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Neutral-current effects in elastic electron-nucleon scattering

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Parity-violating effects are studied in considerable detail for elastic electron-nucleon scattering. Based on a unified Weinberg gauge model properly generalized to include nonstrange hadrons, we derive and discuss corrections to the Rosenbluth formula, and left-right asymmetries of longitudinally polarized electrons as well as nucleons. Any dependence of electron-nucleon scattering on the longitudinal polarization would be evidence of parity violation. The size of such neutral-current effects in general differs from that naively expected on purely dimensional grounds and strongly depends on the nucleon target used.

I. INTRODUCTION

Unified gauge theories of weak and electromagnetic interactions[1] yield striking predictions for leptonic and semileptonic processes.[2] The main emphasis has for obvious reasons so far been put on neutrino-induced reactions. For electron-induced processes the effects of the neutral weak boson (the Z^0 boson) are in general of order $(G/e^2)q^2$, where q^2 is the momentum transfer squared and G is the weak Fermi coupling constant. With the advent of the new generation of accelerators and electron-nucleon storage rings,[3] this neutral weak current could lead to observable effects relative to the electromagnetic background. Possibly high-precision intermediate-energy experiments searching, e.g., for parity violation in elastic electron-nucleon scattering could also yield information on the existence of neutral weak currents.

In this paper we delineate and discuss in considerable detail effects of the Z^0 boson on elastic electron-nucleon scattering. Because of the sharp falloff of the electromagnetic form factors, this reaction is not the best choice for studying such effects. On the other hand, the fundamental importance of elastic electron-nucleon scattering and, parallel to it, the complete lack of understanding of the q^2 behavior of the form factors warrant a detailed investigation of the effects of weak neutral currents. We find corrections to the Rosenbluth formula and parity-violating effects[4] in longitudinal electron (or nucleon) polarization experiments to be about one order of magnitude smaller than present experimental accuracy, which leaves hope that some of these effects will be observable in the not too distant future. The feasibility of high-energy experiments with longitudinally polarized electrons and unpolarized protons at the future colliding electron-proton beam machines (for example, EPIC and SPEAR) has been recently discussed.[5]

Any dependence of electron-nucleon scattering on the longitudinal polarization of the electrons (or nucleons) would be direct evidence of parity violation. Naively, these effects are expected to be of order Gq^2/e^2. Detailed calculations, however, show that this simple picture generally breaks down and that the magnitude of parity-violating effects is strongly dependent on the nucleon target used: Whereas the longitudinal polarization left-right asymmetry for protons turns out to be about one order of magnitude smaller than naively expected, neutron targets in general yield results by at least one order of magnitude larger than those for protons. To make such quantitative predictions, a detailed knowledge of the presently experimentally unknown $Z^0N\bar{N}$ vertex is required. By performing an isospin rotation and making use of the conserved-vector-current (CVC) hypothesis, this vertex can be expressed in terms of the form factors measured in charge-exchange semileptonic (neutrino) reactions and the electromagnetic nucleon form factors.

In Sec. II we briefly discuss the theoretical framework for our calculations and summarize our notation. Section III deals with the corrections to the Rosenbluth formula for unpolarized particles. In Sec. IV we calculate, as a parity-violating effect, left-right asymmetries for electrons polarized parallel and antiparallel to their direction of motion as well as for similarly polarized nucleons. We concentrate on the latter effects since they should vanish for pure electromagnetic interactions, i.e., in standard quantum electrodynamics (QED). For numerical estimates we consider two distinct limits, $-q^2 \ll ME$ with $-q^2 < M^2$ and $M^2 \ll -q^2 \ll M_Z{}^2$, respectively. (M and M_Z denote the mass of the nucleon and Z^0 boson, and E is the laboratory energy of the incoming electron.) Finally, our conclusions are summarized in Sec. V.

II. THEORY

A. Weinberg model of leptons and nonstrange hadrons

If one ignores strange particles altogether, the original SU(2)⊗U(1) Weinberg model for leptons[1] can be straightforwardly generalized to describe the weak and electromagnetic interactions of leptons as well as hadrons.[6] Denoting the usual nucleon doublet by N,

$$N=\begin{pmatrix} p \\ n \end{pmatrix},$$

it is natural to identify[7] $\frac{1}{2}(1-\gamma_5)N$ as a $\vec{\mathrm{T}}_L$ doublet, where the "left-handed isospin" $\vec{\mathrm{T}}_L$ generates the "strong" SU(2) group. The U(1) group is then generated by $T_{3L}-Q$, with Q being the ordinary electric charge operator. The hadronic $V-A$ and electric currents generated by $\vec{\mathrm{T}}_L$ and Q, respectively, are then

$$\vec{\mathcal{J}}^{\,\mu}=\tfrac{1}{2}\bar{N}\gamma^\mu(1-\gamma_5)\vec{\tau}N+\text{meson terms}\,, \tag{2.1}$$

$$J^\mu=\tfrac{1}{2}\bar{N}\gamma^\mu(1+\tau_3)N+\text{ meson terms}\,, \tag{2.2}$$

where the Pauli matrices are denoted by $\vec{\tau}$. Imposing gauge invariance on the total interaction Lagrangian, the theory must involve a triplet $\vec{\mathrm{A}}_\mu$ of intermediate vector bosons associated with $\vec{\mathrm{T}}_L$, and a singlet B_μ associated with $T_{3L}-Q$. The physical (corresponding to a diagonal mass matrix) neutral field Z_μ and photon field A_μ are then given by

$$\begin{aligned} Z_\mu &= \cos\theta_W A_{3\mu}+\sin\theta_W B_\mu\,, \\ A_\mu &= -\sin\theta_W A_{3\mu}+\cos\theta_W B_\mu\,, \end{aligned} \tag{2.3}$$

and their interactions with the hadrons turn out to be[6]

$$(\mathcal{L}_h)_{Z+A}=\tfrac{1}{2}(g^2+g'^2)^{1/2}Z_\mu\mathcal{J}_Z^\mu-eA_\mu J^\mu\,, \tag{2.4}$$

with

$$\mathcal{J}_Z^\mu\equiv\mathcal{J}_3^\mu-2\sin^2\theta_W J^\mu\,. \tag{2.5}$$

The triplet and singlet coupling constants are denoted by g and g', respectively, and the only free parameter of the theory is given by $\sin^2\theta_W=e^2/g^2=g'^2/(g^2+g'^2)$. Since we are only interested in neutral currents, we will not consider explicitly the interactions of the charged vector boson fields $W_\mu^\pm=(A_{1\mu}\mp iA_{2\mu})/\sqrt{2}$. Contrary to similar but nonrenormalizable models[8] of this kind, the masses of the vector-boson fields W_μ and Z_μ are now generated by spontaneous symmetry breaking, where the latter one describes the so-called Z^0 boson with a mass of

$$M_Z=\left(\frac{e^2}{\sqrt{2}G}\right)^{1/2}\Big/\sin2\theta_W\,. \tag{2.6}$$

The weak Fermi coupling constant $G=1.01\times10^{-5}M^{-2}$ (M being the nucleon mass) is related to M_Z and the above coupling constants by $G/\sqrt{2}=(g^2+g'^2)/(8M_Z{}^2)$. Similarly, the interaction of electrons (e^-) with Z_μ and A_μ is given by[1]

$$\begin{aligned}(\mathcal{L}_{e^-})_{Z+A}=&\tfrac{1}{2}(g^2+g'^2)^{-1/2}\\ &\times\bar{e}\gamma^\mu[g'^2(1+\gamma_5)+\tfrac{1}{2}(g'^2-g^2)(1-\gamma_5)]eZ_\mu\\ &-e(\bar{e}\gamma^\mu e)A_\mu\,.\end{aligned} \tag{2.7}$$

Thus, from (2.4) and (2.7), we get for the effective P- and C- violating electron-hadron interaction

$$\begin{aligned}\mathcal{L}_{\text{eff}}=&-\frac{G}{\sqrt{2}}\bar{e}\gamma_\mu(4\sin^2\theta_W-1+\gamma_5)e\,\mathcal{J}_Z^\mu\\ &+(e^2/q^2)(\bar{e}\gamma_\mu e)J^\mu\,,\end{aligned} \tag{2.8}$$

where we approximated the Z propagator by $-1/M_Z{}^2$; this is certainly a good approximation for not too high momentum transfers, since $M_Z\simeq80$ GeV. [For $-q^2\gtrsim M_Z{}^2$ the full Z propagator $(q^2-M_Z{}^2)^{-1}$ has to be used,[9] which corresponds to replacing G in (2.8) and in all subsequent formulas by $G(1-q^2/M_Z{}^2)^{-1}$.]

B. Form factors

Consider elastic electron-nucleon scattering

$$e^-(p,s)+N(P,S)\to e^-(p',s')+N(P',S'),$$

where the symbols in parentheses will be used to label momenta and spins of the corresponding particles. Assuming time-reversal invariance and only first class currents, the form factors for this process may be defined by

$$\begin{aligned}\langle P'|\mathcal{J}_Z^\mu|P\rangle &\equiv \bar u(P',S')\Gamma_Z^\mu u(P,S)\\ &=\bar u(P',S')\left[g_V^0\gamma^\mu-g_A^0\gamma^\mu\gamma_5-f_V^0(P+P')^\mu\right.\\ &\qquad\left.-h_A^0(P-P')^\mu\gamma_5\right]u(P,S)\end{aligned}\tag{2.9}$$

and

$$\begin{aligned}\langle P'|J^\mu|P\rangle &=\bar u(P',S')\left[F_1\gamma^\mu+iF_2\sigma^{\mu\nu}(P'-P)_\nu\right]u(P,S)\\ &=\bar u(P',S')\left[G_M\gamma^\mu-F_2(P+P')^\mu\right]u(P,S),\end{aligned}\tag{2.10}$$

with $G_M=F_1+2MF_2$ and $G_E=F_1+(q^2/2M)F_2$, where $q^2=(P'-P)^2=(p-p')^2$. The form factors g_V^0, g_A^0, f_V^0, h_A^0, and the electromagnetic nucleon form factors in (2.10) are real functions of q^2. The presently unknown $Z^0N\bar N$ form factors in (2.9) can be related to experimentally known ones by using (2.5), keeping in mind that the form factors for $\mathcal{J}_3^\mu$ may be determined by an isospin rotation[10,6] from the form factors for $\mathcal{J}_W^\mu\equiv\mathcal{J}_1^\mu+i\,\mathcal{J}_2^\mu$,

$$\begin{aligned}\langle P_p|\mathcal{J}_W^\mu|P_n\rangle &=u(P_p,S_p)\left[g_V\gamma^\mu-g_A\gamma^\mu\gamma_5-f_V(P_n+P_p)^\mu\right.\\ &\qquad\left.-h_A(P_n-P_p)^\mu\gamma_5\right]u(P_n,S_n),\end{aligned}\tag{2.11}$$

which are measured in charge-exchange semileptonic (neutrino) reactions. According to (2.5) the form factors in (2.9) are then given by

$$\begin{aligned}g_V^0&=\tfrac{1}{2}g_V-2\sin^2\theta_W\,G_M,\\ f_V^0&=\tfrac{1}{2}f_V-2\sin^2\theta_W\,F_2,\\ g_A^0&=\tfrac{1}{2}g_A,\\ h_A^0&=\tfrac{1}{2}h_A,\end{aligned}\tag{2.12}$$

and from the CVC relations[10] we have

$$g_V=G_M^p-G_M^n\ ,\ f_V=F_2^p-F_2^n.\tag{2.13}$$

The form factor h_A^0 never enters the final results of our calculations.

It is commonly assumed that the q^2 dependence of the vector form factors is given by the same function $\mathfrak{F}_V(q^2)$:

$$\frac{G_M(q^2)}{G_M(0)}=\frac{F_1(q^2)}{F_1(0)}=\frac{F_2(q^2)}{F_2(0)}=\frac{g_V(q^2)}{g_V(0)}=\frac{f_V(q^2)}{f_V(0)}=\mathfrak{F}_V(q^2),\tag{2.14}$$

with $\mathfrak{F}_V(0)=1$. Similarly, we may write

$$g_A(q^2)/g_A(0)=\mathfrak{F}_A(q^2),\tag{2.15}$$

with $\mathfrak{F}_A(0)=1$, and based on recent inelastic neutrino-nucleon scattering experiments[11] we may assume that over a limited q^2 range the vector and axial-vector form factors exhibit the same q^2 dependence,

$$\mathfrak{F}_A(q^2)\simeq\mathfrak{F}_V(q^2).\tag{2.16}$$

We shall use these relations later for numerical estimates only. In Table I we summarize the normalizations of all relevant form factors at $q^2=0$.

III. NEUTRAL-CURRENT CORRECTIONS TO THE ROSENBLUTH FORMULA

The cross section for unpolarized elastic electron-nucleon scattering is modified by the Z^0 exchange diagram [Fig. 1(b)]. To lowest order the correction to the Rosenbluth formula[12] is given by the interference of this diagram with the corresponding one-photon-exchange diagram [Fig. 1(a)]. The relevant amplitudes can be read off directly from Eq. (2.8) to be

$$\begin{aligned}\mathfrak{M}^\gamma=&-i\frac{e^2}{q^2}\bar u(p',s')\gamma_\mu u(p,s)\\ &\times\bar u(P',S')\left[G_M\gamma^\mu-F_2(P+P')^\mu\right]u(P,S),\end{aligned}\tag{3.1}$$

$$\begin{aligned}\mathfrak{M}^{Z^0}=&-i\frac{G}{\sqrt2}\bar u(p',s')\gamma_\mu(4\sin^2\theta_W-1+\gamma_5)u(p,s)\\ &\times\bar u(P',S')\Gamma_Z^\mu u(P,S),\end{aligned}\tag{3.2}$$

where Γ_Z^μ is defined in Eq. (2.9). In the nucleon laboratory frame, $P=(M,\vec 0)$, we get for the differential cross section (neglecting the electron mass m)

$$\begin{aligned}\frac{d\sigma}{d\Omega}=\left(\frac{d\sigma}{d\Omega}\right)_M\Bigg[&(F_1^{\rm eff})^2-q^2(F_2^{\rm eff})^2-\frac{q^2}{2M^2}(G_M^{\rm eff})^2\tan^2(\theta/2)\\ &+2\sqrt2\frac{G}{e^2}q^2\frac{E}{M}G_Mg_A^0\tan^2(\theta/2)\Bigg],\end{aligned}\tag{3.3}$$

where E and θ denote the electron's laboratory energy and scattering angle, respectively, and

$$\left(\frac{d\sigma}{d\Omega}\right)_M=\frac{\alpha^2}{4E^2\sin^4(\theta/2)}\frac{\cos^2(\theta/2)}{1+(2E/M)\sin^2(\theta/2)}$$

is the Mott cross section with $\alpha=e^2/4\pi=\frac{1}{137}$. Neglecting terms of order G^2, the effective nucleon form factors are given by

TABLE I. Normalizations of form factors at $q^2=0$. The anomalous magnetic moments are, in units of nuclear magnetons, given by $\mu_p=1.79$ and $\mu_n=-1.91$.

	Proton	Neutron
$F_1(0)$	1	0
$F_2(0)$	$\mu_p/2M$	$\mu_n/2M$
$G_E(0)$	1	0
$G_M(0)$	$1+\mu_p$	μ_n
$g_V^0(0)$	$\frac{1}{2}(1+\mu_p-\mu_n)-2(1+\mu_p)\sin^2\theta_W$	$\frac{1}{2}(1+\mu_p-\mu_n)-2\mu_n\sin^2\theta_W$
$f_V^0(0)$	$\frac{1}{4M}(\mu_p-\mu_n)-\frac{1}{M}\mu_p\sin^2\theta_W$	$\frac{1}{4M}(\mu_p-\mu_n)-\frac{1}{M}\mu_n\sin^2\theta_W$
$g_V^0(0)-2Mf_V^0(0)$	$-\frac{1}{2}(4\sin^2\theta_W-1)$	$\frac{1}{2}$
$g_V(0)$	$1+\mu_p-\mu_n$	
$f_V(0)$	$\frac{1}{2M}(\mu_p-\mu_n)$	
$g_V(0)-2Mf_V(0)$	1	
$g_A(0)$	1.2	
$g_A^0(0)$	0.6	

$$
\begin{aligned}
(F_1^{\mathrm{eff}})^2 &= F_1{}^2-\sqrt{2}\,\frac{G}{e^2}(4\sin^2\theta_W-1)q^2F_1(g_V^0-2Mf_V^0)\,,\\
(F_2^{\mathrm{eff}})^2 &= F_2{}^2-\sqrt{2}\,\frac{G}{e^2}(4\sin^2\theta_W-1)q^2F_2f_V^0\,, \qquad (3.4)\\
(G_M^{\mathrm{eff}})^2 &= G_M{}^2-\sqrt{2}\,\frac{G}{e^2}q^2G_M[(4\sin^2\theta_W-1)g_V^0-g_A^0].
\end{aligned}
$$

These effective form factors differ very little from the purely electromagnetic ones. Even at $-q^2=10^3$ $(\mathrm{GeV}/c)^2$ this difference amounts to a few percent only. Owing to the theoretical uncertainties in the high-q^2 behavior of the electromagnetic form factors, such small corrections remain unobservable. Thus the only relevant deviation from the Rosenbluth formula comes from the last term in Eq. (3.3) with a strongly nonlinear $\tan^2(\theta/2)$ dependence, and which turns out to be independent of the free parameter of the theory. This term

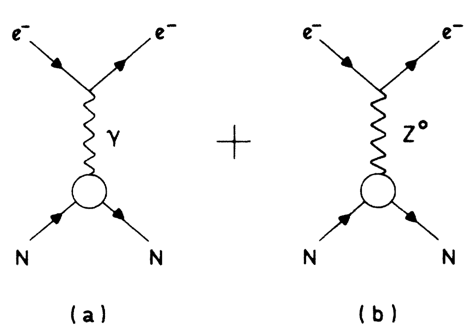

FIG. 1. Lowest-order Feynman diagrams contributing to elastic electron-nucleon scattering.

which is due to the parity-violating Z^0 current is best extracted by plotting $(d\sigma/d\Omega)/(d\sigma/d\Omega)_M$ versus $\tan^2(\theta/2)$. Any deviation from a straight line represents a neutral-current (parity-violating) effect. The normalized slope is given by

$$\xi=1-4\sqrt{2}\,(G/e^2)ME\,G_M g_A^0/(G_M^{\mathrm{eff}})^2\,, \qquad (3.5)$$

where $\xi=1$ corresponds to the Rosenbluth prediction. In Fig. 2 we plot ξ versus θ at various fixed values of q^2 for elastic e^-p scattering. We used Eqs. (2.14), (2.15), and (2.16) together with

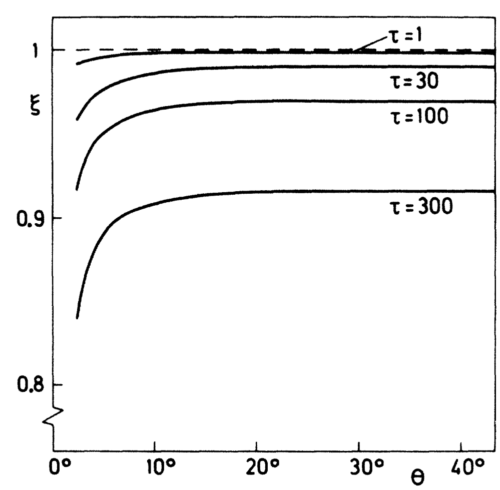

FIG. 2. Variation of the elastic e^-p slope relative to the Rosenbluth slope (----) as a function of the lab scattering angle θ, according to Eq. (3.5), for various fixed values of q^2 with $\tau=-q^2/4M^2$.

Table I, and $\sin^2\theta_W \simeq \frac{1}{3}$ corresponding to the recent CERN-Gargamelle experiment.[13] Whereas the change in the slope is very small for $-q^2 \simeq 3$ $(\mathrm{GeV}/c)^2$, the effect becomes noticeable at $-q^2 \simeq 100$ $(\mathrm{GeV}/c)^2$ and quite large at still higher q^2. The nonlinear effects are especially prominent in the forward direction and should best be searched for there. It should be noted, however, that radiative corrections produce additional modifications of the Rosenbluth formula. Calculations[14] of these corrections ignore strong-interaction effects and extract only leading infrared terms of the two-photon diagrams. The uncertainties due to these approximations are about 1%. Thus, according to Fig. 2, it appears to be difficult to separate neutral-current effects from higher-order radiative corrections at the present available energies.

Therefore neutral currents preferably ought to be looked for in parity-violating effects which do not depend on radiative corrections. This case will be discussed in the subsequent section.

IV. LEFT-RIGHT ASYMMETRIES

A more direct way to study neutral-current effects is through polarization experiments. We consider only left-right asymmetries since they should vanish to all orders in QED and their observation to order αG would constitute direct proof of parity-violating neutral currents. Polarized targets are already in use and polarized electron beams should be available with the newly planned colliding-beam machines.[3,5] We therefore consider both cases.

A. Polarized electrons

We choose the axis of spin quantization along the direction of the incoming electron beam and define the left-right asymmetry as

$$P_B = \left(\frac{d\sigma_R}{d\Omega} - \frac{d\sigma_L}{d\Omega}\right) \Big/ \left(\frac{d\sigma_R}{d\Omega} + \frac{d\sigma_L}{d\Omega}\right). \tag{4.1}$$

The labels refer to right-handed (R) electrons when the electron spin is parallel to the beam direction $\vec{p}$ (positive helicity), and to left-handed (L) electrons when the spin is antiparallel to $\vec{p}$ (negative helicity). The appropriate amplitudes are similar to those in (3.1) and (3.2). To get $\mathfrak{M}_R$ we just replace $u(p,s)$ by $\frac{1}{2}(1+\gamma_5 \not{s}_R)u(p,s)$ in Eqs. (3.1) and (3.2), where s_R is the spin four-vector of positive-helicity electrons. A similar substitution with $s_L = -s_R$ yields $\mathfrak{M}_L$. The total amplitudes with definite helicity are then simply given by $\mathfrak{M}_i^{\gamma+Z^0} = \mathfrak{M}_i^{\gamma} + \mathfrak{M}_i^{Z^0}$, with $i = R, L$. A straightforward calculation then yields for the asymmetry

$$P_B = \frac{|\mathfrak{M}_R^{\gamma+Z^0}|^2 - |\mathfrak{M}_L^{\gamma+Z^0}|^2}{|\mathfrak{M}_R^{\gamma+Z^0}|^2 + |\mathfrak{M}_L^{\gamma+Z^0}|^2}, \tag{4.2}$$

with

$$|\mathfrak{M}_R^{\gamma+Z^0}|^2 - |\mathfrak{M}_L^{\gamma+Z^0}|^2 = -\frac{e^2}{q^2}\frac{G}{\sqrt{2}}\frac{1}{m^2M^2}\{[F_1(g_V^0 - 2Mf_V^0) - q^2F_2f_V^0][4M^2E^2 + 2MEq^2 + M^2q^2] + 2(4\sin^2\theta_W - 1)G_M g_A^0 MEq^2 + \tfrac{1}{2}G_M[g_V^0 + (4\sin^2\theta_W - 1)g_A^0]q^4\} + O(G^2) \tag{4.3}$$

and

$$|\mathfrak{M}_R^{\gamma+Z^0}|^2 + |\mathfrak{M}_L^{\gamma+Z^0}|^2 = \frac{e^4}{q^4}\frac{1}{2m^2M^2}\{(F_1{}^2 - q^2F_2{}^2)[4M^2E^2 + 2MEq^2 + M^2q^2] + \tfrac{1}{2}G_M{}^2q^4\} + O(e^2G, G^2). \tag{4.4}$$

To lowest order, Eq. (4.4) is just twice the square of the Rosenbluth amplitude. To get an idea of the size of P_B we consider two limiting cases:

(i) The limit

$$L_1:\ -\frac{q^2}{4M^2} \ll 1 \text{ and } -q^2 \ll ME, \tag{4.5}$$

appropriate for low- and intermediate-energy experiments. Under these conditions, (4.2) simplifies, for the case of a proton target, to

$$P_B^p \underset{L_1}{\simeq} -\sqrt{2}\,\frac{G}{e^2}q^2(g_V^{0p} - 2Mf_V^{0p})/F_1^p. \tag{4.6}$$

Using Eqs. (2.14) to (2.16) and Table I we obtain

$$P_B^p \underset{L_1}{\simeq} 0.8\times 10^{-4}(4\sin^2\theta_W - 1)(q^2/M^2) \simeq 3\times 10^{-5}(q^2/M^2), \tag{4.7}$$

where again $\sin^2\theta_W \simeq \frac{1}{3}$ was used. Similarly, we obtain for elastic e^-n scattering[15]

$$P_B^n \underset{L_1}{\simeq} +\sqrt{2}\,\frac{G}{e^2} \times q^2\,\frac{[-M^2F_2^n f_V^{0n} + (4\sin^2\theta_W - 1)G_M^n g_A^0(M/2E)]}{(MF_2^n)^2}. \tag{4.8}$$

For a typical low-energy experiment with $E/M = \frac{1}{3}$, say, we expect the left-right asymmetry to be

$$P_B^n \underset{L_1}{\simeq} 1.5\times 10^{-4}(q^2/M^2). \tag{4.9}$$

This is almost an order of magnitude higher than for a proton target. Although such tiny asymmetries are at present beyond experimental accuracy by at least an order of magnitude, there is some hope of observing them. The idea is to choose a reasonably large $-q^2$, but not too large so that the statistics will not suffer from the decreasing form factors. The mere observation that $P_B \neq 0$ and a very rough order-of-magnitude agreement with the predictions would suffice to prove the existence of a neutral current, i.e., parity-violating effects. Thus, using a deuteron target it really would not be necessary to worry about Glauber corrections, for example.

(ii) For high-energy experiments a suitable limit is

$$L_2:\ \ -q^2/M^2 \gg 1 \text{ and } (E/M)(1-\cos\theta)\gg 1. \tag{4.10}$$

Here the asymmetry takes the form

$$P_B \underset{L_2}{\simeq} -\sqrt{2}\,\frac{G}{e^2}q^2\,\frac{\frac{1}{2}G_M[g_V^0-(4\sin^2\theta_W-1)g_A^0]+F_2 f_V^0 M^2\cot^2(\theta/2)}{\frac{1}{2}G_M^2+F_2^2M^2\cot^2(\theta/2)}. \tag{4.11}$$

Specializing to the case of a proton target we obtain, for $\sin^2\theta_W\simeq\frac{1}{3}$,

$$P_B^p \underset{L_2}{\simeq} -1.5\times 10^{-4}\frac{q^2}{M^2}\,\frac{0.41+0.29\cot^2(\theta/2)}{3.89+0.81\cot^2(\theta/2)}. \tag{4.12}$$

For a neutron target the corresponding result is

$$P_B^n \underset{L_2}{\simeq} +1.5\times 10^{-4}\frac{q^2}{M^2}\,\frac{3.26+1.49\cot^2(\theta/2)}{1.82+0.91\cot^2(\theta/2)}. \tag{4.13}$$

Again we find an order-of-magnitude difference between p and n. For example, at $\theta=90°$,

$$P_B^p \underset{L_2}{\simeq} -2.2\times 10^{-5}(q^2/M^2),$$
$$P_B^n \underset{L_2}{\simeq} 2.6\times 10^{-4}(q^2/M^2). \tag{4.14}$$

Thus, at the momentum transfers available at NAL, CERN II, and the next generation of accelerators and eN storage rings,[3] these parity-violating effects grow with q^2 and approach, at characteristic values of $-q^2\simeq 10^3\ (\text{GeV}/c)^2$, the 5% and 30% level for protons and neutrons, respectively. Since high-energy elastic electron-nucleon scattering yields also very interesting information on electromagnetic form factors, these experiments should be carried out to as large $-q^2$ values as possible.

B. Nucleon polarization

The target left-right asymmetry is calculated in the same way as in Sec. IV A. The nucleon target is defined to have positive or negative helicity if it is polarized parallel or antiparallel to the incoming beam, respectively. Again we only have to make a simple replacement in the Rosenbluth amplitudes (3.1) and (3.2), i.e., replace $u(P,S)$ by $\frac{1}{2}(1+\gamma_5 \not{S}_R)u(P,S)$ for the positive-helicity amplitudes. In the negative-helicity amplitudes, S_L appears with $S_L=-S_R$. The nucleon target polarization is then given by

$$P_T=\frac{|\mathcal{T}_R^{\gamma+Z^0}|^2-|\mathcal{T}_L^{\gamma+Z^0}|^2}{|\mathcal{T}_R^{\gamma+Z^0}|^2+|\mathcal{T}_L^{\gamma+Z^0}|^2}, \tag{4.15}$$

where $\mathcal{T}_i^{\gamma+Z^0}$ denotes the total target polarization amplitudes, similar to $\mathfrak{M}_i^{\gamma+Z^0}$. After substituting for these amplitudes one obtains

$$\begin{aligned}|\mathcal{T}_R^{\gamma+Z^0}|^2-|\mathcal{T}_L^{\gamma+Z^0}|^2=-\frac{e^2}{q^2}\frac{G}{\sqrt{2}}\frac{1}{m^2M^2}\Big[&G_M g_V^0\left(2MEq^2+\frac{Mq^4}{2E}+\frac{q^4}{2}\right)-M(F_2g_V^0+G_Mf_V^0)\left(2MEq^2+\frac{Mq^4}{2E}+q^4\right)\\&+(4\sin^2\theta_W-1)G_Mg_A^0\left(4M^2E^2+2MEq^2+\frac{Mq^4}{2E}+M^2q^2+\frac{q^4}{2}\right)\\&-(4\sin^2\theta_W-1)MF_2g_A^0\left(8M^2E^2+6MEq^2+\frac{Mq^4}{2E}+2M^2q^2+q^4\right)\Big]+O(G^2)\end{aligned} \tag{4.16}$$

and $|\mathcal{T}_R^{\gamma+Z^0}|^2+|\mathcal{T}_L^{\gamma+Z^0}|^2$ is, to our approximation, again just given by the Rosenbluth amplitude (4.4). To see the size of possible parity-violating effects we consider again the two limiting cases L_1 and L_2 defined in (4.5) and (4.10).

(i) In the low-q^2 limit L_1 we have for proton targets

$$P_T^p \underset{L_1}{\simeq} -\sqrt{2}\,\frac{G}{e^2}q^2(4\sin^2\theta_W-1)g_A^0/F_1^p \simeq -3\times10^{-5}(q^2/M^2)\,, \tag{4.17}$$

where, for the last line, we used Eqs. (2.14) to (2.16), Table I, and $\sin^2\theta_W\simeq\frac{1}{3}$. For neutron targets the corresponding expression is, keeping in mind that $F_1^n\simeq 0$,

$$P_T^n \underset{L_1}{\simeq} \frac{G}{\sqrt{2}\,e^2}\frac{M}{E}\,q^2[G_M^n g_V^{0n}-MG_M^n f_V^{0n}-MF_2^n g_V^{0n}+(4\sin^2\theta_W-1)(G_M^n g_A^0-3MF_2^n g_A^0)]/(MF_2^n)^2$$
$$=\frac{G}{e^2}\frac{\sqrt{2}}{\mu_n}\frac{M}{E}q^2[\tfrac{1}{2}-(4\sin^2\theta_W-1)g_A^0(0)]\,, \tag{4.18}$$

where in the last line we have made use of (2.14)–(2.16) and Table I. Again taking, for example, $E/M=\frac{1}{3}$, (4.18) predicts

$$P_T^n \underset{L_1}{\simeq} -0.7\times10^{-4}(q^2/M^2) \tag{4.19}$$

for $\sin^2\theta_W\simeq\frac{1}{3}$. Up to a sign (because of our helicity convention), the asymmetries for polarized protons and neutrons are of the same order of magnitude as those for polarized (electron) beams.

(ii) In the high-q^2 limit L_2, as defined in (4.10), the asymmetry becomes

$$P_T \underset{L_2}{\simeq} \sqrt{2}\,\frac{G}{e^2}q^2\,\frac{\frac{1}{2}G_M[g_V^0-(4\sin^2\theta_W-1)g_A^0]}{\frac{1}{2}G_M^2+F_2^2M^2\cot^2(\theta/2)}\,, \tag{4.20}$$

which yields for proton targets, using (2.14)–(2.16), Table I, and $\sin^2\theta_W\simeq\frac{1}{3}$,

$$P_T^p \underset{L_2}{\simeq} +1.5\times10^{-4}\,\frac{q^2}{M^2}\,\frac{0.41}{3.89+0.81\cot^2(\theta/2)}\,. \tag{4.21}$$

Similarly for neutron targets one gets

$$P_T^n \underset{L_2}{\simeq} -1.5\times10^{-4}\,\frac{q^2}{M^2}\,\frac{3.26}{1.82+0.91\cot^2(\theta/2)}\,. \tag{4.22}$$

The asymmetry for the neutron is again an order of magnitude larger than that of the proton. For $\theta=90°$ one obtains, for example,

$$P_T^p \underset{L_2}{\simeq} 1.3\times10^{-5}(q^2/M^2)\,,\qquad P_T^n \underset{L_2}{\simeq} -1.8\times10^{-4}(q^2/M^2)\,. \tag{4.23}$$

These results are again similar to those for a polarized beam and the same discussion applies as at the end of Sec. IV A.

However, it should be emphasized that the numerical predictions for the asymmetries depend solely on the scaling assumptions for the various form factors, Eq. (2.14)–Eq. (2.16), which have been established experimentally only for moderate q^2 values of about $-q^2\lesssim 5$ (GeV/c)^2. Thus the predictions at ultrahigh values of $-q^2$ may well be modified.

V. CONCLUSIONS

In considerable detail we have discussed and calculated neutral-current contributions to the usual lowest-order one-photon-exchange diagram of elastic electron-nucleon scattering. A specific but in itself rather general gauge model for leptonic as well as semileptonic weak and electromagnetic interactions has been used. The size of parity-violating effects turns out to differ significantly from that naively estimated on purely dimensional grounds, and is strongly dependent on the nucleon target used.

As far as nonpolarization experiments are concerned, corrections to the elastic electromagnetic nucleon form factors turn out to be practically negligible even for $-q^2$ out to 10^3 (GeV/c)^2 where they amount to a few percent only. A more serious deviation from the standard Rosenbluth formula comes from a strongly nonlinear term in $\tan^2(\theta/2)$, which turns out to be independent of the free parameter of the model (the Weinberg mixing angle) and arises because of the intrinsic, parity-violating structure of the theory. Thus, with appropriate modifications of the form factors involved, the structure of this correction term is rather model-independent and will be similar in different unified models of weak and electromagnetic interactions. These neutral-current corrections are already noticeable at $-q^2\simeq 50$ (GeV/c)^2 and especially prominent in the near forward direction. Such effects should be taken into account in future high-q^2 and high-precision measurements of electromagnetic nucleon form factors, which are usually extracted from the standard Rosenbluth formula (after radiative corrections have been subtracted).

Considering elastic electron-nucleon scattering processes, neutral-current effects can be best looked for in longitudinal-polarization experiments. For example, *any* dependence of electron-nucleon scattering on the longitudinal polarization of the electrons (or nucleons) would be evidence of parity violation. The size of these effects differs significantly from that naively expected on purely dimensional grounds, and in general they are about one order of magnitude larger for electron-neutron scattering than for electron-proton reactions. Regardless if longitudinally polarized electrons or nucleons are used, the magnitude of these asymmetries stays practically the same. At low and intermediate energies such 10^{-4} effects should be observable in future very-high-statistics experiments. At high energies, these neutral-current (parity-violating) effects approach, at characteristic values of $-q^2 \simeq 10^3$ (GeV/c)^2, the 5% and 30% level for protons and neutrons, respectively. With the next generation of accelerators and electron-nucleon storage rings, such measurements appear to be feasible.

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