certain low value. Nevertheless, it will be seen from Fig. 2(b) that for the uppermost range of u values the "spectator" contribution should clearly exceed the background "production" events.

[16]Another suggestion for the detection of this configuration is given in H. J. Weber, Phys. Rev. C 9, 1771 (1974).

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Calculation of nucleon-nucleon two-pion exchange utilizing theoretical and experimental $\pi\pi$, πN, and eN information*

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The $J = 0,1,2$ $N\bar{N} \to \pi\pi$ amplitudes are evaluated in the physical region of the two pions, $t \geq 4\mu^2$, and used in a dispersion-theoretic calculation of nucleon-nucleon two-pion exchange. Experimental input to the evaluations of the $N\bar{N} \to \pi\pi$ amplitudes is via $\pi\pi$ phase shifts, πN scattering lengths, and πN phase shifts. The effect of varying the $I = J = 0$ $\pi\pi$ phase shift within its present uncertainty is investigated. Experimentally determined nucleon electromagnetic form factors are used in the evaluation of the $J = 1$ $N\bar{N} \to \pi\pi$ amplitude. After adding π and ω exchange to the two-pion-exchange amplitude, the theoretical phase shifts are compared with phase-shift-analysis results. The calculated $^1S_0$ potential is also shown and compared with the phenomenological one of Hamada and Johnston.

I. INTRODUCTION

The dispersion-theoretic treatment of two-pion exchange has been used by several authors[1-5] in a recent wave of attempts to evaluate the intermediate-range nuclear force. In this approach the two-pion-exchange $NN \to NN$ amplitude is obtained[6] from a fixed-energy dispersion relation. Unitarity and crossing determine the absorptive part as a bilinear product of the $N\bar{N} \to \pi\pi$ amplitudes $A^{(\pm)}$, $B^{(\pm)}$ in the low-energy physical region of the two pions. Thus, full exploitation of this approach requires a knowledge of the $N\bar{N} \to \pi\pi$ amplitudes at energies far below physical threshold. In 1970 Nielsen, Petersen, and Pietarinen[7] (NPP) attempted to evaluate the necessary s-, p-, and d-wave $N\bar{N} \to \pi\pi$ amplitudes by analytic continuation from the pion-nucleon physical scattering regions. The resulting s-wave $N\bar{N} \to \pi\pi$ amplitude was used by Chemtob and Riska[2] to calculate nucleon-nucleon phase shifts, giving qualitative agreement with phase-shift analysis. In later work[1] this $N\bar{N} \to \pi\pi$ s-wave amplitude was used to calculate two-pion-exchange NN potentials.

Unitarity of the $N\bar{N} \to \pi\pi$ partial-wave amplitudes implies[8] that their phases are those of $\pi\pi$ scattering, modulo π, for the same angular momenta. In fact, NPP had hoped to learn about the $\pi\pi$ phases through use of this relation. Since that time, however, the $\pi\pi$ s-wave phase shift has been well established experimentally[9-12]: It shows a strong discrepancy with the NPP result. New results for the s- and p-wave $N\bar{N} \to \pi\pi$ partial-wave amplitudes using the $\pi\pi$ phase shifts as *input* have now been given by Nielsen and Oades[13] and by Epstein and McKellar.[4] Surprisingly, the nuclear force calculated from these newer and supposedly more realistic $N\bar{N} \to \pi\pi$ s-wave amplitudes seemed to be in poorer agreement with nucleon-nucleon phase-shift analysis and with phenomenological NN potentials.[4,5]

These circumstances seemed to indicate to us that a very careful evaluation of all input to the two-pion-exchange calculation was needed. In the calculations presented here the nucleon-pole (Born) contributions to $B^{(\pm)}$ have been included exactly, while $A^{(\pm)}$ and the non-Born part of $B^{(\pm)}$ were represented by partial-wave expansions truncated after d waves. By including the d waves we found it unnecessary to rely upon a narrow-width baryon-resonance model for the $J \geq 2$ partial waves as was the case in several previous NN calculations. We have evaluated the s-, p-, and d-wave $N\bar{N} \to \pi\pi$ amplitudes, while enforcing unitarity, with special attention to the effect of uncertainties in the $\pi\pi$ phase-shift input. Subtraction constants required in our dispersion relations were determined using new pion-nucleon scattering lengths and phase shifts as well as fixed-t dispersion relation results.[14] In addition, in evaluating the p-wave amplitudes, we have used experimental information on the nucleon electromagnetic form factors.

II. THE PHASES OF THE $N\bar{N}\to\pi\pi$ AMPLITUDES

The singularities of the $N\bar{N}\to\pi\pi$ helicity amplitudes[8] $f_{\pm}^{J}$ are along the right-hand cut beginning at $t=4\mu^2$ and the left-hand cut at $t=4\mu^2-\mu^4/m^2$. The helicity amplitudes are determined on the left-hand cut for $t>-26\mu^2$ from continuation of πN amplitudes.[8] An extensive table of the resulting s-, p-, and d-wave amplitudes in this left-hand cut region was given by Nielsen.[14]

For $4\mu^2 \leq t \leq 16\mu^2$, on the right-hand cut, unitarity requires[8] that

$$\mathrm{Im} f_{\pm}^{J}(t) = f_{\pm}^{J*}(t) e^{i\delta_J^I} \sin\delta_J^I, \qquad (1)$$

where $\delta_J^I(t)$ is the appropriate $\pi\pi$ scattering phase shift. The upper limit, $t=16\mu^2$, is set by the opening of the 4π channel, but Eq. (1) will continue to be valid up to the value of t where inelasticity becomes appreciable.

The unitarity condition, Eq. (1), is utilized via the Omnès function[15,8]

$$D_J(t) = \exp\left[\frac{-t}{\pi}\int_{4\mu^2}^{\infty} \frac{\delta_J^I(t')}{t'(t'-t-i\epsilon)}\,dt'\right] . \qquad (2)$$

The products $D_J(t) f_{\pm}^{J}(t)$ then have only the left-hand cut, where, as we have seen, the $f_{\pm}^{J}(t)$ are determined from physical πN scattering amplitudes, and a right-hand cut beginning where inelasticity becomes important, at high t.

Recent analyses of pion-production experiments[9,10] have indicated that $I=0$ s-wave $\pi\pi$ scattering is essentially elastic up to the $K\bar{K}$ threshold and that p-wave scattering is elastic to about the same energy. Thus, the unitarity conditions, Eq. (1), for $J=0$ and $J=1$ should be valid up to $t\simeq 50\mu^2$.

The large amount of attention given to the $\pi\pi$ s-wave amplitude has now given a good qualitative picture of the phase shift δ_0^0 for $t<50\mu^2$. Although various analyses each have small quoted uncertainties, they do maintain some disagreement with one another. *For example*, the values of δ_0^0 from the Berkeley[9] energy-dependent analysis and those from the CERN-Munich[10] energy-independent analysis, both shown in Fig. 1, show disagreement of their central values by about 11° below $t=m_\rho^2\simeq 30\mu^2$. However, their quoted uncertainties are each about ± 4°.

Both of the above-cited experimental analyses, and the K_{e4} decay results from Geneva-Saclay,[11] show a somewhat larger δ_0^0 than that of the Morgan-Shaw[16] analysis, which utilizes analyticity but less experimental $\pi\pi$ information, although the

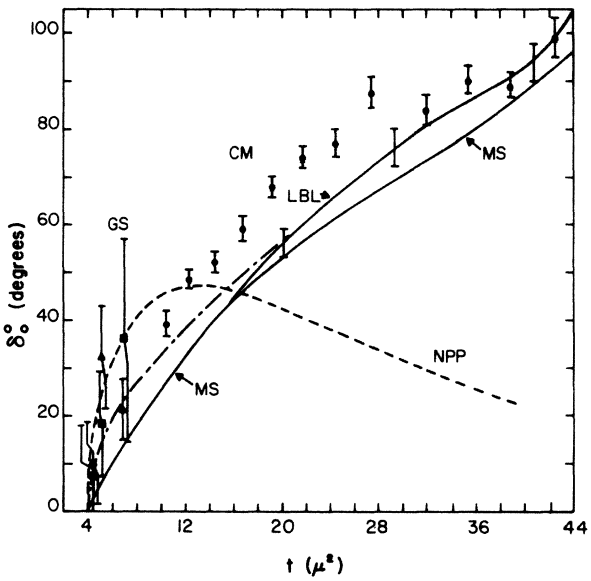

FIG. 1. The $I=J=0$ $\pi\pi$ phase shift δ_0^0. Shown are the Morgan-Shaw result (MS, Ref. 16), the Berkeley result (LBL, Ref. 9), the CERN-Munich results (CM, Ref. 10) (for which half of the quoted points are shown), the Geneva-Saclay results (GS, Ref. 11), and the NPP results (Ref. 7). The dash-dot curve, constructed to agree with GS and LBL, is discussed in the text.

scattering length found in the Morgan-Shaw analysis ($0.16\mu^{-1}$) is in excellent agreement with that found by Pennington and Protopopescu[17] ($0.15\mu^{-1}$). The Pennsylvania[18] K_{e4} decay results appear to be consistent with both the Morgan-Shaw and Geneva-Saclay results. In what follows we first use the Morgan-Shaw δ_0^0 and then indicate the effect of using a larger value.

The p-wave $\pi\pi$ phase shift δ_1^1 is known with greater certainty because of the strong ρ resonance. The phase shift δ_1^1 is well represented, for $t<50\mu^2$, by the expression

$$\left(\frac{\nu}{\nu+1}\right)^{1/2} \cot\delta_1^1 = \frac{(1-0.1536\nu)(1+0.00076\nu)}{0.035\nu} , \qquad (3)$$

where $\nu\mu^2$ is the center-of-mass pion momentum squared. This form was used by Morgan and Shaw[16] but with a smaller width than that used here.[9] The scattering length used, $0.035\mu^{-3}$, is in very good agreement with a recent determination.[17]

The d-wave $\pi\pi$ phase shift δ_2^0 is certainly less well known but all analyses find it to be small. The expression used here is[16]

$$\left(\frac{\nu}{\nu+1}\right)^{1/2} \cot\delta_2^0 = \frac{(1-0.0524\nu)(1+0.204\nu+0.0015\nu^2)}{0.0015\nu^2} . \qquad (4)$$

This $\pi\pi$ phase shift is found to be of minor importance in the nuclear force.

III. THE MAGNITUDE OF THE $N\bar{N}\to\pi\pi$ AMPLITUDE f_+^0

For the determination of the *magnitude* of the f_+^0 amplitude, we first wrote a twice-subtracted dispersion relation for $D_0(t)f_+^0(t)/(t-4m^2)$:

$$\frac{D_0(t)f_+^0(t)}{t-4m^2}=\frac{(t-4\mu^2)f_+^0(0)}{16m^2\mu^2}+\frac{tD_0(4\mu^2)f_+^0(4\mu^2)}{16\mu^2(\mu^2-m^2)}+\frac{t(t-4\mu^2)}{\pi}\int_{-\infty}^{a}\frac{D_0(t')\mathrm{Im}f_+^0(t')dt'}{t'(t'-4\mu^2)(t'-4m^2)(t'-t)}\,, \tag{5}$$

where $a\equiv4\mu^2-\mu^4/m^2$. The non-Born part of the input $\mathrm{Im}f_+^0(t'<a)$ was taken from Nielsen's[14] work and added to the Born term.[19] For the $t=0$ subtraction constant, $\mathrm{Re}f_+^0(0)$ was found by Furuichi and Watanabe[20] to be -2.8μ. However, this value assumed[21] $A^{(+)}(\nu=0,\,t=0)=24.6\mu^{-1}$, which is in disagreement with other determinations.[14,22-24] An improved value was used here, described later. In the $t=4\mu^2$ subtraction constant, the value of $f_+^0(4\mu^2)$ must be known accurately because of its large size. The value currently available in the literature[24] was obtained from fixed-t dispersion relations which depended on a continuation of the pion-nucleon partial-wave amplitudes to the branch cut at $t=4\mu^2$, where formal convergence ceases; it is therefore open to question. An alternate procedure is used here.

IV. DETERMINATION OF THE πN AMPLITUDES AT $\nu=t=0$

Fixed-t dispersion relations for

$$A^{(+)}(\nu,t)$$

and

$$A'^{(+)}(\nu,t)=A^{(+)}(\nu,t)+\frac{\nu}{1-t(4m^2)^{-1}}B^{(+)}(\nu,t)\,,$$

where $\nu=(s-u)/(4m)$, have been used to find $A^{(+)}$ and the derivative of $A^{(+)}(0,t)$ at $t=0$. With these quantities we have been able to find $f_+^0(4\mu^2)$ as described later and to improve the $f_+^0(0)$ of Ref. 20. Both $A^{(+)}$ and $A'^{(+)}$ satisfy dispersion relations of the form

$$A(\nu,t)=A(\nu_c,t)+\frac{2(\nu^2-\nu_c^2)}{\pi}\int_{\nu_c}^{\infty}\frac{\nu'\,\mathrm{Im}A(\nu',t)}{(\nu'^2-\nu_c^2)(\nu'^2-\nu^2)}d\nu'\,, \tag{6}$$

where $\nu_c=\mu+t/4m$. The necessary subtraction functions are written in terms of the isospin-even scattering lengths at the point $\nu=\nu_c$ as

$$A^{(+)}(\mu,0)=8\pi\left[\frac{2m+\mu}{4m}a_0^{(+)}+\mu m(a_{1+}^{(+)}-a_{1-}^{(+)})\right], \tag{7a}$$

$$\frac{\partial}{\partial t}A'^{(+)}(\nu_c(t),t)\bigg|_{t=0}=\frac{4\pi(m+\mu)}{m}\left(\frac{a_0^{(+)}}{8m^2}+\tfrac{1}{2}a_{1-}^{(+)}+a_{1+}^{(+)}\right). \tag{7b}$$

The sensitivity to $a_0^{(+)}$ is small in each case. The value used here is[25] $a_0^{(+)}=\frac{1}{3}(-0.014\pm0.005)\mu^{-1}$. The largest uncertainty in using Eq. (6) comes from the p-wave scattering lengths, for which we used a weighted average of experimental results:

$$a_{1+}^{(+)}-a_{1-}^{(+)}=\tfrac{1}{3}(0.571\pm0.010)\mu^{-3},$$
$$a_{1+}^{(+)}+\tfrac{1}{2}a_{1-}^{(+)}=\tfrac{1}{3}(0.310\pm0.010)\mu^{-3}.$$

The first of these combinations is consistent with the very accurate determination of Bugg, Carter, and Carter[25] as well as older analyses.[22,23] For the second combination both Hamilton and Woolcock[22] and Höhler *et al.*[23] found very nearly this value.

The imaginary parts of the amplitudes $A^{(+)}$ and $A'^{(+)}$ were obtained from πN phase shifts. The results for the on-shell amplitudes at $\nu=0$ were rather insensitive to which of several sets[26] was used. The results found[27] are

$$A^{(+)}(0,0)=(25.9\pm0.5)\mu^{-1},$$
$$\frac{\partial}{\partial t}A^{(+)}(0,t)\bigg|_{t=0}=(1.16\pm0.05)\mu^{-3}.$$

Both values are consistent with other determinations.[14,22-24] Combining the first value with the results of Ref. 20, we now obtain

$$\mathrm{Re}f_+^0(0)=-2.4\mu\,.$$

V. THE $J=0$ SUBTRACTION CONSTANT AT $t=4\mu^2$

The function $D_0(t)A^{(+)}(0,t)$ has no s-wave contribution to the nearby part of its right-hand cut where the $\pi\pi$ s-wave amplitude is elastic. The imaginary part of this function can therefore be evaluated on the nearby right-hand cut using the d-wave helicity amplitudes yet to be described, and assuming that the $J\geq4$ helicity amplitudes are adequately given by the nucleon pole. Neglecting $\mathrm{Im}f_+^2$, the twice-subtracted dispersion relation then reads

$$D_0(t)A^{(+)}(0,t)=A^{(+)}(0,0)+t\left[\frac{\partial}{\partial t'}D_0(t')A^{(+)}(0,t')\right]\bigg|_{t'=0}+I_L+\frac{t^2}{\pi}\int_{4\mu^2}^{\infty}\frac{-|D_0(t')|A_{J\geq2}^{(+)}(0,t')\sin\delta_0^0(t')}{t'^2(t'-t)}dt'\,, \tag{8}$$

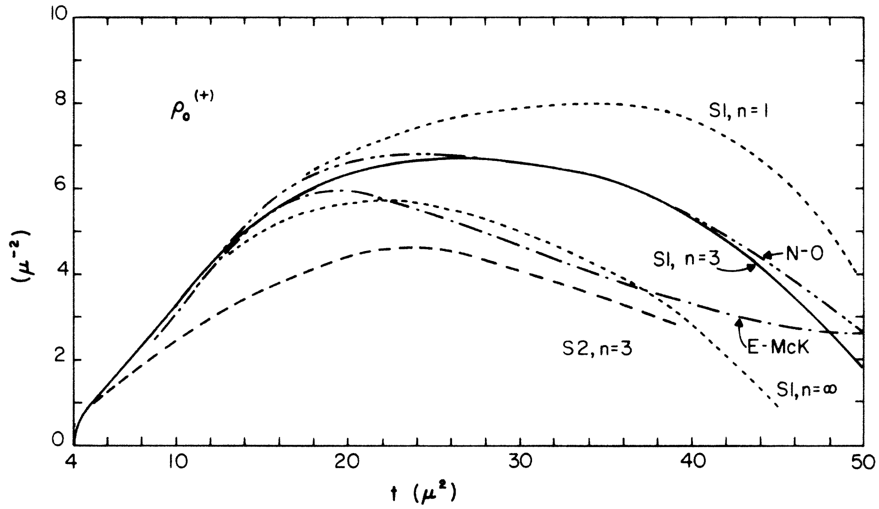

FIG. 2. The $J=0$ spectral function $\rho_0^{(+)}(t)=8\pi[(t-4\mu^2)/t]^{1/2}(4m^2-t)^{-2}|f_+^0(t)|^2$. Results are shown for various inputs to Eq. (5). Also shown are the Nielsen-Oades (N-O, Ref. 13) and the Epstein-McKellar (E-McK, Ref. 4) results.

where

$$A_{J\geq 2}^{(+)}(0,t)=A^{(+)}(0,t)-16\pi(4m^2-t)^{-1}f_+^0(t)\,.$$

The contribution from the distant left-hand cut at $t=-27\mu^2$ is denoted I_L. Because of its remoteness and the two subtractions, it is expected that only the leading edge of this cut is important. The following parameterization was used to represent the left-hand cut:

$$D_0(t)\,\mathrm{Abs}\,A^{(+)}(0,t)=\alpha(-t-4m\mu)^{1/2}\,. \qquad (9)$$

Comparing Eq. (8) with Nielsen's[14] fixed-t dispersion relation results, at negative t, fixed $\alpha\simeq 29\mu^{-2}$ when the Morgan-Shaw δ_0^0 was used. Using Eq. (8) with the Morgan-Shaw δ_0^0 to continue to $t=4\mu^2$, where $A^{(+)}$ is purely s-wave in the $N\bar{N}\to\pi\pi$ channel, we found[28]

$$f_+^0(4\mu^2)=(114\pm 2)\mu\,.$$

The uncertainty quoted here does not include that due to the uncertainty in the low-t δ_0^0 input.

Shown in Fig. 1 is a δ_0^0 constructed to agree with the Geneva-Saclay[11] K_{e4} decay results at $t<8\mu^2$ and to join smoothly with the Berkeley[9] δ_0^0 at $t\simeq 20\mu^2$. When this phase shift, which we shall refer to as GS + LBL, was used in Eq. (8) we found $\alpha\simeq 29\mu^{-2}$ and a larger amplitude at $t=4\mu^2$:

$$f_+^0(4\mu^2)=(118\pm 2)\mu\,.$$

The magnitude $|f_+^0(t)|$ and the $J=0$ spectral function, which directly enters the NN fixed-energy dispersion relation and is proportional to $|f_+^0(t)|^2$, was then calculated from Eq. (5). We have found that the results from Eq. (5) for $t\gtrsim 20\mu^2$ are somewhat sensitive to the phase of f_+^0 above $50\mu^2$, where it is not known. In Fig. 2 results are shown

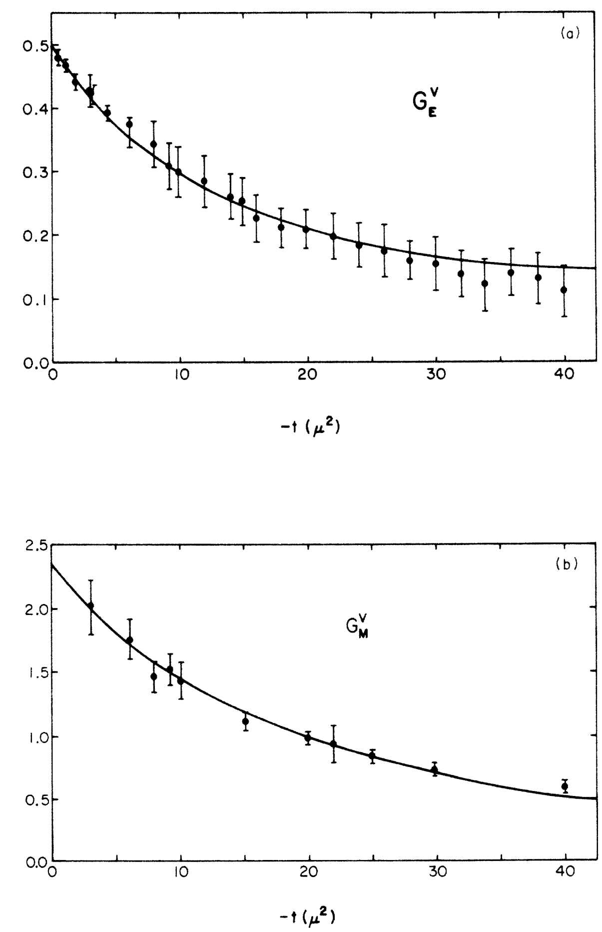

FIG. 3. The nucleon electromagnetic form factors $G_E^V=F_1^V+(t/2m)F_2^V$ and $G_M^V=F_1^V+2mF_2^V$, with F_i^V from Eq. (12) and Γ from Eq. (11). Data are as in Ref. 29.

for several assumed forms of the f_+^0 phase above $t=50\mu^2$, all of which have the general form

$$\delta(t)=(50\mu^2/t)^n\,\delta(50\mu^2)\,. \qquad (10)$$

For clarity, calculations using the Morgan-Shaw δ_0^0, $\mathrm{Re}\,f_+^0(0)=-2.4\mu$, and $f_+^0(4\mu^2)=114\mu$ are designated S1. Since the S1 curves for various $n>0$ are very similar at $t\lesssim 20\mu^2$ they all produce similar NN amplitudes for the states and energies to be shown here. When the calculation was performed with the GS + LBL δ_0^0 the resulting $|f_+^0(t)|$ was considerably smaller for $t>6\mu^2$. This result, which used $\mathrm{Re}\,f_+^0(0)=-2.4\mu$ and $f_+^0(4\mu^2)=118\mu$, is designated S2. We have shown the S2 result using $n=3$.

Also given in Fig. 2 are the results found by Nielsen and Oades[13] using the Morgan-Shaw δ_0^0 and those found by Epstein and McKellar.[4] Although Nielsen and Oades used $n=1$ in Eq. (10), their analytic continuation of $D_0f_+^0/(t-4m^2)$ from $t<4\mu^2$ to $4\mu^2<t<50\mu^2$ would be expected to be less sensitive to the $t>50\mu^2$ input phase.

VI. THE p-WAVE AMPLITUDES $f_\pm^1$

It is convenient to work with the amplitudes[8]

$$\Gamma_1=\frac{4m}{t-4m^2}\left(f_+^1-\frac{t}{4\sqrt{2}m}\,f_-^1\right),$$

$$\Gamma_2=\frac{2}{t-4m^2}\left(-f_+^1+\frac{m}{\sqrt{2}}\,f_-^1\right).$$

For these amplitudes we used the following dispersion relation:

$$D_1(t)\Gamma_i(t)=\Gamma_{iB}(t)+\frac{t_0-t}{t_0}\,\tilde{\Gamma}_i(0)+\frac{t}{t_0}\,[D_1(t_0)\Gamma_i(t_0)-\Gamma_{iB}(t_0)]$$
$$+\frac{t(t-t_0)}{\pi}\int_{-\infty}^{a}\frac{D_1(t')\mathrm{Im}\,\tilde{\Gamma}_i(t')+[D_1(t')-1]\mathrm{Im}\Gamma_{iB}(t')}{t'(t'-t_0)(t'-t)}\,dt' \qquad (11)$$

where $\tilde{\Gamma}_i\equiv\Gamma_i-\Gamma_{iB}$. The dispersion relation for the Born term Γ_{iB} has been subtracted from the relation for Γ_i to obtain a less rapidly varying integrand near $t'=a$. The values $\tilde{\Gamma}_1(0)=0.017\mu^{-2}$ and $\tilde{\Gamma}_2(0)=-0.0185\mu^{-3}$ were obtained from Ref. 14. We chose to take $t_0=30\mu^2\simeq m_\rho^2$, and determined the constants $\Gamma_i(t_0)$ from the nucleon electromagnetic form factors as follows.

Attempts[29] to explain the behavior of the nucleon form factors suggest that the isovector form factors at small $t<0$ (spacelike) are determined via analyticity by the two-pion intermediate state ($N\bar{N}\to 2\pi\to\gamma$) at $t>4\mu^2$. The form factors[30] F_1^V and F_2^V have the following representations[8]:

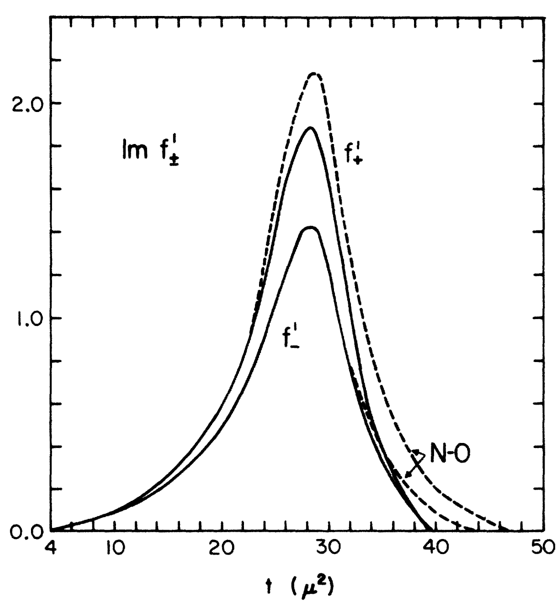

FIG. 4. The imaginary parts of the p-wave helicity amplitudes f_+^1 (upper solid and dashed curves) and f_-^1 (lower curves) in units where $\mu=1$. The solid curves are from Eq. (11); the dashed curves are from Nielsen and Oades, Ref. 13.

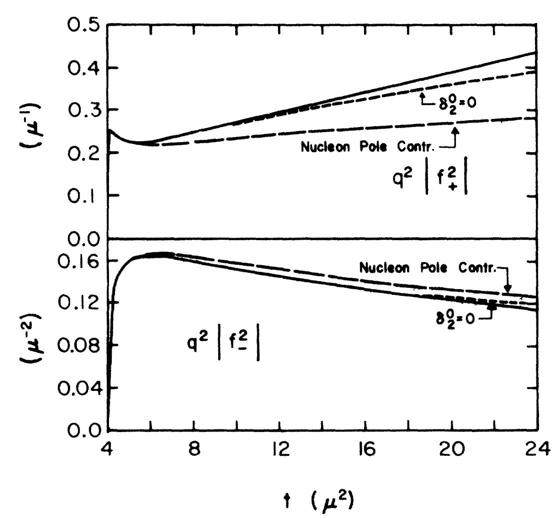

FIG. 5. The d-wave helicity amplitude magnitudes $|f_\pm^2|$ multiplied by $q^2=t/4-\mu^2$. The solid curves are from once-subtracted Omnès relations. The short-dashed curves were obtained by setting $D_2=1$. The long-dashed curves are the Born (nucleon pole) contributions.

$$F_i^V(t)=F_i^V(0)-\frac{t}{\pi}\int_{4\mu^2}^{\infty}\left[\frac{(t'-4\mu^2)^3}{64t'}\right]^{1/2}\times\frac{F_\pi{}^*(t')\Gamma_i(t')}{t'(t'-t)}\,dt', \qquad (12)$$

where $F_1^V(0)=\frac{1}{2}$ and $F_2^V(0)=1.853/(2m)$. We used $1/D_1(t)$ for $F_\pi(t)$, the pion form factor.[8,30]

We have found without detailed fitting that the small-$|t|$ experimental form factor data are very well reproduced (Fig. 3) with the following values of $|\Gamma_i(30\mu^2)|$:

$$|\Gamma_1(30\mu^2)|=0.121\mu^{-2},$$

$$|\Gamma_2(30\mu^2)|=0.0544\mu^{-3}.$$

The result (Fig. 4) for $\mathrm{Im} f_-^1$ is in good agreement with that of Nielsen and Oades,[13] while $\mathrm{Im} f_+^1$ is somewhat lower near resonance.

VII. THE d-WAVE AMPLITUDES $f_\pm^2$

Using a once-subtracted dispersion relation with the Born amplitude removed, subtractions were made at both $t_0=\mu^2$ and $t_0=3\mu^2$ giving very similar results. The subtraction constants $\bar{f}_\pm^2(t_0)$ were obtained from Ref. 14. The results (Fig. 5) show a significant non-Born contribution to f_+^2, while f_-^2 is close to the Born amplitude, in agreement with NPP.[7]

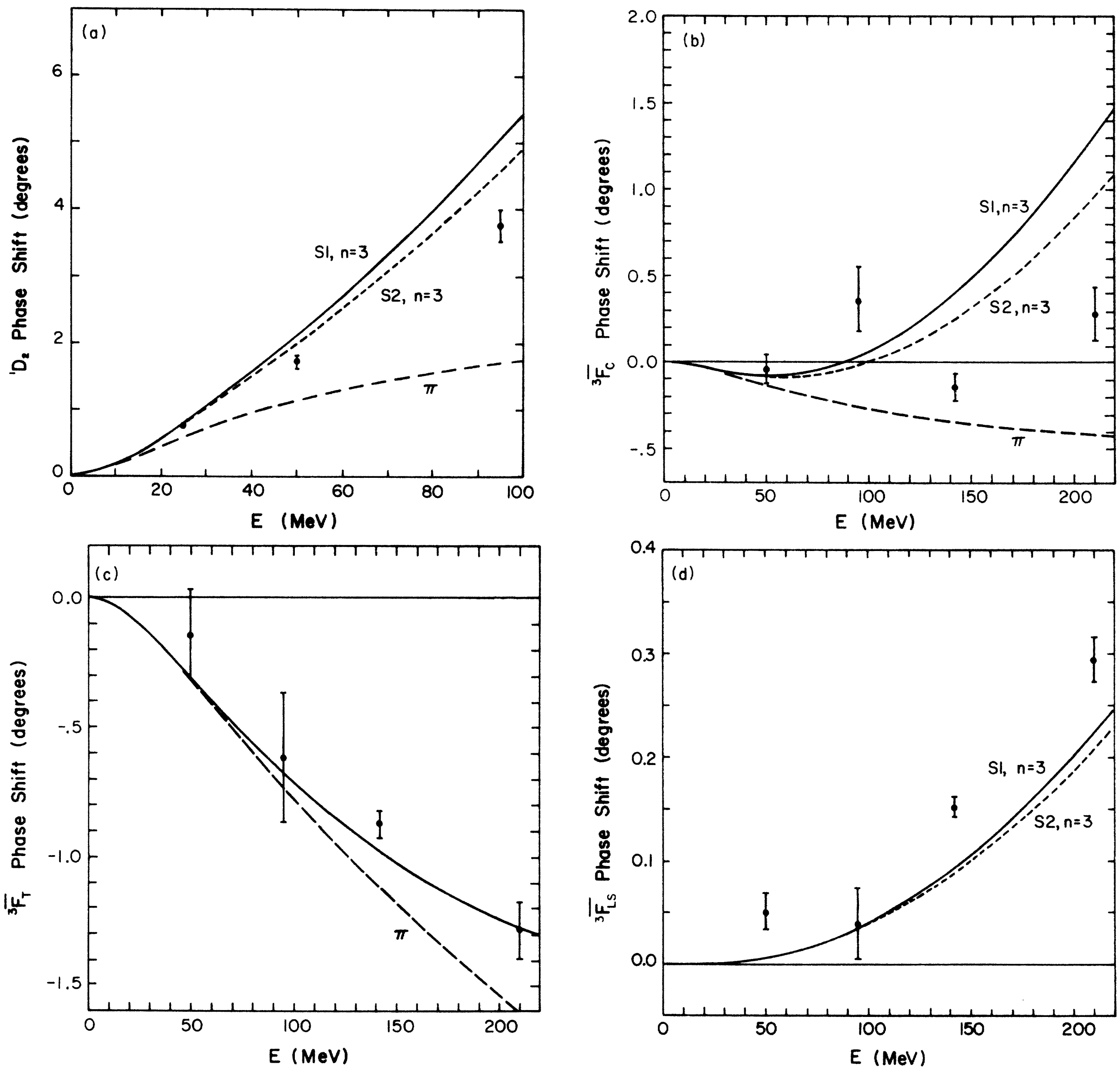

FIG. 6. *(Continued on following page.)*

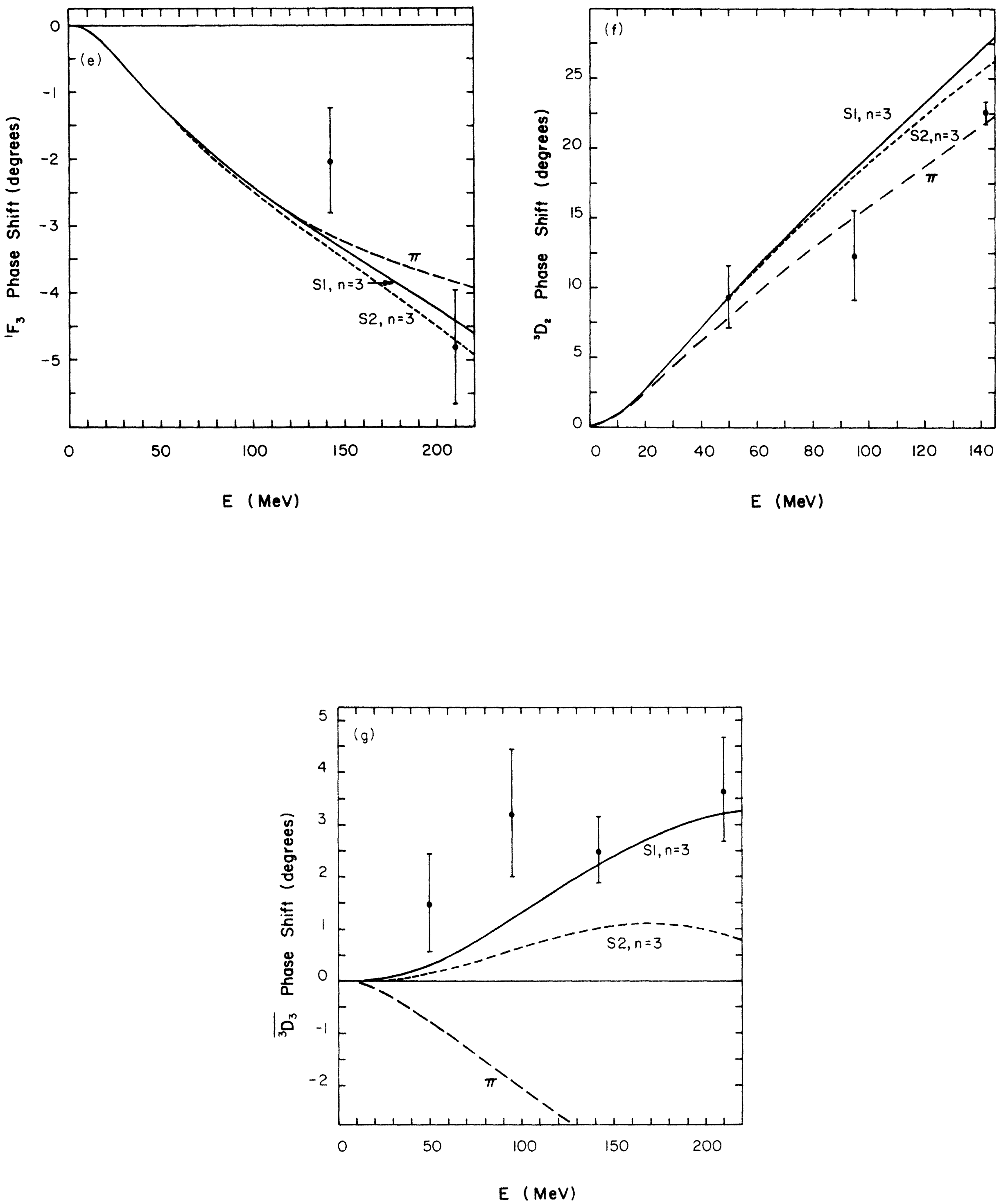

FIG. 6. The *NN* nuclear-bar phase shifts as a function of lab kinetic energy for two sets of input. The data points are from the single-energy phase-shift analyses of M. H. Mac Gregor, R. A. Arndt, and R. M. Wright, Phys. Rev. 182, 1714 (1969). A small correction has been applied to the phase-shift-analysis $^1D_2$ value to remove the effect of the Coulomb force. Also shown are the one-pion-exchange phase shifts. In all cases the theoretical phase shifts are defined via the real part of the scattering amplitude with no unitarization imposed. The F-wave phase-shift combinations are $^3\overline{F}_c = (5\,^3\overline{F}_2 + 7\,^3F_3 + \frac{9}{21}\,^3\overline{F}_4)$, $^3\overline{F}_T = -\frac{5}{112}\,(4\,^3\overline{F}_2 - 7\,^3F_3 + 3\,^3\overline{F}_4)$, $^3\overline{F}_{LS} = -(20\,^3\overline{F}_2 + 7\,^3F_3 - 27\,^3\overline{F}_4)/168$.

It is interesting to see the effect of neglecting the right-hand cut (setting $D_2 = 1$) because of the uncertainty in the d-wave $\pi\pi$ phase shift. Fig. 5 shows that the right-hand cut has a larger effect on f_+^2 than on f_-^2. Even so, neglecting this $J=2$ cut has a negligible effect on the two-pion-exchange amplitude to be shown here.

VIII. THE NUCLEON-NUCLEON AMPLITUDE

The two-pion-exchange amplitude was calculated from the $N\bar{N}\to\pi\pi$ amplitudes, as described in the Introduction, and added to a one-pion-exchange pole with $g^2/4\pi = 14.4$ and an ω-exchange pole with $g_\omega^2/4\pi = 4.7$, $f_\omega/g_\omega = -0.12$. The phase shifts representing this NN amplitude are shown in Fig. 6. Results are given for two different values of the $J=0$ spectral function, which we recall is proportional to $|f_+^0(t)|^2$. The input to the curves labeled S1, $n=3$ included the Morgan-Shaw $I=J=0$ $\pi\pi$ phase shift δ_0^0 and the $t > 50\mu^2$ f_+^0 phase given by $n=3$ in Eq. (10). For the curves labeled S2, $n=3$ the input included the δ_0^0 constructed to agree with the Geneva-Saclay[11] K_{e4} decay results and the Berkeley[9] results (GS+LBL), and $n=3$. The $L=3$ phase shifts are given as linear combinations of the ^3F_J phase shifts to approximately separate the central, tensor, and spin-orbit components of the $T=1$ amplitude. The tensor component is independent of $|f_+^0|$.

The phase shifts indicate that the theoretically calculated $T=0,1$ triplet central and $T=1$ singlet components are too attractive at higher energies. As shown in Fig. 7, the calculated $T=1$ singlet potential is in fact more attractive than the Hamada-Johnston phenomenological potential.[31] The excess attraction in the phase shifts and potential is reduced if one uses a smaller $|f_+^0(t)|$, such as that which would be introduced by an enhancement of the $\pi\pi$ phase shift δ_0^0 in the $4-10\mu^2$ region. This is indeed the region in which the experimental values are less certain.[11,18]

The effect of shifting from S1 to S2 is particularly large on the $^3\bar{D}_3$ phase shift. This phase shift is sensitive to the high-t spectral function input, which has not been convincingly determined. This sensitivity to high-t input indicates that the $^3\bar{D}_3$ may also have a significant contribution from three-pion exchange. The situation of the $T=0$ singlet state ($^1F_3$) is unclear; perhaps it has a little less repulsion than is in the theoretical amplitude. The $T=1$ spin-orbit component is insufficiently attractive with either form of δ_0^0 input.

The $\pi\pi$ p-wave exchange contribution is believed to have been accurately included. Neglect of higher mass states in the electromagnetic form factors may have had some influence on our Γ_i but the effect in the *intermediate-range NN* force is expected to be small. Whether one uses our Γ_i results or those of Nielsen and Oades[13] makes negligible difference in the NN amplitude shown here.

The high-t three-pion-exchange contribution was represented here by the ω pole with coupling constants which are rather uncertain. The g_ω^2 value that we used is consistent with the quark-model prediction $g_\omega^2 = 9g_\rho^2$; the f_ω/g_ω value is found by assuming the simplest ω-pole dominance of the isoscalar nucleon form factors. Larger coupling constants, such as those required in recent one-boson-exchange fits[32] to the NN data, would simultaneously increase the spin-orbit repulsion and decrease the central attraction. However, the high mass of the ω meson requires a very large value of g_ω^2 in order to subtantially influence the intermediate-range NN force.[33]

While two-pion exchange is certainly the dominant intermediate-range correction to one-pion exchange, our understanding of the intermediate-range force is still not complete. We have shown that the two-pion-exchange amplitude is very dependent on the low-t $\pi\pi$ phase shift δ_0^0. A more accurate determination of the low-t δ_0^0 is essential to reducing the uncertainty in the two-pion-exchange amplitude. The calculation of low-t three-pion-exchange effects may also be helpful in understanding the remaining differences between experiment and present theory.

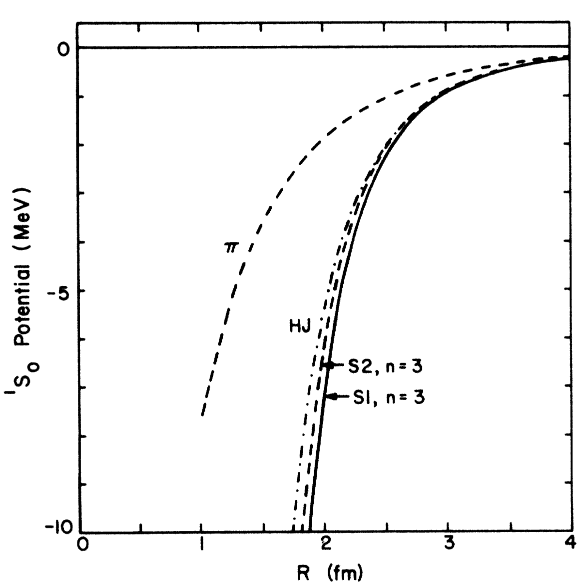

FIG. 7. The $^1S_0$ potential for two sets of δ_0^0 input. The Hamada-Johnston (HJ, Ref. 30) and one-pion-exchange potentials are also shown.

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