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Dispersion calculation of the transition form factor $F_{\pi\omega\gamma}(t)$ with cut contributions*

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In this paper we study the t dependence of the $\gamma\pi\omega$ vertex functions on the basis of partial-wave dispersion relations and unitarity. The right-hand cut is approximated by the 2π contribution and the left-hand cut by the nearest s- and u-channel poles. The electromagnetic $\pi\omega$ transition form factor is calculated as a function of t in the timelike and spacelike region and is compared with predictions of the ρ-dominance model and with recent experimental data for $e^+e^- \to \omega\pi^0 \to \pi^+\pi^-\pi^0\pi^0$ near threshold. The influence of the left-hand cut and the finite width of the ρ resonance is explicitly shown.

I. INTRODUCTION

The construction of electron-positron storage rings has opened a new field in high-energy physics. Interest is centered on $e^\pm$-$e^\pm$ interactions, both in annihilation and in scattering processes. In the high-energy region these reactions are accompanied by hadron production.

Experimental results from storage rings are already available, especially from the Orsay, Novosibirsk, and Frascati rings.[1-5] These give an idea what pion and kaon form factors look like in the timelike region and also yield annihilation cross sections for multimesonic final states (where the latter can partly be understood as quasi-two-body states).

The description of e^+e^- annihilation into hadronic two-body or quasi-two-body states leads in the one-photon approximation—which will be one of our basic assumptions—to the definition of form factors and transition form factors in timelike region. The kinematic structure of such reactions has been given in detail by Kramer and Walsh.[6]

The best known example, from the theoretical as well as the experimental point of view, is that of the two-pion final state which reduces to the description of the pion form factor. This problem has been studied many times, and we refer, for instance, to the calculations of Frazer and Fulco,[7] Gounaris and Sakurai, Schwarz, Aubrecht, Renard, and Bonneau and Martin.[8] $K\bar{K}$ production has been analyzed, for instance, by Renard.[8,9]

The next step in e^+e^- annihilation is the production of three-pion final states. The case of the $\pi^+\pi^-\pi^0$ channel has been analyzed by KW in a model based on $\rho^0\pi^0$ production. They also discuss 4π production by resonance formation using a vector-dominance model as has been done by other authors.[10]

We shall concentrate on the production channel $\pi^+\pi^-\pi^0\pi^0$. In contrast with KW and others, we study the influence of the left-hand cut and subsequent finite-width corrections on the resulting transition form factors.

Finite-width corrections in connection with analyticity have been the subject of many discussions in the past especially in studying pion and kaon form factors.[8,9] There exists also a dynamical model for the reaction $e^+e^- \to \rho^0 \to \pi^0 + \omega$ ($\pi^+\pi^-\pi^0$) due to Renard.[11] He assumes Breit-Wigner shapes for ρ^0 and the decaying ω meson. His results reduce to that of KW if one goes to the zero-width limit.

For energies at which resonance production should dominate the annihilation cross section, many channels are opening successively. The uncertainty of the decay width of the several possible resonances leads to nearly unknown coupling parameters. KW have discussed the dependence of the annihilation cross sections on these couplings and concluded that within an energy region from threshold up to 2.5-GeV e^+e^- center-of-mass energy the dominant contributions should be given by the channels $\pi^0\omega$, πA_1, and $\rho^0\epsilon$.

Recent experiments[12] at ADONE about F_π favor the Gounaris-Sakurai approximation[8] after K^+K^- pairs have been discriminated from $\pi^+\pi^-$ pairs in $e^+e^- \to \pi^+\pi^-$ (K^+K^-). This means that the 2π contribution in the unitarity relation is the dominant one. It seems reasonable to us to start with this approximation for other form factors. In this paper we shall treat only the $\pi^0\omega$ channel, which is the best defined of all resonant configurations among $\pi^+\pi^-\pi^0\pi^0$ because of fairly well-known couplings. In this channel one is left with one form factor only. The $\rho^0\epsilon$ channel shall be the subject of later calculations.

In the next section we start with the unitarity relation which leads to an expression for the imaginary part of the helicity form factor. This is directly related to $\operatorname{Im} F_{\pi\omega\gamma}(t)$. In the following section we describe a dispersion method from which we obtain the partial-wave helicity amplitude for $\pi\pi \to \pi\omega$.

In Sec. IV we consider in some details the partial-wave amplitude. For this we have to decide how to handle the left-hand-cut contributions which enter the dispersion relation. One possibility is to describe these contributions by a constant N function or a simple pole ansatz for this function within an N/D method. This has been done by Frazer and Fulco,[7] Gounaris and Sakurai,[8] Renard, and Bonneau and Martin,[8] for the elastic $\pi\pi$ and the $\pi\pi \to \pi\omega$ partial-wave amplitude, respectively, within a coupled analysis. Another possibility is to approximate the left-hand cut by the corresponding one-particle exchange terms. This description has been chosen by Frazer and Fulco[7] and Höhler *et al.*[13] for the $\pi\pi \to N\bar{N}$ amplitude, and by Aubrecht[8] for the elastic $\pi\pi$ amplitude. We shall use the latter method and assume one-particle-exchange dominance for the left-hand cut. This means that the couplings of the ρ meson to $\pi\omega$ and $\pi\pi$ fully determine the input. We believe this to be a reasonable prescription within the energy region of interest.

In Sec. V we summarize the results for $F_{\pi\omega\gamma}(t)$ and also give the annihilation cross section for $e^+e^- \to \pi^0\omega$. In Sec. VI we discuss the numerical results and compare them with the pole-dominance approximation. Finally we end with a short conclusion.

II. UNITARITY RELATION FOR STRUCTURE FUNCTIONS

The unitarity relation in the 2π approximation for a vertex function corresponding to Fig. 1 reads

$$\operatorname{Im}[\epsilon^\mu_{\lambda_\gamma}\langle p_b, p_a|j_\mu(0)|0\rangle] = -\frac{(2\pi)^4}{2}\int\int\frac{d^3q_1}{2q_1^0}\frac{d^3q_2}{2q_2^0}\,\delta^{(4)}(q_1+q_2-p_b-p_a)\langle p_b, p_a|T|q_1, q_2\rangle^*\epsilon^\mu_{\lambda_\gamma}\langle q_1, q_2|j_\mu(0)|0\rangle, \tag{2.1}$$

where $\epsilon^\mu_{\lambda_\gamma}$ is the polarization vector of the virtual photon and j_μ is the electromagnetic current. Furthermore, we have

$$(2\pi)^3\langle q_1, q_2|j_\mu(0)|0\rangle = (q_1-q_2)_\mu F_\pi(Q^2), \tag{2.2}$$

with $Q = q_1+q_2$ being the four-momentum of the virtual photon. $\langle p_b, p_a|T|q_1, q_2\rangle^*$ is the matrix element for the transition $\pi(q_1)+\pi(q_2) \to b(p_b)+a(p_a)$.

We go to the rest frame of the virtual photon and choose the center-of-mass momenta as $\vec{q} = \vec{q}(\theta, 0)$, $\vec{p} = \vec{p}(\theta_n, \phi_n)$. Attaching the four-momentum p_b to the ω meson with polarization vector $\epsilon_{\lambda_\omega}(p_b)$ we define the helicity amplitude for $\pi(q_1)+\pi(q_2) \to \omega(p_b, \lambda_\omega)+\pi(p_a)$ by

$$T_{\lambda_\omega}(t, \Omega_{nf}) = (2\pi)^6\langle\omega(p_b, \lambda_\omega), \pi(p_a)|T|\pi(q_1), \pi(q_2)\rangle,$$

where $t = Q^2$ and Ω_{nf} is related to the angles between $\vec{p}$ and $\vec{q}$.

With this and an expansion of the helicity amplitude in the series of rotation functions $d^J_{mm'}(\theta)$ (see

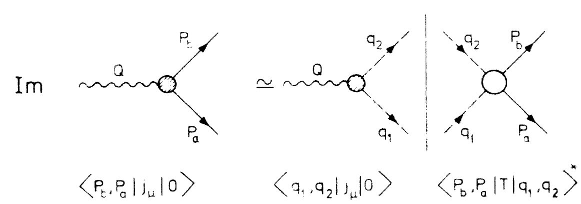

FIG. 1. Unitarity relation for 2π intermediate state.

Ref. 14) we get from (2.1) and (2.2)

$$\mathrm{Im}[\epsilon^{\mu}_{\lambda_\gamma}\langle\omega(p_b,\lambda_\omega),\pi(p_a)|j_\mu(0)|0\rangle] = -\frac{1}{(2\pi)^4}\frac{|\vec{q}|^2}{2\sqrt{t}}F_\pi(t)T^{1*}_{\lambda_\omega}(t)d^{1}_{\lambda_\gamma\lambda_\omega}(\theta), \quad (2.3)$$

where the $T^1_{\lambda_\omega}(t)$ are the $I=J=1$ partial-wave helicity amplitudes for the transition $\pi^+ + \pi^- \to \pi^0 + \omega$.

The number of independent helicity amplitudes is fixed by parity conservation.[15] This leads to $T^1_{\lambda_\omega}(t) = -T^1_{-\lambda_\omega}(t)$, so that there remains only one independent amplitude which by (2.3) corresponds to a single structure function $F_{\pi\omega\gamma}(t)$.

For $F_{\pi\omega\gamma}(t)$ we make the ansatz

$$(2\pi)^3J(\lambda_\gamma,\lambda_\omega) = (2\pi)^3\epsilon^{\mu}_{\lambda_\gamma}\langle\omega(p_b,\lambda_\omega),\pi^0(p_a)|j_\mu(0)|0\rangle = i\epsilon_{\mu\tau\rho\sigma}\epsilon^{\mu}_{\lambda_\gamma}\epsilon^{\tau *}_{\lambda_\omega}(p_b)p_b^{\rho}Q^{\sigma}F_{\pi\omega\gamma}(t), \quad (2.4)$$

which is justified by the Lorentz-transformation properties of the matrix element and has been given by KW in Eq. (3.4).

On the other hand, we conclude from the transformation property of the state

$$j_{\lambda_\gamma}(0)|0\rangle \equiv \epsilon^{\mu}_{\lambda_\gamma}j_\mu(0)|0\rangle$$

(see Ref. 6) in the rest system of the virtual photon

$$(2\pi)^3J(\lambda_\gamma,\lambda_\omega) = (2\pi)^3\langle|\vec{p}|,\theta,\lambda_\omega|j_{\lambda_\gamma}(0)|0\rangle = \hat{\Gamma}(|\vec{p}|,\lambda_\omega)d^{1}_{\lambda_\gamma\lambda_\omega}(\theta), \quad (2.5)$$

which defines the helicity structure functions $\hat{\Gamma}(|\vec{p}|,\lambda_\omega) \equiv \Gamma(t,\lambda_\omega)$. Comparing this with (2.4) we obtain after some kinematics

$$\Gamma(t,\lambda_\omega) = -\Gamma(t,-\lambda_\omega) = -\lambda_\omega|\vec{p}|\sqrt{t}\,F_{\pi\omega\gamma}(t), \quad (2.6)$$

and finally from (2.3)

$$\mathrm{Im}\,\Gamma(t,\lambda_\omega) = -\frac{|\vec{q}|^2}{4\pi\sqrt{t}}F_\pi(t)T^{1*}_{\lambda_\omega}(t). \quad (2.7)$$

III. PARTIAL-WAVE DISPERSION RELATION

Next we shall develop a dispersion relation which gives the $I=J=1$ partial wave for $\pi\pi\to\pi\omega$ in terms of the Gounaris-Sakurai D function, normalized to $D(0)=1$.

The Gounaris-Sakurai model leads to a generalized effective range approximation for the $I=J=1$ $\pi\pi$ phase shift which defines the corresponding elastic $\pi\pi$ scattering amplitude $b_{\pi\pi}(t)$. One obtains the following representation for the pion form factor (using hereafter $m_\pi \equiv \mu = 1$):

$$F_\pi(t) = \frac{b_{\pi\pi}(t)}{b_{\pi\pi}(0)} = \frac{1}{D(t)} = \frac{f(m_\rho^2)}{m_\rho^2 - t + g(t) - im_\rho\Gamma_\rho(t)}, \quad (3.1)$$

with $b_{\pi\pi}(0)$ real and

$$f(m_\rho^2) = m_\rho^2\left(1+\delta\frac{\Gamma_\rho}{m_\rho}\right),$$

$$g(t) = \frac{m_\rho^2\Gamma_\rho}{q_\rho^3}[k(t) - k(m_\rho^2) - (t-m_\rho^2)k'(m_\rho^2)], \quad (3.2)$$

$$k(t) = \frac{2q^3}{\pi\sqrt{t}}\ln(q+\tfrac{1}{2}\sqrt{t}).$$

δ is known as the finite-width correction and is determined by $F_\pi(0)=1$. The function $g(t)$ is of second order in $(t-m_\rho^2)$ near the ρ-meson pole and $\Gamma_\rho(t)$ is the usual energy-dependent width

$$\Gamma_\rho(t) = \frac{m_\rho}{\sqrt{t}}\left(\frac{q}{q_\rho}\right)^3\Gamma_\rho, \qquad q_\rho = q(m_\rho^2), \quad q = |\vec{q}| \quad (3.3)$$

in terms of the physical ρ width $\Gamma_\rho = \Gamma_{\rho\to 2\pi}$.

The starting point for the determination of the $I=J=1$ partial-wave amplitude $t^1 \equiv t^{11}_{\pi\pi\to\pi\omega}$ is the approximated unitarity relation $\mathrm{Im}\,t^1 = -(q^3/\sqrt{t})b^*_{\pi\pi}t^1$. On the right-hand cut (R) $b^{-1}_{\pi\pi}$ by definition has the same phase as the D function. Therefore $\mathrm{Im}_R[D(t)t^1(t)]=0$ and we can write down an integral representation for $D(t)t^1(t)$ using

$$\mathrm{Im}_R[D(t)t^{1,R}(t)] = -\mathrm{Im}_R[D(t)t^{1,L}(t)] = -t^{1,L}\,\mathrm{Im}_R D(t). \quad (3.4)$$

From this there results the following once-subtracted dispersion relation:

$$t^1(t) - t^{1,L}(t) = \frac{1}{D(t)}\left[a - \frac{(t-t_0)}{\pi}\int_4^\infty \frac{dt'\,t^{1,L}(t')\,\mathrm{Im}\,D(t')}{(t'-t_0)(t'-t)}\right]. \quad (3.5)$$

The index L characterizes the left-hand-cut contributions and we have dropped the index R in front of $D(t')$ because there exists only a right-hand discontinuity for this function. We have chosen a once-subtracted form because $T^1_{\lambda\omega}(t)$ does not converge rapidly enough; it is related to $t^1(t)$ by Eq. (4.6) below.

IV. PARTIAL-WAVE AMPLITUDES FOR $\pi+\pi\to\pi+\omega$

According to (3.5) the helicity partial-wave amplitudes are defined—up to the constant a—by their left-hand-cut contributions. As mentioned in Sec. I, we require these contributions to be given by the pole terms only, which we describe in all channels by ρ-meson exchange. We assume that the exchange of higher mesons like ρ' and g can be neglected.

From parity conservation the general form of the helicity amplitude is given by

$$T_{\lambda_\omega}(t,\theta)=i\epsilon_{\mu\tau\rho\sigma}\epsilon^{\mu *}_{\lambda_\omega}(p_b)q_1^\tau q_2^\rho p_b^\sigma A(s,t,u), \qquad (4.1)$$

with the single (totally symmetric) invariant function $A(s,t,u)$. Choosing a system in which the c.m. momentum $\vec{q}$ is pointed in the z direction and $\vec{p}_b=\vec{p}$ is characterized by the angle θ, we get from (4.1)

$$T_{\lambda_\omega}(t,\theta)=|\vec{p}|\,|\vec{q}|\sqrt{t}\,A(s,t,u)d^1_{01}(\theta),$$

where we have used $d^1_{01}(\theta)=(2)^{-1/2}\sin\theta$. Partial-wave projection leads for $J=1$ to

$$T^1_{\lambda_\omega}(t)=-\tfrac{1}{3}\lambda_\omega|\vec{p}|\,|\vec{q}|\sqrt{t}\,(A_2-A_0),\quad \lambda_\omega=\pm 1, \qquad (4.2)$$

where we have defined

$$A_J(t)=\tfrac{1}{2}\int d\cos\theta\, d^J_{00}(\theta)A(s,t,u).$$

The pole approximation to the invariant amplitude reads

$$A^{\text{pole}}(s,t,u):=A^{\text{pole},L}+A^{\text{pole},R}$$
$$=-2g_1g_2\left(\frac{1}{s-m_\rho^2}+\frac{1}{u-m_\rho^2}+\frac{1}{t-m_\rho^2}\right), \qquad (4.3)$$

where the left-hand contribution $A^{\text{pole},L}$ is given by the first two terms only and we have defined

$$g_1=g_{\rho^0\pi^+\pi^-},\quad g_2=g_{\rho^0\pi^0\omega}.$$

In the t-channel c.m. system we have

$$t=4(\vec{q}^{\,2}+1),$$
$$s=2-p_a^0\sqrt{t}+2|\vec{p}|\,|\vec{q}|\cos\theta, \qquad (4.4)$$
$$u=-t+1+m_\omega^2+p_a^0\sqrt{t}-2|\vec{p}|\,|\vec{q}|\cos\theta,$$

with

$$|\vec{p}|=\left\{\frac{[t-(m_\omega+1)^2][t-(m_\omega-1)^2]}{4t}\right\}^{1/2},$$
$$p_a^0=\frac{t-m_\omega^2+1}{2\sqrt{t}}.$$

This leads to the following projections:

$$A_J^{\text{pole},L}=\frac{g_1g_2}{|\vec{p}|\,|\vec{q}|}[1+(-1)^J]Q_J(Z),$$
$$A_J^{\text{pole},R}=\frac{2g_1g_2}{m_\rho^2-t}\delta_{J,0}. \qquad (4.5)$$

The $Q_J(Z)$ are Legendre functions of second kind with argument

$$Z=\frac{t+2m_\rho^2-m_\omega^2-3}{4|\vec{p}|\,|\vec{q}|}.$$

In order to remove kinematic singularities and threshold factors from $T^1_{\lambda_\omega}(t)$ define

$$\lambda_\omega t^1(t)=\frac{2}{|\vec{p}|\,|\vec{q}|\sqrt{t}}T^1_{\lambda_\omega}(t). \qquad (4.6)$$

With the projection (4.5) this leads to

$$t^{1,\text{pole},L}(t)=\tfrac{4}{3}g_1g_2\frac{[Q_0(Z)-Q_2(Z)]}{|\vec{p}|\,|\vec{q}|}, \qquad (4.7a)$$

$$t^{1,\text{pole},R}(t)=\tfrac{4}{3}g_1g_2\cdot\frac{1}{m_\rho^2-t}, \qquad (4.7b)$$

where the representation (4.7a) is only valid in $4\leq t\leq(m_\omega-1)^2$ and $(m_\omega+1)^2\leq t\leq\infty$. Between these $\vec{p}^{\,2}$ becomes negative and we have to replace

$$t^{1,\text{pole},L}(t)=\frac{2g_1g_2}{|\vec{p}_-|\,|\vec{q}|}\left[(Z_-^2+1)\arctan\frac{1}{Z_-}-Z_-\right], \qquad (4.7c)$$

where

$$|\vec{p}_-|=\left\{\frac{[(m_\omega+1)^2-t][t-(m_\omega-1)^2]}{4t}\right\}^{1/2}$$

and $Z_-=Z(|\vec{p}_-|)$.

The dispersion relation (3.5) still contains an unknown subtraction constant a. If we choose $t_0=m_\rho^2$ and require $t^{1,R}(t)$ to be dominated near the ρ pole by the Born term of the direct channel, a comparison of (3.5) with (4.7b) yields

$$a=\frac{\tfrac{4}{3}g_1g_2}{f(m_\rho^2)},$$

where we have used Eq. (3.1).

Incorporating the foregoing assumptions, we get the dispersion relation

$$t^1(t)=\frac{1}{f(m_\rho^2)D(t)}\left[\tfrac{4}{3}g_1g_2+f(m_\rho^2)t^{1,\text{pole},L}(t)\,\text{Re}D(t)+\frac{m_\rho(t-m_\rho^2)}{\pi}\,\text{P}\int_4^\infty\frac{dt'\,t^{1,\text{pole},L}(t')\Gamma_\rho(t')}{(t'-m_\rho^2)(t'-t)}\right], \qquad (4.8)$$

where P stands for the principal value.

V. TRANSITION FORM FACTOR AND ANNIHILATION CROSS SECTION

Now we are able to give $\text{Im}F_{\pi\omega\gamma}(t)$ in terms of the partial wave $t^1(t)$, using Eqs. (2.6), (2.7), and (4.6):

$$\text{Im}F_{\pi\omega\gamma}(t)=\frac{1}{8\pi}\frac{|\vec{q}|^3}{\sqrt{t}}F_\pi(t)t^{1*}(t). \qquad (5.1)$$

With the known t dependence of the right-hand side we estimate the asymptotic behavior of $\text{Im}F_{\pi\omega\gamma}(t)$ which results in $(t\ln t)^{-1}$. Therefore we may de-

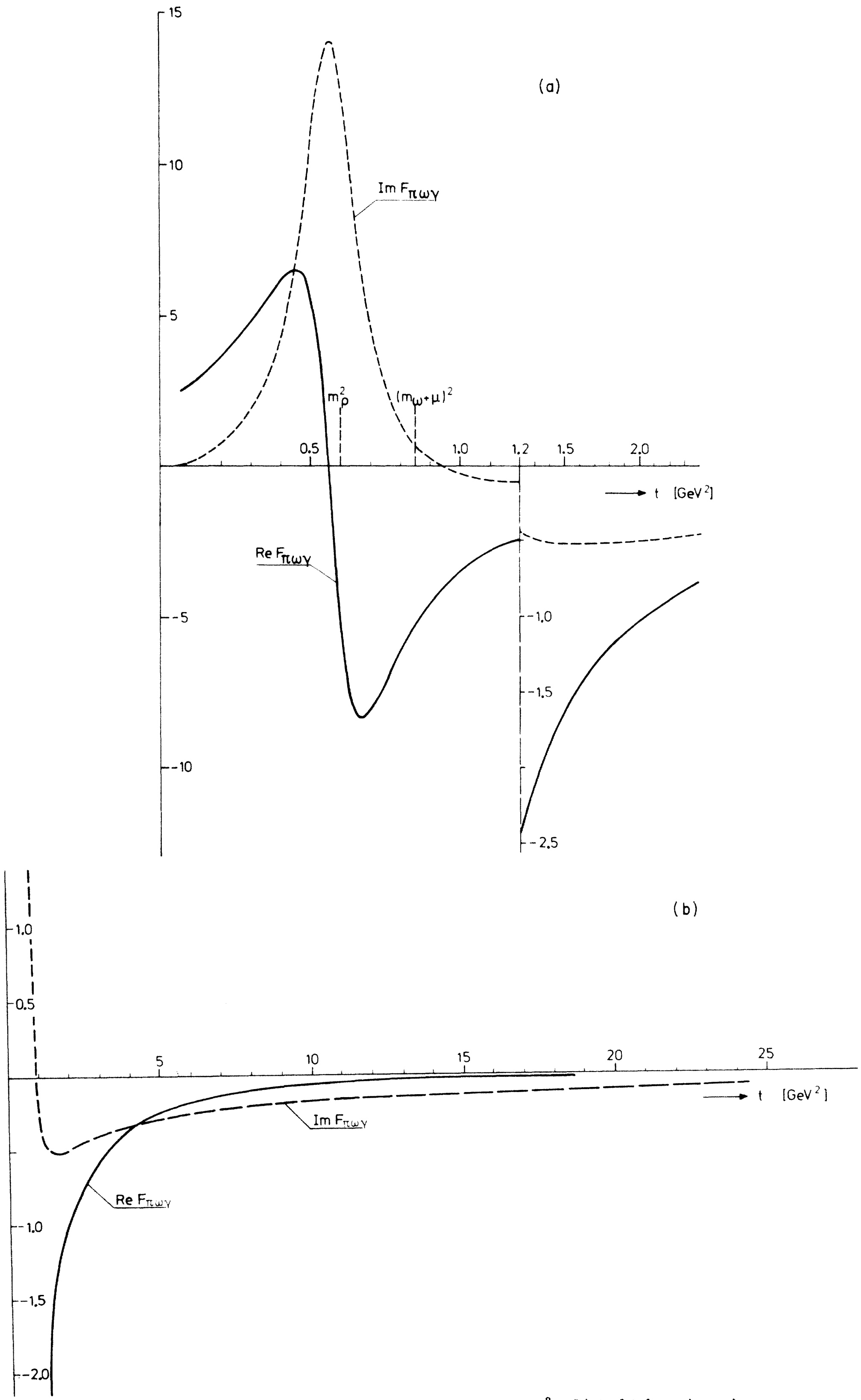

FIG. 2. Im $F_{\pi\omega\gamma}$ and Re $F_{\pi\omega\gamma}$ (a) for $t \leq 2.5$ GeV^2; (b) in higher-t region.

fine $\text{Re}F_{\pi\omega\gamma}(t)$ by the unsubtracted dispersion relation

$$\text{Re}F_{\pi\omega\gamma}(t)=\frac{1}{\pi}\text{P}\int_4^\infty \frac{dt'\,\text{Im}F_{\pi\omega\gamma}(t')}{t'-t}\,. \qquad (5.2)$$

The annihilation cross section $\sigma_{e^+e^-\to\pi^0\omega}$ is, in the one-photon approximation, related to $|F_{\pi\omega\gamma}(t)|$:

$$\sigma_{e^+e^-\to\pi^0\omega}=\frac{4\pi\alpha^2}{3}\frac{|\vec{p}|^3}{m_\omega^2 t^{3/2}}|F_{\pi\omega\gamma}(t)|^2\,, \qquad (5.3)$$

where α is the fine structure constant. Equation (5.3) differs by a factor m_ω^{-2} from the KW result, their Eq. (3.6), because we deal with dimensionless form factors and couplings throughout this paper.

Finally, we are interested in the comparison of our results with those corresponding to vector dominance. Reducing our model to the zero-width approximation, we get

$$F_{\pi\omega\gamma}(t)=\frac{g_{\rho\omega\pi}}{g_\rho}\frac{m_\rho^2}{m_\rho^2-t}\,, \qquad (5.4)$$

which agrees with Eq. (5.2) of KW. Within the Gounaris-Sakurai framework $g_\rho=g_{\rho\to e^+e^-}$ is related to $g_{\rho\pi\pi}$ by

$$g_\rho=g_{\rho\pi\pi}\left(1+\delta\frac{\Gamma_\rho}{m_\rho}\right)^{-1}. \qquad (5.5)$$

VI. NUMERICAL RESULTS

Input quantities for the numerical calculations are the couplings, mass, and width of the ρ meson.

The values of the couplings depend somewhat on the method of their calculation. For $g_{\rho\pi\pi}$, we start with a fit to the Orsay F_π data by Lefrançois[16] which leads to $m_\rho=775\pm7.3$ MeV, $\Gamma_{\rho\to2\pi}=149\pm23$ MeV. From these a finite-width correction $\delta=0.48$ is obtained. From $\Gamma_{\rho\to2\pi}$ we calculate

$$g_{\rho\pi\pi}=5.96\,. \qquad (6.1)$$

For $g_{\rho\omega\pi}$ we refer to a representation for $\Gamma_{\omega\to3\pi}$ given by Goldberg and Srivastava[17] which is a specific version of the Gell-Mann–Sharp–Wagner

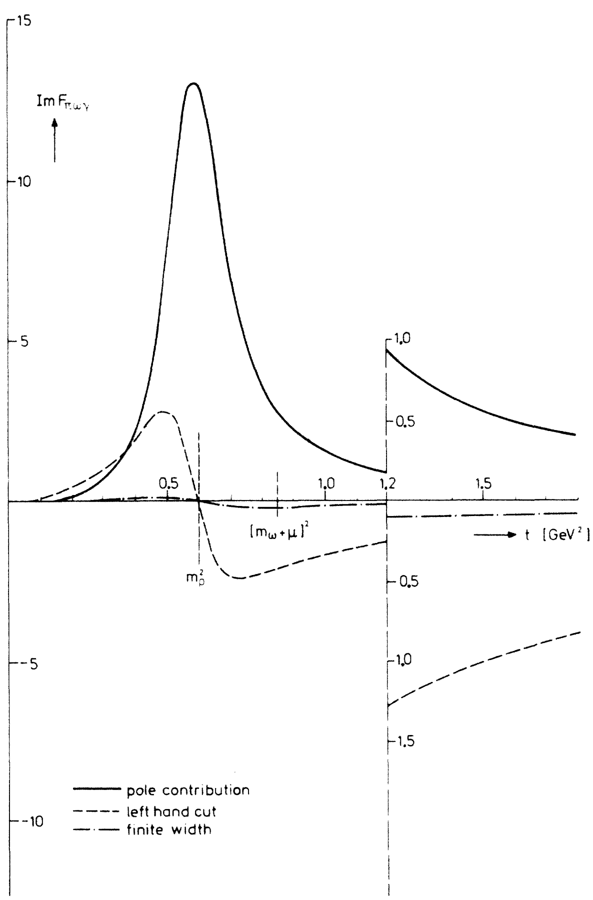

FIG. 3. Different contributions to Im $F_{\pi\omega\gamma}$. ——— pole contribution: $\frac{4}{3}g_1g_2/[f(m_\rho^2)D(t)]$. - - - - left-hand cut: $t^{1,\text{pole},L}(t)\,[\text{Re}D(t)]/D(t)$. -·-·- finite width: integral contribution of Eq. (4.8).

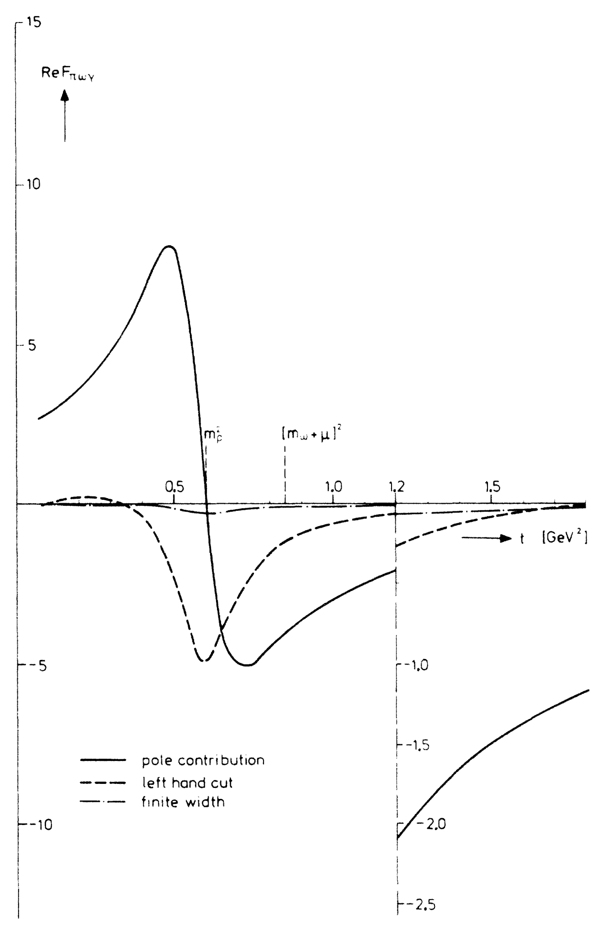

FIG. 4. Different contributions to Re $F_{\pi\omega\gamma}$. ——— pole contribution: $\frac{4}{3}g_1g_2/[f(m_\rho^2)D(t)]$. - - - - left-hand cut: $t^{1,\text{pole},L}(t)\,[\text{Re}D(t)]/D(t)$. -·-·- finite width: integral contribution of Eq. (4.8).

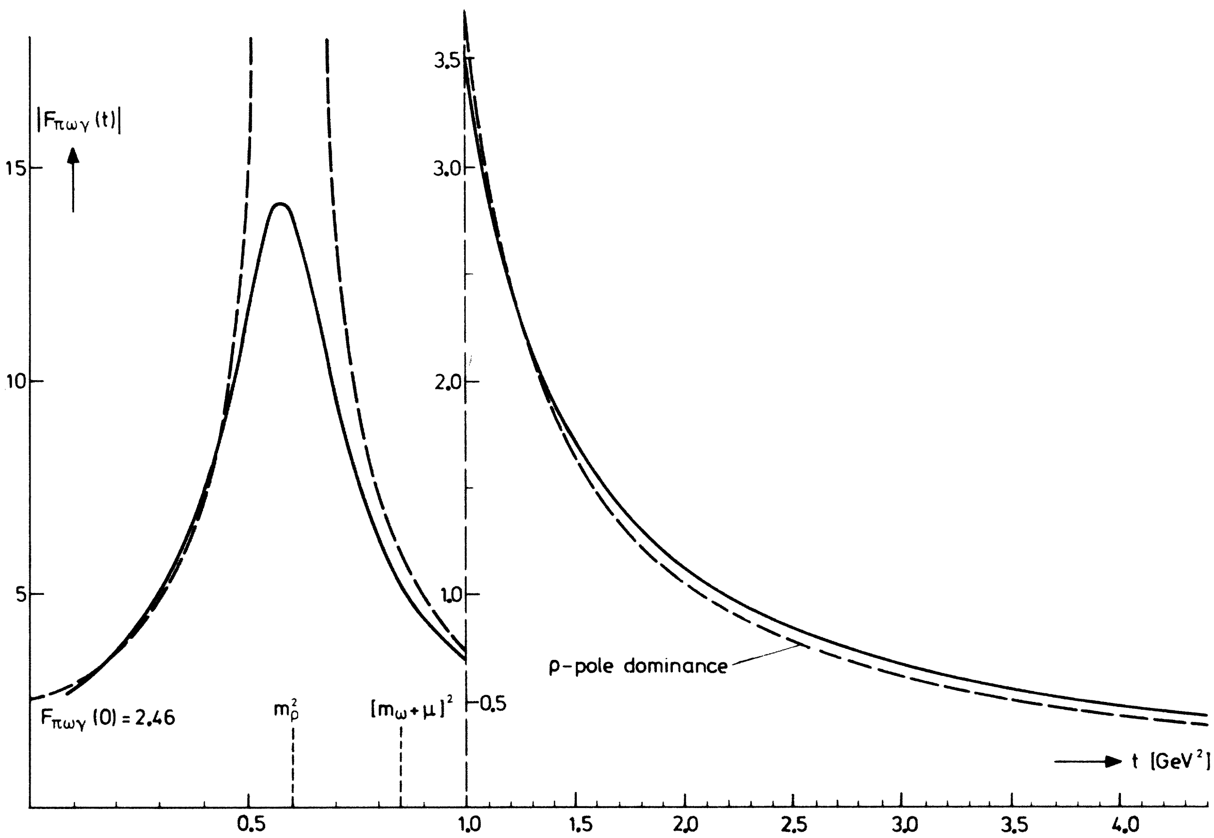

FIG. 5. $|F_{\pi\omega\gamma}|$ for $t \geq 0$ compared with ρ dominance.

model. Using $\Gamma_{\omega\to 3\pi} = 8.77 \pm 0.45$ MeV and $g_{\rho\pi\pi}$ from (6.1) we calculate

$$g_{\rho\omega\pi} = 13.42 .$$

This coupling gives somewhat too large a value for $\Gamma_{\omega\to\pi^0\gamma}$. The value $F_{\pi\omega\gamma}(0) = g_{\rho\omega\pi}/g_\rho = 2.46$, which follows from (5.4), can be compared with the numerical result from the finite-width method.

The results from the numerical integration of $\mathrm{Im}F_{\pi\omega\gamma}$ and $\mathrm{Re}F_{\pi\omega\gamma}$ are shown in Fig. 2. $\mathrm{Re}F_{\pi\omega\gamma}(t)$ is obtained from (5.2), where we have cut off the dispersion integral at $t' = 1250 m_\pi^2$. The insensitivity of the result to this cutoff has been checked. Increasing it by a factor of 10 changes the results for $|F_{\pi\omega\gamma}(t)|$ in the physical region only by 1%.

In the physical region for $e^+e^- \to \pi^0\omega$, $\mathrm{Re}F_{\pi\omega\gamma}$ dominates $\mathrm{Im}F_{\pi\omega\gamma}$, say up to 3 GeV². Above this limit, higher intermediate states could become important in the unitarity relation, so our results are uncertain.

Figures 3 and 4 present the different contributions to the structure functions from the various terms in $t^1(t)$. The main correction to the so-called "pole term" $[a/D(t)]$ is due to the left-hand-cut contributions, whereas the integral term (which is really the finite-width correction) changes the result negligibly.

Figure 5 shows $|F_{\pi\omega\gamma}(t)|$ in the timelike region compared with the ρ-pole dominance model. The difference is noticeable only at t values near the ρ mass, reflecting the structure given in Figs. 3 and 4. Extrapolating $|F_{\pi\omega\gamma}(t)|$ to $t=0$ we get $F_{\pi\omega\gamma}(0) = 2.2$, which shows nearly a 10% agreement with the above-mentioned value. This agreement seems to us quite satisfactory because the calculation of the coupling constant $g_{\rho\omega\pi}$ that was used is highly sensitive to the input parameters Γ_ρ and m_π. This is pointed out by Schwarz and Aubrecht,[18] where the first author presents also a thorough discussion of the $\pi\pi \to \pi\omega$ case.

In Fig. 6, the dependence of $|F_{\pi\omega\gamma}(t)|$ on spacelike t values is shown. Finally, we give in Fig. 7 the annihilation cross section, again compared with the pure pole contribution. The left-hand cut and the finite-width corrections shift the maximum a little towards larger t values and enhance it by about 5%, which leads to $\sigma = 10.25$ nb at $W = \sqrt{t} = 1.14$ GeV. Just above threshold for $\pi^0\omega$ production we may compare our result with an experimental point given by Cosme *et al.*[4] They get at $W = 990$ MeV, $\sigma_{e^+e^-\to\pi^+\pi^-\pi^0\pi^0} = (1.1 \pm 0.5)\times 10^{-32}$

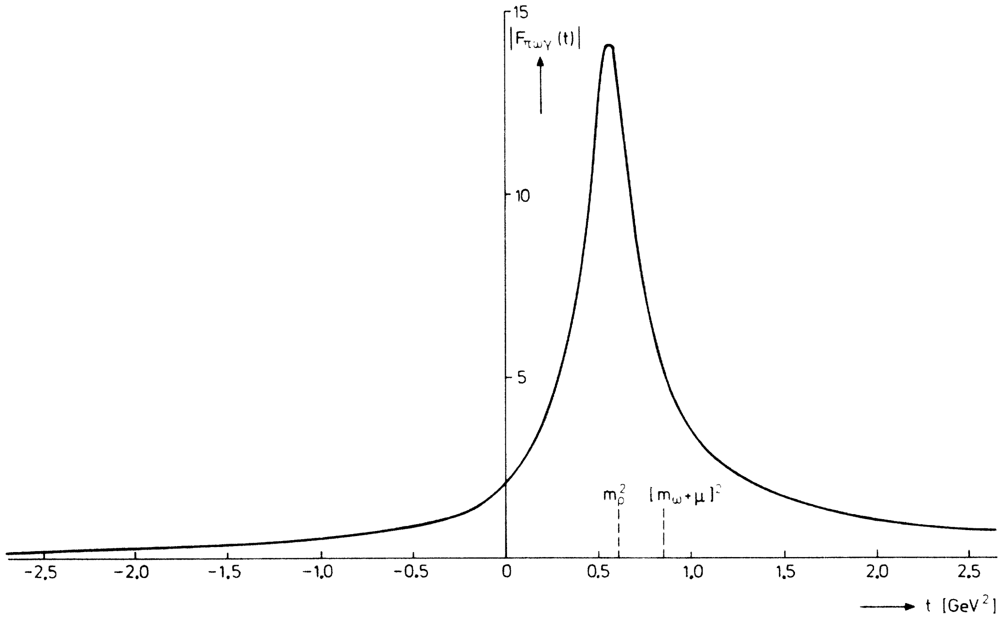

FIG. 6. $|F_{\pi\omega\gamma}|$ for $-2.5\ \mathrm{GeV}^2 \leq t \leq +2.5\ \mathrm{GeV}^2$.

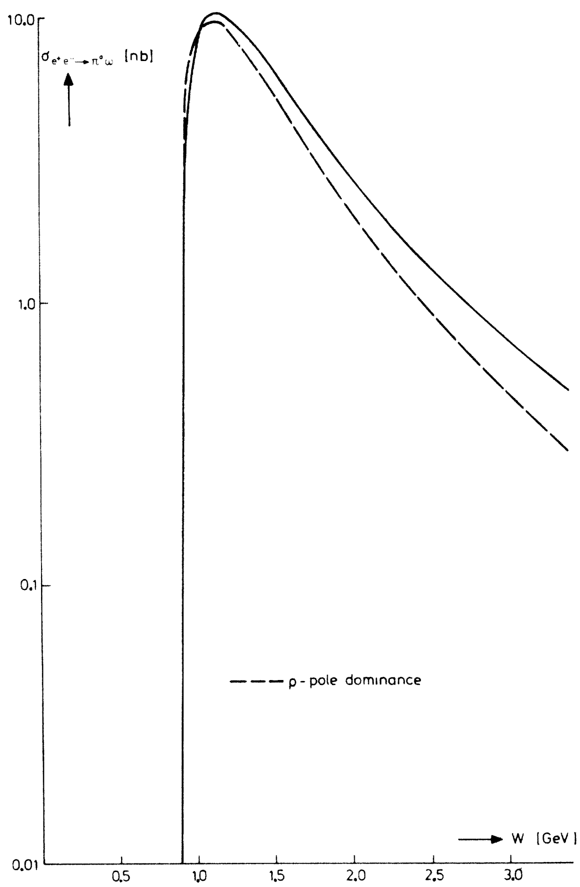

FIG. 7. Annihilation cross section compared with ρ dominance.

cm^2. This value for σ is compatible within the quoted errors with our $\sigma = 0.62 \times 10^{-32}\ \mathrm{cm}^2$.

VII. CONCLUSION

We have studied the influence of the left-hand cut and of finite-width corrections to the complex structure function of the $\gamma\pi\omega$ vertex and thereby to the annihilation cross section for the reaction $e^+e^- \to \pi^0\omega$.

The starting point for this analysis was the unitarity relation in the 2π approximation, which led to a relation between $\mathrm{Im}F_{\pi\omega\gamma}(t)$, the pion form factor $F_\pi(t)$, and the transition amplitude for $\pi\pi \to \pi\omega$. Both functions depend on the D function of elastic $\pi\pi$ scattering and thus contain the influence of the finite width of the ρ. A further assumption, the pole approximation to the left-hand-cut contributions of the transition amplitude, should work within not-too-far timelike regions.

The influence of the left-hand cut on $\mathrm{Im}F_{\pi\omega\gamma}$ is noticeable below the physical $\pi^0\omega$ threshold and also changes the pole contribution in the physical region ($\sqrt{t} > m_\pi + m_\omega$). This leads to negative values for $\mathrm{Im}F_{\pi\omega\gamma}$, which becomes comparable to $\mathrm{Re}F_{\pi\omega\gamma}$ near $t = 4\ \mathrm{GeV}^2$.

The influence of the left-hand cut and the finite-width correction on $|F_{\pi\omega\gamma}|$ are appreciably different from the vector-dominance model predictions only below the $\pi^0\omega$ threshold. Therefore the cross section has, at maximum, only a five-percent correction.

The model under consideration should be reasonable only for t values for which $\mathrm{Re}F_{\pi\omega\gamma} \gg \mathrm{Im}F_{\pi\omega\gamma}$. Beyond this, higher intermediate states in the unitarity relation are probably not negligible. Thus we believe the underlying approximations only for center-of-mass energies up to 2 GeV.

An open question within this framework is that of other possible resonance configurations in the $\pi^+\pi^-\pi^0\pi^0$ final state. According to the discussions given by KW, the channels $\rho^0\epsilon$ and πA_1 at least should be studied for additional corrections. A difficulty in including these channels is that the relevant coupling constants are not all known. Therefore, at this time, only the dependence of the results on the unknown parameters can be studied. We propose to extend our model to the $\rho^0\epsilon$ channel subsequently.

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