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Search for the rare decay $K^+ \to \pi^+ e^+ e^-$

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A search for the rare decay $K^+ \to \pi^+ e^+ e^-$ has been carried out at the Lawrence Berkeley Laboratory using a sparkostrictive wire-chamber spectrometer. Analysis programs identified events with three tracks or with two tracks of which one was an electron and the other a positron. Analysis of the three-track events yields a branching ratio $\Gamma(K^+ \to \pi^+ e^+ e^-)/\Gamma(K^+ \to \text{all}) < 1.7\times 10^{-6}$ (90% confidence). The two-track events, analyzed by searching for sparks on a possible pion track, yield a limit of $\Gamma(K^+ \to \pi^+ e^+ e^-)/\Gamma(K^+ \to \text{all}) < 2.7\times 10^{-7}$ (90% confidence). The latter value implies that the coupling constant for a vector neutral current must be about three orders of magnitude smaller than the coupling constant for the charged vector current.

I. INTRODUCTION

The reaction $K^+ \to \pi^+ e^+ e^-$ is one of a series of unobserved K^+ decays having a lepton pair of total charge zero. While the rates for these decays are known to be very low, there are many examples of kaon and baryon decays with a lepton pair consisting of one charged lepton and a neutrino. The low rates for the neutral-current modes suggest that there is a selection rule for semileptonic decays requiring $\Delta Q \neq 0$. The weak interaction would then appear to be sensitive to lepton charge, which was once thought to be a purely electromagnetic property.

Even if the $\Delta Q \neq 0$ rule is valid, the $K^+_{\pi ee}$ decay is expected to occur via a combination of weak and electromagnetic interactions in which, for example, a virtual photon couples to a neutral lepton pair. Theoretical estimates[1a-10] for such a mode typically yield branching ratios of 10^{-6} to 10^{-7}. (See Table I.) The decay rate could also be influenced by a second-order weak interaction

TABLE I. Summary of calculated branching ratios for $K^+\to\pi^+e^+e^-$ decay.

Ref.	Branching ratio R	Process
1(b)	10^{-7}	e.m.
1(d)	$\sim10^{-6}$	e.m.
1(e)*	3×10^{-7}	e.m.
1(f)	5×10^{-6}	e.m.
1(g)	3×10^{-7}	e.m., quark model
1(h)**	$<5\times10^{-7}$	Vector neutral current
1(i)	$>1.4\times10^{-10}$	e.m., vector dominance
1(j)	$\sim10^{-7}-10^{-8}$	Triplet IVB model
1(k)	$10^{-4}\left(\frac{m_K}{m_W}\right)^2$	Scalar W's
1(l)	$\sim10^{-6}-10^{-7}$	e.m.
1(m)***	$(3.1\pm1.2)\times10^{-7}$	e.m., PCAC
1(n)	$\sim10^{-6}-10^{-7}$	e.m., SU_3 pole model

*With $\Gamma_\rho=146$ MeV.
**With $\Gamma(K_2^0\to e^+e^-)/\Gamma(K_2^0\to\text{all})<1.6\times10^{-9}$ (Ref. 5).
***With $\gamma^2=\Gamma(K^+\to\pi^+\pi^0\gamma\text{-direct})/\Gamma(K^+\to\pi^+\pi^0\gamma\text{-all})=0.061\pm0.24$ (Ref. 12).

which interfered with the weak-electromagnetic process. Such predictions, as well as any nonlocality of the second-order diagrams, could be studied if a significant number of $K^+_{\pi ee}$ events were found.

The lowest limit reported thus far on the $K^+_{\pi ee}$ branching ratio is near the top of the range where the decay might be expected to occur:

$$\frac{\Gamma(K^+\to\pi^+e^+e^-)}{\Gamma(K^+\to\text{all})}<8.8\times10^{-7}$$

(90% confidence; see Ref. 2).

The limits for two related decay modes are

$$\frac{\Gamma(K^+\to\pi^+\mu^+\mu^-)}{\Gamma(K^+\to\text{all})}<2.4\times10^{-6}$$

(90% confidence; see Ref. 3),

$$\frac{\Gamma(K^+\to\pi^+\nu\bar\nu)}{\Gamma(K^+\to\text{all})}<5.6\times10^{-7}$$

(90% confidence; see Ref. 4).

The neutral-current decays of K^0 mesons are also of great interest and have recently been extensively investigated.[5]

II. APPARATUS

The experimental arrangement is shown in Fig. 1. A separated K^+ beam of 1.475 GeV/c and $\Delta p/p=\pm3\%$[6a,b,c] was incident on a large sparkostrictive wire-chamber spectrometer,[7] and decay electrons were distinguished from pions and muons by two 1-atm-Freon Cherenkov counters,[8] each of about 10 m^3 active volume.

The K^+ beam[6a-6c] was produced by the external proton beam from the Berkeley Bevatron and utilized (1) a single 4.5-m separator with 600-kV electrostatic field and tunable magnetic field, (2) a cylindrical beam Cherenkov counter filled with Freon 13 at 11.2 atm and used to separate further the π^+ and K^+ constituents, (3) various scintillation counters to define the beam geometry, timing, and K^+ decay volume, (4) quadrupoles and bending magnets to focus and momentum-analyze the beam, and (5) three magnetostrictive spark chambers to determine the momenta of individual charged particles. The over-all length of the K^+ beam was 17.7 m, about 1.7 K^+ decay lengths at 1.475 GeV/c. The typical yield was 30 000 K^+ per Bevatron pulse of 0.5×10^{12} protons on target. Comparable numbers of π^+ and protons were also present in the raw K^+ signal, but these did not contribute significantly to the final data sample. The momentum resolution for individual beam particles was ± 1.5%.

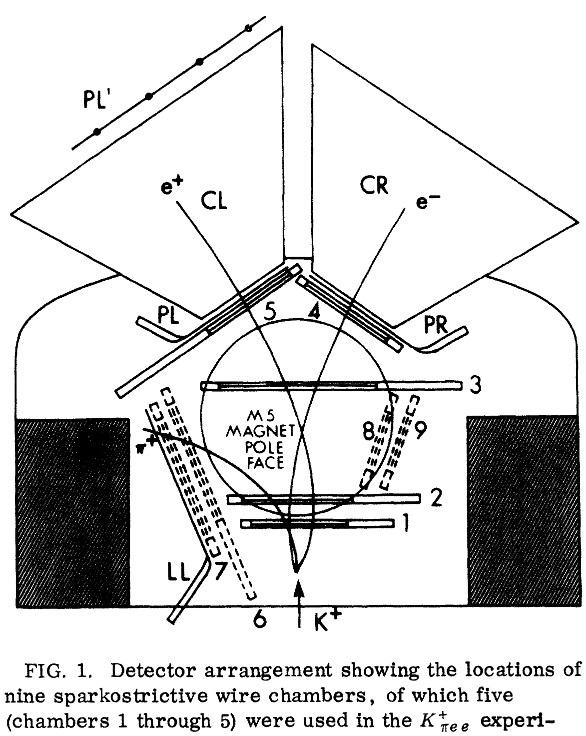

FIG. 1. Detector arrangement showing the locations of nine sparkostrictive wire chambers, of which five (chambers 1 through 5) were used in the $K^+_{\pi ee}$ experiment. The positions of the scintillator-counter hodoscopes (LL, PL, PL', and PR, which was a double plane) and Cherenkov counters (CL and CR) used in the trigger are also indicated. The $K^+_{\pi ee}$ trigger required an incident K^+ in coincidence with a forward-going electron and a forward-going positron. The scale is indicated by the $M5$ magnet pole face, which was 152 cm in diameter.

As indicated in Fig. 1, the sparkostrictive wire-chamber spectrometer[7] consisted of nine chambers, of which only five were involved in the $K^+_{\pi ee}$ search. The chambers ranged in active width-times-height from 91 cm × 81 cm to 152 cm × 91 cm and were positioned between the 152-cm-diameter pole faces of the Lawrence Berkeley Laboratory *M*5 magnet, which had a 120-cm gap and a 6-kG central field in this application. The grid wires were spaced 1 mm apart and on most chambers were oriented with wires running vertically and at $\pm 30°$ angles to the vertical to provide three redundant coordinates per track. The root-mean-square spatial resolution was typically 1.6 mm.[7,9] The K^+ mass and full width at half-maximum, reconstructed from 986 events of the type $K^+ \to \pi^+\pi^+\pi^-$ were, respectively, 495 MeV and 12 MeV.[9]

The $K^+_{\pi ee}$ trigger required an incident K^+ in time coincidence with a forward-going electron and a forward-going positron. Forward-going particles were signaled by scintillator-hodoscope planes PL and PL', located before and after the left Cherenkov counter CL, and PR (a double plane) located before the right Cherenkov counter CR, as shown in Fig. 1. Light from $e^\pm$ incident on the Cherenkov counters was focused on four 60 AVP photomultipliers. The upper and lower halves of CL were isolated by a septum, and the 60 AVP signals were individually recorded on magnetic tape, producing, in effect, a four-quadrant hodoscope.

The Cherenkov counter efficiency measured[8] with incident $e^\pm$ having well-defined trajectories was $(92.5\pm 0.7)\%$. This dropped to 66–85% for the $K^+_{\pi ee}$ data sample, for which the phase volume of the decay electrons exceeded the nominal $25°\times 25°$ acceptance of each counter by more than a factor of 2. The probability for a $\pi^\pm$ to simulate an $e^\pm$ was also measured[8] with both incident and decay $\pi^\pm$ and was found to be less than 1% in each case.

In the $K^+_{\pi ee}$ experiment, 3.6×10^{10} K^+ candidates were recorded during 5.6×10^5 beam pulses. Approximately $0.9\times 10^9 K^+$ decays were detected, yielding 320 734 triggers with a final e^+ and e^-. These triggers were analyzed for possible $K^+_{\pi ee}$ events.

III. DATA ANALYSIS

A. Four stages

The analysis of events selected by the e^+e^- trigger was carried out in four stages. In stage 1, individual sparks were reconstructed as four-wire, three-wire, and in certain cases,[6b] two-wire fits in a particular chamber. Sparks from successive chambers were matched in stage 2 to form tracks having three or four sparks per track. This yielded about 10 000 events with three prongs and about 48 000 events with two prongs. Usually, the two-prong events occurred because a decay pion missed all three of the downstream chambers (chambers 3, 4, 5 in Fig. 1). In some cases, however, a decay pion passed through these chambers but was not detected due to chamber inefficiencies.[7]

In stage 3, the tracks were fitted to a common vertex and their vector momenta determined. Kinematic fitting and hypothesis testing of the two-prong and three-prong events were carried out in stage 4 with a program called HASH.[10]

B. Three-prong events

Of the kinematically fitted three-prong events, 2631 were consistent with the two-constraint reaction

$$K^+ \to \pi^+ + \pi^0, \quad \pi^0 \to \gamma + e^+ + e^-, \quad (2C)$$

i.e., with "$K^+_{\pi 2}$-Dalitz" decay. A plot of the number of events versus e^+e^- effective mass $M_{e^+e^-}$ is shown for this sample in Fig. 2 and is in good agreement with Monte Carlo calculations. A Monte Carlo calculation was also made for the $K^+_{\pi ee}$ mode. It happens that the decay distributions are quite similar for either an electromagnetic interaction or a vector neutral-current interaction.[1c,11] The reason is that the electromagnetic interaction is also vector, and gauge invariance demands that the form factor be proportional to q^2, which just

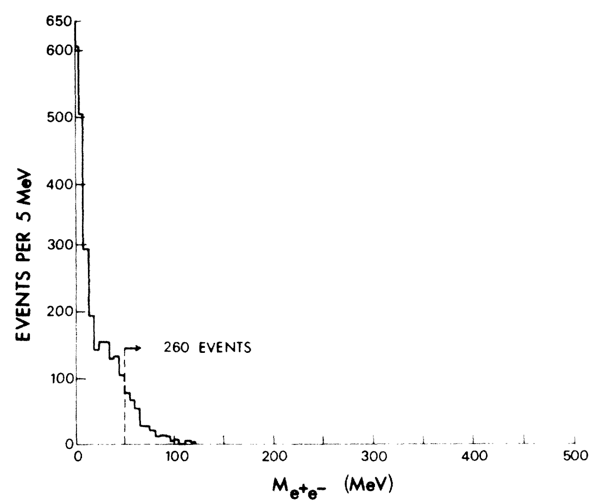

FIG. 2. Three-prong events from HASH satisfying the two-constraint $K^+_{\pi 2}$-Dalitz hypothesis. The sample shown consists of 2631 fits with χ^2 less than 13, corresponding to about the 1% probability level. Of these, only 260 have $M_{e^+e^-}$ greater than 50 MeV.

cancels the $1/q^2$ from the photon propagator. If the form factors remaining after account has been taken of gauge invariance and Lorentz invariance are assumed to be constant, then the decay spectra for the above two interactions are identical and somewhat different from phase space. A phase-space distribution was actually used for the Monte Carlo determination of the effect of the various cuts made in this experiment. Figure 3 shows the Monte Carlo $M_{e^+e^-}$ distribution implied by phase space. A correction factor was then applied to the sample remaining after the $M_{e^+e^-}$ cut to make it consistent with the above-mentioned constant form factor distributions. This amounted to 5% and 13% for the two-prong and three-prong events, respectively. The vast majority of the simulated $K^+_{\pi ee}$ events satisfied $M_{e^+e^-} > 50$ MeV, whereas only 10% of the observed $K^+_{\pi 2}$-Dalitz decays survived such a cut.

Less than 50 of the fitted three-prong events were kinematically consistent with the four-constraint τ decay

$$K^+ \to \pi^+ + \pi^- + \pi^+ . \quad (4C)$$

The χ^2 cutoffs for $K^+_{\pi 2}$-Dalitz and τ fits were 13 and 10, respectively, corresponding to about the 1% level in probability. Only events that did not satisfy the $K^+_{\pi 2}$-Dalitz or the τ hypotheses were further considered as $K^+_{\pi ee}$ candidates. The χ^2 for a $K^+_{\pi ee}$ fit in HASH was required to be less than 17, corresponding to a probability of about 0.2%.

Only eleven three-prong events with $M_{e^+e^-}$ greater than 40 MeV survived the deletion of $K^+_{\pi 2}$-Dalitz and τ events and also satisfied the $K^+_{\pi ee}$ fit in HASH. Likely sources of these events were unidentified $K^+_{\pi 2}$-Dalitz decays, as well as Dalitz decays of π^0's associated with the K^+_{e3} and $K^+_{\pi 3}$ reactions

$$K^+ \to \pi^0 + e^+ + \nu ,$$

$$K^+ \to \pi^+ + \pi^0 + \pi^0 .$$

The K^+_{e3}-Dalitz mode was particularly effective in generating e^+e^- combinations with large $M_{e^+e^-}$. The $K^+_{\pi 2}$- and $K^+_{\pi 3}$-Dalitz decays could survive the mass cut $M_{e^+e^-} > 40$ MeV if the e^+ and π^+ assignments were ambiguous, due, for example, to both positive tracks hitting the same Cherenkov quadrant. Only six of the eleven events have the "other" e^+e^- invariant mass combination ($e^+\pi^+$ or e^+e^+ reversed) greater than 30 MeV. A cut was therefore made at $M_{e^+e^-}$ (other) > 50 MeV.

Further discrimination against the $K^+_{\pi 2}$-Dalitz background was obtained by requiring $K^+_{\pi \mathrm{MM}} > 180$ MeV, where $K^+_{\pi \mathrm{MM}}$ signifies the missing mass recoiling against the positive track when this track is assumed to be a π^+. The effect of such a cut for this one-constraint, quasi-two-body decay can be seen in Fig. 4, where the event distribution versus $K^+_{\pi \mathrm{MM}}$ is plotted for the three-prong sample. A final cut was made of $M_{e^+e^-} > 75$ MeV. These last two cuts eliminated the remaining five events.

The effective number of K^+ decays for the three-prong data sample was estimated from the number of $K^+_{\pi 2}$-Dalitz decays actually observed and was 1.65×10^6. This value was increased by the factor 1.047 to account for the relative $K^+_{\pi ee}/K^+_{\pi 2}$-Dalitz triggering efficiency. The correction factors for the data cuts were also calculated by a Monte

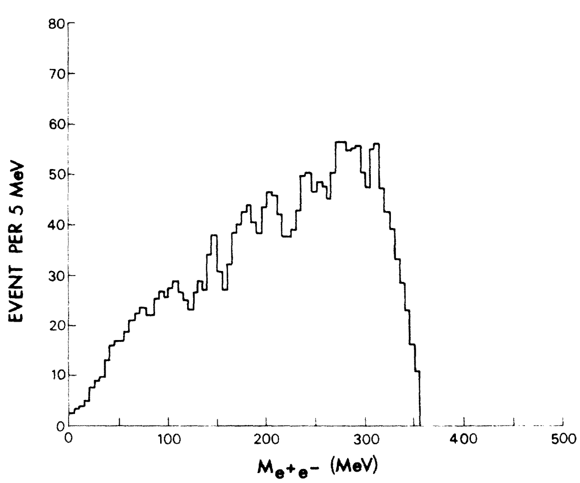

FIG. 3. Three-prong $M_{e^+e^-}$ distribution obtained by Monte Carlo calculation of 2312 phase-space $K^+_{\pi ee}$ decays. Of these, 91.6% had $M_{e^+e^-}$ greater than 75 MeV.

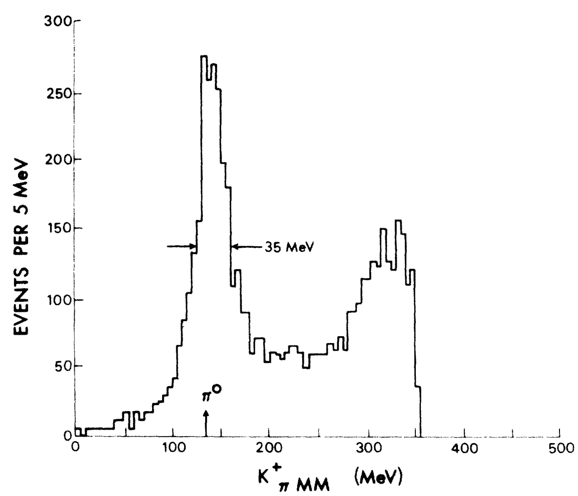

FIG. 4. Distribution of missing mass opposite the positive track, assumed to be a π^+, in the decay $K^+ \to \pi^+ +$ (missing mass). No cut on $M_{e^+e^-}$ has yet been made. Thus most of the 5452 events in this three-prong sample are $K^+_{\pi 2}$-Dalitz decays. Approximately 97% of the $K^+_{\pi ee}$ events were expected, by Monte Carlo calculation, to have $K^+_{\pi \mathrm{MM}}$ larger than 180 MeV.

Carlo technique and were as follows:

(1) $M_{e^+e^-} > 75$ MeV: 0.808;

(2) $M_{e^+e^-}$ (other) > 50 MeV: 0.975;

(3) $K^+_{\pi MM}$ of positive track > 180 MeV: 0.97.

No more efficiencies had to be taken into account. The final number of effective K^+ decays was 1.50×10^6, resulting in a limit from the three-prong events only of

$$\frac{\Gamma(K^+ \to \pi^+ e^+ e^-)}{\Gamma(K^+ \to \text{all})} < 1.73\times 10^{-6} \quad (90\% \text{ confidence}).$$

C. Two-prong events

In order to remove the large number of Dalitz pairs as soon as possible, a mass cut was applied early in stage 3 to the two-prong output of stage 2. At the beginning of stage 3, an estimate was made of the momentum and direction of each track, as a prelude to the complete track fitting. Events were rejected if $M^{\text{est}}_{e^+e^-} > 50$ MeV, where $M^{\text{est}}_{e^+e^-}$ was computed from the estimated track data. A check on e^+ identification was also made at this point.[6b] Such a cut was expected to eliminate 93% of the $K^+_{\pi 2}$-Dalitz triggers. After stage 3 track fitting, 5603 $K^+_{\pi ee}$ candidates in the form (K^+, e^+, e^-, missing π^+) resulted from the original stage 2 output of about 48 000.

Following stage 3, the missing mass recoiling against the positive track, assumed to be a pion, was calculated. This $K^+_{\pi MM}$ distribution, plotted in Fig. 5, indicates that only about 5% of the surviving two-prong events have $K^+_{\pi MM}$ in the vicinity of 135 MeV, the π^0 mass. Thus the $K^+_{\pi 2}$-Dalitz contamination was small at this point, and the identification of positrons worked well.

The missing mass MM_{rec} recoiling against the e^+ and e^- was also calculated, and this distribution is shown in Fig. 6. Evidently, events tended to have MM_{rec} much larger than the π^+ mass. In fact, only 652 of the 5603 events have MM_{rec} between 100 MeV and 180 MeV, consistent with the π^+ from $K^+_{\pi ee}$ decay. A highly effective cut on MM_{rec} could have been applied at this stage; instead, the 5603 events were fitted in HASH stage 4 to the one-constraint hypothesis

$$K^+ \to e^+ + e^- + (\text{missing } \pi^+). \quad (1C)$$

This resulted in 1850 candidates with χ^2 less than 10, a very loose constraint.

The next step in the two-prong analysis was unusual and took advantage of the fact that the solid angles subtended at the K^+ decay volume by chambers 1 and 2 were much larger than those subtended by chambers 3, 4, and 5. In particular, the probability for a π^+ from a $K^+_{\pi ee}$ phase-space decay to hit chambers 1 and 2 was greater than 80%. The trajectory of the missing π^+ was thus calculated for each of the 1850 two-prong $K^+_{\pi ee}$ fits to determine if and where that π^+ would intercept any of the five $K^+_{\pi ee}$ chambers. About 1200 events had at least one spark in the neighborhood of the calculated π^+ trajectory.

There were about 100 events with three or even

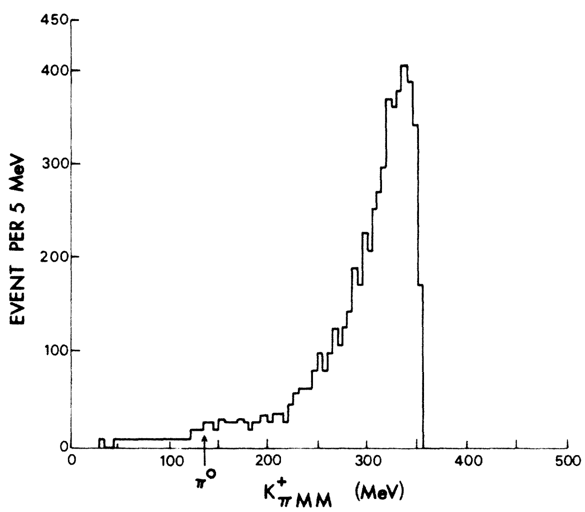

FIG. 5. $K^+_{\pi MM}$ distribution for 5549 events in the two-prong data sample. A cut, $M_{e^+e^-} > 50$ MeV, has already been applied, thus eliminating 93% of the $K^+_{\pi 2}$-Dalitz triggers. Only about 5% of the events shown have $K^+_{\pi MM}$ in the vicinity of 135 MeV, the π^0 mass.

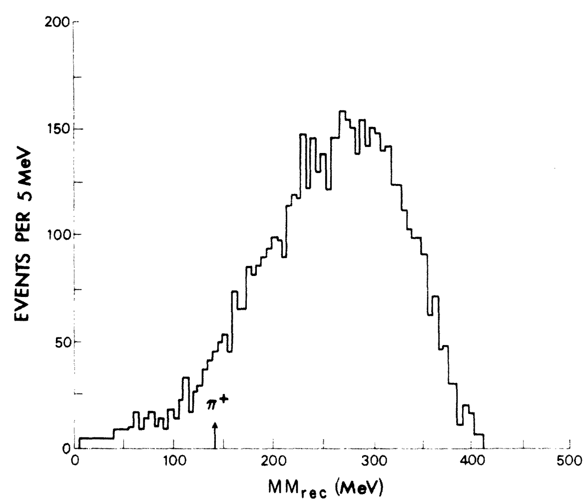

FIG. 6. Two-prong MM_{rec} distribution for the reaction $K^+ \to e^+ + e^- +$(missing mass). A cut, $M_{e^+e^-} > 50$ MeV, has already been applied to eliminate 93% of the $K^+_{\pi 2}$-Dalitz triggers. Only 652 of the remaining 5603 events shown here have MM_{rec} between 100 MeV and 180 MeV, the vicinity of the π^+ mass. All of the events in this sample were subsequently tested with the one-constraint hypothesis $K^+ \to e^+ + e^- +$(missing π^+).

four sparks for the pion, such sparks defining tracks which stage 2 had missed.

All 1200 events were passed through stage 3 (geometric reconstruction) a second time to determine whether the new "track" was compatible with the two tracks originally found. Of the 1200 candidates, 523 had a vertex consistent with all three tracks. When there were two or more sparks on the third track, a meaningful test could be made, and the momentum vector determined. In cases where there was only one spark on a possible third track, the dip angle was fairly well determined, but the momentum and azimuth were restricted only to a locus of points in P-ϕ space. The primary effect of the subsequent HASH fit was then to reject events whose expected pion dip was far from the fitted dip or whose expected P and ϕ did not form a point near the locus determined by the one spark.

The 523 survivors from stage 3 were fitted in HASH stage 4 to both the $K^+_{\pi ee}$ - and $K^+_{\pi 2}$-Dalitz hypotheses. Events were not rejected from the $K^+_{\pi ee}$ sample if they fitted $K^+_{\pi 2}$-Dalitz constraints; rather, they were retained provided the χ^2 for the $K^+_{\pi ee}$ fit itself was less than 17, corresponding to about the 0.2% probability level. There were 144 events that satisfied this requirement, of which 28 had two or three sparks along the supposed pion track.

Concentrating first on the 28 two-spark and three-spark events, a cut was made on $\mathrm{MM}_{\mathrm{rec}}$ from 100 to 170 MeV, which left 11 events, all in the Dalitz region of the $M_{e^+e^-}$ plot. In particular, 9 of the 11 had a $K^+_{\pi 2}$-Dalitz χ^2 below 4.6, for which the probability was greater than 10%. The two remaining events had spark locations that differed from the pion intercepts calculated in the one-constraint $K^+_{\pi ee}$ fit by more than three times the root-mean-square error. When these were eliminated, no candidates remained with more than one spark on the pion track.

Turning now to the 116 candidates with only one spark along the third "track," a cut was first made on $\mathrm{MM}_{\mathrm{rec}}$ from 115 to 170 MeV, leaving 37 events. By demanding that the single spark be a three-wire or four-wire fit, thereby decreasing the possibility that the spark was purely noise, this was reduced to 15 events. Of these, 12 had a spark for the assumed π^+ that was too far from the projected position (χ^2 probability for a fit less than 1.8%). One of the remaining three events had a positive track that was almost certainly a beam K^+. The second had a χ^2 from the second HASH $K^+_{\pi ee}$ fit of 15, which was beyond the 1% level. The last event was eliminated by requiring $M_{e^+e^-}$ (other) > 50 MeV, as in the case of the three-prong sample. Thus none of the two-prong candidates survived.

Just as in the three-prong case, Dalitz pairs were used to monitor the system efficiency for the two-prong sample. Some 45 800 such pairs were estimated to be present among the 48 000 two-prong candidates. Monte Carlo calculations indicated that 76% of these pairs were associated with the $K^+_{\pi 2}$-Dalitz mode with a net branching ratio of 2.43×10^{-3}. The effective number of K^+ decays for the two-prong sample was thus

$$E_K(2\text{-pr}) = 45\,800\times 0.76/(2.43\times 10^{-3}) = 1.44\times 10^{7}\,.$$

The relative e^+e^- triggering efficiency for a $K^+_{\pi ee}$ decay with the pion in chamber 2 and for a $K^+_{\pi 2}$-Dalitz decay with a pion anywhere was 0.853. The corrections associated with the $M_{e^+e^-}$ and $\mathrm{MM}_{\mathrm{rec}}$ cuts were also calculated by a Monte Carlo technique and were

(1) $M_{e^+e^-} > 50$ MeV: 0.912;

(2a) 100 MeV < $\mathrm{MM}_{\mathrm{rec}}$ < 170 MeV (two-spark cut): 0.922;

(2b) 115 MeV < $\mathrm{MM}_{\mathrm{rec}}$ < 170 MeV (one-spark cut): 0.85 .

The probability that at least two sparks would be found along a third track was determined from the wire-chamber efficiencies and from the Monte Carlo acceptances to be 0.344. The probability that one and only one spark would be found along a third track was 0.247, giving a combined probability of 0.591 for at least one spark.

When two or more sparks were required along the pion track, the above correction factors and effective number of K^+ decays yielded a branching ratio limit of

$$\frac{\Gamma(K^+\to\pi^+e^+e^-)}{\Gamma(K^+\to \mathrm{all})} < 4.73\times 10^{-7} \quad \text{(90\% confidence) (two or more sparks).}$$

This is within a factor of 2 of the best previous limit, which was obtained in a bubble-chamber experiment.[2] The formal result of this experiment, obtained by requiring at least one spark on the pion track, is

$$\frac{\Gamma(K^+\to\pi^+e^+e^-)}{\Gamma(K^+\to \mathrm{all})} < 2.73\times 10^{-7} \quad \text{(90\% confidence)}\,.$$

This is lower than the best previous result[2] by a factor of 3. If the branching ratio had been exactly 10^{-6}, the experiment would have yielded 9 ± 3 events, which would have been easily visible, especially since more than half of them would have had two or more sparks for the pion.

IV. DISCUSSION

The upper limit obtained in this experiment is tantalizingly close to the branching ratios calculated by a number of workers. It was assumed in most of these calculations that the decay is due to a coupling between weak and electromagnetic interactions. Table I summarizes the results of these calculations. In particular, Pakvasa and Simmons used PCAC (partially conserved axial-vector current) to relate $K^+_{\pi ee}$ decay to $K^+ \to \pi^\pm \pi^0 \gamma$-direct.[1m] Recent data for the latter decay mode[12] (see footnote to Table I) yield the result $(3.1 \pm 1.2) \times 10^{-7}$ for the branching ratio of $K^+_{\pi ee}$. This is very close to our upper limit.

Finally, our upper limit can be related to a neutral-current coupling constant. Two assumptions are needed: (a) that the neutral-current interaction has the usual $V-A$ form, and (b) that it is the dominant interaction in $K^+_{\pi ee}$ decay. Using assumption (a), one can write for the interaction Hamiltonian

$$H(\text{neutral}) = \frac{G}{\sqrt{2}} \sum_l g_l J_\lambda \, l i \gamma_\mu (1 + i\gamma_5) + \text{H.c.},$$

where

$l = \nu_e, e^-, \nu_\mu, \mu^-$,

J_λ = hadronic weak current (vector only for $K^+_{\pi ee}$),

g_l = coupling constant to be determined by experiment.

The neutral hadronic weak current can be related to the charged hadronic weak current using SU_2 invariance,

$$\langle \pi^+ | J_\lambda^{(6+i7)} | K^+ \rangle = \sqrt{2} \langle \pi^0 | J_\lambda^{(4+i5)} | K^+ \rangle .$$

This gives the following relation between the branching ratios:

$$\frac{R(K^+ \to \pi^+ e^+ e^-)}{R(K^+ \to \pi^0 e^+ \nu)} \cong \frac{2|g(e^+e^-)|^2}{|g(e^+\nu)|^2} ,$$

where $|g(e^+\nu)| = \sin\theta_V$, θ_V = the vector Cabibbo angle, and $g(e^+\nu)$ is the coupling corresponding to the charged current. Putting in our upper limit of 2.7×10^{-7} for the $K^+_{\pi ee}$ branching ratio and 4.85×10^{-2} for $K^+_{\pi^0 e^+ \nu}$,[13] one obtains

$$\frac{|g(e^+e^-)|}{|g(e^+\nu)|} < 1.7 \times 10^{-3} .$$

The calculated branching ratios listed in Table I indicate that $K^+_{\pi ee}$ decay may occur electromagnetically just below the experimental upper limit. The amplitudes for electromagnetic decay might then be larger than, or comparable with, the amplitudes for a neutral-current decay, in violation of our assumption (b). Destructive interference could suppress the ratio of $|g(e^+e^-)/g(e^+\nu)|$ derived from this experiment, leading to an erroneous result. Nevertheless, the conclusion that the ratio is quite small must be correct.

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Intranuclear cascading in photographic emulsion at accelerator and cosmic-ray energies*

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The data on the interactions of cosmic-ray protons in nuclear emulsion have been analyzed in light of the results of recent exposures of emulsion to accelerator protons. Plots of $\langle \log_{10} \tan\theta \rangle$ as a function of the number of evaporation prongs, N_h, display an energy-independent behavior which is exploited to determine the energies of cosmic-ray events. It is found that the energies quoted in the literature are generally too high. The average charged multiplicity is linearly related to N_h from 30 GeV to 5 TeV. While a model of nuclear cascading based on direct production is consistent with these data from 30 GeV to 500 GeV, a model proceeding through an intermediate state gives better agreement at cosmic-ray energies.

I. INTRODUCTION

The suggestion that the study of proton-nucleus interactions might provide insight into proton-proton processes is currently generating considerable interest.[1-3] It has been recognized that proton interactions in photographic emulsion are particularly useful in this regard since the observed number of heavily ionizing evaporation prongs, N_h, provides information on the number of secondary interactions occurring in the intranuclear cascade.[4-6]

In this paper we exploit this idea to compare two classes of models to the proton-emulsion multiplicity expressed as functions of N_h, atomic number A, and the primary energy E. We shall adopt the terminology of Fishbane and Trefil[2] in referring to the classes of models as "independent particle models" (IPM's) and "coherent production models" (CPM's). In the IPM, secondary particles are produced directly, while in the CPM, production proceeds through an intermediate state. We have developed a simplified IPM suitable for comparison with proton-emulsion data. Gottfried's[3] "energy flux cascade" (EFC) displays many CPM features and has already been cast in a form suitable for our analysis. Comparison with the Fishbane and Trefil models is possible in some instances.

Fortunately, proton-emulsion interactions exhibit some relatively simple characteristic properties which are suitable for modeling. The regularities in the accelerator data from 30 to 200 GeV are striking. It is possible to exploit these regularities to determine the energies of higher-energy cosmic-ray interactions. The cosmic-ray data analyzed in this manner are quite consistent and sufficiently accurate to provide a good test of the models in their present stage of development.