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New scaling variable and early scaling in single-particle inclusive distributions for hadron-hadron collisions

E. Yen
Department of Physics, Michigan State University, East Lansing, Michigan 48823
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We suggest that the invariant cross section for $a+b\to c+X$ should be plotted in terms of $(\tilde{x}, P_\perp)$, where $\tilde{x}=2E^*/\sqrt{s}$ and E^* is the energy of particle c in the center-of-mass system. We have shown for processes with exotic $ab\bar{c}$ [such as $p+p\to(\pi^\pm, K^\pm, \bar{p})+X$] that the invariant cross sections $Ed^3\sigma/d^3p\to f(\tilde{x}, P_\perp)$ at $P_\perp=0.2$, 0.4, and 0.8 GeV/c over an energy range of $P_{\text{inc}}=12-1500$ GeV/c. Furthermore, scaling in terms of $(\tilde{x}, P_\perp)$ provides a natural connection between the small-$P_\perp$ and large-$P_\perp$ regions. Predictions on the single-particle distribution when both $P_\parallel^*$ and $P_\perp^*$ are large are also presented.

I. INTRODUCTION

Interest in the study of inclusive single-particle distributions has been generated mainly by the suggestion of Benecke, Chou, Yang, and Yen[1] and Feynman[2] that *for finite and fixed* $P_\perp$ the invariant cross section $E\,d^3\sigma/d^3p$ for the process $a+b\to c+X$ should approach a limiting function of x and $P_\perp$ at high energies, i.e.,

$$\lim_{\substack{s\to\infty \\ x,\,P_\perp \text{ fixed}}} E\frac{d^3\sigma}{d^3p}\to f(x,P_\perp)\,, \tag{1}$$

where $x=P_\parallel^*/P_0^*\simeq P_\parallel^*/(\frac{1}{2}s)^{1/2}$ is the usual Feynman scaling variable, and $P_\parallel^*$, $P_\perp^*$ are the longitudinal and transverse momenta of c in the center-of-mass system, respectively.

The rate at which the scaling limit is approached for $a+b\to c+X$ in the fragmentation region has been studied by Chan, Hsue, Quigg, and Wang[3] based on Mueller's generalized optical theorem[4] and Regge phenomenology. They concluded that the scaling limit should be reached rather early if $ab\bar{c}$ is exotic. Recent CERN ISR data,[5-10] however, indicate that for processes $p+p\to\bar{p}+X$ and $p+p\to K^\pm+X$ the approach to the limit is extremely slow, even though for these processes $ab\bar{c}$ is exotic.

Recent experiments[11,12] also indicate that at 90° in the center-of-mass system (i.e., $x\simeq 0$) the invariant cross section for $a+b\to c+X$ *for large* $P_\perp$ is not given by the naive extrapolation of Eq. (1) but rather depends on both $(s,P_\perp)$. A convenient set of variables is $(x_\perp, P_\perp)$, i.e.,

$$E\frac{d^3\sigma}{d^3p}\to f(x_\perp,P_\perp)\,,$$

where

$$x_\perp=\frac{P_\perp}{P_0^*}\simeq\frac{2P_\perp}{\sqrt{s}}\,. \tag{2}$$

In this paper we suggest, in the spirit of the "correspondence principle" of Bjorken and Kogut,[13] that *for all* $P_\perp$ the invariant cross section $E\,d^3\sigma/d^3p$ for the process $a+b\to c+X$ should approach a limiting function of $\tilde{x}$ and $P_\perp$ at high energies, i.e.,

$$\lim_{\substack{s\to\text{large} \\ \tilde{x},\,P_\perp \text{ fixed}}} E\frac{d^3\sigma}{d^3p}\to f(\tilde{x},P_\perp)\,, \tag{3}$$

where

$$\begin{aligned}\tilde{x}&=\frac{E^*}{E_0^*}\simeq\frac{2E^*}{\sqrt{s}}\\ &=\left[x^2+\frac{4(P_\perp{}^2+m^2)}{s}\right]^{1/2}\\ &=\left(x^2+x_\perp{}^2+\frac{4m^2}{s}\right)^{1/2}\end{aligned}$$

and E^* and m are the center-of-mass energy and mass of c, respectively. This variable $\tilde{x}$ has also been used by Kinoshita and Noda.[14]

Equation (3) not only naturally incorporates both Eqs. (1) and (2), but also makes a definite statement about single-particle distributions when both $P_\parallel^*$ and $P_\perp$ are large. Furthermore, for processes $a+b\to c+X$ with exotic $ab\bar{c}$, $E\,d^3\sigma/d^3p\to f(\tilde{x},P_\perp)$ at relatively low energy.

II. EARLY SCALING

We shall use the single-particle distribution[5-10,15-21] for processes (1) $p+p\to\pi^-+X$, (2) $p+p\to\pi^++X$, (3) $p+p\to K^-+X$, (4) $p+p\to K^++X$, and (5) $p+p\to\bar{p}+X$ in the energy range $P_{\text{inc}}=12-1500$ GeV/c. The reasons for studying these processes are the following: (i) Extensive data exist for these processes over sufficiently large energy range, and (ii) according to Chan, Hsue, Quigg, and Wang,[3] these processes should exhibit early scaling.

In Figs. 1–5, we have plotted $E\,d^3\sigma/d^3p$ vs $\tilde{x}$ for these processes with $P_\perp$ fixed at 0.2, 0.4, and 0.8

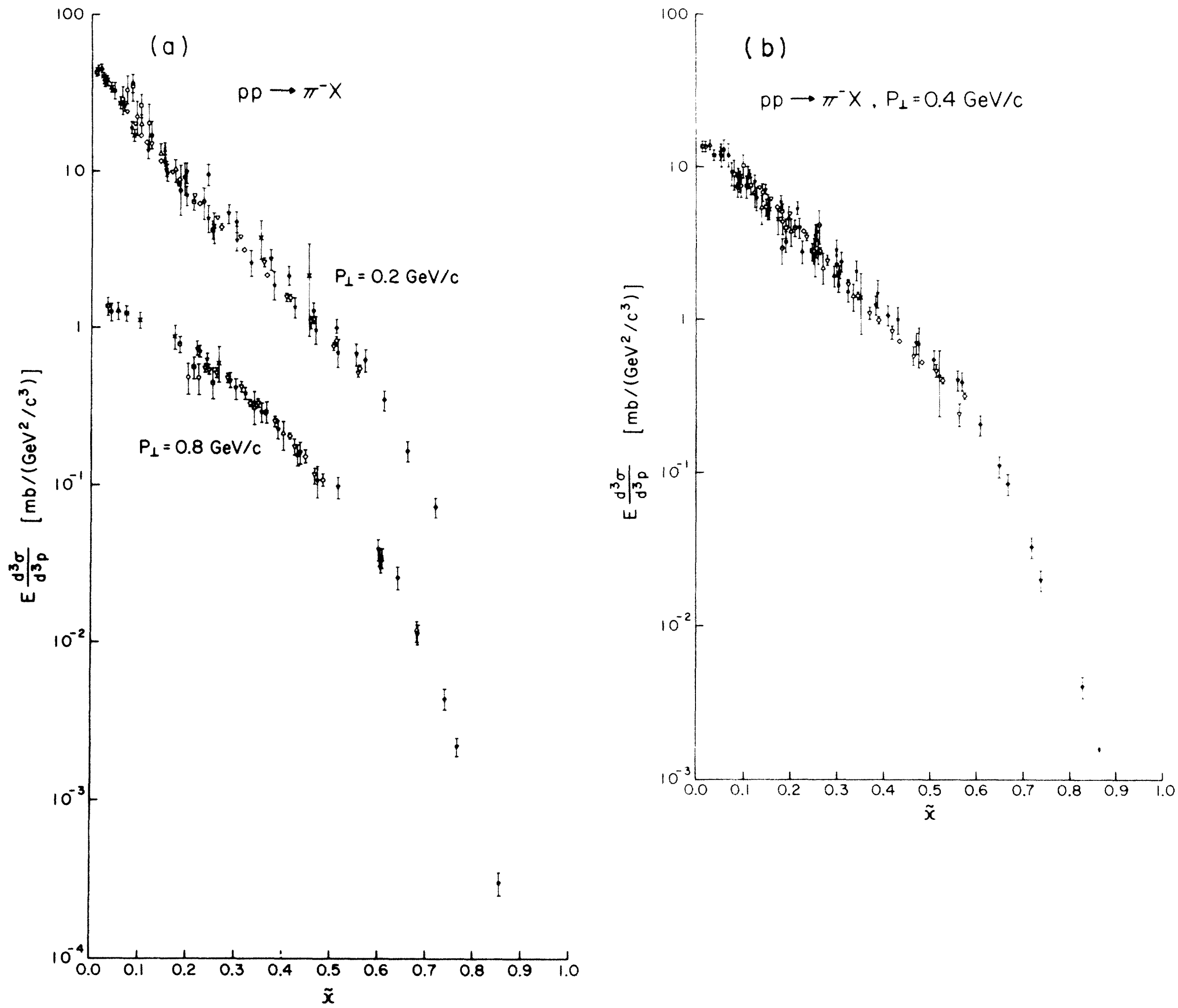

FIG. 1. $Ed^3\sigma/d^3p$ vs $\tilde{x}$ for $p+p\to\pi^-+x$. (a) $P_\perp=0.2$ and 0.8 GeV/c; (b) $P_\perp=0.4$ GeV/c. The notations for the experimental points of Figs. 1–5 and 7 are as follows: ◇ $P_{inc}=12$ GeV/c, Refs. 17, 18; ◆ $P_{inc}=19.2$ GeV/c, Ref. 16; ▽ $P_{inc}=24$ GeV/c, Ref. 17, 18; ▼ $P_{inc}=24$ GeV/c, Ref. 16; ✱ $P_{inc}=28.5$ GeV/c, Ref. 19; × $P_{inc}=205$ GeV/c, Ref. 20; ⊙ $P_{inc}\simeq 225$ GeV/c, Ref. 8; ✶ $P_{inc}\sim 285$ GeV/c, Ref. 7; ⊖ ~ 303 GeV/c, Ref. 21; △ $P_{inc}\sim 500$ GeV/c, Ref. 8; ◭ $P_{inc}\sim 500$ GeV/c, Ref. 7; ○ $P_{inc}\sim 1100$ GeV/c, Ref. 8; ◓ $P_{inc}\sim 1100$ GeV/c, Ref. 7; □ $P_{inc}\sim 1500$ GeV/c, Ref. 8; ◧ $P_{inc}\sim 1500$ GeV/c, Ref. 7; ⬒ $P_{inc}\sim 1500$ GeV/c, Ref. 6; ▲ $P_{inc}\sim 1000$ and 1500 GeV/c, Ref. 10; × $P_{inc}\sim 285$ GeV/c, Ref. 5; ▲ $P_{inc}\sim 500$ GeV/c, Ref. 5; ● $P_{inc}\sim 1100$ GeV/c, Ref. 5; ■ $P_{inc}\sim 1500$ GeV/c, Ref. 5; ◑ $P_{inc}\sim 1100$ GeV/c, Ref. 9.

GeV/c and $P_{inc}=12-1500$ GeV/c. The errors are either read directly from various references or obtained by adding quadratically the quoted statistical and systematic errors.

It can be seen that the data fall on top of each other (if they overlap) or join smoothly to each other (if they do not overlap), indicating that $E\,d^3\sigma/d^3p$ is indeed a function of $(\tilde{x},P_\perp)$ over the energy range $P_{inc}=12-1500$ GeV/c. One should compare these figures with those presented in Ref. 5, where $E\,d^3\sigma/d^3p$ is plotted against x at $P_\perp=0.4$ GeV/c. Notice in particular that $E\,d^3\sigma/d^3p$ for $p+p\to\bar{p}+X$ rises by almost an order of magnitude between $P_{inc}\sim 19$ GeV/c and ISR energies—hardly early scaling in terms of $(x,P_\perp)$.

There are very few inclusive data with $P_{inc}<12$ GeV/c. In Fig. 6, we have plotted $E\,d^3\sigma/d^3p$ vs $\tilde{x}$ at $P_\perp=0.2$, 0.4, and 0.8 GeV/c using the limited bubble chamber data[15] for $p+p\to\pi^++X$ at 6.6 GeV/c. Although there is no reason to expect scaling to hold at such a low energy, it is amusing to note, by comparing Figs. 2 and 6, that the data are consistent with scaling in terms of $(\tilde{x},P_\perp)$.

Recently, there have been indications[22,23] that σ_{tot} for pp collision is rising in the CERN ISR energy region. One should, probably, modify the

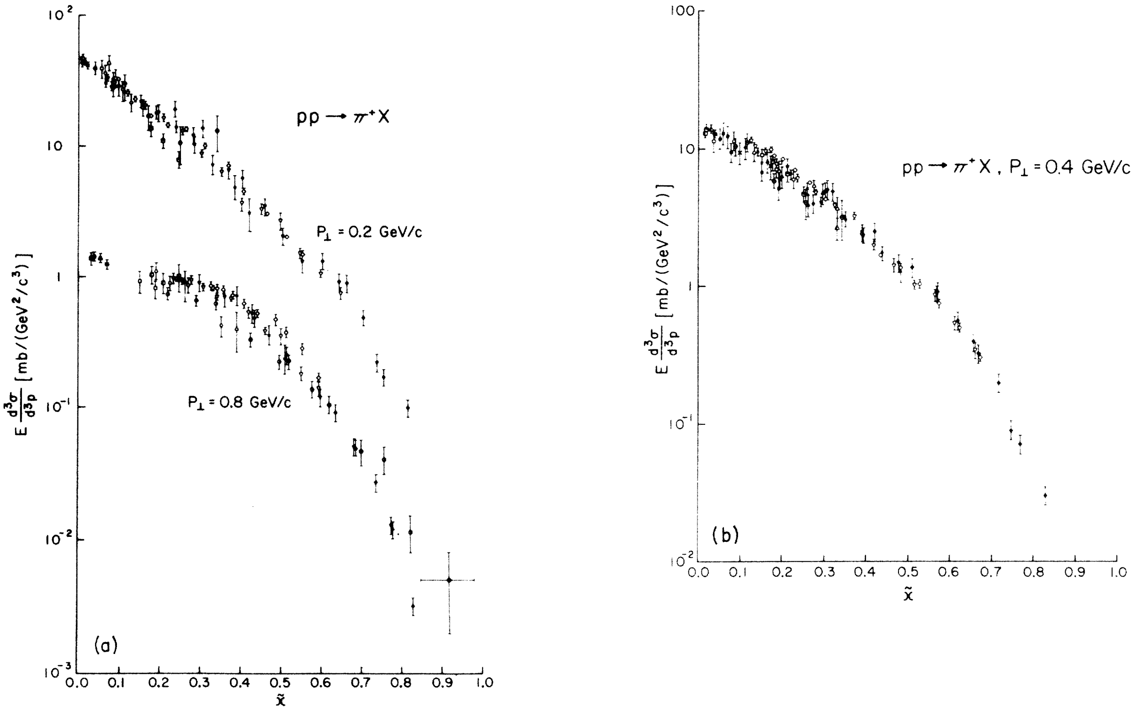

FIG. 2. $E d^3\sigma/d^3p$ vs $\tilde{x}$ for $p+p \to \pi^+ + x$. (a) $P_\perp = 0.2$ and 0.8 GeV/c; (b) $P_\perp = 0.4$ GeV/c.

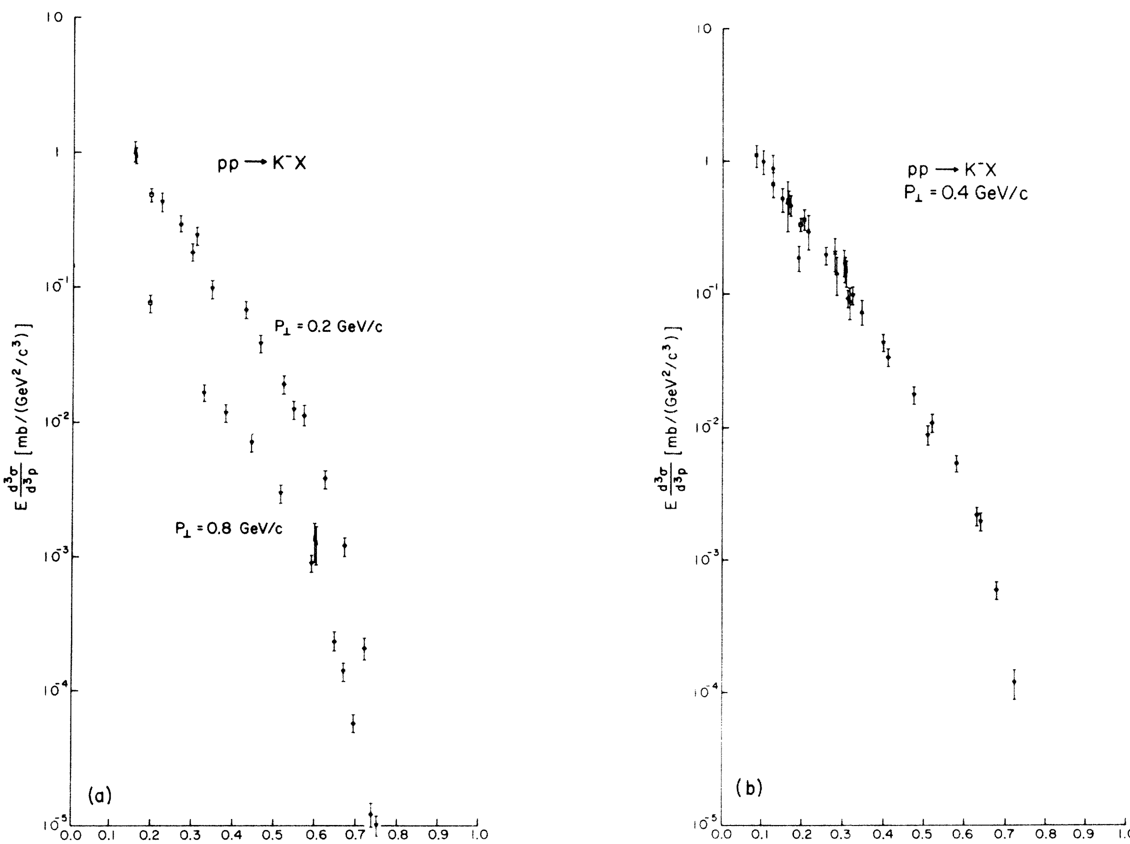

FIG. 3. $Ed^3\sigma/d^3p$ vs $\tilde{x}$ for $p+p \to K^- + x$. (a) $P_\perp = 0.2$ and 0.8 GeV/c; (b) $P_\perp = 0.4$ GeV/c.

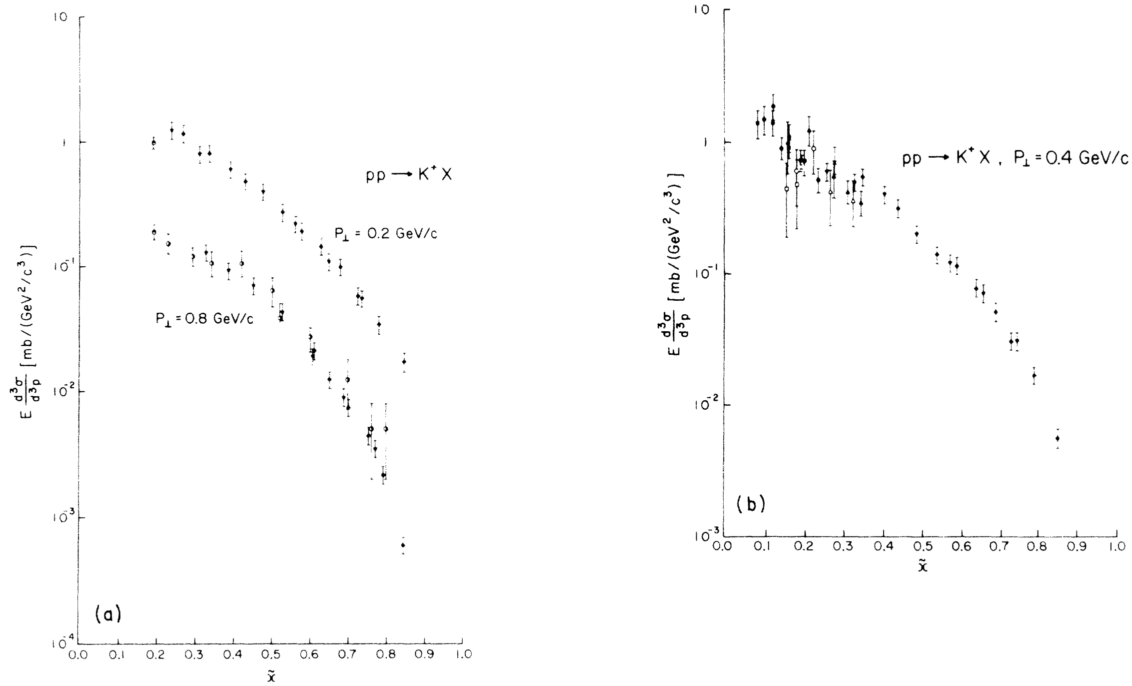

FIG. 4. $Ed^3\sigma/d^3p$ vs $\tilde{x}$ for $p+p \rightarrow K^+ + x$. (a) $P_\perp = 0.2$ and 0.8 GeV/c; (b) $P_\perp = 0.4$ GeV/c.

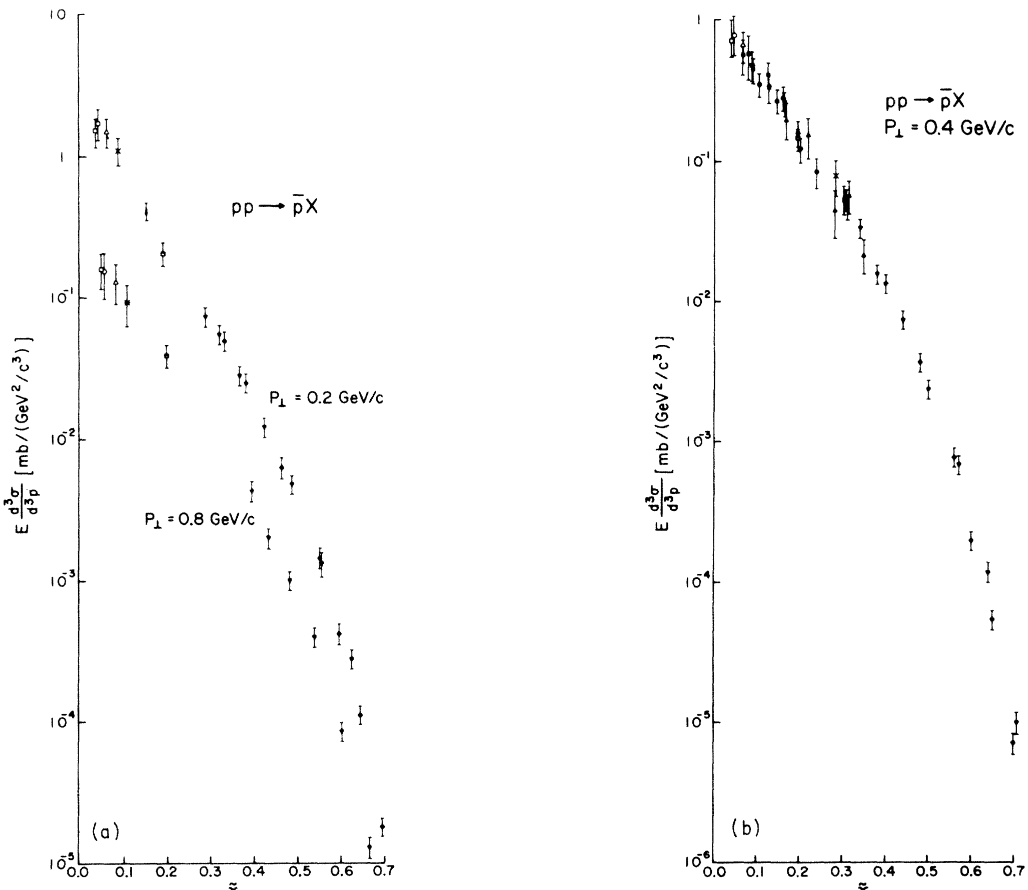

FIG. 5. $Ed^3\sigma/d^3p$ vs $\tilde{x}$ for $p+p \rightarrow \bar{p} + x$. (a) $P_\perp = 0.2$ and 0.8 GeV/c; (b) $P_\perp = 0.4$ GeV/c.

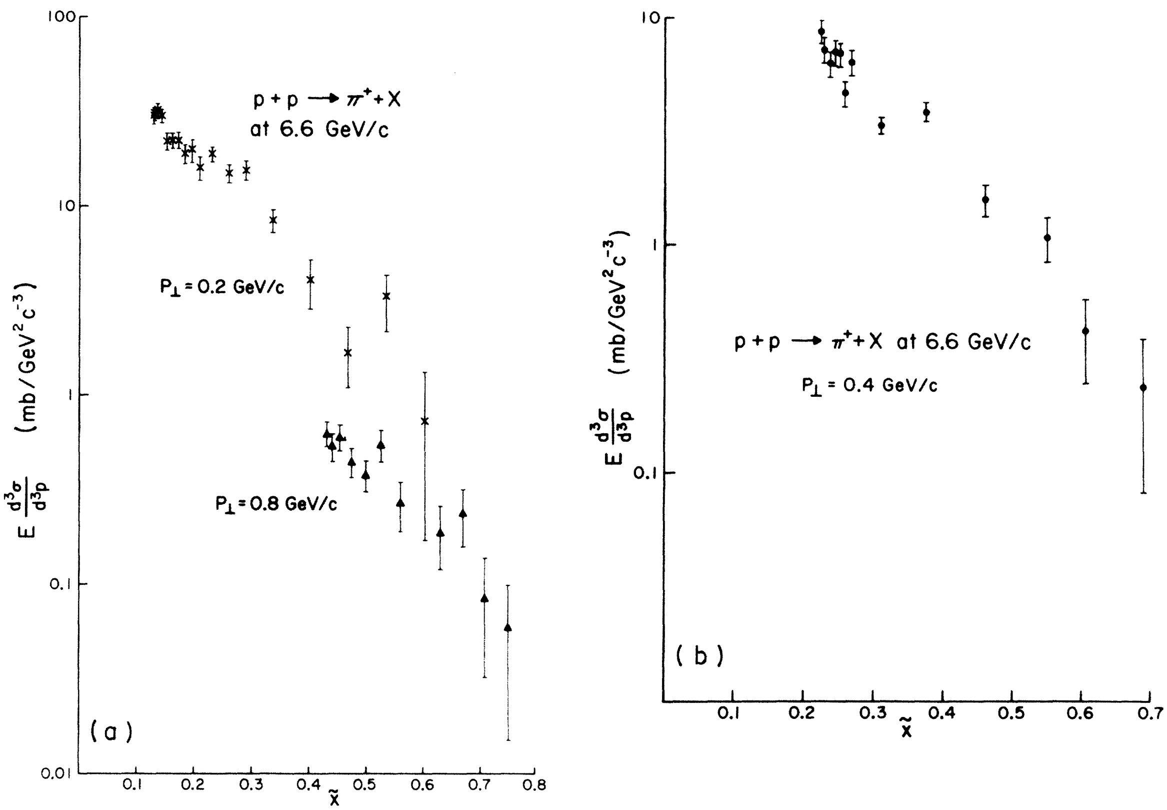

FIG. 6. $Ed^3\sigma/d^3p$ vs $\tilde{x}$ for $p+p\to\pi^+ + x$. (a) $P_\perp = 0.2$ and 0.8 GeV/c at $P_{\rm inc} = 6.6$ GeV/c. The data are from Ref. 15; (b) $P_\perp = 0.4$ GeV/c at $P_{\rm inc} = 6.6$ GeV/c. The data are from Ref. 15.

scaling hypothesis to

$$\lim_{\substack{s\ \text{large}\\ \tilde{x},\,P_\perp\ \text{fixed}}} \frac{1}{\sigma_{\rm tot}} E\frac{d^3\sigma}{d^3p} \to f(\tilde{x},P_\perp)\,. \qquad (4)$$

However, given that $\sigma_{\rm tot}$ varies less than 10% in the energy range we have studied and the uncertainty in the inclusive data, this modification will have very little effect on our conclusion.

For completeness, we have plotted in Fig. 7 the invariant cross section for $p+p\to p+X$ as a function of $\tilde{x}$ with $P_\perp$ fixed at 0.2, 0.4, and 0.8 GeV/c. For this channel $ab\bar{c}$ is not exotic. Owing to the uncertainty in the data and differences among data taken at the same energy, it is difficult to extract the energy dependence for this process. However, by comparing Fig. 7 with Fig. 18 of Ref. 18 and Fig. 1 of Ref. 10, it can be seen that the shapes for the single-particle distributions vary much less from energy to energy when they are plotted in terms of $(\tilde{x},P_\perp)$ than when they are plotted in terms of $(x,P_\perp)$.

Therefore, we believe it is useful to plot the data at finite energies in terms of $(\tilde{x},P_\perp)$. Furthermore, in terms of these variables, at least for processes with exotic $ab\bar{c}$ the single-particle distributions exhibit early scaling.

III. CONSEQUENCE OF EARLY SCALING IN TERMS OF $(\tilde{x},P_\perp)$

(i) It has been noted by Albrow[24] that the s dependence of $E\,d^3\sigma/d^3p$, when plotted in terms of $(x,P_\perp)$, is steeper in the central region near $x=0$ and near the phase-space boundary than it is in the medium-x region. This can be readily understood if the cross section scales in terms of $(\tilde{x},P_\perp)$ because (a) in the central region $\tilde{x}$ differs the most from x, and (b) the invariant cross section varies very fast as a function of $\tilde{x}$ near the phase-space boundary, so that a slight difference between $\tilde{x}$ and x will produce a large effect in $E\,d^3\sigma/d^3p$.

(ii) It is important to note that $x=0$ does not correspond to $\tilde{x}=0$ for finite s. Instead, $x=0$ corresponds to

$$\tilde{x} = \frac{2(P_\perp{}^2+m^2)^{1/2}}{\sqrt{s}}\,.$$

Therefore, for fixed $P_\perp$ and m^2, the $\tilde{x}$ value that corresponds to $x=0$ decreases as s increases. If the single-particle distribution scales in terms of

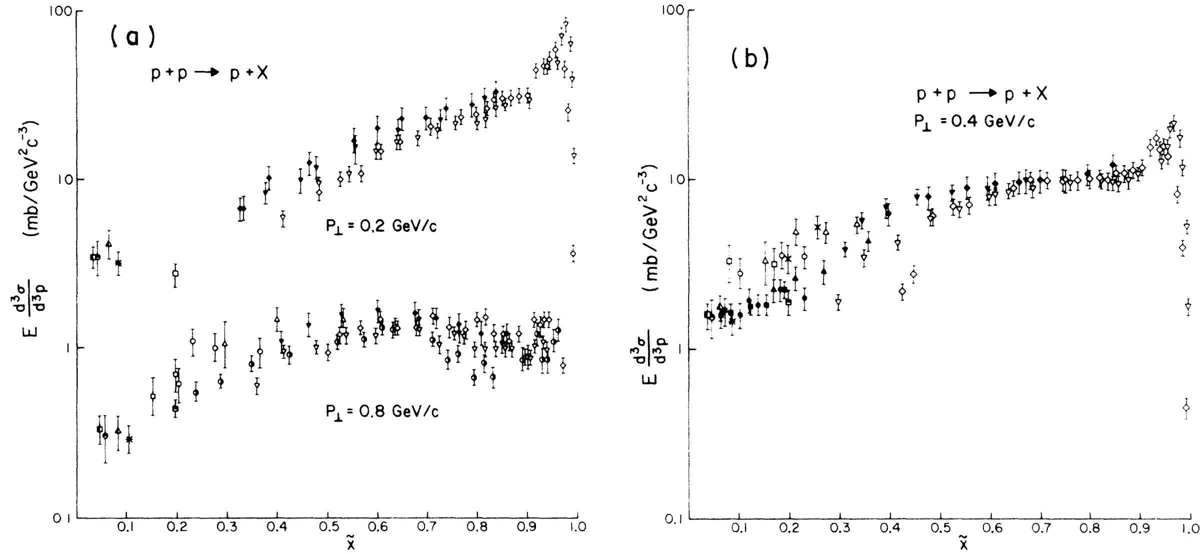

FIG. 7. $Ed^3\sigma/d^3p$ vs $\tilde{x}$ for $p+p\to p+x$. (a) $P_\perp = 0.2$ and 0.8 GeV/c; (b)$P_\perp = 0.4$ GeV/c.

$(\tilde{x}, P_\perp)$ and is a decreasing function of $\tilde{x}$ (such as $pp\to\pi^\pm X$, $pp\to K^\pm X$, $pp\to\bar{p}X$), then the invariant cross section $E\,d^3\sigma/d^3p$ at $x=0$ should increase with energy. That is, for these processes, the limit should be approached *from below* at $x=0$.

(iii) At $\tilde{x}=0$, almost all arguments tend to predict that the invariant cross section for particle and antiparticle should approach the same limit. With the new ISR data, the amount of extrapolation needed to reach $\tilde{x}=0$ is indeed very small. From Figs. 1–4 it can be seen that this conjecture is indeed supported by the data for $pp\to\pi^\pm X$ and $pp\to K^\pm X$. Comparing Figs. 5 and 7, it is obvious that data for $pp\to\bar{p}X$ and $pp\to pX$ also support this conjecture.

IV. FURTHER TESTS OF SCALING IN TERMS OF $(\tilde{x}, P_\perp)$

So far, we have shown that $E\,d^3\sigma/d^3p \to f(\tilde{x}, P_\perp)$ for processes with $ab\bar{c}$ and $P_\perp$ fixed at 0.2, 0.4, 0.8 GeV/c. For large $P_\perp$ (i.e., $P_\perp > 1.5$ GeV/c), there are only data at 90° in the center-of-mass system (i.e., $P_\parallel^* = 0$). Since

$$\tilde{x} = \left[\frac{4(P_\parallel^{*2} + P_\perp{}^2 + m^2)}{s}\right]^{1/2},$$

data at $P_\parallel^* = 0$ alone will not be sufficient to test Eq. (3).

If $E\,d^3\sigma/d^3p \to f(\tilde{x}, P_\perp)$ is valid, then

$$E\frac{d^3\sigma}{d^3p}(s_1, P_\parallel^*, P_\perp) = E\frac{d^3\sigma}{d^3p}(s_2, P_\parallel^* = 0, P_\perp) \to f(\tilde{x}, P_\perp)\,, \qquad (5)$$

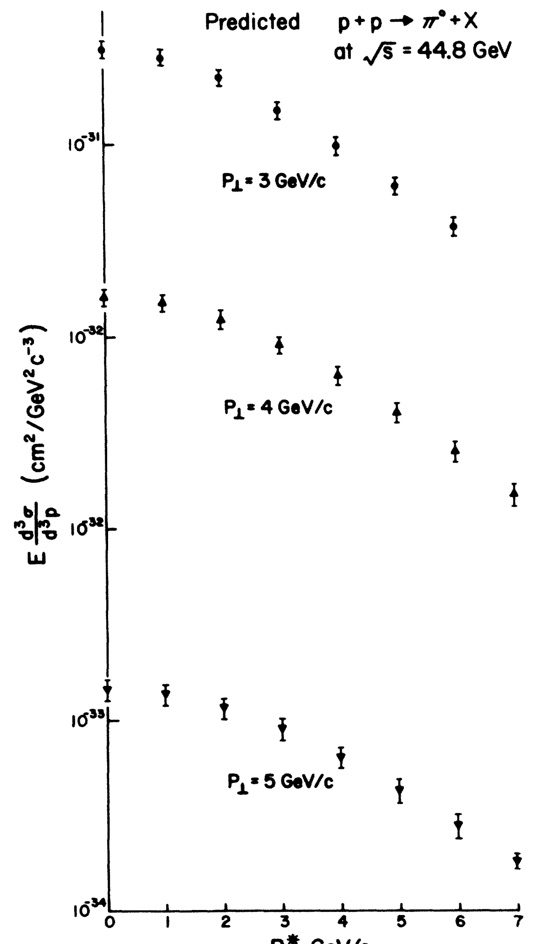

FIG. 8. Prediction of $Ed^3\sigma/d^3p$ vs $P_\parallel^*$ for $p+p\to\pi^0+x$ at $\sqrt{s} = 44.8$ GeV/c and $P_\perp = 3$, 4, and 5 GeV/c. The error is deducted from the error given in Ref. 11 which is statistical only.

with

$$\bar{x} = \left[\frac{4(P_\parallel^{*2} + P_\perp^{\,2} + m^2)}{s_1}\right]^{1/2} = \left[\frac{4(P_\perp^{\,2} + m^2)}{s_2}\right]^{1/2} \tag{6}$$

and s_1, s_2 related by

$$\frac{s_1}{s_2} = \frac{P_\parallel^{*2} + P_\perp^{\,2} + m^2}{P_\perp^{\,2} + m^2}. \tag{7}$$

Therefore, if $E\, d^3\sigma/d^3p \to f(\bar{x}, P_\perp)$ we can use data at 90° in the center-of-mass system at various energies to predict $E\, d^3\sigma/d^3p$ everywhere. As an illustration, we have used the data[11,25] on $p+p \to \pi^0 + X$ at 90° in the center-of-mass system to predict $E\, d^3\sigma/d^3p$ for this process with $P_\parallel^* = 1-7$ GeV/c and $P_\perp^* = 3, 4, 5$ GeV/c at $\sqrt{s} = 44.8$ GeV ($P_{\text{inc}} \sim 1060$ GeV/c). The results are given in Fig. 8. It would be most interesting to check this prediction.

V. REMARKS

(i) The $\bar{x}$ variable is not contrived. We did not introduce any artificial parameter; $\bar{x}$ is just the fractional energy of the particle in the center-of-mass system. As a matter of fact, if c is the leading proton in a proton-proton collision, then $\bar{x}$ is just the elasticity parameter used in cosmic-ray physics.

(ii) If we fix $P_\perp$ and let $s \to \infty$, then obviously we recover the scaling hypothesis of Feynman and Yang. We consider that the single-particle distribution $E\, d^3\sigma/d^3p$ can be expressed as a function of $(\bar{x}, P_\perp)$ over a wide range of energies as an indirect evidence that Feynman-Yang scaling would be valid [with a possible modification mentioned in Eq. (4)] at fixed $P_\perp$ and $s \to \infty$.

(iii) As we have already mentioned, as $\bar{x} \to 0$, the single-particle distributions for particle and antiparticle production seem to approach each other. Therefore, we would expect that at 90° center of mass (i.e., $P_\parallel^* = 0$) and fixed $P_\perp$, the ratio of particle and antiparticle production should approach 1 as $s \to \infty$.

(iv) Equation (3) is also applicable for the case when *both* $P_\perp$ and $P_\parallel^*$ are large.

ACKNOWLEDGMENTS

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[25]We have used the fitted formula

$$E\frac{d^3\sigma}{d^3p} = AP_\perp^{-n} e^{-bP_\perp/\sqrt{s}}$$

with $A = (1.54 \pm 0.10)\times 10^{-26}$, $n = 8.24 \pm 0.05$, and $b = 26.10 \pm 0.5$ given in Ref. 11.

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Dispersion-relation calculation of the πN elastic amplitude A' for $0.20 \leqslant -t \leqslant 0.40$ (GeV/c)^2 and $0.87 \leqslant p \leqslant 20.0$ GeV/c

J. A. McClure
Department of Physics, Georgetown University, Washington, D. C. 20007

L. E. Pitts
Department of Physics, Mary Washington College, Fredericksburg, Virginia 22401
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A method previously presented in which dispersion relations were written for the logarithm of the scattering amplitude is extended to larger values of momentum transfer. The earlier technique was applied to πN elastic scattering but was restricted to $-t \lesssim 0.24$ (GeV/c)^2 because of the crossing of the t-dependent threshold energy and the nucleon pole. That limitation is here removed by writing the dispersion relations for the amplitude with its pole removed, and the thus extended method is again applied to πN scattering. The amplitudes $A'_{\pm}$ calculated at pion lab momenta of 2, 4, 8, 12, 15, 20, and 30 GeV/c for $-t = 0.20$, 0.25, 0.30, 0.35, 0.40 (GeV/c)^2 are presented.

I. INTRODUCTION

In a previous paper[1] (herein referred to as I) we applied the Hilbert transforms

$$\text{Im}\, F(\nu, t) = -\frac{1}{\pi}\,\text{P}\int \frac{\text{Re}\, F(\nu', t)d\nu'}{\nu' - \nu} \tag{1}$$

to the logarithm of the pion-nucleon elastic scattering amplitude $A' = |A'|e^{i\phi}$,

$$F = \ln A' = \ln|A'| + i\,\phi\,, \tag{2}$$

to obtain a dispersion relation giving the phase of this amplitude in terms of its magnitude