PHYSICAL REVIEW D VOLUME 10, NUMBER 2 15 JULY 1974

Secondary trajectories in Reggeon field theories

Henry D. I. Abarbanel
California Institute of Technology, Pasadena, California 91109 and National Accelerator Laboratory, Batavia, Illinois 60510

R. L. Sugar*
National Accelerator Laboratory, Batavia, Illinois 60510†
(Received 15 April 1974)

We study the corrections to any secondary Regge trajectory arising from repeated exchange of the Pomeranchuk singularity. Using renormalization-group methods we are able to determine the form of all Pomeron, Reggeon Green's functions in the neighborhood of $J = \alpha(0) = 1$ for the Pomerons and $J = \alpha_R(0) \approx 1/2$ for the Reggeons. Starting with bare linear trajectories for both Pomerons and Reggeons we establish how these are modified by a triple-Pomeron and a two-Reggeon-Pomeron coupling. In an expansion of the theory around $D = 4$ space dimensions ($D = 2$ is where physics takes place), we find three allowed stable points of the renormalization-group equations in the infrared [$J \to 1$ or $\alpha_R(0)$] limit. For each of these we study the renormalized Reggeon trajectories and the structure of the Green's functions.

I. INTRODUCTION

The t-channel exchange of Reggeons like the P', ρ, A_2, etc. has a firm place in hadronic physics folklore as providing both the correct large-s description of amplitudes involving quantum-number exchange and the corrections, down by approximately $\sqrt{s}$, to diffraction scattering governed by the Pomeranchuk singularity. The fact that the Pomeron has J-plane intercept $\alpha(0) = 1$ leads to the well-known observation[1] that t-channel exchange of a Reggeon with intercept $\alpha_R(0)$ and any number of Pomerons yields a series of branch points in J at $\alpha_R(0)$. This piling up of singularities in J means that the structure of the Reggeon exchange in the neighborhood of $J = \alpha_R(0)$ and $t = 0$ could be modified from the usual simple pole which is phenomenologically so attractive.

In this paper we will employ renormalization-group techniques to discuss the corrections to a Reggeon arising from any number of Pomeron exchanges and interactions. We will proceed along the lines laid down in previous work on interacting Reggeons[2,3] by first establishing a field theory for Pomerons and Reggeons. Although our methods will clearly be applicable to a variety of different situations (some of which we will discuss in our concluding section), we confine our detailed calculations to a theory where the bare (noninteracting) J-plane physics consists of two poles linear in t

$$\alpha_0(t) = \alpha_0 + \alpha_0' t \tag{1}$$

and

$$\alpha_{R0}(t) = \alpha_{R0} + \alpha_{R0}' t . \tag{2}$$

Then we choose for the interaction a triple-Pomeron coupling[2,3] and an R-R-P coupling (see Fig. 1) which preserves the number of Reggeons. This last choice is motivated by our desire to concentrate on the neighborhood of $J = \alpha_R(0)$, the renormalized intercept of the Reggeon singularity, and t near zero. Interactions which allow production of Reggeons with $\alpha_R(0) < 1$ give rise to singularities lying further to the left in the J plane and are not of any particular interest to us.

Perhaps at this juncture it will be useful to recall a similar problem from quantum electrodynamics: the alteration of a charged boson propagator, $(m_0^2 - p^2)^{-1}$, taking into account all interactions with a massless photon. In studying this problem near $p^2 = m^2$, the renormalized mass, it makes eminent sense to neglect all three-and-more-boson intermediate states in the calculation; of course, all photon states are to be treated. Since by changing the names (Pomeron → massless photon; Reggeon → boson) this situation is made qualitatively identical to the Reggeon problem, it is useful to recall the results of the electrodynamics calculation. The bare propagator changes from $(m_0^2 - p^2)^{-1}$ into $(m^2 - p^2)^{-1+\kappa}$ (in most gauges) where κ is the anomalous dimension. We can expect very much the same consequence of the interacting Reggeon problem; namely, a bare propagator $[J - \alpha_{R0}(t)]^{-1}$ will change into $[J - \alpha_R(t)]^{-1-\kappa}$. [This will result in a change from $s^{\alpha_R(t)}$ in the Reggeon-exchange amplitude into $s^{\alpha_R(t)} (\ln s)^\kappa$.] There are, of course, differences in detail between the Reggeon problem and electrodynamics. First, with photons one has a gauge-dependent Green's function to compute. Second, Pomerons can interact directly without the intermediary of other Reggeons. Photons do not convert into other photons in the absence of intermediate-charged-boson pairs, which we have ar-

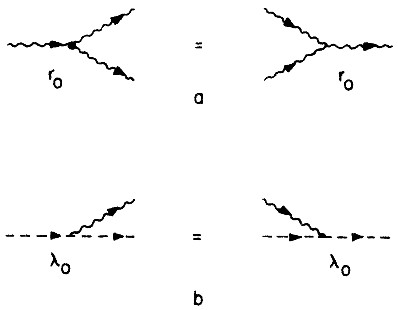

FIG. 1. (a) The bare triple-Pomeron vertex. It has strength r_0. (b) The bare Reggeon-Reggeon-Pomeron vertex. It conserves Reggeon number and has strength λ_0. Dashed lines are Reggeons; wiggly lines are Pomerons.

gued one ought to neglect. The Pomeron problem, therefore, is slightly more complicated.

Our calculation takes advantage of the fact that in $D=4$ space dimensions of the Reggeon space, the theory we will set up in the next section possesses a scale invariance. The physics of real interacting Reggeons takes place at $D=2$. We follow the lead of Refs. 2 and 3 by carrying out our calculations in a general number of dimensions and then expanding the results about $D=4$. This procedure is justified, as in the strictly Pomeron case, by the existence of a renormalized dimensionless coupling constant, which is of order $(D-4)^{1/2}$ and which is the appropriate expansion parameter for the problem. Actually we find three possible values of the dimensionless *RRP* coupling constant which could, in principle, govern the behavior of the interacting Green's functions near $J=\alpha_R(0)$ and $t=0$. Each of the three coupling constants is of order $(4-D)^{1/2}$ and is a stable point (infrared) of the renormalization-group equations. To choose among the three solutions requires principles in addition to the formal structure of the renormalization-group program. We will indicate our preference at the appropriate moment, but it will be clear we are adding to the rules of the game.

The problem of Pomeron corrections to a Reggeon exchange has been treated before in the literature. Basically there are two approaches:

(1) One pretends that the Reggeon exchange and the Pomeron exchange are two scattering "potentials." Then by some eikonal or impact-parameter-absorption technique[4,5] one takes an infinite number of Pomeron exchanges into account while treating the Reggeon in first-order perturbation theory. Interactions between the Pomerons and Reggeons are neglected. A Reggeon-exchange graph as in Fig. 2 is included in these calculations. Typically higher-order Pomeron exchanges contribute corrections $(-1)^n s^{\alpha_R}/(\ln s)^n$ from these considerations. A Reggeon interaction graph such as Fig. 3 is not evaluated by these absorptive procedures; we will include it.

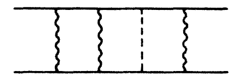

FIG. 2. A Pomeron-exchange graph which is included in the usual eikonal or absorption treatment of Pomeron corrections to Reggeon exchange.

(2) As part of their program to study Reggeon interactions via the Schwinger-Dyson equations of Reggeon field theories, Gribov and his collaborators[6] have explicitly considered the theory in this paper. Beginning with the linear trajectory in Eq. (2), they argue that either the *RRP* coupling must be zero and the renormalized pole is determined by an *RRPP* coupling to be

$$\alpha_R(t)=\alpha_R(0)+\alpha_R{}'t-Ct/(\ln t)^3 \tag{3}$$

or the linear trajectory changes form dramatically to

$$\alpha_R(t)=\alpha_R(0)\pm i\kappa\sqrt{-t} \tag{4}$$

for $t<0$.

Although we have not yet studied the former case, it is reminiscent of the results in the pure Pomeron problem when only a quartic interaction is present.[7] In the work in this paper where we do examine the latter case, we do not find anything resembling the $\sqrt{-t}$ trajectory of Gribov *et al.* We do find that for each of the possible stable points of our equations there is a Regge trajectory of the form

$$\alpha_R(t)=\alpha_R(0)+\alpha_R{}'t^p\,, \tag{5}$$

but the power p is either one or deviates from it by a small, computable number.

The solution, Eq. (4), of Gribov *et al.* is nevertheless extremely intriguing, for it is the kind of result one might expect not from an initial linear trajectory, but from an initial trajectory of $\sqrt{-t}$ form itself, interacting with a Pomeron whose singularity is something like

$$[(J-1)^2+a^2t]^n\,, \tag{6}$$

with a and n some constants. In the present work we have not examined this attractive theory. The linear trajectories appeared both attractive enough and indeed are intricate enough in detail to provide

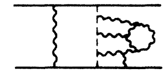

FIG. 3. An example of a graph omitted by eikonal methods.

a substantial discussion in themselves. We hope to return to this problem, coupled with the consideration of pure Pomerons with the structure of Eq. (6).[8]

The plan of this paper will be to introduce the Reggeon-Pomeron field theory in Sec. II. Much of the background for this section is to be found in Refs. 2 and 3 and, for the uninitiated, in the classical work of Gribov.[9] In this section we will also establish the renormalization-group equations and obtain the scaling laws for the full Green's functions. The infrared behavior [$J\to\alpha_R(0)$ or $\alpha(0)$, t small] is the subject of Sec. III. Section IV will present what information one can extract from the combination of scaling laws derived from the solution of the renormalization-group equations and the expansion about 4 space dimensions suggested by the fact that the effective renormalized coupling constants are of order $(4-D)^{1/2}$. A final section is devoted to a brief discussion of our results and some thoughts on future investigations.

II. FIELD THEORY FOR INTERACTING REGGEONS AND POMERONS; RENORMALIZATION-GROUP EQUATIONS

In a Reggeon field theory a noninteracting Reggeon is taken to be a quasiparticle with an energy-momentum relation

$$E(\vec{q}) = 1 - \alpha(\vec{q}), \tag{7}$$

where $\alpha(t=-|\vec{q}|^2)$ is the ordinary Regge trajectory, and the energy is in general 1-angular momentum. The noninteracting theory clearly depends on what $\alpha(\vec{q})$ one begins with. We shall study in this paper a theory where both the bare Pomeron and bare Reggeon have linear trajectories given respectively by

$$\alpha_0(t) = \alpha_0 + \alpha_0' t \tag{8}$$

and

$$\alpha_{R0}(t) = \alpha_{R0} + \alpha_{R0}' t , \tag{9}$$

or

$$E(\vec{q}) = \alpha_0' \vec{q}^2 + \Delta_0 \tag{10}$$

and

$$E_R(\vec{q}) = \alpha_{R0}' \vec{q}^2 + \Delta_{R0} , \tag{11}$$

where

$$\Delta_0 = 1 - \alpha_0 \tag{12}$$

and

$$\Delta_{R0} = 1 - \alpha_{R0} \tag{13}$$

are the mass gaps for the two bare trajectories. The action which describes this situation is

$$A_0 = \int d^D x\, dt \left[\tfrac{1}{2} i \phi^\dagger(\vec{x},t) \frac{\overleftrightarrow{\partial}}{\partial t} \phi(\vec{x},t) - \alpha_0' \nabla\phi^\dagger \cdot \nabla\phi - \Delta_0 \phi^\dagger\phi + \tfrac{1}{2} i X^\dagger(\vec{x},t) \frac{\overleftrightarrow{\partial}}{\partial t} X(\vec{x},t) - \alpha_{R0}' \nabla X^\dagger \cdot \nabla X - \Delta_{R0} X^\dagger X \right], \tag{14}$$

with a field $\phi(\vec{x},t)$ for the Pomeron and a field $X(\vec{x},t)$ for the Reggeon. These fields are defined in D space dimensions $\vec{x}$. Physics takes place at $D=2$; it will be convenient to leave D free for now.

The interaction is described by a Lagrangian density

$$\mathcal{L}_I(\vec{x},t) = -\tfrac{1}{2} i r_0 [\phi^\dagger(\vec{x},t)^2 \phi(\vec{x},t) + \phi^\dagger(\vec{x},t)\phi^2(\vec{x},t)] - i\lambda_0 X^\dagger(\vec{x},t) X(\vec{x},t) [\phi^\dagger(\vec{x},t) + \phi(\vec{x},t)] + \delta\phi^\dagger\phi + \delta_R X^\dagger X . \tag{15}$$

The first two terms in the interaction Lagrangian describe a triple-Pomeron coupling and a Reggeon-number-conserving Reggeon-Reggeon-Pomeron interaction. The last two quantities are mass counterterms to be determined order by order in perturbation theory in r_0 and λ_0. The two coupling constants r_0 and λ_0 are real as is dictated by the general signature analysis of Gribov. This last remark means immediately that the one-Reggeon–one-Pomeron cut occurs with a negative sign with respect to the one-Reggeon-pole term in an expansion in λ_0. This is strongly supported by phenomenological analyses of quantum-number exchange processes.

Since the Reggeon number operator is conserved, we may make a phase change on the field operator $X(\vec{x},t)$,

$$X(\vec{x},t) \to e^{i\Delta_{R0} t} X(\vec{x},t), \tag{16}$$

which leaves the commutation relation

$$[X^\dagger(\vec{x},t), X(\vec{y},t)] = \delta^D(\vec{x}-\vec{y}) \tag{17}$$

and the interaction unmodified and replaces $E=1-J$ for a Reggeon by

$$E - \Delta_{R0} = \alpha_{R0} - J \equiv \mathcal{E} . \tag{18}$$

We will, therefore, discuss this theory in terms of $E_i = 1 - J_i$ for Pomerons and $\mathcal{E}_i = \alpha_R(0) - J_i$ for Reggeons. If Reggeon number were not conserved, this little trick would fail.

Our objects of study will be the Green's functions for n Pomerons $+\,k$ Reggeons $\to m$ Pomerons $+\,k$ Reggeons. They are defined by

$$G^{(n,m;k)}(E_i,\vec{q}_i,\mathcal{E}_j,\vec{p}_j)\delta\left(\sum_{i=1}^{n}E_i+\sum_{j=1}^{k}\mathcal{E}_j-\sum_{i=1}^{m}E_i'-\sum_{j=1}^{k}\mathcal{E}_j'\right)\delta^D\left(\sum_{i=1}^{n}\vec{q}_i+\sum_{j=1}^{k}\vec{p}_j-\sum_{i=1}^{m}\vec{q}_i'-\sum_{j=1}^{k}\vec{p}_j'\right)$$
$$=\int\prod_{i=1}^{n}d^Dx_i\,dt_i\prod_{i=1}^{m}d^Dx_i'\,dt_i'\prod_{j=1}^{k}d^Dy_j\,d\tau_j\,d^Dy_j'\,d\tau_j'$$
$$\times\exp\{i(\vec{x}_i\cdot\vec{q}_i-E_i t_i-\vec{x}_i'\cdot\vec{q}_i'+E_i't_i'+\vec{y}_j\cdot\vec{p}_j-\mathcal{E}_j\tau_j-\vec{y}_j'\cdot\vec{p}_j'+\mathcal{E}_j'\tau_j')\}$$
$$\times\langle 0|\,T[\phi^\dagger(\vec{x}_1,t_1)\cdots\phi^\dagger(\vec{x}_n,t_n)\phi(\vec{x}_1',t_1')\cdots\phi(\vec{x}_m',t_m')$$
$$\times X^\dagger(\vec{y}_1,\tau_1)\cdots X^\dagger(\vec{y}_k,\tau_k)X(\vec{y}_1',\tau_1')\cdots X(\vec{y}_k',\tau_k')]\,|0\rangle \qquad (19)$$

(see Fig. 4). The Pomeron Green's functions $G^{(n,m;0)}$ will be unaffected by the Reggeon field, so the results in Refs. 2 and 3 for those quantities may be carried over directly. Our task here is to study the Green's functions for $k>0$.

From the action we may extract the Feynman graph rules to be used in evaluating $G^{(n,m;k)}$ in a perturbation series in r_0 and λ_0. They are as follows:

(1) Draw all topologically distinct diagrams with arrows indicating the direction of propagation of Reggeons and Pomerons.

(2) $\int d^Dq\,dE$ around each loop.

(3) At each triple-Pomeron vertex put a factor of $r_0/(2\pi)^{(D+1)/2}$.

(4) At each RRP vertex put a factor of $\lambda_0/(2\pi)^{(D+1)/2}$.

(5) For each mass-renormalization counterterm use a factor of $i\delta$ or $i\delta_R$.

(6) For each Pomeron of momentum $\vec{q}$ and energy E use the bare propagator

$$G_0^{(1,1;0)}(E,\vec{q}^{\,2})=i(E-\alpha_0'\vec{q}^{\,2}+i\epsilon)^{-1}.$$

(7) For each Reggeon of momentum $\vec{p}$ and energy $\mathcal{E}+\Delta_R$ use the bare propagator

$$G_0^{(0,0;1)}(\mathcal{E},\vec{p}^{\,2})=i(\mathcal{E}-\alpha_{R0}'\vec{p}^{\,2}+i\epsilon)^{-1}.$$

(8) For each two-Pomeron loop with both momenta in the same direction, multiply by $\frac{1}{2}$.

(9) Conserve energy and momentum at all vertices.

Our equations will all be expressed in terms of the connected proper vertex functions $\Gamma^{(n,m;k)}$ which are defined by amputating the external legs of the connected Green's functions $G^{(n,m;k)}_{\text{connected}}$:

$$\Gamma^{(n,m;k)}(E_i,\vec{q}_i,\mathcal{E}_j,\vec{p}_j)$$
$$=\prod_{i=1}^{n}G^{(1,1;0)}(E_i,\vec{q}_i^{\,2})^{-1}\prod_{i=1}^{m}G^{(1,1;0)}(E_i',\vec{q}_i'^{\,2})^{-1}\prod_{j=1}^{k}G^{(0,0;1)}(\mathcal{E}_j,\vec{p}_j^{\,2})^{-1}G^{(0,0;1)}(\mathcal{E}_j',\vec{p}_j'^{\,2})^{-1}G^{(n,m;k)}_{\text{connected}}(E_i,\vec{q}_i,\mathcal{E}_j,\vec{p}_j). \qquad (20)$$

One may now proceed to evaluate the $\Gamma^{(n,m;k)}$ as a function of the parameters α_0', α_{R0}', r_0, and λ_0 to whatever order is desired. These quantities will be renormalized by the interaction and take on new values α', α_R', r, and λ. We shall always determine the counterterms δ and δ_R so that the singularities of the renormalized inverse propagators pass through $J=1$ at $\vec{q}^{\,2}=0$ and $J=\alpha_R(0)$ at $\vec{p}^{\,2}=0$. We guarantee this by

$$\Gamma^{(1,1;0)}(E,\vec{q}^{\,2})\big|_{E=0;\,\vec{q}^{\,2}=0}=0 \qquad (21)$$

and

$$\Gamma^{(0,0;1)}(\mathcal{E},\vec{p}^{\,2})\big|_{\mathcal{E}=0;\,\vec{p}^{\,2}=0}=0. \qquad (22)$$

The renormalization procedure replaces the unrenormalized fields $\phi(\vec{x},t)$ and $X(\vec{x},t)$ by $Z^{-1/2}\phi$ and $Z_R^{-1/2}X$, respectively. This takes an unrenormalized vertex function $\Gamma_U^{(n,m;k)}$ into its renormalized counterpart $\Gamma^{(n,m;k)}$ via

$$\Gamma^{(n,m;k)}(E_i,\vec{q}_i,\mathcal{E}_i,\vec{p}_i,r,\lambda,\alpha',\alpha_R',E_N)$$
$$=Z^{(n+m)/2}Z_R^{\,k}$$
$$\times\Gamma_U^{(n,m;k)}(E_i,\vec{q}_i,\mathcal{E}_i,\vec{p}_i,r_0,\lambda_0,\alpha_0',\alpha_{R0}',\Lambda). \qquad (23)$$

Here $E_n>0$ is a point where we choose to normalize the vertex functions Γ in order to define the renormalized parameters r, λ, α', and α_R'. As explained in Ref. 3 it replaces the renormalized mass gap $\Delta=1-\alpha(0)$. The parameter Λ in Γ_U is

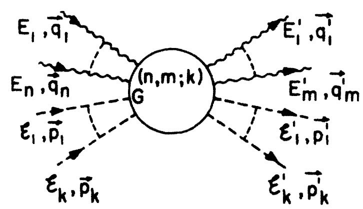

FIG. 4. The full Reggeon-Pomeron Green's function.

a cutoff one may wish to use to regularize the integrals appearing in the unrenormalized theory.

We define the renormalized parameters, which are functions of E_N, by the following conditions on the vertex functions:

$$\frac{\partial}{\partial E} i\Gamma^{(1,1;0)}(E,\vec{q}^2)\Big|_{E=-E_N;\,\vec{q}^2=0} = 1\,, \quad (24)$$

$$\frac{\partial}{\partial \mathcal{E}} i\Gamma^{(0,0;1)}(\mathcal{E},\vec{p}^2)\Big|_{\mathcal{E}=-E_N;\,\vec{p}^2=0} = 1\,, \quad (25)$$

$$\frac{\partial}{\partial \vec{q}^2} i\Gamma^{(1,1;0)}(E,\vec{q}^2)\Big|_{E=-E_N;\,\vec{q}^2=0} = \alpha'(E_N)\,, \quad (26)$$

$$\frac{\partial}{\partial \vec{p}^2} i\Gamma^{(0,0;1)}(\mathcal{E},\vec{p}^2)\Big|_{\mathcal{E}=-E_N;\,\vec{p}^2=0} = -\alpha_R'(E_N)\,, \quad (27)$$

$$\Gamma^{(1,2;0)}(E_1,\vec{q}_1,\ldots,E_3,\vec{q}_3)\,|_{E_1=2E_2=2E_3=-E_N;\,\vec{q}_i=0} = r(E_N)/(2\pi)^{(D+1)/2}\,, \quad (28)$$

and

$$\Gamma^{(0,1;1)}(\mathcal{E}_1,\vec{p}_1,\mathcal{E}_1',\vec{p}_1',E_1',\vec{q}_1')\,|_{\mathcal{E}_1=2\mathcal{E}_1'=2E_1'=-E_N\,\vec{p}_1=\vec{p}_1'=\vec{q}_1'=0} = \lambda(E_N)/(2\pi)^{(D+1)/2}\,. \quad (29)$$

These parameters are not directly the actual slope of a renormalized trajectory or a renormalized coupling. They are simply a set of normalization numbers which serve to parameterize the renormalized vertex functions.

We choose to replace the constants $\alpha_R'(E_N)$, $r(E_N)$, and $\lambda(E_N)$ by the dimensionless quantities

$$g(E_N) = \frac{r(E_N)}{[\alpha'(E_N)]^{D/4}} E_N^{D/4-1}\,, \quad (30)$$

which plays an important role in the Pomeron problem,

$$g_R(E_N) = \frac{\lambda(E_N) E_N^{D/4-1}}{\{\frac{1}{2}[\alpha'(E_N)+\alpha_R'(E_N)]\}^{D/4}} \quad (31)$$

and

$$\nu(E_N) = \alpha_R'(E_N)/\alpha'(E_N)\,. \quad (32)$$

Expressing the $\Gamma^{(n,m;k)}$ in terms of these allows us to write

$$\Gamma^{(n,m;k)}(E_i,\vec{q}_i,\mathcal{E}_j,\vec{p}_j,g,g_R,\alpha',\nu,E_N) = E_N(E_N/\alpha')^{(2-n-m-2k)D/4}\,\psi_{n,m;k}\left(\frac{E_i}{E_N},\frac{\mathcal{E}_i}{E_N},\left(\frac{\alpha'}{E_N}\right)^{1/2}\vec{q}_i,\left(\frac{\alpha'}{E_N}\right)^{1/2}\vec{p}_j,g,g_R,\nu\right), \quad (33)$$

using ordinary dimensional analysis. This last observation means

$$\Gamma^{(n,m;k)}(\xi E_i,\vec{q}_i,\xi\mathcal{E}_j,\vec{p}_j,g,g_R,\alpha',\nu,E_N) = \xi\Gamma^{(n,m;k)}\left(E_i,\vec{q}_i,\mathcal{E}_j,\vec{p}_j,g,g_R,\frac{\alpha'}{\xi},\nu,\frac{E_N}{\xi}\right)\,, \quad (34)$$

which will prove useful shortly.

The equations of the renormalization group follow from the observation that the unrenormalized vertex functions cannot know about E_N, so

$$E_N\frac{\partial}{\partial E_N}\Gamma_U^{(n,m;k)} = 0\,. \quad (35)$$

This translates, using (23), into the constraint on $\Gamma^{(n,m;k)}$

$$\left[E_N\frac{\partial}{\partial E_N} + \beta\frac{\partial}{\partial g} + \beta_R\frac{\partial}{\partial g_R} + \zeta\frac{\partial}{\partial \alpha'} + \sigma\frac{\partial}{\partial \nu} - \tfrac{1}{2}(n+m)\gamma - k\gamma_R\right] \times\Gamma^{(n,m;k)}(E_i,\vec{q}_i,\mathcal{E}_j,\vec{p}_j,g,g_R,\alpha',\nu,E_N) = 0\,, \quad (36)$$

where the various renormalization-group functions are defined by

$$\beta = E_N\frac{\partial}{\partial E_N} g\,, \quad (37)$$

$$\beta_R = E_N\frac{\partial}{\partial E_N} g_R\,, \quad (38)$$

$$\gamma = E_N\frac{\partial}{\partial E_N}\ln Z\,, \quad (39)$$

$$\gamma_R = E_N\frac{\partial}{\partial E_N}\ln Z_R\,, \quad (40)$$

$$\zeta = E_N\frac{\partial}{\partial E_N}\alpha'\,, \quad (41)$$

and

$$\sigma = E_N\frac{\partial}{\partial E_N}\nu\,, \quad (42)$$

all derivatives to be taken at fixed r_0, λ_0, α_0', α_{R0}', Λ, and dimension D.

Several of these functions are dependent only on the Pomeron parameters g and α', namely, β, γ, and ζ/α'. Indeed since they are dimensionless, they are functions of g only. The evaluation of these functions has been carried out in Ref. 3. Our problem is to discuss the additional dimensionless functions β_R, γ_R, and σ which depend on g, g_R, and ν.

Utilizing the scaling property of the $\Gamma^{(n,m;k)}$ in (34) we are able to trade off a derivative with respect to E_N for a derivative with respect to the scale factor ξ:

$$\left[\xi\frac{\partial}{\partial\xi}-\beta(g)\frac{\partial}{\partial g}-\beta_R\frac{\partial}{\partial g_R}-\alpha'\left(1-\frac{\zeta}{\alpha'}\right)\frac{\partial}{\partial\alpha'}-\sigma\frac{\partial}{\partial\nu}+\tfrac{1}{2}(n+m)\gamma(g)+k\gamma_R-1\right]$$
$$\times\Gamma^{(n,m;k)}(\xi E_i,\vec{q}_i,\xi\mathcal{E}_j,\vec{p}_j,g,g_R,\alpha',\nu,E_N)=0\,. \tag{43}$$

The solution of this equation is

$$\Gamma^{(n,m;k)}(\xi E_i,\vec{q}_i,\xi\mathcal{E}_j,\vec{p}_j,g,g_R,\alpha',\nu,E_N)=\Gamma^{(n,m;k)}(E_i,\vec{q}_i,\mathcal{E}_j,\vec{p}_j,\bar{g}(-t),\bar{g}_R(-t),\bar{\alpha}'(-t),\bar{\nu}(-t),E_N)$$
$$\times\exp\left\{\int_{-t}^{0}dt'[1-\tfrac{1}{2}(n+m)\gamma(\bar{g}(t'))-k\gamma_R(\bar{g}(t'),\bar{g}_R(t'),\bar{\nu}(t'))]\right\}, \tag{44}$$

where $t=\ln\xi$. $\bar{g}(t)$, $\bar{g}_R(t),\ldots$ are solutions of the characteristic equations

$$\frac{d\bar{g}(t)}{dt}=-\beta(\bar{g}(t)), \tag{45}$$

$$\frac{1}{\bar{\alpha}'(t)}\frac{d\bar{\alpha}'(t)}{dt}=1-\frac{\zeta(\bar{\alpha}'(t),\bar{g}(t))}{\bar{\alpha}'(t)}=z(\bar{g}(t)), \tag{46}$$

$$\frac{d\bar{g}_R(t)}{dt}=-\beta_R(\bar{g}(t),\bar{g}_R(t),\bar{\nu}(t)), \tag{47}$$

and

$$\frac{d\bar{\nu}(t)}{dt}=-\sigma(\bar{g}(t),\bar{g}_R(t),\bar{\nu}(t)), \tag{48}$$

with the boundary conditions

$$\bar{g}(0)=g,\ \bar{g}_R(0)=g_R,\ \bar{\alpha}'(0)=\alpha',\ \text{and}\ \bar{\nu}(0)=\nu\,. \tag{49}$$

Without knowing the detailed form of the β's, γ's, etc., we, of course, cannot solve for the effective couplings $\bar{g}(t)$, slope $\bar{\alpha}'(t)$, and slope ratio $\bar{\nu}(t)$. Knowing the renormalization-group functions in detail is tantamount to solving the whole Reggeon field theory. To make any progress one must evaluate these functions in perturbation theory. Many of the deductions about the form of $\Gamma^{(n,m;k)}$ may transcend perturbation theory, but one is forced to begin there. If one is lucky, then the effective coupling constants $\bar{g}(-t)$ and $\bar{g}_R(-t)$ entering in (44) will be small in the limit of interest ($\xi\to 0$, $t=\ln\xi\to-\infty$). If that is the case, then perturbation theory will receive an *a posteriori* justification.

III. INFRARED PROPERTIES OF THE RENORMALIZATION-GROUP EQUATIONS

In the discussion of the purely Pomeron field theory[2,3] it was suggested that an expansion of the theory around $D=4$ space dimensions would provide a method for estimating the Pomeron Green's functions to high accuracy. Indeed the effective coupling $\bar{g}(-t)$ was shown to retreat to a small coupling, of order $(4-D)^{1/2}$, as $t\to-\infty$. This behavior indicated that perturbation theory might be reliable for determining the infrared behavior of the Green's functions.

In particular it was found that the functions $\gamma(g)$ and ζ/α' were

$$\gamma(g)=-2Kg^2 \tag{50}$$

and

$$\zeta/\alpha'=-Kg^2, \tag{51}$$

where

$$K=\left(\frac{\pi}{2}\right)^{D/2}\frac{\Gamma(3-D/2)}{4(2\pi)^D}. \tag{52}$$

To leading order in $\epsilon=4-D$ the function $\beta(g)$ was determined to be

$$\beta(g)=-g(\tfrac{1}{4}\epsilon-6Kg^2), \tag{53}$$

whose zero at

$$g_1^{\,2}=\epsilon/24K \tag{54}$$

determines the infrared behavior of Pomeron Green's functions.

Our immediate aim here will be to locate zeros in β_R which have $d\beta/dg_R>0$ and will thus determine the infrared behavior of the Pomeron-Reggeon Green's functions. We must first determine the various renormalization-group functions in perturbation theory. To this end we begin with the lowest-order correction to the Reggeon propagator $\Gamma^{(0,0;1)}$ as shown in Fig. 5:

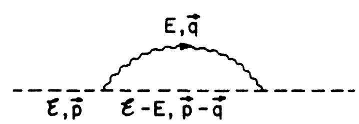

FIG. 5. The lowest-order correction to the Reggeon propagator.

$$i\Gamma_U^{(0,0;1)}(\mathcal{E},\vec{p}^2)=\mathcal{E}-\alpha_{R0}'\vec{p}^2-\frac{i\lambda_0^2}{(2\pi)^{D+1}}\int d^Dq\,dE\,\frac{i}{E-\alpha_0'\vec{q}^2+i\epsilon}\,\frac{i}{\mathcal{E}-E-\alpha_{R0}'(\vec{p}-\vec{q})^2+i\epsilon} \tag{55}$$

$$=\mathcal{E}-\alpha_{R0}'\vec{p}^2-\frac{\lambda_0^2}{[\frac{1}{2}(\alpha_0'+\alpha_{R0}')]^{D/2}}\,\frac{4K}{(2-\frac{1}{2}D)(1-\frac{1}{2}D)}\left(\frac{\alpha_0'\alpha_{R0}'}{\alpha_0'+\alpha_{R0}'}\vec{p}^2-\mathcal{E}\right)^{D/2-1}. \tag{56}$$

Utilizing our normalization conditions this yields

$$\frac{1}{Z_R}=1-\frac{4K}{2-\frac{1}{2}D}\,\frac{\lambda_0^2}{[\frac{1}{2}(\alpha_0'+\alpha_{R0}')]^{D/2}}E_N^{D/2-2}, \tag{57}$$

$$\gamma_R=-4Kg_R^2, \tag{58}$$

$$\alpha_R'(E_N)=\alpha_{R0}'\left(1+\frac{4K}{2-D/2}g_R^2\frac{\nu}{1+\nu}\right), \tag{59}$$

and

$$\sigma=-\frac{K\nu}{1+\nu}[4g_R^2\nu-g^2(1+\nu)]. \tag{60}$$

It is clear at this stage that the presence of the two dimensionless parameters g_R and $\nu=\alpha_R'/\alpha'$ will complicate life a bit.

In order to obtain β_R we evaluate the graphs shown in Fig. 6 at the normalization point given in Eq. (29). After some algebra we find for β_R from these graphs

$$\beta_R=-g_R\left[\frac{\epsilon}{4}+Kg^2\frac{\nu}{1+\nu}-4Kg_R^2\left(\frac{\nu}{1+\nu}\right)^2-\frac{4Kg_Rg(3+\nu)}{1+\nu}\right]. \tag{61}$$

We now desire to extract from the characteristic equations (45)–(48) the large-t behavior of $\bar{g}(t)$, $\bar{\alpha}'(t)$, $\bar{g}_R(t)$, and $\bar{\nu}(t)$. In other words, we must search for the fixed points of these equations. We have already indicated that the fixed point of the equation for $\bar{g}(t)$ occurs at $g=g_1$. We may use this value of g in the equations for $\bar{g}_R(t)$ and $\bar{\nu}(t)$ when analyzing their fixed points. They then become a pair of coupled equations in two variables. To analyze them it is convenient to introduce the matrix

$$\underline{A}(g_R,\nu)=\begin{bmatrix}\frac{\partial\beta_R}{\partial g_R} & \frac{\partial\beta_R}{\partial\nu}\\[2ex] \frac{\partial\sigma}{\partial g_R} & \frac{\partial\sigma}{\partial\nu}\end{bmatrix}. \tag{62}$$

Then the necessary and sufficient conditions for the point (g_{R1},ν_1) to be a stable fixed point are that[10]

$$\beta_R(g_{R1},\nu_1)=\sigma(g_{R1},\nu_1)=0 \tag{63}$$

and

$$\mathrm{Re}\lambda_i>0, \tag{64}$$

where the λ_i are the eigenvalues of $\underline{A}(g_{R1},\nu_1)$. To see this we note that in the neighborhood of (g_{R1},ν_1) Eqs. (47) and (48) can be linearized and written

$$\frac{d\vec{Z}}{dt}=-\underline{A}(g_{R1},\nu_1)\vec{Z}, \tag{65}$$

with

$$\vec{Z}=\begin{bmatrix}Z_1\\ Z_2\end{bmatrix}=\begin{bmatrix}\bar{g}_R(t)-g_{R1}\\ \bar{\nu}(t)-\nu_1\end{bmatrix}. \tag{66}$$

There will always be a matrix $\underline{B}$ such that either

$$\underline{BAB}^{-1}=\begin{bmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{bmatrix} \tag{67}$$

or

$$\underline{BAB}^{-1}=\begin{bmatrix}\lambda & 0\\ 1 & \lambda\end{bmatrix}, \tag{68}$$

and in the latter case $\lambda_1=\lambda_2=\lambda$. Now write $\vec{W}=\underline{B}\vec{Z}$ and find

$$W_1(t)=W_1(0)e^{-\lambda_1 t} \tag{69}$$

and

$$W_2(t)=W_2(0)e^{-\lambda_2 t}, \tag{70}$$

when (67) holds, or

$$W_1(t)=W_1(0)e^{-\lambda t} \tag{71}$$

and

$$W_2(t)=[W_2(0)-tW_1(0)]e^{-\lambda t}, \tag{72}$$

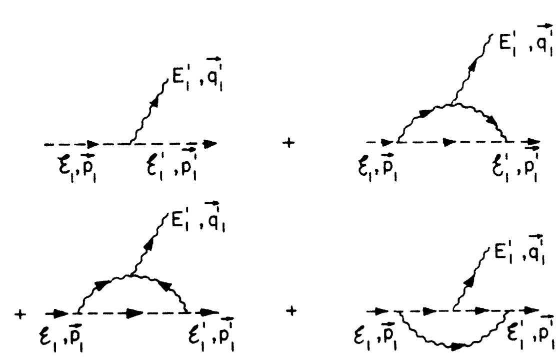

FIG. 6. The graphs needed to evaluate the renormalization-group functions in lowest order of perturbation theory.

when (68) is true. In either case there is a stable fixed point, $W_i(t)\to\infty$, only if the eigenvalues have positive real parts.

Three stable fixed points emerge from this analysis[11]:

Case I.

$$g=g_1,\quad g_{R1}=g_1/2, \quad \text{and} \quad \nu_1=0; \tag{73}$$

Case II.

$$g=g_1,\quad g_{R1}=g_+=g_1(\sqrt{2}-\tfrac{1}{2}), \quad \text{and} \quad \nu_1=\infty; \tag{74}$$

Case III.

$$g=g_1,\quad g_{R1}=g_-=-g_1(\sqrt{2}+\tfrac{1}{2}), \quad \text{and} \quad \nu_1=\infty. \tag{75}$$

Notice that in each case the fixed point of the dimensionless coupling constants is of order $\sqrt{\epsilon}$. With precisely the same force as in the Pomeron problem one may argue here that for small values of ϵ the use of perturbation theory to calculate the renormalization-group functions is quite justified. Furthermore, for small ϵ, perturbation theory may be used to give a reliable estimation of $\Gamma^{(n,m;k)}$ on the right-hand side of (44) in the regime $\xi\to 0$. In other words, we have a perturbation scheme to calculate the renormalized Pomeron-Reggeon Green's functions in the neighborhood of $J=\alpha(0)=1$ for Pomerons and $J=\alpha_R(0)$ for Reggeons. Whether or not the actual value of ϵ needed for physics, namely $\epsilon=2$, is indeed small enough, is certainly an open question at this juncture.

We are now prepared to solve the characteristic equations for large t. To leading order in $\xi=e^t$ one finds

$$\bar{g}(-t)\underset{t\to-\infty}{\longrightarrow} g_1, \tag{76}$$

$$\bar{\alpha}'(-t)\underset{t\to-\infty}{\longrightarrow}\alpha' C_\alpha \xi^{-z(g_1)}, \tag{77}$$

as in the straight Pomeron problem, and

$$\bar{g}_R(-t)\underset{t\to-\infty}{\longrightarrow} g_{R1}, \tag{78}$$

$$\bar{\nu}(-t)\underset{t\to-\infty}{\longrightarrow}\nu C_\nu \xi^{\omega(g_1)}. \tag{79}$$

In these formulas C_α and C_ν are constants which depend on the renormalized parameters ν, g, and g_R. Their evaluation is outlined in Ref. 3.

The explicit values of $z(g_1)$ and $\omega(g_1)$ are

$$z(g_1)=1+Kg_1^2=1+\tfrac{1}{24}\epsilon, \tag{80}$$

and for Case I

$$g_{R1}=g_1/2 \tag{81}$$

and

$$\omega(g_1)=Kg_1^2=\tfrac{1}{24}\epsilon, \tag{82}$$

while for Case II and Case III

$$g_{R1}=g_\pm \tag{83}$$

and

$$\omega(g_1)=-K(4g_\pm^2-g_1^2) \tag{84}$$

$$=-\tfrac{1}{6}\epsilon(2\mp\sqrt{2}). \tag{85}$$

With these results in hand we can investigate the constraints on $\Gamma^{(n,m;k)}$ which follow from the solution to the renormalization-group equation as given in Eq. (44). First note that

$$\exp\int_{-t}^{0} dt'[1-\tfrac{1}{2}(n+m)\gamma(\bar{g}(t'))-k\gamma_R(\bar{g}(t'),\bar{g}_R(t'),\bar{\nu}(t'))]\underset{t\to-\infty}{\longrightarrow}(C_\gamma)^{(n+m)/2}(C_{\gamma_R})^k\,\xi^{1-(n+m)\gamma(g_1)/2-k\gamma_R(g_1,g_{R1},\nu_1)}, \tag{86}$$

where C_γ and C_{γ_R} are two more constants depending on g, g_R, and ν. Now we learn that for small E_i and $\mathcal{E}_j$ and fixed $\vec{q}_i$ and $\vec{p}_j$

$$\Gamma^{(n,m;k)}(E_i,\vec{q}_i,g,g_R,\alpha',\nu,E_N)$$

$$=(C_\gamma)^{(n+m)/2}(C_{\gamma_R})^k E_N\left(\frac{-E}{E_N}\right)^{1-(n+m)\gamma(g_1)/2-k\gamma_R(g_1,g_{R1},\nu_1)-z(g_1)D(2-m-n-2k)/4}$$

$$\times\left(\frac{E_N}{C_\alpha\alpha'}\right)^{D(2-n-m-2k)/4}\Phi_{n,m;k}\left(\frac{E_i}{-E},\left(\frac{-E}{E_N}\right)^{-z(g_1)}\frac{\alpha'\vec{q}_i\cdot\vec{q}_j}{E_N},\nu C_\nu\left(\frac{-E}{E_N}\right)^{\omega(g_1)},g_1,g_{R1}\right), \tag{87}$$

where we have introduced the total energy

$$E=\sum_{j=1}^{n} E_i+\sum_{j=1}^{k} \mathcal{E}_j \tag{88}$$

as a scaling variable. These scaling laws follow immediately from the solution to the renormalization-group equations combined with the dimensional analysis contained in Eq. (33).

Unfortunately these scaling rules are much less powerful than those which obtain in the Pomeron field theories.[3] There the total energy E entered the unknown scaling function only in conjunction with the inner product of momentum vectors. Here, however, because of the behavior of $\nu(\xi)$ as $\xi\to 0$, it enters in a second fashion. This means that we can conclude neither very strong general results about a renormalized Reggeon trajectory from the form of $\Gamma^{(0,0;1)}$ nor, strictly speaking, the strong general scaling laws for Reggeon-exchange amplitudes—both these nice features were present in the Pomeron problem.

This, by the way, appears to be the essential point of departure of our results from the work of Gribov, Levin, and Migdal.[6] Although their methods are quite different, it would appear that they did not take sufficient account of the role that the dimensionless ratio $\nu=\alpha_R'/\alpha'$ plays in the analysis.

We are able to go slightly beyond the present stage by combining the scaling result in Eq. (87) with the expansion in ϵ. The next section is devoted to this.

IV. THE RENORMALIZED REGGEON TRAJECTORY

In this section we will concentrate on the renormalized inverse Reggeon propagator $\Gamma^{(0,0;1)}$. In order to obtain the scaling function $\Phi_{0,0;1}$ as a power series in ϵ it is necessary to obtain solutions to the characteristic equations which are valid for all values of t, not just for $t\to\infty$. Fortunately, we are free to choose the initial conditions so as to simplify the calculation.

First, consider Case I where $\nu_1=0$. Here it is convenient to start with the initial values $g=g_1$, $g_R=\frac{1}{2}g_1$, and $\nu\ll 1$. Then working to leading order in ϵ and ν, one finds for all values of t

$$\bar{\nu}(t)=\nu e^{-Kg_1^2t}, \tag{89}$$

$$\bar{g}_R(t)=\tfrac{1}{2}g_1+\tfrac{1}{2}g_1\nu(e^{-Kg_1^2t}-e^{-6Kg_1^2t}). \tag{90}$$

For Cases II and III where $\nu_1=\infty$, it is convenient to take the initial values to be $g=g_1$, $g_R=g_\pm$, and $\nu\gg 1$. Then to leading order in ϵ and $1/\nu$

$$\bar{\nu}(t)=\nu e^{-\omega(g_1)t}-\frac{4Kg_\pm^2}{\omega(g_1)}(1-e^{-\omega(g_1)t}), \tag{91}$$

$$\bar{g}_R(t)=g_\pm-\frac{g_\pm D}{\nu}(e^{-\mu(g_1)t}-e^{-\omega(g_1)t}), \tag{92}$$

where $\omega(g_1)$ is given in Eq. (84) and

$$\mu(g_1)=4K(2g_\pm^2-g_1g_\pm), \tag{93}$$

$$D=K(8g_\pm^2-8g_1g_\pm-g_1^2). \tag{94}$$

In all cases $\bar{g}(t)=g_1$ and $\bar{\alpha}'(t)=\alpha' e^{-z(g_1)t}$.

It is clear that with these values for the characteristic functions Eq. (87) can be written in the form

$$i\Gamma^{0,0;1}(\mathcal{E},\vec{p}^2,g_1,g_{R1},\nu,E_N)=E_N\exp\left\{\int_{-t}^{0}dt'(1-\gamma_R)\right\}\left[\Phi_0\left(\frac{\bar{\alpha}'(-t)\vec{p}^2}{E_N}\bar{\nu}(-t)\right)+\epsilon\Phi_1\left(\frac{\bar{\alpha}'(-t)\vec{p}^2}{E_N},\bar{\nu}(-t)\right)+O(\epsilon^2)\right] \tag{95}$$

for all values of t. Since g_1 and g_{R1} are of order $\epsilon^{1/2}$ one can obtain Φ_0 and Φ_1 by calculating $i\Gamma^{0,0;1}$ to second order in perturbation theory. From Eqs. (55)–(59) we see that

$$i\Gamma^{0,0;1}(\mathcal{E},\vec{p}^2,g_1,g_{R1},\nu,E_N)=\mathcal{E}-\alpha'\nu\vec{p}^2+4Kg_{R1}^2\left(\frac{\alpha'\nu}{1+\nu}\vec{p}^2-\mathcal{E}\right)\left[1-\ln\left(\frac{\alpha'\nu\vec{p}^2}{(1+\nu)E_N}-\frac{\mathcal{E}}{E_N}\right)\right]. \tag{96}$$

For values of $t=\ln(-E/E_N)$ for which $\epsilon|t|\ll 1$ one can equate powers of ϵ in Eqs. (95) and (96) to find

$$\Phi_0(x,y)=-(1+xy), \tag{97}$$

$$\epsilon\Phi_1(x,y)=4g_{R1}^2\left(1+\frac{xy}{1+y}\right)\left[1-\ln\left(1+\frac{xy}{1+y}\right)\right]. \tag{98}$$

Then by considering values of t for which $\epsilon|t|\gg 1$ one sees that to order ϵ

$$i\Phi_{0,0;1}\left(\frac{\bar{\alpha}'(-t)\vec{p}^2}{E_N},\bar{\nu}(-t)\right)=\Phi_0\left(\frac{\bar{\alpha}'(-t)\vec{p}^2}{E_N},\bar{\nu}(-t)\right)+\epsilon\Phi_1\left(\frac{\alpha'(-t)\vec{p}^2}{E_N},\bar{\nu}(-t)\right). \tag{99}$$

The properties of the Reggeon trajectory can now be read off. In Case I, where $\nu_1 = 0$, there is a zero of $\Gamma^{0,0;1}$ when $xy+1$ is close to zero, so

$$\alpha_R(-\vec{p}^2) = \alpha_R(0) - \vec{p}^2(C_\alpha C_\nu \alpha_R') + O(\epsilon p^4). \qquad (100)$$

That is, the trajectory remains linear but has its slope renormalized. This is an attractive result. In Cases II and III the renormalized pole appears at

$$\alpha_R(-\vec{p}^2) = \alpha_R(0) + E_N\left(\frac{C_\alpha C_\nu \alpha_R' \vec{p}^2}{E_N(1-4Kg_\pm^2)}\right)^{1/(1+4Kg_\pm^2)}. \qquad (101)$$

In this case, the trajectory is not analytic at $t = -\vec{p}^2 = 0$. In fact it has an infinite slope there. This less attractive result is very similar to the behavior of the Pomeron trajectory in the pure Pomeron field theory. Which of the three fixed points actually controls the infrared behavior of the theory depends, of course, on the input values of g, g_R, and ν.

V. DISCUSSION

In this paper we have examined an interacting field theory of Pomerons and Reggeons to discover how their interaction modifies the input form for the Reggeon singularity in the angular momentum plane. We considered in detail here the case where the input for both the Pomeron and the Reggeon was a simple linear trajectory. Our main result is that in the infrared limit [Pomeron angular momenta tend to one; Reggeon angular momenta tend to $\alpha_R(0)$; all t's small] there are three stable fixed points to the renormalization-group equations evaluated in lowest order of perturbation theory for the triple-Pomeron and Reggeon-Reggeon-Pomeron couplings. The instruction that this may be a sufficient order of perturbation in which to operate comes from the observation that each of the coupling constants at the stable points is small in Reggeon space dimensions near four. All our results then were cast in terms of an expansion about $D=4$.

For each of these stable points we were able to derive general scaling laws for the renormalized Green's functions. These do not coincide with the scaling rules found by Gribov, Levin, and Midgal.[6] The essential reason for this appears to be their neglect of the behavior of the ratio of renormalized Reggeon and Pomeron slopes in the infrared regime.

By exploiting these scaling forms and utilizing our ϵ expansion, we found the renormalized Reggeon trajectory for each stable point. In one case, this yielded a linear trajectory with a renormalized slope. In the other two cases, however, the trajectory was more severely modified and acquired an infinite slope at $t=0$. One's natural preference is for the linear trajectory with small corrections. Unfortunately, since in the pure Pomeron problem[2,3] the trajectory developed a cusp, one is forced to contemplate the singular alternatives.

The scaling rules derived here are not sufficiently strong to enable us to predict the behavior of amplitudes involving Reggeon exchange as was possible for the Pomeranchuk singularity. By treating the problem in the ϵ expansion, however, we can make some statement about the zero momentum transfer amplitudes. From the graph in Fig. 7, which gave the leading behavior in s for the Pomeron case, we discover for a Reggeon of signature τ_R

$$T_{AB}(s,0) \underset{s\to\infty}{\simeq} \beta_A\beta_B(e^{-i\pi\alpha_R(0)} + \tau_R)\, s^{\alpha_R(0)}(\ln s)^{\gamma_R}, \qquad (102)$$

where $\gamma_R = \frac{1}{12}$ when the trajectory is linear and $\gamma_R = (9 \pm 4\sqrt{2})/12$ in the other cases. The corrections to this leading term are certainly something that we have little confidence in. However, it seems quite safe to conjecture that they are (a) negative in sign at $t=0$ and (b) down by $(\ln s)^{-p}$ where $p \approx \frac{1}{2}$ as in the Pomeron case. Estimates based on rather cavalier treatment of the scaling laws bear this out.

We would like to end on a guardedly optimistic note. The calculations presented here will clearly be modified in detail, but are rather the same in procedure for any Reggeon field theory consideration of Pomeron corrections to a given Reggeon exchange. Our detailed conclusions in this paper have rested rather heavily on the expansion of the theory about $D=4$ dimensions. Recent calculations to higher order in $\epsilon = 4-D$ in the Pomeron problem[12] indicate that the convergence of such an expansion is problematic. This suggests one look for theories which are effectively free in the infrared limit[7,8] and require no ϵ expansion. An attractive example of such a renormalization-group bootstrap is found in Ref. 8 and interestingly

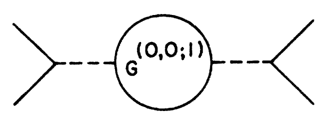

FIG. 7. The leading contribution to the $AB \to AB$ elastic amplitude due to Reggeon exchange corrected by Pomeron interactions.

enough has much the same general appearance as the results of Gribov *et al.*[6] It seems very worthwhile to examine the possibility that a $\sqrt{-t}$ type trajectory can be realized by Reggeons in interaction with Pomerons of the same variety. We shall return to this matter.

ACKNOWLEDGMENTS

This work was carried out in the main while H.D.I.A. enjoyed the hospitality of the high-energy theory group at Caltech, and R.L.S. enjoyed that of the theory group at NAL.

---

*Visitor from the University of California at Santa Barbara, Santa Barbara, California 93106.

†Operated by Universities Research Association Inc. under contract with the United States Atomic Energy Commission.

[1]D. Amati, A. Stanghellini, and S. Fubini, Nuovo Cimento 26, 896 (1962).

[2]A. A. Migdal, A. M. Polyakov, and K. A. Ter-Martirosyan, ITEP (Moscow) Report No. 102 (unpublished).

[3]H. D. I. Abarbanel and J. B. Bronzan, Phys. Rev. D 9, 2397 (1974).

[4]R. C. Arnold, Phys. Rev. 153, 1523 (1967); S. C. Frautschi and B. Margolis, Nuovo Cimento 56A, 1155 (1968).

[5]J. B. Bronzan and R. L. Sugar, Phys. Rev. D 8, 3409 (1973).

[6]V. N. Gribov, E. M. Levin, and A. A. Migdal, Yad. Fiz. 12, 173 (1970) [Sov. J. Nucl. Phys. 12, 93 (1971)].

[7]H. D. I. Abarbanel and J. B. Bronzan, Phys. Rev. D 9, 3302 (1974).

[8]H. D. I. Abarbanel, J. Bartels, and J. W. Dash (unpublished).

[9]V. N. Gribov, Zh. Eksp. Teor. Fiz. 53, 654 (1967) [Sov. Phys.—JETP 26, 414 (1968)].

[10]This assumes $\underline{A}$ is not identically zero; for us, it is not. For a general discussion of coupled characteristic equations see S. L. Lefschetz, *Differential Equations: Geometric Theory* (Interscience, New York, 1957).

[11]Notice that the zero in σ at $\nu/(1+\nu)=g^2/4g_R{}^2$ corresponds to a saddle point ($\lambda_1 > 0$, $\lambda_2 < 0$) rather than a stable fixed point.

[12]M. Baker (private communication); J. B. Bronzan and J. W. Dash (unpublished).