{-1.5}$	20^{+6}_{-5}	$\Sigma_c^{++,0}\pi^{-,+}, D^0p$	$\Sigma_c^{(*)++,0}\pi^{-,+}, D^0p, \Sigma_c^{(*)+}\pi^0, D^+N$

TABLE II. Some possible J^P assignments of Λ_c.

Resonances	Ref. [8]	Ref. [9]	Refs. [10–13]	Ref. [14]	Ref. [3]	Refs. [16–18]	Refs. [19,20]	Ref. [21]
Λ_c	$\frac{1}{2}^+$	$\frac{1}{2}^+$	$\frac{1}{2}^+$	$\frac{1}{2}^+$	$\frac{1}{2}^+$	$\frac{1}{2}^+$	$\frac{1}{2}^+$	$\frac{1}{2}^+$
$\Lambda_c(2595)$	$\frac{1}{2}^-$	$\frac{1}{2}^-$	$\frac{1}{2}^-$	$\frac{1}{2}^-$	$\frac{1}{2}^-$	$\frac{1}{2}^-$	$\frac{1}{2}^-$	$\frac{1}{2}^-$
$\Lambda_c(2625)$	$\frac{3}{2}^-$	$\frac{3}{2}^-$	$\frac{3}{2}^-$	$\frac{3}{2}^-$	$\frac{3}{2}^-$	$\frac{3}{2}^-$	$\frac{3}{2}^-$	$\frac{3}{2}^-$
$\Lambda_c(2765)$ ($\Sigma(2765)$)	$\frac{1}{2}^{+*}$	$\frac{1}{2}^{+*}$	...	$\frac{1}{2}^{+*}$	...	$\frac{1}{2}^{+*}$	...	...
$\Lambda_c(2860)$	...	...	...	...	...	$\frac{3}{2}^+$	...	...
$\Lambda_c(2880)$	...	$\frac{5}{2}^+$	$\frac{5}{2}^+$	$\frac{5}{2}^+$	...	$\frac{5}{2}^+$	$\frac{3}{2}^+$	$\frac{5}{2}^+$
$\Lambda_c(2940)$	...	$(\frac{3}{2}^+, \frac{5}{2}^-)$	...	$\frac{1}{2}^-$	...	...	$\frac{5}{2}^+$	$\frac{1}{2}^-, \frac{3}{2}^\pm, \frac{5}{2}^-$

to compute the Okubo-Zweig-Iizuka (OZI)-allowed hadronic decay widths of hadrons [34–37]. There have also been attempts to connect the phenomenological $^3P_0$ model and QCD [38–40]. The $^3P_0$ model has been employed to study the strong decays of Λ_c baryons [10,41–44]. In addition to the computation of strong decay widths, the dynamics and structure of the Λ_c baryons have also been explored in these references. However, these studies only analyzed one Λ_c baryon or a few observed Λ_c baryons. The Λ_c baryons have not been systematically analyzed in the $^3P_0$ model.

In this work, all of the observed Λ_c states except for $\Lambda_c(2286)^+$ will be systematically examined as the $1P$-wave, $1D$-wave, or $2S$-wave Λ_c baryons from their strong decay properties in the $^3P_0$ model. We pay particular attention to their internal structure, especially the ρ-mode and λ-mode excitations.

The paper is organized as follows. In Sec. II the $^3P_0$ model is briefly introduced, and some notations of heavy baryons and related parameters are indicated. We present our numerical results and analyses in Sec. III. In the last section, we give our conclusions and discussions.

II. THE $^3P_0$ MODEL, SOME NOTATIONS, AND PARAMETERS

The $^3P_0$ model is also known as a quark-pair creation model. It was first proposed by Micu [34] and further developed by Yaouanc *et al.* [35–37]. The basic idea of this model is that a quark-antiquark pair $q\bar{q}$ is created from the QCD vacuum with quantum numbers $J^{PC} = 0^{++}$, and then the created quark and antiquark recombine with the quarks from the initial hadron A to form two daughter hadrons B and C [34]. The decays follow the OZI rule. For baryon decays, one quark of the initial baryon regroups with the created antiquark to form a meson, and the other two quarks regroup with the created quark to form a daughter baryon. There are three recombination processes,

$$A(q_1q_2q_3) + P(q_4\bar{q}_5) \to B(q_1q_4q_2) + C(q_3\bar{q}_5), \quad (1)$$

$$A(q_1q_2q_3) + P(q_4\bar{q}_5) \to B(q_1q_4q_3) + C(q_2\bar{q}_5), \quad (2)$$

$$A(q_1q_2q_3) + P(q_4\bar{q}_5) \to B(q_4q_2q_3) + C(q_1\bar{q}_5), \quad (3)$$

which are shown in Fig. 1, where each quark is numbered for a convenience. The two-body hadronic decay width Γ for a baryon A into B and C final states follows as in the $^3P_0$ model [37,41–45],

$$\Gamma = \pi^2 \frac{|\vec{p}|}{m_A^2} \sum_{JL} |\mathcal{M}^{JL}|^2 = \pi^2 \frac{|\vec{p}|}{m_A^2} \frac{1}{2J_A+1} \sum_{M_{J_A}M_{J_B}M_{J_C}} |\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}|^2. \quad (4)$$

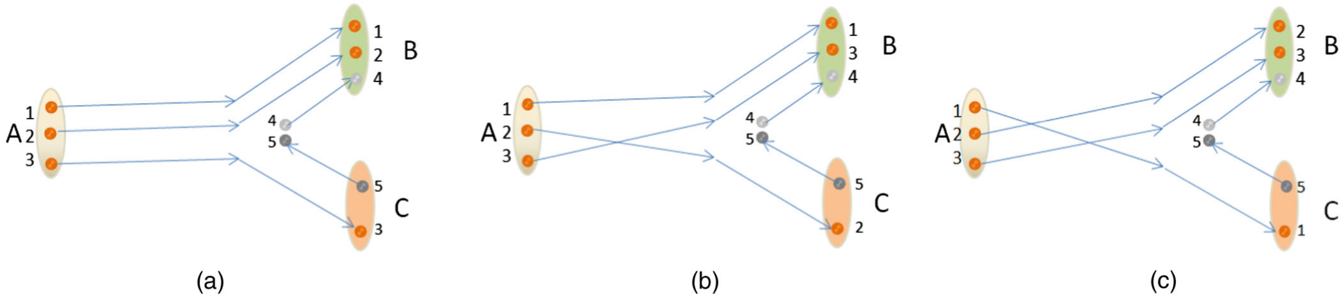

FIG. 1. Baryon decay process of $A \to B + C$ in the ${}^3P_0$ model. A is the initial baryon, and B and C are the final baryon and meson, respectively. (a) for Eq. (1), (b) for Eq. (2) and (c) for Eq. (3).

with $J = J_B + J_C$, $J_A = J_B + J_C + L$, and $M_{J_A} = M_{J_B} + M_{J_C}$. The partial-wave amplitude $\mathcal{M}^{JL}$ is related to the helicity amplitude $\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}$ via a Jacob-Wick formula [46]. In the equation, $\vec{p}$ is the momentum of the daughter baryon in A's center-of-mass frame,

$$|\vec{p}| = \frac{\sqrt{[m_A^2 - (m_B - m_C)^2][m_A^2 - (m_B + m_C)^2]}}{2m_A}, \tag{5}$$

where m_A and J_A are the mass and total angular momentum of the initial baryon A, respectively. m_B and m_C are the masses of the final hadrons. The helicity amplitude $\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}$ reads [10,41,42,44]

$$\begin{aligned}\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}} = & -F\gamma\sqrt{8E_AE_BE_C}\sum_{M_{\rho_A}}\sum_{M_{L_A}}\sum_{M_{\rho_B}}\sum_{M_{L_B}}\sum_{M_{S_1},M_{S_3},M_{S_4},m}\langle J_{l_A}M_{J_{l_A}}S_3M_{S_3}|J_AM_{J_A}\rangle\langle L_{\rho_A}M_{L_{\rho_A}}L_{\lambda_A}M_{L_{\lambda_A}}|L_AM_{L_A}\rangle \\ & \times\langle L_AM_{L_A}S_{12}M_{S_{12}}|J_{l_A}M_{J_{l_A}}\rangle\langle S_1M_{S_1}S_2M_{S_2}|S_{12}M_{S_{12}}\rangle\langle J_{l_B}M_{J_{l_B}}S_3M_{S_3}|J_BM_{J_B}\rangle\langle L_{\rho_B}M_{L_{\rho_B}}L_{\lambda_B}M_{L_{\lambda_B}}|L_BM_{L_B}\rangle \\ & \times\langle L_BM_{L_B}S_{14}M_{S_{14}}|J_{l_B}M_{J_{l_B}}\rangle\langle S_1M_{S_1}S_4M_{S_4}|S_{14}M_{S_{14}}\rangle\langle 1m;1-m|00\rangle\langle S_4M_{S_4}S_5M_{S_5}|1-m\rangle \\ & \times\langle L_CM_{L_C}S_CM_{S_C}|J_CM_{J_C}\rangle\langle S_2M_{S_2}S_5M_{S_5}|S_CM_{S_C}\rangle\langle\varphi_B^{1,4,3}\varphi_C^{2,5}|\varphi_A^{1,2,3}\varphi_0^{4,5}\rangle\times I^{M_{L_A},m}_{M_{L_B},M_{L_C}}(\vec{p}).\end{aligned} \tag{6}$$

Both the conservation of the total angular momentum of hadron and the conservation of the angular momentum of the light quarks are indicated explicitly in the equation. The factor $F = 2$ when both of the quarks in C have isospin $\frac{1}{2}$, and $F = 1$ when one of the two quarks in C has isospin 0.

In the last equation, the matrix $\langle\varphi_B^{1,4,3}\varphi_C^{2,5}|\varphi_A^{1,2,3}\varphi_0^{4,5}\rangle$ of the flavor wave functions φ_i $(i = A, B, C, 0)$ can also be presented in terms of Clebsch-Gordan coefficients of the isospin as follows [37,41,45]:

$$\langle\varphi_B^{1,4,3}\varphi_C^{2,5}|\varphi_A^{1,2,3}\varphi_0^{4,5}\rangle = \mathcal{F}^{(I_A;I_BI_C)}\langle I_BI_B^3I_CI_C^3|I_AI_A^3\rangle, \tag{7}$$

with

$$\mathcal{F}^{(I_A;I_BI_C)} = f\cdot(-1)^{I_{13}+I_C+I_A+I_2}\left[\frac{1}{2}(2I_C+1)(2I_B+1)\right]^{1/2} \times\begin{Bmatrix} I_{13} & I_B & I_4 \\ I_C & I_2 & I_A\end{Bmatrix}, \tag{8}$$

where $f = (\frac{2}{3})^{1/2}$ for a created $u\bar{u}$ or $d\bar{d}$ quark pair, and $f = -(\frac{1}{3})^{1/2}$ for a created $s\bar{s}$ quark pair. I_A, I_B, and I_C represent the isospins of the initial baryon, the final baryon, and the final meson, respectively. I_{13}, I_2, and I_4 denote the isospins of the relevant quarks. For example, the flavor matrix elements for $\Lambda_c^+ \to \Sigma_c^{++,+,0}\pi^{-,0,+}$ and $\Lambda_c^+ \to D^+n/D^0p$ are $\sqrt{1/6}$ and $\sqrt{1/3}$, respectively.

The space integral follows as

$$\begin{aligned}I^{M_{L_A},m}_{M_{L_B},M_{L_C}}(\vec{p}) = & \int d\vec{p}_1d\vec{p}_2d\vec{p}_3d\vec{p}_4d\vec{p}_5 \\ & \times\delta^3(\vec{p}_1+\vec{p}_2+\vec{p}_3-\vec{p}_A)\delta^3(\vec{p}_4+\vec{p}_5) \\ & \times\delta^3(\vec{p}_1+\vec{p}_4+\vec{p}_3-\vec{p}_B)\delta^3(\vec{p}_2+\vec{p}_5-\vec{p}_C) \\ & \times\Psi_B^*(\vec{p}_1,\vec{p}_4,\vec{p}_3)\Psi_C^*(\vec{p}_2,\vec{p}_5) \\ & \times\Psi_A(\vec{p}_1,\vec{p}_2,\vec{p}_3)y_{1m}\left(\frac{\vec{p}_4-\vec{p}_5}{2}\right)\end{aligned} \tag{9}$$

with a simple harmonic oscillator wave function for the baryons [8,41,47],

$$\Psi(\vec{p}) = N\Psi_{n_\rho L_\rho M_{L_\rho}}(\vec{p}_\rho)\Psi_{n_\lambda L_\lambda M_{L_\lambda}}(\vec{p}_\lambda), \tag{10}$$

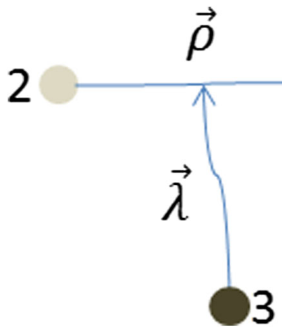

FIG. 2. Definitions of the Jacobi coordinates $\vec{\rho}$ and $\vec{\lambda}$. Quarks 1 and 2 are the light quarks, and quark 3 is the heavy (charmed or bottomed) quark.

where N represents a normalization coefficient of the total wave function. Explicitly,

$$\Psi_{nLM_L}(\vec{p}) = \frac{(-1)^n(-i)^L}{\beta^{3/2}}\sqrt{\frac{2n!}{\Gamma(n+L+\frac{3}{2})}}\left(\frac{\vec{p}}{\beta}\right)^L \exp\left(-\frac{\vec{p}^2}{2\beta^2}\right) \times L_n^{L+1/2}\left(\frac{\vec{p}^2}{\beta^2}\right) Y_{LM_L}(\Omega_p), \quad (11)$$

where $L_n^{L+1/2}(\frac{\vec{p}^2}{\beta^2})$ denotes the Laguerre polynomial function, and $Y_{LM_L}(\Omega_p)$ is a spherical harmonic function. The relation between the solid harmonic polynomial $y_{LM}(\vec{p})$ and $Y_{LM_L}(\Omega_{\vec{p}})$ is $y_{LM}(\vec{p}) = |\vec{p}|^L Y_{LM_L}(\Omega_p)$.

In order to describe three-body systems, a center-of-mass motion and a two-body system of internal relative motion in the Jacobi coordinates [48] are usually employed. As displayed in Fig. 2, $\vec{\rho}$ is the relative coordinate between the two light quarks (quarks 1 and 2), and $\vec{\lambda}$ is the relative coordinate between the center of mass of the two light quarks and the charmed quark.

In the quark model with heavy-quark symmetry [9,10,12,13,21,43–45], there is one 1S-wave Λ_c, seven P-wave Λ_c, two 2S-wave Λ_c, and 17 D-wave Λ_c states. The internal angular momentum of the 1S-wave, 1P-wave, and 2S-wave Λ_c states are presented in Table III, where $\tilde{\Lambda}^{\rho'}_{c0}(\frac{1}{2}^+)$ and $\tilde{\Lambda}^{\lambda'}_{c0}(\frac{1}{2}^+)$ denote the radial excitation of a ρ mode and a λ mode, respectively. The internal angular momenta of the 1D-wave Λ_c state are presented in Table IV.

In these tables, L_ρ denotes the orbital angular momentum between the two light quarks, L_λ denotes the orbital angular momentum between the charm quark and the two-light-quark system, and S_ρ denotes the total spin of the two light quarks. L is the total orbital angular momentum of L_ρ and L_λ ($L = L_\rho + L_\lambda$), and J_l is the total angular momentum of L and S_ρ ($J_l = L + S_\rho$). J is the total angular momentum of the baryons ($J = J_l + \frac{1}{2}$). In $\tilde{\Lambda}^L_{cJ_l}$ ($\tilde{\Sigma}^L_{cJ_l}$), a superscript L denotes the total angular orbital momentum, a tilde indicates $L_\rho = 1$, and no tilde indicates $L_\rho = 0$. More details about the notations can be found in Refs. [10,43,44,47]

In the $^3P_0$ model, the $q\bar{q}$ quark pair created from the vacuum may be $u\bar{u}$, $d\bar{d}$, or $s\bar{s}$. So far, there is no sign of $s\bar{s}$ quark-pair creation (which is forbidden by the mass threshold of final states) in the observed strong decay channels of Λ_c states. In addition to the masses, the decay widths, experimentally observed strong decay channels, and theoretically predicted strong decay channels of all of the Λ_c states are also given in Table I. The masses of relevant mesons and baryons involved in our calculation are presented in Table V [1].

TABLE III. Quantum numbers of 1S-wave, 1P-wave, and 2S-wave excited Λ_c states.

N	Assignment	n_ρ	n_λ	J	J_l	L_ρ	L_λ	L	S_ρ
1	$\Lambda_{c0}(\frac{1}{2}^+)$	0	0	$\frac{1}{2}$	0	0	0	0	0
2	$\Lambda_{c1}(\frac{1}{2}^-)$	0	0	$\frac{1}{2}$	1	0	1	1	0
3	$\Lambda_{c1}(\frac{3}{2}^-)$	0	0	$\frac{3}{2}$	1	0	1	1	0
4	$\tilde{\Lambda}_{c0}(\frac{1}{2}^-)$	0	0	$\frac{1}{2}$	0	1	0	1	1
5	$\tilde{\Lambda}_{c1}(\frac{1}{2}^-)$	0	0	$\frac{1}{2}$	1	1	0	1	1
6	$\tilde{\Lambda}_{c1}(\frac{3}{2}^-)$	0	0	$\frac{3}{2}$	1	1	0	1	1
7	$\tilde{\Lambda}_{c2}(\frac{3}{2}^-)$	0	0	$\frac{3}{2}$	2	1	0	1	1
8	$\tilde{\Lambda}^1_{c2}(\frac{5}{2}^-)$	0	0	$\frac{5}{2}$	2	1	0	1	1
9	$\tilde{\Lambda}^{\rho'}_{c0}(\frac{1}{2}^+)$	1	0	$\frac{1}{2}$	0	0	0	0	0
10	$\tilde{\Lambda}^{\lambda'}_{c0}(\frac{1}{2}^+)$	0	1	$\frac{1}{2}$	0	0	0	0	0

TABLE IV. Quantum numbers of the 1D-wave excited Λ_c state.

N	Assignment	n_ρ	n_λ	J	J_l	L_ρ	L_λ	L	S_ρ
1	$\Lambda_{c2}(\frac{3}{2}^+)$	0	0	$\frac{3}{2}$	2	0	2	2	0
2	$\Lambda_{c2}(\frac{5}{2}^+)$	0	0	$\frac{5}{2}$	2	0	2	2	0
3	$\hat{\Lambda}_{c2}(\frac{3}{2}^+)$	0	0	$\frac{3}{2}$	2	2	0	2	0
4	$\hat{\Lambda}_{c2}(\frac{5}{2}^+)$	0	0	$\frac{5}{2}$	2	2	0	2	0
5	$\check{\Lambda}^1_{c0}(\frac{1}{2}^+)$	0	0	$\frac{1}{2}$	0	1	1	1	1
6	$\check{\Lambda}^1_{c1}(\frac{1}{2}^+)$	0	0	$\frac{1}{2}$	1	1	1	1	1
7	$\check{\Lambda}^1_{c1}(\frac{3}{2}^+)$	0	0	$\frac{3}{2}$	1	1	1	1	1
8	$\check{\Lambda}^1_{c2}(\frac{3}{2}^+)$	0	0	$\frac{3}{2}$	2	1	1	1	1
9	$\check{\Lambda}^1_{c2}(\frac{5}{2}^+)$	0	0	$\frac{5}{2}$	2	1	1	1	1
10	$\check{\Lambda}^0_{c1}(\frac{1}{2}^+)$	0	0	$\frac{1}{2}$	1	1	1	0	1
11	$\check{\Lambda}^0_{c1}(\frac{3}{2}^+)$	0	0	$\frac{3}{2}$	1	1	1	0	1
12	$\check{\Lambda}^2_{c1}(\frac{1}{2}^+)$	0	0	$\frac{1}{2}$	1	1	1	2	1
13	$\check{\Lambda}^2_{c1}(\frac{3}{2}^+)$	0	0	$\frac{3}{2}$	1	1	1	2	1
14	$\check{\Lambda}^2_{c2}(\frac{3}{2}^+)$	0	0	$\frac{3}{2}$	2	1	1	2	1
15	$\check{\Lambda}^2_{c2}(\frac{5}{2}^+)$	0	0	$\frac{5}{2}$	2	1	1	2	1
16	$\check{\Lambda}^2_{c3}(\frac{5}{2}^+)$	0	0	$\frac{5}{2}$	3	1	1	2	1
17	$\check{\Lambda}^2_{c3}(\frac{7}{2}^+)$	0	0	$\frac{7}{2}$	3	1	1	2	1

TABLE V. Masses of mesons and baryons involved in the decays [1].

State	Mass (MeV)	State	Mass (MeV)
$\pi^\pm$	139.570	$\Sigma_c(2520)^{++}$	2518.41
π^0	134.977	$\Sigma_c(2520)^{+}$	2517.5
$K^\pm$	493.677	$\Sigma_c(2520)^{0}$	2518.48
K^0	497.611	$\Sigma_c(2455)^{++}$	2453.97
Λ_c^+	2286.46	$\Sigma_c(2455)^{+}$	2452.9
D^0	1864.84	$\Sigma_c(2455)^{0}$	2453.75
D^+	1869.59	$\Sigma_c(2765)^{++}$	2766.6
$\Sigma_c(2765)^+$	2766.6	$\Sigma_c(2765)^{0}$	2766.6
$\Sigma_c(2800)^+$	2792	...	...

The parameters are chosen as follows. The dimensionless pair-creation strength γ is usually fitted with different values in some references, which may result in large changes to the numerical decay widths. However, the variation of γ does not change the branching fraction ratios or relevant ratios. In this paper $\gamma = 13.4$ is employed, as in Refs. [10,41–44]. $\beta_{\lambda,\rho} = 600$ MeV is used for the $1S$-wave baryon wave functions, $\beta_{\lambda,\rho} = 300$–$500$ MeV is used for the P-wave baryon wave functions, and $\beta_{\lambda,\rho} = 200$–$400$ MeV is used for the $2S$-wave and D-wave baryon wave functions. These $\beta_{\lambda,\rho}$ values are consistent with those in Refs. [10,41,49–51]. $\beta = 400$ MeV is used for the harmonic oscillator wave functions of π/K mesons and $\beta = 600$ MeV is used for the D meson [10,41,49–51].

III. STRONG DECAYS OF Λ_c STATES

A. $\Lambda_c(2595)$ and $\Lambda_c(2625)$

$\Lambda_c(2595)^+$ and $\Lambda_c(2625)^+$ were first discovered by the ARGUS Collaboration at the e^+e^- storage ring DORIS II at DESY [52], and subsequently confirmed by the E687 [53] and CLEO [54] collaborations.

$\Lambda_c^+\pi\pi$ and its submode $\Sigma_c(2455)\pi$ are the only allowed strong decays of $\Lambda_c(2595)^+$. $\Lambda_c^+\pi\pi$ results from a two-step process, $\Lambda_c(2595) \to \Sigma_c(2455)\pi$ with $\Sigma_c(2455) \to \Lambda_c\pi$, and a direct $\Lambda_c^+\pi\pi$ three-body decay with a fraction of about $18 \pm 10\%$. The branching fractions $\Gamma(\Lambda_c(2595)^+ \to \Sigma_c^{++}\pi^-)/\Gamma_{\rm total} = 24 \pm 7\%$ and $\Gamma(\Lambda_c(2595)^+ \to \Sigma_c^0\pi^+) = 24 \pm 7\%$ [1].

$\Lambda_c^+\pi\pi$ and its submode $\Sigma_c(2455)\pi$ are also the only allowed strong decays of $\Lambda_c(2625)^+$. In contrast to $\Lambda_c(2595)^+$, the branching fraction of the direct three-body decay mode $\Lambda_c^+\pi\pi$ of $\Lambda_c(2625)^+$ is large, while the branching fraction $\Gamma(\Lambda_c(2625)^+ \to \Sigma_c^{++}\pi^-)/\Gamma_{\rm total}$ or $\Gamma(\Lambda_c(2625)^+ \to \Sigma_c^0\pi^+)/\Gamma_{\rm total}$ is less than 5% [1], which means that the decay width $\Gamma(\Lambda_c(2625)^+ \to \Sigma_c^{++}\pi^-)$ or $\Gamma(\Lambda_c(2625)^+ \to \Sigma_c^0\pi^+)$ is less than 0.05 MeV.

$\Lambda_c(2595)^+$ and $\Lambda_c(2625)^+$ are believed to be the low-lying P-wave Λ_c states, and form a doublet $\Lambda_{c1}(\frac{1}{2}^-, \frac{3}{2}^-)$ [9,11,14]. Their J^P are assumed to be $\frac{1}{2}^-$ and $\frac{3}{2}^-$, respectively [1]. In our analyses, all of the hypotheses in which $\Lambda_c(2595)^+$ and $\Lambda_c(2625)^+$ are the low-lying $1P$-wave, $2S$-wave, and $1D$-wave charmed baryons are examined. In Table VI, the numerical results of the decay widths of $\Lambda_c(2595)^+$ as the $1P$-wave and $2S$-wave states are given. Similar numerical results for $\Lambda_c(2625)^+$ are presented in Table VII. In Tables VIII and IX, the numerical results of the decay widths of $\Lambda_c(2595)^+$ and $\Lambda_c(2625)^+$ as D-wave charmed baryons are given, respectively. In these tables, some branching ratios are also given.

From Table VI, we can see that $\Lambda_c(2595)^+$ is not a $\Lambda_{c1}(\frac{3}{2}^-)$, $\tilde\Lambda_{c0}(\frac{1}{2}^-)$, $\tilde\Lambda_{c1}(\frac{3}{2}^-)$, $\tilde\Lambda_{c2}(\frac{3}{2}^-)$, or $\tilde\Lambda_{c2}(\frac{5}{2}^-)$ state for a vanishing total decay width (denoted by "0" in the table) or an approximately vanishing total decay width (denoted by "≈0" in the table). $\Lambda_c(2595)^+$ is not a $2S$-wave excitation, $\tilde\Lambda_{c0}(\frac{1}{2}^+)$ or $\tilde\Lambda_{c0}(\frac{1}{2}^+)$, for a much lower predicted branching fraction $B = \Gamma(\Sigma_c^{++}\pi^-)/\Gamma_{\rm total}$. The predicted total decay width is much smaller in comparison with experimental data.

From Table VIII, we can see that neither the branching ratios nor the total decay widths are consistent with experimental measurements. Therefore, $\Lambda_c(2595)^+$ is not

TABLE VI. Decay widths (MeV) of $\Lambda_c(2595)^+$ as $1P$-wave and $2S$-wave charmed baryons. $\mathcal{B} = \Gamma(\Sigma_c^{++}\pi^-)/\Gamma_{\rm total}$.

N	$\Lambda_{cJ_l}(J^P)$	$\Sigma_c^{++}\pi^-$	$\Sigma_c^0\pi^+$	$\Sigma_c^+\pi^0$	$\Gamma_{\rm total}$	$\mathcal{B}$
1	$\Lambda_{c1}(\frac{1}{2}^-)$	2.10–3.70	2.22–3.93	4.21–7.46	8.53–15.09	24.52–24.62%
2	$\Lambda_{c1}(\frac{3}{2}^-)$	≈0	≈0	≈0	≈0	≈0
3	$\tilde\Lambda_{c0}(\frac{1}{2}^-)$	0	0	0	0	...
4	$\tilde\Lambda_{c1}(\frac{1}{2}^-)$	12.57–22.22	13.33–23.56	25.27–44.75	51.17–90.53	24.54–24.57%
5	$\tilde\Lambda_{c1}(\frac{3}{2}^-)$	≈0	≈0	≈0	≈0	≈0
6	$\tilde\Lambda_{c2}(\frac{3}{2}^-)$	≈0	≈0	≈0	≈0	≈0
7	$\tilde\Lambda_{c2}(\frac{5}{2}^-)$	≈0	≈0	≈0	≈0	≈0
8	$\tilde\Lambda_{c0}^{\rho'}(\frac{1}{2}^+)$	7.61×10^{-4}–2.33×10^{-3}	9.06×10^{-4}–2.77×10^{-3}	6.20×10^{-3}–1.91×10^{-2}	7.87×10^{-3}–2.42×10^{-2}	9.63–9.67%
9	$\tilde\Lambda_{c0}^{\lambda'}(\frac{1}{2}^+)$	1.58×10^{-3}–4.38×10^{-3}	1.88×10^{-3}–5.21×10^{-3}	1.30×10^{-2}–3.58×10^{-2}	1.65×10^{-2}–4.54×10^{-2}	9.58–9.65%

TABLE VII. Decay widths (MeV) of $\Lambda_c(2625)^+$ as $1P$-wave and $2S$-wave charmed baryons. $\mathcal{B} = \Gamma(\Sigma_c^{++}\pi^-)/\Gamma_{\text{total}}$.

N	$\Lambda_{cJ_l}(J^P)$	$\Sigma_c^{++}\pi^-$	$\Sigma_c^0\pi^+$	$\Sigma_c^+\pi^0$	Γ_{total}	$\mathcal{B}$
1	$\Lambda_{c1}(\frac{1}{2}^-)$	10.98–19.67	11.03–19.75	11.73–21.06	33.74–60.48	32.53–32.54%
2	$\Lambda_{c1}(\frac{3}{2}^-)$	$2.75\times10^{-3}-0.33\times10^{-2}$	$2.80\times10^{-3}-0.33\times10^{-2}$	$3.91\times10^{-3}-0.47\times10^{-2}$	$9.46\times10^{-3}-1.13\times10^{-2}$	29.07–29.20%
3	$\tilde{\Lambda}_{c0}(\frac{1}{2}^-)$	0	0	0	0	⋯
4	$\tilde{\Lambda}_{c1}(\frac{1}{2}^-)$	65.88–118.01	66.15–118.50	70.38–126.35	202–363	32.51–32.61%
5	$\tilde{\Lambda}_{c1}(\frac{3}{2}^-)$	$4.13\times10^{-3}-0.49\times10^{-2}$	$4.21\times10^{-3}-0.50\times10^{-2}$	$5.87\times10^{-3}-0.70\times10^{-2}$	$1.42\times10^{-2}-1.69\times10^{-2}$	28.99–29.06%
6	$\tilde{\Lambda}_{c2}(\frac{3}{2}^-)$	$7.43\times10^{-3}-0.89\times10^{-2}$	$7.57\times10^{-3}-0.90\times10^{-2}$	$1.06\times10^{-2}-0.13\times10^{-1}$	$2.56\times10^{-2}-3.09\times10^{-2}$	28.80–29.02%
7	$\tilde{\Lambda}^1_{c2}(\frac{5}{2}^-)$	$3.30\times10^{-3}-0.39\times10^{-2}$	$3.36\times10^{-3}-0.40\times10^{-2}$	$4.69\times10^{-3}-0.56\times10^{-2}$	$1.14\times10^{-2}-1.35\times10^{-2}$	28.89–28.94%
8	$\tilde{\Lambda}^{\rho'}_{c0}(\frac{1}{2}^+)$	$8.18\times10^{-2}-0.27$	$8.27\times10^{-2}-0.27$	0.10–0.33	0.26–0.87	31.03–31.46%
9	$\tilde{\Lambda}^{\lambda'}_{c0}(\frac{1}{2}^+)$	0.18–0.49	0.18–0.50	0.23–0.61	0.59–1.60	30.51–30.63%

a D-wave excitation of Λ_c. Accounting for the branching fractions $B = \Gamma(\Sigma_c^{(++)}\pi^{(-)})/\Gamma_{\text{total}}$, $\Lambda_c(2595)^+$ is possibly a $1P$-wave $\Lambda_{c1}(\frac{1}{2}^-)$ or $\tilde{\Lambda}_{c1}(\frac{1}{2}^-)$. Comparing the predicted decay widths with experimental data and taking into account the uncertainties from parameters, $\Lambda_c(2595)^+$ prefers a $1P$-wave $\Lambda_{c1}(\frac{1}{2}^-)$ rather than a $\tilde{\Lambda}_{c1}(\frac{1}{2}^-)$.

From Table VII, we can see that $\Lambda_c(2625)^+$ is not a $\Lambda_{c1}(\frac{1}{2}^-)$, $\tilde{\Lambda}_{c1}(\frac{1}{2}^-)$, or $\tilde{\Lambda}_{c0}(\frac{1}{2}^-)$ state for a large predicted decay width or a vanishing $\Sigma_c^{++}\pi^-$ mode. For $\Lambda_c(2625)^+$, $\Sigma_c(2455)\pi$ are the only two-body decay modes of this state, and the branching fraction of the direct three-body decay mode $\Lambda_c^+\pi\pi$ is large, so it is impossible to understand this state only from the branching fraction of these two-body strong decay modes. However, the predicted masses of $\tilde{\Lambda}_{c1}(\frac{3}{2}^-)$, $\tilde{\Lambda}_{c2}(\frac{3}{2}^-)$, $\tilde{\Lambda}_{c2}(\frac{5}{2}^-)$, the $2S$-wave excitations, and the $1D$-wave excitations are much higher than that of $\Lambda_c(2625)^+$ [14,18,21]. Accounting for this fact, $\Lambda_c(2625)^+$ is not one of these charmed baryons. In short, $\Lambda_c(2625)^+$ is possibly a P-wave $\Lambda_{c1}(\frac{3}{2}^-)$ charmed baryon.

In the given configurations of $\Lambda_c(2595)^+$ and $\Lambda_c(2625)^+$, there is a λ-mode excitation while there is not a ρ-mode excitation. The two light quarks inside couple with total spin $S_\rho = 0$. $\Lambda_c(2595)^+$ and $\Lambda_c(2625)^+$ make a doublet $\Lambda_{c1}(\frac{1}{2}^-, \frac{3}{2}^-)$.

TABLE VIII. Decay widths (MeV) of $\Lambda_c(2595)^+$ as D-wave excitations. The branching fraction ratio $\mathcal{B} = \Gamma(\Lambda_c(2595)^+ \to \Sigma_c^{++}\pi^-)/\Gamma_{\text{total}}$.

N	$\Lambda_{cJ_l}(J^P)$	$\Sigma_c^{++}\pi^-$	$\Sigma_c^0\pi^+$	$\Sigma_c^+\pi^0$	Γ_{total}	$\mathcal{B}$
1	$\Lambda_{c2}(\frac{3}{2}^+)$	2.96×10^{-5}–4.11×10^{-4}	3.52×10^{-5}–4.88×10^{-4}	2.42×10^{-4}–3.36×10^{-3}	3.07×10^{-4}–4.26×10^{-3}	9.64–9.65%
2	$\Lambda_{c2}(\frac{5}{2}^+)$	≈0	≈0	≈0	≈0	≈0
3	$\hat{\Lambda}_{c2}(\frac{3}{2}^+)$	2.66×10^{-4}–3.70×10^{-3}	3.17×10^{-4}–4.40×10^{-3}	2.17×10^{-3}–3.02×10^{-2}	2.75×10^{-3}–3.83×10^{-2}	9.66–9.67%
4	$\hat{\Lambda}_{c2}(\frac{5}{2}^+)$	≈0	≈0	≈0	≈0	≈0
5	$\check{\Lambda}^1_{c0}(\frac{1}{2}^+)$	0	0	0	0	⋯
6	$\check{\Lambda}^1_{c1}(\frac{1}{2}^+)$	0	0	0	0	⋯
7	$\check{\Lambda}^1_{c1}(\frac{3}{2}^+)$	≈0	≈0	≈0	≈0	≈0
8	$\check{\Lambda}^1_{c2}(\frac{3}{2}^+)$	≈0	≈0	≈0	≈0	≈0
9	$\check{\Lambda}^1_{c2}(\frac{5}{2}^+)$	≈0	≈0	≈0	≈0	≈0
10	$\check{\Lambda}^0_{c1}(\frac{1}{2}^+)$	6.41×10^{-4}–9.08×10^{-3}	7.62×10^{-4}–1.08×10^{-2}	5.23×10^{-3}–7.43×10^{-2}	6.63×10^{-3}–9.42×10^{-2}	9.64–9.67%
11	$\check{\Lambda}^0_{c1}(\frac{3}{2}^+)$	1.60×10^{-4}–2.27×10^{-3}	1.91×10^{-4}–2.70×10^{-3}	1.31×10^{-3}–1.86×10^{-2}	1.66×10^{-3}–2.36×10^{-2}	9.62–9.64%
12	$\check{\Lambda}^2_{c1}(\frac{1}{2}^+)$	1.18×10^{-4}–1.64×10^{-3}	1.41×10^{-4}–1.95×10^{-3}	9.67×10^{-4}–1.34×10^{-2}	1.23×10^{-3}–1.70×10^{-2}	9.59–9.65%
13	$\check{\Lambda}^2_{c1}(\frac{3}{2}^+)$	2.96×10^{-5}–4.11×10^{-4}	3.52×10^{-5}–4.88×10^{-4}	2.42×10^{-4}–3.36×10^{-3}	3.07×10^{-4}–4.26×10^{-3}	9.64–9.65%
14	$\check{\Lambda}^2_{c2}(\frac{3}{2}^+)$	2.66×10^{-4}–3.70×10^{-3}	3.17×10^{-4}–4.40×10^{-3}	2.17×10^{-3}–3.02×10^{-2}	2.75×10^{-3}–3.83×10^{-2}	9.66–9.67%
15	$\check{\Lambda}^2_{c2}(\frac{5}{2}^+)$	≈0	≈0	≈0	≈0	≈0
16	$\check{\Lambda}^2_{c3}(\frac{5}{2}^+)$	≈0	≈0	≈0	≈0	≈0
17	$\check{\Lambda}^2_{c3}(\frac{7}{2}^+)$	≈0	≈0	≈0	≈0	≈0

TABLE IX. Decay widths (MeV) of $\Lambda_c(2625)^+$ as D-wave excitations. $\mathcal{B}$ represents the ratio of branching fractions, $\Gamma(\Lambda_c(2625)^+ \to \Sigma_c^{++}\pi^-)/\Gamma_{\text{total}}$.

N	$\Lambda_{cJ_l}(J^P)$	$\Sigma_c^{++}\pi^-$	$\Sigma_c^0\pi^+$	$\Sigma_c^+\pi^0$	Γ_{total}	$\mathcal{B}$
1	$\Lambda_{c2}(\frac{3}{2}^+)$	3.26×10^{-3}–4.59×10^{-2}	3.30×10^{-3}–4.64×10^{-2}	4.01×10^{-3}–5.66×10^{-2}	1.06×10^{-2}–1.49×10^{-1}	30.75–30.83%
2	$\Lambda_{c2}(\frac{5}{2}^+)$	≈0	≈0	≈0	≈0	≈0
3	$\hat{\Lambda}_{c2}(\frac{3}{2}^+)$	2.94×10^{-2}–0.41	2.97×10^{-2}–0.42	3.61×10^{-2}–0.51	9.52×10^{-2}–1.34	30.60–30.88%
4	$\hat{\Lambda}_{c2}(\frac{5}{2}^+)$	≈0	≈0	≈0	≈0	≈0
5	$\check{\Lambda}^1_{c0}(\frac{1}{2}^+)$	0	0	0	0	…
6	$\check{\Lambda}^1_{c1}(\frac{1}{2}^+)$	0	0	0	0	…
7	$\check{\Lambda}^1_{c1}(\frac{3}{2}^+)$	≈0	≈0	≈0	≈0	≈0
8	$\check{\Lambda}^1_{c2}(\frac{3}{2}^+)$	≈0	≈0	≈0	≈0	≈0
9	$\check{\Lambda}^1_{c2}(\frac{5}{2}^+)$	≈0	≈0	≈0	≈0	≈0
10	$\check{\Lambda}^0_{c1}(\frac{1}{2}^+)$	7.08×10^{-2}–1.02	7.16×10^{-2}–1.03	8.70×10^{-2}–1.25	0.23–3.30	30.86–30.91%
11	$\check{\Lambda}^0_{c1}(\frac{3}{2}^+)$	1.77×10^{-2}–0.25	1.79×10^{-2}–0.26	2.18×10^{-2}–0.31	5.74×10^{-2}–0.82	30.49–30.84%
12	$\check{\Lambda}^2_{c1}(\frac{1}{2}^+)$	1.31×10^{-2}–0.18	1.32×10^{-2}–0.19	1.61×10^{-2}–0.23	4.24×10^{-2}–0.60	30.00–30.89%
13	$\check{\Lambda}^2_{c1}(\frac{3}{2}^+)$	3.26×10^{-3}–4.59×10^{-2}	3.30×10^{-3}–4.64×10^{-2}	4.01×10^{-3}–5.66×10^{-2}	10.57×10^{-3}–14.89×10^{-2}	30.83–30.84%
14	$\check{\Lambda}^2_{c2}(\frac{3}{2}^+)$	2.94×10^{-2}–40.41	2.97×10^{-2}–0.42	3.61×10^{-2}–0.51	9.52×10^{-2}–1.34	30.60–30.88%
15	$\check{\Lambda}^2_{c2}(\frac{5}{2}^+)$	≈0	≈0	≈0	≈0	≈0
16	$\check{\Lambda}^2_{c3}(\frac{5}{2}^+)$	≈0	≈0	≈0	≈0	≈0
17	$\check{\Lambda}^2_{c3}(\frac{7}{2}^+)$	≈0	≈0	≈0	≈0	≈0

B. $\Lambda_c(2765)$ (or $\Sigma_c(2765)$)

$\Lambda_c(2765)^+$ [or $\Sigma_c(2765)^+$] is a broad state first observed in $\Lambda_c^+\pi^-\pi^+$ channel by the CLEO Collaboration [55]. However, nothing is known about its J^P, nor whether it is a Λ_c or a Σ_c. $\Lambda_c(2765)^+$ [or $\Sigma_c(2765)^+$] was suggested as a first orbital excitation of Λ_c with $J^P=\frac{1}{2}^+$ [8], $J^P=\frac{1}{2}^-$ [19], or $J^P=\frac{3}{2}^+$ [3,56]. $\Lambda_c(2765)^+$ [or $\Sigma_c(2765)^+$] was suggested as a first orbital $1P$ excitation of Σ_c with $J^P=\frac{1}{2}^-$ [14] or $J^P=\frac{3}{2}^-$ [14,18,57]. $\Lambda_c(2765)^+$ [or $\Sigma_c(2765)^+$] was also suggested as a first radial $2S$ excitation of Λ_c with $J^P=\frac{1}{2}^+$ in a relativistic flux tube model [16] and a hypercentral constituent quark model [58].

In this subsection, the possible assignments of $\Lambda_c(2765)$ [or $\Sigma_c(2765)$] as the $1P$-wave, $2S$-wave, and $1D$-wave charmed baryon with isospin $I=0$ are examined. When $\Lambda_c(2765)$ [or $\Sigma_c(2765)$] is assigned in these configurations, the relevant hadronic decay widths are calculated in the $^3P_0$ model and are shown in Table X.

From Table X, accounting for the fact that $\Lambda_c(2595)^+$ and $\Lambda_c(2625)^+$ have been assigned as $\Lambda_{c1}(\frac{1}{2}^-)$ and $\Lambda_{c1}(\frac{3}{2}^-)$, respectively, we can see that $\Lambda_c(2765)^+$ [or $\Sigma_c(2765)$] is

TABLE X. Decay widths (MeV) of $\Lambda_c(2765)^+$ as $1P$-wave and $2S$-wave charmed baryons with isospin $I=0$. $\mathcal{R}=\Gamma(\Sigma_c(2520)^{++,0}\pi^{-,+})/\Gamma(\Sigma_c(2455)^{++,0}\pi^{-,+})$.

N	$\Lambda_{cJ_l}(J^P)$	$\Sigma_c^{++,0,+}(2455)\pi^{-,+,0}$	$\Sigma_c^{++,0,+}(2520)\pi^{-,+,0}$	Γ_{total}	$\mathcal{R}$
1	$\Lambda_{c1}(\frac{1}{2}^-)$	104.0–203.0	0.6–0.8	105.0–204.0	0.004–0.006
2	$\Lambda_{c1}(\frac{3}{2}^-)$	1.5–1.9	76.0–142.0	78.0–144.0	50.42–75.14
3	$\tilde{\Lambda}_{c0}(\frac{1}{2}^-)$	0	0	0	⋯
4	$\tilde{\Lambda}_{c1}(\frac{1}{2}^-)$	622.0–1218.0	0.9–1.1	623.0–1219.0	0.001–0.001
5	$\tilde{\Lambda}_{c1}(\frac{3}{2}^-)$	2.3–2.9	455.0–848.0	457.0–851.0	201.33–299
6	$\tilde{\Lambda}_{c2}(\frac{3}{2}^-)$	4.1–5.2	0.8–1.0	4.9–6.2	0.20–0.20
7	$\tilde{\Lambda}_{c2}(\frac{5}{2}^-)$	1.8–2.3	1.3–1.6	3.1–3.9	0.68–0.70
8	$\tilde{\Lambda}^{\rho'}_{c0}(\frac{1}{2}^+)$	4.5–21.9	4.0–15.6	8.6–37.5	0.71–0.88
9	$\tilde{\Lambda}^{\lambda'}_{c0}(\frac{1}{2}^+)$	16.2–38.8	11.2–28.5	27.4–67.2	0.68–0.73

not a P-wave Λ_c. Otherwise, $\Lambda_c(2765)^+$ [or $\Sigma_c(2765)$] has an extremely small or extremely large decay width. Except for the total decay width, the strong decay behaviors of the $2S$-wave $\tilde{\Lambda}^{\rho'}_{c0}(\frac{1}{2}^+)$ (ρ-mode excitation) and $\tilde{\Lambda}^{\lambda'}_{c0}(\frac{1}{2}^+)$ (λ-mode excitation) are very similar, and it is difficult to distinguish them through their strong decays. Given the theoretical and experimental uncertainties, $\Lambda_c(2765)^+$ [or $\Sigma_c(2765)$] may be a $2S$-wave $\tilde{\Lambda}^{\rho'}_{c0}(\frac{1}{2}^+)$ or $\tilde{\Lambda}^{\lambda'}_{c0}(\frac{1}{2}^+)$.

In Table XI we list the relevant hadronic decay widths when $\Lambda_c(2765)$ [or $\Sigma_c(2765)$] is assumed to be a $1D$-wave baryon with isospin $I = 0$. From this table, we can see that the predicted total decay widths are around the measured one in several configurations. That is to say, $\Lambda_c(2765)$ [or $\Sigma_c(2765)$] is possibly a D-wave charmed baryon. However, there are no accurate measurements of the total decay width of $\Lambda_c(2765)$ [or $\Sigma_c(2765)$], and there are no measurements of any of the branching fractions or branching ratios of its decay channel. In fact, it is not suitable to draw a confirmative conclusion in terms of such less information of $\Lambda_c(2765)$ [or $\Sigma_c(2765)$].

C. $\Lambda_c(2860)$, $\Lambda_c(2880)$, and $\Lambda_c(2940)$

The newly reported Λ_c baryon $\Lambda_c(2860)^+$ was first observed by the LHCb Collaboration in the D^0p channel [33]. The mass and width of $\Lambda_c(2860)^+$ were measured. The mass of $\Lambda_c(2860)^+$ is consistent with the predictions for an orbital D-wave Λ_c excitation with $J^P = \frac{3}{2}^+$ [12,18]. In particular, the quantum numbers of $\Lambda_c(2860)^+$ were found to be $J^P = \frac{3}{2}^+$, while the other quantum numbers were excluded with a significance of more than 6 standard deviations [33].

$\Lambda_c(2880)^+$ was first observed by the CLEO Collaboration in $\Lambda_c^+\pi^-\pi^+$ [55] and confirmed by the *BABAR* Collaboration in the D^0p channel [59]. From an analysis of angular distributions in $\Lambda_c(2880)^+ \to \Sigma_c(2455)^{0,++}\pi^{+,-}$ decays and the measured $R = \Gamma(\Sigma_c(2520)\pi)/\Gamma(\Sigma_c(2455)\pi) = 0.225 \pm 0.062 \pm 0.0255$, the preferred quantum numbers of the $\Lambda_c(2880)^+$ state were constrained to $J^P = \frac{5}{2}^+$ by the Belle Collaboration [60]. Recently, the LHCb Collaboration studied the spectrum of excited Λ_c states that decay into the D^0p channel and measured the mass and width of $\Lambda_c(2880)^+$. The preferred spin of $\Lambda_c(2880)^+$ was found to be $\frac{5}{2}$, and the spin assignments $\frac{1}{2}$ and $\frac{3}{2}$ were excluded [33].

$\Lambda_c(2940)^+$ was first observed by the *BABAR* Collaboration in the D^0p invariant mass distribution [59]. The spin parity of $\Lambda_c(2940)^+$ was constrained to $J^P = \frac{3}{2}^-$ by the LHCb Collaboration [33], although other solutions with spins of $\frac{1}{2}$ to $\frac{7}{2}$ cannot be excluded.

$\Lambda_c(2860)^+$ was interpreted as a D-wave charmed baryon with $J^P = \frac{3}{2}^+$ [17,22]. In particular, $\Lambda_c(2860)^+$ and $\Lambda_c(2880)^+$ are supposed to form a D-wave doublet $[\frac{3}{2}^+, \frac{5}{2}^+]$ [17].

$\Lambda_c(2880)^+$ was once assigned quantum numbers $J^P = \frac{1}{2}^-$ or $J^P = \frac{3}{2}^-$ [57], and it was also interpreted as a D-wave

TABLE XI. Decay widths (MeV) of $\Lambda_c(2765)^+$ as D-wave excitations. $\mathcal{R} = \Gamma(\Lambda_c(2765)^+ \to \Sigma_c(2520)^{++,0}\pi^{-,+})/\Gamma(\Lambda_c(2765)^+ \to \Sigma_c(2455)^{++,0}\pi))$.

N	$\Lambda_{cJ_l}(J^P)$	$\Sigma_c^{++,0,+}(2455)\pi^{-,+,0}$	$\Sigma_c^{++,0,+}(2520)\pi^{-,+,0}$	Γ_{total}	$\mathcal{R}$
1	$\Lambda_{c2}(\frac{3}{2}^+)$	2.2×10^{-1}–3.4	1.8×10^{-2}–2.6×10^{-1}	2.4×10^{-1}–3.6	0.07–0.08
2	$\Lambda_{c2}(\frac{5}{2}^+)$	1.4×10^{-3}–1.3×10^{-2}	1.1×10^{-1}–1.6	1.1×10^{-1}–1.6	78.14–121.00
3	$\hat{\Lambda}_{c2}(\frac{3}{2}^+)$	2.0–30.6	0.2–2.3	2.2–33.0	0.08–0.08
4	$\hat{\Lambda}_{c2}(\frac{5}{2}^+)$	1.2×10^{-2}–1.2×10^{-1}	1.0–13.9	1.0–14.0	77.98–119.89
5	$\check{\Lambda}^1_{c0}(\frac{1}{2}^+)$	0	0	0	⋯
6	$\check{\Lambda}^1_{c1}(\frac{1}{2}^+)$	0	0	0	⋯
7	$\check{\Lambda}^1_{c1}(\frac{3}{2}^+)$	≈0	≈0	≈0	≈0
8	$\check{\Lambda}^1_{c2}(\frac{3}{2}^+)$	≈0	≈0	≈0	≈0
9	$\check{\Lambda}^1_{c2}(\frac{5}{2}^+)$	≈0	≈0	≈0	≈0
10	$\check{\Lambda}^0_{c1}(\frac{1}{2}^+)$	4.8–75.8	1.0–14.3	5.8–90.1	0.19–0.19
11	$\check{\Lambda}^0_{c1}(\frac{3}{2}^+)$	1.2–18.9	2.4–35.7	3.6–54.7	1.87–1.95
12	$\check{\Lambda}^2_{c1}(\frac{1}{2}^+)$	0.9–13.6	0.2–2.6	1.1–16.2	0.19–0.20
13	$\check{\Lambda}^2_{c1}(\frac{3}{2}^+)$	0.2–3.4	0.4–6.4	0.7–9.8	1.88–1.92
14	$\check{\Lambda}^2_{c2}(\frac{3}{2}^+)$	2.0–30.6	0.2–2.3	2.2–3.0	0.08–0.08
15	$\check{\Lambda}^2_{c2}(\frac{5}{2}^+)$	5.4×10^{-3}–5.2×10^{-2}	1.0–13.9	1.0–14.0	175.64–270.03
16	$\check{\Lambda}^2_{c3}(\frac{5}{2}^+)$	6.2×10^{-3}–5.9×10^{-2}	8.8×10^{-4}–8.2×10^{-2}	7.1×10^{-3}–6.7×10^{-2}	0.13–0.14
17	$\check{\Lambda}^2_{c3}(\frac{7}{2}^+)$	3.5×10^{-3}–3.3×10^{-2}	1.2×10^{-3}–1.1×10^{-2}	4.7×10^{-2}–4.4×10^{-2}	0.32–0.33

state with $J^P = \frac{3}{2}^+$ [19,20]. In most references [9–14, 16–18,21,61] $\Lambda_c(2880)^+$ was conjectured as an excited charmed baryon with $J^P = \frac{5}{2}^+$, although its structure differed throughout these works.

In addition to an S-wave D^*N molecular state interpretation [25–32], $\Lambda_c(2940)^+$ was interpreted as an excited charmed baryon with different J^P quantum numbers, as shown in Table II.

In order to check all of the possible charmed baryons candidates, $\Lambda_c(2860)^+$, $\Lambda_c(2880)$ and $\Lambda_c(2940)$ are studied as the $1P$-wave, $2S$-wave and $1D$-wave states in detail in the $^3P_0$ model. Their OZI-allowed two-body strong decay channels are given and the relevant decay widths are estimated. Their decay widths as $1P$-wave and $2S$-wave charmed baryons are presented in Tables XII–XIV. Their decay widths as $1D$-wave charmed baryons are presented in Tables XV–XVII.

From Tables XII and XV, we can see that there are two suitable P-wave assignments [$\tilde{\Lambda}_{c2}(\frac{3}{2}^-)$ and $\tilde{\Lambda}_{c2}(\frac{5}{2}^-)$] for $\Lambda_c(2860)^+$, which has an observable D^0p mode and a $\Gamma_{\rm total}$ that is comparable to experimental data. There are also two D-wave assignments [$\check{\Lambda}^2_{c1}(\frac{1}{2}^+)$ or $\check{\Lambda}^2_{c1}(\frac{3}{2}^+)$] suitable for $\Lambda_c(2860)^+$ for the same reason. If the experimental constraint $J^P = \frac{3}{2}^+$ for $\Lambda_c(2860)^+$ is true [33], then $\Lambda_c(2860)^+$ can only be the D-wave $\check{\Lambda}^2_{c1}(\frac{3}{2}^+)$. In this case, the branching ratio $R = \Gamma(\Sigma_c(2520)^{++,0}\pi^{-,+})/\Gamma(\Sigma_c(2455)^{++,0}\pi^{-,+}) = 2.8$–$3.0$, and $\Lambda_c(2860)^+$ has a total decay width $\Gamma = 3.8$–59.6 MeV. For the purpose of identifying $\Lambda_c(2860)^+$, it is very important to measure the branching ratio $R = \Gamma(\Sigma_c(2520)^{++,0}\pi^{-,+})/\Gamma(\Sigma_c(2455)^{++,0}\pi^{-,+})$.

From Table XIII, the observation of a D^0p mode, the measured branching ratio $R = \Gamma(\Sigma_c(2520)\pi)/\Gamma(\Sigma_c(2455)\pi) = 0.225 \pm 0.062 \pm 0.0255$ and the total decay width indicate that $\Lambda_c(2880)^+$ is not a $1P$-wave or $2S$-wave charmed baryon [the $1P$-wave $\tilde{\Lambda}_{c2}(\frac{3}{2}^-)$ assignment has a comparable R but a much larger predicted total decay width in comparison with experimental data]. From Table XVI, the observation of a D^0p mode and the measured

TABLE XII. Decay widths (MeV) of $\Lambda_c(2860)^+$ as $1P$-wave and $2S$-wave charmed baryons. $\mathcal{R} = \Gamma(\Sigma_c(2520)^{++,0}\pi)/\Gamma(\Sigma_c(2455)^{++,0}\pi)$.

N	$\Lambda_{cJ_l}(J^P)$	D^0P	D^+N	$\Sigma_c^{++,0,+}(2455)$ $\times\pi^{-,+,0}$	$\Sigma_c^{++,0,+}(2520)$ $\times\pi^{-,+,0}$	$\Gamma_{\rm total}$	$\mathcal{B}1$	$\mathcal{B}2$	$\mathcal{R}$
1	$\Lambda_{c1}(\frac{1}{2}^-)$	0	0	125.0–265.0	5.0–6.3	130.0–271.0	64.2–65.1%	1.5–2.5%	0.02–0.04
2	$\Lambda_{c1}(\frac{3}{2}^-)$	0	0	6.9–9.0	115.0–227.9	122.0–236.9	2.5–3.7%	62.8–64.0%	16.9–25.6
3	$\tilde{\Lambda}_{c0}(\frac{1}{2}^-)$	373.0–726.0	366.0–704.0	0	0	739.0–1430.0	…	…	…
4	$\tilde{\Lambda}_{c1}(\frac{1}{2}^-)$	0	0	748.0–1591.0	7.5–9.5	755.0–1600.0	66.0–66.3%	0.4–0.7%	0.006–0.010
5	$\tilde{\Lambda}_{c1}(\frac{3}{2}^-)$	0	0	10.3–13.5	679.0–1353.0	689.0–1367.0	0.7–1.0%	65.6–65.9%	64.84–101.20
6	$\tilde{\Lambda}_{c2}(\frac{3}{2}^-)$	2.0–2.5	1.5–1.8	18.5–24.3	6.7–8.5	28.7–37.2	42.5–43.1%	15.1–15.4%	0.35–0.36
7	$\tilde{\Lambda}_{c2}(\frac{5}{2}^-)$	2.0–2.5	1.5–1.8	8.2–10.8	10.5–13.3	22.2–28.4	24.5–25.1%	30.7–31.0%	1.22–1.26
8	$\tilde{\Lambda}^{\rho'}_{c0}(\frac{1}{2}^+)$	0	0	7.8–57.4	11.2–60.0	19.0–117.0	27.2–32.5%	33.8–33.9%	1.04–1.44
9	$\tilde{\Lambda}^{\lambda'}_{c0}(\frac{1}{2}^+)$	0	0	45.5–97.7	45.3–105.3	90.7–203.0	31.9–31.9%	33.0–34.3%	0.99–1.07

TABLE XIII. Decay widths (MeV) of $\Lambda_c(2880)^+$ as $1P$-wave and $2S$-wave charmed baryons. The branching fractions $\mathcal{B}1 = \Gamma(\Lambda_c(2880)^+ \to \Sigma_c(2455)^{++,0}\pi^{-,+})/\Gamma_{\rm total}$ and $\mathcal{B}2 = \Gamma(\Lambda_c(2880)^+ \to \Sigma_c(2520)^{++,0}\pi^{-,+})/\Gamma_{\rm total}$. $\mathcal{R}$ represents the ratio of $\mathcal{B}2/\mathcal{B}1$.

N	$\Lambda_{cJ_l}(J^P)$	D^0P	D^+N	$\Sigma_c^{++,0,+}(2455)$ $\times\pi^{-,+,0}$	$\Sigma_c^{++,0,+}(2520)$ $\times\pi^{-,+,0}$	$\Gamma_{\rm total}$	$\mathcal{B}1$	$\mathcal{B}2$	$\mathcal{R}$
1	$\Lambda_{c1}(\frac{1}{2}^-)$	0	0	126.0–276.0	7.7–9.9	134.0–286.0	62.8–64.4%	2.3–3.8%	0.04–0.06
2	$\Lambda_{c1}(\frac{3}{2}^-)$	0	0	9.6–12.8	123.0–248.0	133.0–261.0	3.2–4.8%	61.7–63.3%	12.89–19.60
3	$\tilde{\Lambda}_{c0}(\frac{1}{2}^-)$	383.0–784.0	384.0–777.0	0	0	767.0–1561.0	…	…	…
4	$\tilde{\Lambda}_{c1}(\frac{1}{2}^-)$	0	0	756–1657	11.59–14.91	768–1672	65.63–66.07%	0.59–0.99%	0.0088–0.015
5	$\tilde{\Lambda}_{c1}(\frac{3}{2}^-)$	0	0	14.4–19.2	720.0–1468.0	734.0–1487.0	0.9–1.3%	65.4–65.8%	33.28–77.34
6	$\tilde{\Lambda}_{c2}(\frac{3}{2}^-)$	5.1–6.5	4.2–5.4	26.0–34.6	10.4–13.4	45.7–59.9	37.5–38.1%	14.7–15.0%	0.39–0.40
7	$\tilde{\Lambda}_{c2}(\frac{5}{2}^-)$	5.1–6.5	4.2–5.4	11.5–15.4	16.2–20.9	37.0–48.2	20.6–21.1%	28.5–28.8%	1.35–1.40
8	$\tilde{\Lambda}^{\rho'}_{c0}(\frac{1}{2}^+)$	0	0	8.2–71.3	13.2–79.5	21.4–151.0	25.6–31.3%	34.8–41.0%	1.11–1.60
9	$\tilde{\Lambda}^{\lambda'}_{c0}(\frac{1}{2}^+)$	0	0	57.6–119.7	61.1–138.0	119.0–258.0	30.8–32.2%	34.0–35.4%	1.06–1.15

TABLE XIV. Decay widths (MeV) of $\Lambda_c(2940)^+$ as $1P$-wave and $2S$-wave charmed baryons. $\mathcal{R} = B(\Lambda_c(2940)^+ \to \Sigma_c(2520)\pi/B(\Lambda_c(2940)^+ \to \Sigma_c(2455)\pi)$.

N	$\Lambda_{cJ_l}(J^P)$	D^0P	D^+N	$\Sigma_c^{++,0,+}(2455)$ $\times\pi^{-,+,0}$	$\Sigma_c^{++,0,+}(2520)$ $\times\pi^{-,+,0}$	$\Sigma_c^{++,0,+}(2765)$ $\times\pi^{-,+,0}$	$\Sigma_c^{++}\pi^0$	Γ_{total}	$\mathcal{R}$
1	$\Lambda_{c1}(\frac{1}{2}^-)$	0	0	122–288	17.89–23.75	0	0	140–312	0.08–0.14
2	$\Lambda_{c1}(\frac{3}{2}^-)$	0	0	18.69–25.79	135–287	0	0	154–313	7.29–11.20
3	$\tilde{\Lambda}_{c0}(\frac{1}{2}^-)$	329–769	338–777	0	0	0.60–13.68	0.07–1.59	682–1547	⋯
4	$\tilde{\Lambda}_{c1}(\frac{1}{2}^-)$	0	0	734–1731	26.83–35.63	3.46–14.12	0.10–0.63	776–1770	0.02–0.04
5	$\tilde{\Lambda}_{c1}(\frac{3}{2}^-)$	0	0	28.04–38.68	772–1667	0.86–5.54	0.24–1.11	807–1707	27.75–43.42
6	$\tilde{\Lambda}_{c2}(\frac{3}{2}^-)$	18.32–25.15	16.60–22.64	50.45–69.61	24.16–32.06	2.20–3.91	0.21–0.32	114–152	0.46–0.48
7	$\tilde{\Lambda}_{c2}(\frac{5}{2}^-)$	18.32–25.15	16.60–22.64	22.42–30.94	37.57–49.97	1.84–2.61	0.23–0.39	97.91–131	1.61–1.67
8	$\tilde{\Lambda}^{\rho'}_{c0}(\frac{1}{2}^+)$	0	0	7.95–109	16.76–136	0	0	35.56–245	1.25–2.05
9	$\tilde{\Lambda}^{\lambda'}_{c0}(\frac{1}{2}^+)$	0	0	92.52–177	110–230	0	0	275–407	1.18–1.30

TABLE XV. Decay widths (MeV) of $\Lambda_c(2860)^+$ as D-wave excitations. The branching ratios $\mathcal{B}1 = \Gamma(\Lambda_c(2860)^+ \to \Sigma_c(2455)^{++,0}\pi^{-,+})/\Gamma_{\text{total}}$ and $\mathcal{B}2 = \Gamma(\Lambda_c(2860)^+ \to \Sigma_c(2520)^{++,0}\pi^{-,+})/\Gamma_{\text{total}}$. $\mathcal{R}$ represents the ratio of $\mathcal{B}2/\mathcal{B}1$.

N	$\Lambda_{cJ_l}(J^P)$	D^0P	D^+N	$\Sigma_c^{++,0,+}(2455)\pi^{-,+,0}$	$\Sigma_c^{++,0,+}(2520)\pi^{-,+,0}$	Γ_{total}	$\mathcal{B}1$	$\mathcal{B}2$	$\mathcal{R}$
1	$\Lambda_{c2}(\frac{3}{2}^+)$	0	0	0.5–8.1	0.1–1.0	0.5–9.1	59.3–61.5%	7.0–7.7%	0.09–0.13
2	$\Lambda_{c2}(\frac{5}{2}^+)$	0	0	0.01–0.11	0.4–5.5	0.4–5.6	1.2–1.9%	64.8–64.9%	23.97–52.27
3	$\hat{\Lambda}_{c2}(\frac{3}{2}^+)$	0	0	4.4–73.1	0.6–8.6	5.0–81.7	58.8–59.3%	7.0–7.7%	0.12–0.13
4	$\hat{\Lambda}_{c2}(\frac{5}{2}^+)$	0	0	0.1–1.0	3.2–49.4	3.2–50.4	1.2–1.3%	64.4–64.9%	33.71–52.72
5	$\check{\Lambda}^1_{c0}(\frac{1}{2}^+)$	0	0	0	0	0	⋯	⋯	⋯
6	$\check{\Lambda}^1_{c1}(\frac{1}{2}^+)$	0	0	0	≈0	≈0	⋯	⋯	⋯
7	$\check{\Lambda}^1_{c1}(\frac{3}{2}^+)$	0	0	≈0	0	≈0	⋯	⋯	⋯
8	$\check{\Lambda}^1_{c2}(\frac{3}{2}^+)$	≈0	≈0	≈0	≈0	0	⋯	⋯	⋯
9	$\check{\Lambda}^1_{c2}(\frac{5}{2}^+)$	≈0	≈0	≈0	≈0	≈0	⋯	⋯	⋯
10	$\check{\Lambda}^0_{c1}(\frac{1}{2}^+)$	1.4–21.5	1.2–18.4	10.7–182.0	3.2–50.8	16.4–272.0	43.1–44.2%	12.3–12.8%	0.3–0.30
11	$\check{\Lambda}^0_{c1}(\frac{3}{2}^+)$	1.4–21.5	1.2–18.4	2.7–45.4	7.9–126.0	13.2–211.0	13.4–14.3%	39.3–39.9%	2.75–2.98
12	$\check{\Lambda}^2_{c1}(\frac{1}{2}^+)$	1.0–15.5	0.9–13.2	2.0–32.5	0.6–9.1	4.4–70.3	29.5–30.7%	8.6–8.6%	0.28–0.29
13	$\check{\Lambda}^2_{c1}(\frac{3}{2}^+)$	1.0–15.5	0.9–13.2	0.5–8.1	1.5–22.8	3.8–59.6	8.4–9.0%	25.2–25.3%	2.80–3.00
14	$\check{\Lambda}^2_{c2}(\frac{3}{2}^+)$	0	0	4.4–73.1	0.5–8.4	4.9–81.5	59.1–59.5%	6.8–7.3%	0.11–0.12
15	$\check{\Lambda}^2_{c2}(\frac{5}{2}^+)$	0	0	0.04–0.43	3.14–49.32	3.18–49.75	1–1%	65–66%	76–117
16	$\check{\Lambda}^2_{c3}(\frac{5}{2}^+)$	0.004–0.03	0.002–0.02	0.05–0.49	0.02–0.15	0.08–0.69	39–46%	12–13%	0.30–0.31
17	$\check{\Lambda}^2_{c3}(\frac{7}{2}^+)$	0.004–0.03	0.002–0.02	0.03–0.27	0.02–0.20	0.06–0.52	29–34%	22–25%	0.72–0.75

$R=\Gamma(\Sigma_c(2520)\pi)/\Gamma(\Sigma_c(2455)\pi)=0.225\pm0.062\pm0.0255$ indicate that $\Lambda_c(2880)^+$ may be a $\check{\Lambda}^0_{c1}(\frac{1}{2}^+)$, $\check{\Lambda}^2_{c1}(\frac{1}{2}^+)$, or $\check{\Lambda}^2_{c3}(\frac{5}{2}^+)$. Accounting for the much larger predicted total decay widths of $\check{\Lambda}^0_{c1}(\frac{1}{2}^+)$ and $\check{\Lambda}^2_{c1}(\frac{1}{2}^+)$ in comparison with experimental data, $\Lambda_c(2880)^+$ is possibly the D-wave $\check{\Lambda}^2_{c3}(\frac{5}{2}^+)$.

From Tables XIV and XVII, there are three P-wave assignments [$\tilde{\Lambda}_{c0}(\frac{1}{2}^-)$, $\tilde{\Lambda}_{c2}(\frac{3}{2}^-)$, $\tilde{\Lambda}_{c2}(\frac{5}{2}^-)$] and six D-wave assignments [$\check{\Lambda}^0_{c1}(\frac{1}{2}^+)$, $\check{\Lambda}^0_{c1}(\frac{3}{2}^+)$, $\check{\Lambda}^2_{c1}(\frac{1}{2}^+)$, $\check{\Lambda}^2_{c1}(\frac{3}{2}^+)$, $\check{\Lambda}^2_{c3}(\frac{5}{2}^+)$, and $\check{\Lambda}^2_{c3}(\frac{7}{2}^+)$], which have an observable D^0p mode. Accounting for the total decay width and given the theoretical and experimental uncertainties, $\Lambda_c(2940)^+$ is possibly the P-wave $\tilde{\Lambda}_{c2}(\frac{3}{2}^-)$ or $\tilde{\Lambda}_{c2}(\frac{5}{2}^-)$, it is also possibly the D-wave $\check{\Lambda}^2_{c3}(\frac{5}{2}^+)$ or $\check{\Lambda}^2_{c3}(\frac{7}{2}^+)$. For the $\tilde{\Lambda}_{c2}(\frac{3}{2}^-)$ and $\tilde{\Lambda}_{c2}(\frac{5}{2}^-)$ assignments, the predicted total decay widths (151.9 and 130.7 MeV) are bigger than the measured one. For the $\check{\Lambda}^2_{c3}(\frac{5}{2}^+)$ and $\check{\Lambda}^2_{c3}(\frac{7}{2}^+)$ assignments, the predicted total decay widths (4.5 and 3.9 MeV) are smaller than the measured one. However, the branching ratios $R = \Gamma(\Sigma_c(2520)^{++,0}\pi^{-,+})/\Gamma(\Sigma_c(2455)^{++,0}\pi^{-,+})$ are largely different between the two P-wave assignments or two D-wave assignments. Obviously, the measurement of the branching ratio $R = \Gamma(\Sigma_c(2520)^{++,0}\pi^{-,+})/\Gamma(\Sigma_c(2455)^{++,0}\pi^{-,+})$ is also very important for the identification of $\Lambda_c(2940)^+$.

TABLE XVI. Decay widths (MeV) of $\Lambda_c(2880)^+$ as D-wave excitations. The branching ratios $\mathcal{B}1 = \Gamma(\Lambda_c(2880)^+ \to \Sigma_c(2455)^{++,0}\pi^{-,+})/\Gamma_{\text{total}}$ and $\mathcal{B}2 = \Gamma(\Lambda_c(2880)^+ \to \Sigma_c(2520)^{++,0}\pi^{-,+})/\Gamma_{\text{total}}$. $\mathcal{R}$ represents the ratio of $\mathcal{B}2/\mathcal{B}1$.

N	$\Lambda_{cJ_l}(J^P)$	D^0P	D^+N	$\Sigma_c^{++,0,+}(2455)\pi^{-,+,0}$	$\Sigma_c^{++,0,+}(2520)\pi^{-,+,0}$	Γ_{total}	$\mathcal{B}1$	$\mathcal{B}2$	$\mathcal{R}$
1	$\Lambda_{c2}(\frac{3}{2}^+)$	0	0	0.6–9.8	0.1–0.8	0.7–11.0	58.5–58.8%	7.6–8.3%	0.13–0.14
2	$\Lambda_{c2}(\frac{5}{2}^+)$	0	0	0.0–0.2	0.5–7.1	0.5–7.3	1.5–2.3%	63.8–64.7%	27.62–41.98
3	$\hat{\Lambda}_{c2}(\frac{3}{2}^+)$	0	0	5.1–87.8	0.7–7.5	5.9–99.2	58.1–58.8%	7.6–8.2%	0.13–0.14
4	$\hat{\Lambda}_{c2}(\frac{5}{2}^+)$	0	0	0.2–1.5	4.0–63.9	4.1–65.4	1.5–2.4%	63.8–64.7%	26.99–42.25
5	$\check{\Lambda}^1_{c0}(\frac{1}{2}^+)$	0	0	0	0	0	…	…	…
6	$\check{\Lambda}^1_{c1}(\frac{1}{2}^+)$	0	0	≈0	≈0	≈0	…	…	…
7	$\check{\Lambda}^1_{c1}(\frac{3}{2}^+)$	0	0	≈0	0	≈0	…	…	…
8	$\check{\Lambda}^1_{c2}(\frac{3}{2}^+)$	≈0	≈0	≈0	≈0	≈0	…	…	…
9	$\check{\Lambda}^1_{c2}(\frac{5}{2}^+)$	0	0	≈0	≈0	≈0	…	…	…
10	$\check{\Lambda}^0_{c1}(\frac{1}{2}^+)$	2.2–35.6	2.0–32.3	12.5–219.0	4.0–65.5	20.6–352.2	40.2–41.2%	12.3–12.8%	0.21–0.32
11	$\check{\Lambda}^0_{c1}(\frac{3}{2}^+)$	2.2–35.6	2.0–32.3	3.1–54.7	10.0–164.0	17.3–286.5	12.0–12.7%	37.9–8.3%	2.99–3.18
12	$\check{\Lambda}^2_{c1}(\frac{1}{2}^+)$	1.6–25.5	1.5–23.2	2.3–39.1	0.7–12.0	6.1–99.5	25.1–26.1%	7.8–7.9%	0.30–0.32
13	$\check{\Lambda}^2_{c1}(\frac{3}{2}^+)$	1.6–25.5	1.5–23.2	0.6–9.8	1.8–29.4	5.5–87.8	7.0–7.4%	22.2–22.4%	3.00–3.21
14	$\check{\Lambda}^2_{c2}(\frac{3}{2}^+)$	0	0	5.1–87.9	0.7–10.9	5.8–98.8	58.5–59.0%	7.3–7.9%	0.12–0.14
15	$\check{\Lambda}^2_{c2}(\frac{5}{2}^+)$	0	0	0.1–0.7	4.0–63.6	4.0–64.3	0.7–1.1%	64.9–65.0%	60.23–96.34
16	$\check{\Lambda}^2_{c3}(\frac{5}{2}^+)$	0.0–0.1	0.0–0.1	0.1–0.8	0.0–0.3	0.1–1.3	38.2–39.8%	13.8–14.1%	0.35–0.36
17	$\check{\Lambda}^2_{c3}(\frac{7}{2}^+)$	0.0–0.1	0.0–0.1	0.0–0.4	0.0–0.4	0.1–1.0	27.2–28.8%	23.3–25.0%	0.86–0.87

TABLE XVII. Decay widths (MeV) of $\Lambda_c(2940)^+$ as D-wave excitations. $\mathcal{R} = \Gamma(\Sigma_c(2520)^{++,0}\pi)/\Gamma(\Sigma_c(2455)^{++,0}\pi)$.

N	$\Lambda_{cJ_l}(J^P)$	D^0P	D^+N	$\Sigma_c^{++,0,+}(2455)$ $\times\pi^{-,+,0}$	$\Sigma_c^{++,0,+}(2520)$ $\times\pi^{-,+,0}$	$\Sigma_c^{++,0,+}(2765)$ $\times\pi^{-,+,0}$	$\Sigma_c'^{++}\pi^0$	Γ_{total}	$\mathcal{R}$
1	$\Lambda_{c2}(\frac{3}{2}^+)$	0	0	0.8–13.8	0.1–2.2	0	0	0.9–15.9	0.16–0.18
2	$\Lambda_{c2}(\frac{5}{2}^+)$	0	0	0.0–0.4	0.7–11.4	0	0	0.7–11.8	16.42–27.00
3	$\hat{\Lambda}_{c2}(\frac{3}{2}^+)$	0	0	6.7–124.0	1.2–19.4	0	0	8.0–143.2	0.16–0.18%03
4	$\hat{\Lambda}_{c2}(\frac{5}{2}^+)$	0	0	0.4–3.9	6.1–72.7	0	0	6.4–106.6	16.79–26.20
5	$\check{\Lambda}^1_{c0}(\frac{1}{2}^+)$	0	0	0	0	0	0	0	…
6	$\check{\Lambda}^1_{c1}(\frac{1}{2}^+)$	0	0	0	≈0	44.1–72.4	≈0	44.1–72.4	…
7	$\check{\Lambda}^1_{c1}(\frac{3}{2}^+)$	0	0	≈0	0	0.01–0.02	8.01–12.46	8.03–12.47	…
8	$\check{\Lambda}^1_{c2}(\frac{3}{2}^+)$	0	0	≈0	≈0	0.01–0.03	≈0	0.01–0.03	…
9	$\check{\Lambda}^1_{c2}(\frac{5}{2}^+)$	≈0	≈0	≈0	≈0	0.004–0.013	≈0	0.004–0.013	…
10	$\check{\Lambda}^0_{c1}(\frac{1}{2}^+)$	3.6–66.0	3.5–63.2	16.5–309.0	6.1–105.0	0.3–5.0	0.001–0.003	34.6–543.0	0.34–0.37
11	$\check{\Lambda}^0_{c1}(\frac{3}{2}^+)$	3.6–66.0	3.5–63.2	4.1–77.4	15.1–263.0	0.0–8.7	0.0–1.0	34.9–470.0	3.40–3.67
12	$\check{\Lambda}^2_{c1}(\frac{1}{2}^+)$	2.6–46.9	2.5–45.0	3.0–55.0	1.1–18.8	37.5–48.9	≈0	58.1–203.3	0.34–0.37
13	$\check{\Lambda}^2_{c1}(\frac{3}{2}^+)$	2.6–46.9	2.5–45.0	0.8–13.8	2.8–47.0	0.01–0.05	6.6–9.3	18.0–159.3	3.41–3.68
14	$\check{\Lambda}^2_{c2}(\frac{3}{2}^+)$	0	0	6.7–124.0	1.1–18.0	0.0–0.2	≈0	8.0–142.0	0.15–0.17
15	$\check{\Lambda}^2_{c2}(\frac{5}{2}^+)$	0	0	0.2–1.8	6.0–102.0	0.002–0.007	≈0	6.2–104.0	37.12–58.38
16	$\check{\Lambda}^2_{c3}(\frac{5}{2}^+)$	0.1–0.9	0.1–0.8	0.2–2.0	0.1–0.9	≈0	≈0	0.5–4.5	0.44–0.46
17	$\check{\Lambda}^2_{c3}(\frac{7}{2}^+)$	0.1–0.9	0.1–0.8	0.1–1.1	0.1–1.1	≈0	≈0	0.37–3.90	1.05–1.10

IV. CONCLUSIONS AND DISCUSSIONS

In this work, we briefly reviewed the studies of the observed $\Lambda_c(2595)^+$, $\Lambda_c(2625)^+$, $\Lambda_c(2765)^+$ [or $\Sigma_c(2765)^+$], $\Lambda_c(2860)^+$, $\Lambda_c(2880)^+$ and $\Lambda_c(2940)^+$ states. We studied the OZI-allowed strong decay features of all of these Λ_c states in the $^3P_0$ model. Possible $1P$, $1D$, and $2S$ assignments of these observed Λ_c states were examined. Their possible quantum numbers J^P and internal structure were given based on our numerical results.

For $\Lambda_c(2595)^+$ and $\Lambda_c(2625)^+$, $\Sigma_c(2455)\pi$ are their only two-body decay modes. The branching fraction of the direct three-body decay mode $\Lambda_c^+\pi\pi$ is not large for $\Lambda_c(2595)^+$ but it is large for $\Lambda_c(2625)^+$, so it is impossible to understand $\Lambda_c(2625)^+$ only from the branching fraction of the two-body strong decay modes. Accounting for the theoretical predictions of the masses of excited Λ_c states, $\Lambda_c(2595)^+$ and $\Lambda_c(2625)^+$ are possibly the $1P$-wave charmed baryons $\Lambda_{c1}(\frac{1}{2}^-)$ and $\Lambda_{c1}(\frac{3}{2}^-)$, respectively. The predicted branching fractions and decay widths are consistent with experimental data.

$\Lambda_c(2765)^+$ [$\Sigma_c(2765)^+$] is not the $1P$-wave charmed baryon. It is possibly the $2S$-wave or $1D$-wave charmed baryon. The strong decay behaviors of the two $2S$-wave (λ-mode excitation and ρ-mode excitation) baryons are similar, and it is difficult to distinguish them through their strong decays. So far, little experimental information about $\Lambda_c(2765)^+$ [$\Sigma_c(2765)^+$] has been obtained, and there is insufficient information to understand $\Lambda_c(2765)^+$ [$\Sigma_c(2765)^+$].

$\Lambda_c(2860)^+$ is not a $2S$-wave charmed baryon, but it may be the P-wave $\tilde{\Lambda}_{c2}(\frac{3}{2}^-)$ or $\tilde{\Lambda}_{c2}(\frac{5}{2}^-)$, or the D-wave $\check{\Lambda}_{c1}^2(\frac{1}{2}^+)$ or $\check{\Lambda}_{c1}^2(\frac{3}{2}^+)$. If $\Lambda_c(2860)^+$ has $J^P=\frac{3}{2}^+$, it is possibly the D-wave $\check{\Lambda}_{c1}^2(\frac{3}{2}^+)$ with total decay $\Gamma=3.8$–59.6 MeV. In this case, the predicted branching ratio $R=\Gamma(\Sigma_c(2520)\pi)/\Gamma(\Sigma_c(2455)\pi)=2.8$–$3.0$. The measurement of R will be very important for the identification of $\Lambda_c(2860)^+$.

$\Lambda_c(2880)^+$ is not a $1P$-wave or $2S$-wave charmed baryon, but it may be a D-wave $\check{\Lambda}_{c3}^2(\frac{5}{2}^+)$ with $\Gamma_{\rm total}=0.1$–1.3 MeV. The predicted branching ratio $R=\Gamma(\Sigma_c(2520)\pi)/\Gamma(\Sigma_c(2455)\pi)=0.35$–$0.36$, which is consistent with the measured $R=\Gamma(\Sigma_c(2520)\pi)/\Gamma(\Sigma_c(2455)\pi)=0.225\pm0.062\pm0.0255$.

$\Lambda_c(2940)^+$ is possibly the P-wave $\tilde{\Lambda}_{c2}(\frac{3}{2}^-)$ or $\tilde{\Lambda}_{c2}(\frac{5}{2}^-)$, it is possibly the D-wave $\check{\Lambda}_{c3}^2(\frac{5}{2}^+)$ or $\check{\Lambda}_{c3}^2(\frac{7}{2}^+)$. The branching ratios $R=\Gamma(\Sigma_c(2520)\pi)/\Gamma(\Sigma_c(2455)\pi)$ are largely different between these assignments, which could be employed to experimentally distinguish them in the future.

From Tables III and IV, the two light quarks couple with spin $S_\rho=0$ or spin $S_\rho=1$ in different configurations. In the assignments consistent with experimental data, the two light quarks couple with spin $S_\rho=0$ in $\Lambda_c(2595)^+$ and $\Lambda_c(2625)^+$. The two light quarks couple with spin $S_\rho=1$ in $\Lambda_c(2860)^+$, $\Lambda_c(2880)^+$, and $\Lambda_c(2940)^+$. In $\Lambda_c(2765)^+$ [or $\Sigma_c(2765)^+$], $S_\rho=0$ and $S_\rho=1$ are both possible.

High excited assignments such as the $2P$-wave [62] or $2D$-wave charmed baryons have not been examined for these Λ_c states, and relevant calculation and analyses have not been performed using the $^3P_0$ model. Other higher excitation assignments for these Λ_c states may be possible. There are some uncertainties in the $^3P_0$ model. The main uncertainties result from the uncertainties of the parameters γ and β. These uncertainties may result in some large uncertainties in the numerical results of decay widths. However, the predicted branching ratios depend weakly on the parameters.

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APPENDIX: FLAVOR WAVE FUNCTIONS OF BARYONS AND MESONS

The flavor wave functions of baryons and mesons involved in our study are the same as those in Ref. [3]:

$$\Lambda_c^+=\frac{1}{\sqrt{2}}(ud-du)c,\qquad \Sigma_c^{++}=uuc,$$
$$\Sigma_c^+=\frac{1}{\sqrt{2}}(ud+du)c,\qquad \Sigma_c^0=ddc,$$
$$p=\frac{1}{\sqrt{2}}(du-ud)u,\qquad \pi^+=u\bar{d},$$
$$n=\frac{1}{\sqrt{2}}(du-ud)d,\qquad \pi^-=d\bar{u},$$
$$\pi^0=\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d}),\qquad D^+=\bar{d}c,\qquad D^0=\bar{u}c.$$

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