are distinct. This would be the case if states having different SU(3)″ symmetries (singlets, triplets, etc.) are allowed to occur with finite energy. In this sense the octet-gluon model belongs *a priori* to the distinguishable case, but it will not if all nonsinglet states are somehow forbidden, a situation speculated or desired by some people.

[30]C. G. Callan and D. Gross, Phys. Rev. Lett. 22, 156 (1969).

[31]N. Cabibbo and R. Gatto, Phys. Rev. 124, 1577 (1961).

This result may not asymptotically hold in a gauge theory which incorporates electromagnetism.

[32]A recent report from SLAC [B. Richter, in proceedings of the American Physical Society Meeting, Chicago, 1974 (unpublished)] strengthens the suspicion that R may indeed be rising with energy. If this is confirmed, our estimate of μ in Sec. IV will become less meaningful.

[33]See for example, G. R. Farrar and J. L. Rosner, Phys. Rev. D 7, 2747 (1973).

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Electron-electron scattering: Spectral forms for the invariant amplitudes to order e^4*

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The five invariant amplitudes for electron-electron scattering to order e^4 are calculated using the causal methods of source theory. The basis set employed, which is free of kinematic singularities and zeros, consists of the Fermi invariants. The amplitudes are expressed in terms of double-spectral forms with accompanying single-spectral forms which are determined by analysis of causal forward scattering. The radiative corrections required for the analysis of polarization experiments are contained in these amplitudes.

I. INTRODUCTION

Quite a number of fourth-order processes have been calculated in quantum electrodynamics, including the magnetic moment of the electron,[1] photon-photon scattering,[2] and Compton scattering.[3,4] And some aspects of fermion-fermion scattering[5,6] have been considered, among which is an investigation of the hard-photon corrections[7] to electron-electron scattering. Except for the unpolarized differential cross section,[8] the corresponding polarization calculations for electron-electron (positron) scattering have not been previously done. Theoretically, it is of interest to know the invariant amplitudes and to have them available for use in higher-order processes. Besides, more can be learned of the role played by dynamics in the choice of a basis for the invariant amplitudes. Experimentally, with the advent of colliding beams, a more detailed investigation of electron-electron scattering is possible. In particular, the asymmetry parameters[9] derived from the invariant amplitudes can be compared with experiment. Also, the determination of production cross sections for hadronic states requires an accurate knowledge of the purely electrodynamic processes.

Electron-positron scattering has been considered by McEnnan[10] within a context that attempts to move beyond perturbation theory, to include the effects of bound states and to eliminate the dependence on an artificial photon mass. Barbieri *et al.*[11] have used dispersive techniques for the "box diagram" in their recent calculation of the magnetic moment and charge radius of the electron. However, explicit expressions for the invariant amplitudes or helicity cross sections have not been presented.

The purpose of this paper is to calculate the electron-electron scattering to order e^4 and to present the invariant amplitudes in spectral form. An appropriate choice of a complete set of spinor basis for these amplitudes is considered in Sec. II. The general approach that we will use is the causal methodology of source theory.[12] We start the calculation in Sec. III by considering four virtual electron sources that are causally related (see Fig. 1). The removal of the causal restrictions (space-time generalization) and the application to free external particles (mass extrapolation) yields the invariant amplitudes in double-spectral form. To this must be added possible contact terms which are themselves single-spectral forms. The latter are determined by comparison with the causal forward scattering amplitudes for the photon-exchange and electron-exchange channels which are given in Secs. IV and V, respectively. The contact terms are determined in Sec. VI. The contributions due to vertex and vacuum-polarization insertions are considered in Sec. VII, while Sec. VIII contains a few concluding remarks. Appendixes A, B, and C deal with the integrals occurring in Secs. III, IV, and V, respectively. Appendix D is concerned with a test of consistency for the invariant amplitudes.

II. BASIS

Straightforward calculation yields eight types of spinor functions. They are the five Fermi invariants[13] (P_j denote momenta)

$$F_j(P_1,P_{1'};P_2,P_{2'})\equiv\psi_1(-P_1)\gamma^0 O_j\psi_{1'}(-P_{1'})\times\psi_2(P_2)\gamma^0 O_j\psi_{2'}(P_{2'}), \quad (1)$$

where

$$O_j=\left\{\frac{i}{\sqrt{2}}\sigma^{\mu\nu},\gamma^\mu,1,\gamma^\mu\gamma_5,\gamma_5\right\}, \quad (2)$$

together with [suppressing momentum labels which are as in Eq. (1)]

$$G_2=\psi\gamma^0\frac{1}{m}\gamma P_-\psi\psi\gamma^0\psi+\psi\gamma^0\psi\psi\gamma^0\frac{1}{m}\gamma Q_-\psi, \quad (3)$$

$$G_3=\psi\gamma^0\frac{1}{m}\gamma P_-\psi\psi\gamma^0\frac{1}{m}\gamma Q_-\psi, \quad (4)$$

$$G_4=\psi\gamma^0\frac{1}{m}\gamma_5\gamma P_-\psi\psi\gamma^0\frac{1}{m}\gamma_5\gamma Q_-\psi, \quad (5)$$

where

$$P_-=\tfrac{1}{2}(P_{2'}-P_2),\quad Q_-=\tfrac{1}{2}(P_1-P_{1'}). \quad (6)$$

We will denote the Fermi invariants of Eqs. (1) and (2) by T, V, S, A, and P, respectively.

It can be seen by the application of C, P, and T symmetries that there are only five invariant amplitudes for which we must choose a basis. Of the above eight functions almost any five are linearly independent and thus can be used. One such basis which has been considered is the Goldberger-Nambu-Oehme (GNO) set[14] comprised of G_2, G_3, G_4, S, and P.

There are a number of conditions an acceptable basis has to satisfy. These are:

(1) The basis must be free of kinematic singularities and zeros.[5]

(2) The resulting double- and single-spectral forms must converge.

(3) The causal forward scattering amplitudes cannot be singular.

(4) The contact terms determined by considering either forward or backward scattering in the electron channel must be the same.

It is to be noted that only the first condition involves kinematics and any linearly independent combination of our basic eight functions satisfies it. The question of whether the last three conditions are satisfied is determined by dynamics. It is possible that the vector coupling of electrodynamics picks out a different set than a pseudoscalar coupling (meson exchange) theory. It is not even known if a single set would satisfy all the conditions for every order of perturbation theory because of this crucial dependence on dynamics.

We have considered a large number of sets and quite a few are not allowed because of the above conditions. The GNO set, for instance, has singularities in the forward amplitudes for the photon channel (Sec. IV). However, this set has been successfully used in the calculation of NN scattering.[5,14] This then is an example of the interrelationship between dynamics and basis sets. The set comprised of S, P, A, G_2, and G_3 has singularities in the electron channel (Sec. V). The set employing S, P, A, T, and G_3 has no singularities, but it fails the consistency check of condition (4) (Appendix D).

The Fermi set satisfies all the conditions. In order to obtain the simplest possible spectral weight functions and contact terms we choose the following linear combination of the Fermi set as our basis:

$$\Gamma_i=V-T,\ V,\ P,\ S-P,\ A+S-P. \quad (7)$$

As a notation, we have for $i=1$, for example [momenta suppressed, cf. Eq. (1)],

$$\Gamma_1\equiv\psi\gamma^0\Gamma_1\psi\psi\gamma^0\Gamma_1\psi$$
$$\equiv\psi\gamma^0\gamma^\mu\psi\psi\gamma^0\gamma_\mu\psi+\tfrac{1}{2}\psi\gamma^0\sigma^{\mu\nu}\psi\psi\gamma^0\sigma_{\mu\nu}\psi. \quad (8)$$

Having chosen our basis we must eliminate G_2, G_3, and G_4 in terms of the Γ_i or equivalently the F_j. The momentum variables as presented in Eq. (1) can be combined to form three orthogonal vectors

$$S=P_1+P_{1'}=P_2+P_{2'} \quad (9)$$

(not to be confused with the scalar S),

$$T=P_1-P_2=P_{2'}-P_{1'} \quad (10)$$

(not to be confused with the tensor T),

$$U=P_1-P_{2'}=P_2-P_{1'}, \quad (11)$$

which satisfy

$$S^2+T^2+U^2+4m^2=0, \quad (12)$$

when the electrons are on the mass shell. [Throughout the paper, the variables S, T, and U will always be defined by reference to Eq. (1).] Introducing a fourth orthogonal vector,

$$L^\mu = \epsilon^{\mu\nu\lambda\kappa} S_\nu T_\lambda U_\kappa \,, \tag{13}$$

and using the relations

$$\gamma^\mu = \frac{1}{S^2}\gamma S S^\mu + \frac{1}{T^2}\gamma T T^\mu + \frac{1}{U^2}\gamma U U^\mu + \frac{1}{L^2}\gamma L L^\mu \,,$$

$$\gamma_5 = \frac{1}{L^2}\gamma L \gamma S \gamma T \gamma U \,, \tag{14}$$

$$L^2 = -S^2 T^2 U^2 \,,$$

we find[5]

$$m^2 G_2 = \tfrac{1}{4} S^2 T + \tfrac{1}{4}(T^2 - U^2)(S-P) + m^2 V \,, \tag{15}$$

$$m^2 G_3 = \tfrac{1}{4}(T^2 - U^2) V - \tfrac{1}{4} S^2 A - m^2 P \,, \tag{16}$$

$$m^2 G_4 = \tfrac{1}{4}(U^2 + T^2)(V - T) + \tfrac{1}{4}(U^2 - T^2)(P - S - A) \,. \tag{17}$$

It must be emphasized that the elimination of the dependent spinor functions is done under the appropriate causal conditions.

III. DOUBLE-SPECTRAL FORM

The causal process with which we begin is depicted in Fig. 1. The physical process is as follows: An extended (virtual) electron source (η_2) emits a free electron and a free photon. In the intermediate stage, the electron is absorbed and a free photon emitted by an extended electron source (η_3) while the photon emitted by η_2 is absorbed and a free electron emitted by another extended electron source ($\eta_{3'}$). Finally, the newly produced electron and photon are detected by an extended electron source (η_1). Under the causal arrangement there is another process which is just like the one above except with the intermediate sources ($\eta_3, \eta_{3'}$) interchanged. The second process is to be understood to accompany all the calculations of this section [except in the final result, Eq. (33)].

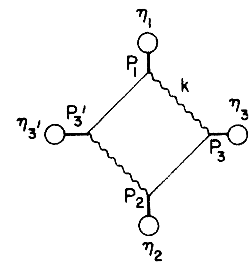

FIG. 1. "Diamond" process leading to double-spectral form.

All physical information is contained in the vacuum amplitude which, for this process, is given by[12,15]

$$\langle 0_+ | 0_- \rangle = e^4 \int \frac{(dP_1)}{(2\pi)^4}\frac{(dP_3)}{(2\pi)^4}\frac{(dP_{3'})}{(2\pi)^4}\frac{(dP_2)}{(2\pi)^4}(2\pi)^4 \times \delta(P_1 - P_2 - P_3 - P_{3'}) J \langle N \rangle \,, \tag{18}$$

where

$$J = \int (dk)\, \delta\big((P_1 - k)^2 + m^2\big)\, \delta\big((P_3 - k)^2 + m^2\big) \times \delta\big((P_1 - P_{3'} - k)^2 + \mu^2\big)\, \delta(k^2 + \mu^2) \tag{19}$$

and

$$\langle N \rangle = \langle \psi_1(-P_1)\gamma^0 \gamma^\nu [m - \gamma(P_1 - k)]\gamma^\mu \psi_{3'}(P_{3'})\psi_3(P_3)\gamma^0 \gamma_\nu [m - \gamma(k - P_3)]\gamma_\mu \psi_2(P_2)\rangle \,. \tag{20}$$

Here, J describes the fact that the internal particles are on-shell (the photon has mass μ), while the notation $\langle\ \rangle$ stands for an averaged value, that is, J has been factored out. The kinematic vectors are given in Eqs. (9)–(11) with

$$P_{1'} \to -P_{3'} \,, \quad P_2 \to P_3 \,, \quad P_{2'} \to P_2 \,,$$

and we can define two variables, x and y, as

$$S^2 = -m^2 y \,, \quad T^2 = -m^2(x+4) \,, \quad U^2 = m^2(x+y) \,. \tag{21}$$

The expression for U^2 actually contains additional terms that involve the factor $P_j^2 + m^2$. Since we will eventually be interested in on-shell momenta, we can simply make the mass extrapolation directly. This is admissible because no singularities or ambiguities are encountered in this process. Using standard techniques[4,12,16] the calculation of J yields

$$J = -\frac{i}{2m^4}\frac{1}{\sqrt{\Delta}} \,, \tag{22}$$

$$\Delta = y(x+4)[x(y - 4\lambda^2) - 4\lambda^4] \,, \tag{23}$$

where $\mu = m\lambda$. Here is the only place where mass extrapolation is not trivial. The factor of $-i$ results from this extrapolation as discussed in Ref. 16. Using the integrals presented in Appendix A and Eqs. (14)–(17), we find [cf. Eqs. (1) and (8) for notation]

$$\langle N \rangle = m^2 \sum_{i=1}^{5} h_i \Gamma_i (P_1, -P_{3'}; P_3, P_2) \,, \tag{24}$$

where

$$h_1 = -\frac{1}{2}\frac{y(x+2)}{\delta}[(x+2)y - 2x] - \frac{1}{2}\frac{\lambda^2}{\delta}[(x+4)y^2 - (3x^2 + 12x + 16)y + 8x(x+2)] + 2\frac{\lambda^4}{\delta}[(x+3)y - 3x - 8] \,, \tag{25}$$

$$h_2 = -\frac{1}{x+y}[(x+1)y + 2x(x+2)] + \frac{1}{2}\frac{\lambda^2}{x+y}(y-4x-12) - \frac{\lambda^4}{x+y}, \quad (26)$$

$$h_3 = \frac{4y}{x+y} - \frac{\lambda^2}{y(x+y)}(12y+8x) - \frac{8\lambda^4}{y(x+y)}, \quad (27)$$

$$h_4 = \frac{3y}{x+y} + \frac{1}{2}\frac{\lambda^2}{x+y}(y-2x-12) - \frac{3\lambda^4}{x+y}, \quad (28)$$

$$h_5 = \frac{1}{2}\frac{y}{\delta}[(x^2+2x-4)y + 2x(x^2+4x+2)] + \frac{\lambda^2}{\delta}[(4x+8)y - 2x(x^2+4x+4)] + \frac{\lambda^4}{\delta}[(x+6)y - x^2 - 6x - 16], \quad (29)$$

and

$$\delta = (x+4)(x+y)^2 . \quad (30)$$

We have retained the complete dependence on the photon mass as the amplitudes are very sensitive to it, particularly for scattering in the forward direction.

We now make a space-time generalization of Eq. (18) by observing that there are two independent mass excitations presented in Fig. 1, that associated with

$$-S^2 = -(P_2+P_3)^2 = m^2 y > 4\lambda^2 , \quad (31)$$

and that associated with

$$-T^2 = -(P_2+P_{3'})^2 = m^2(x+4) > 4m^2 . \quad (32)$$

Therefore, the space-time-generalized vacuum amplitude is[16]

$$\begin{aligned}\langle 0_+|0_-\rangle = {} & i\frac{\alpha^2}{2m^2}\int (dz)(d\eta)(d\zeta)\,d(m^2x)\,d(m^2y)\,\frac{1}{\sqrt{\Delta}}\,\Delta_+(\eta, m^2y)\,\Delta_+(\zeta, m^2(x+4))\sum_{i=1}^{5} h_i G_i(z,\eta,\zeta) \\ & + i\frac{\alpha^2}{2m^2}\int (dz)(d\eta)\,d(m^2y)\,\Delta_+(\eta,m^2y)\,\frac{(y-4\lambda^2)^{1/2}}{y^{1/2}}\,\frac{1}{(y-4)^2}\sum_{i=1}^{5}\chi_i G_i(z,\eta,\zeta=0) \\ & + i\frac{\alpha^2}{2m^2}\int (dz)(d\zeta)\,d(m^2x)\,\Delta_+(\zeta,m^2(x+4))\,\frac{1}{x^{1/2}(x+4)^{1/2}}\sum_{i=1}^{5}\bar{\chi}_i G_i(z,\eta=0,\zeta) ,\end{aligned} \quad (33)$$

where

$$G_i(z,\eta,\zeta) = \psi[z+\tfrac{1}{2}(\zeta+\eta)]\gamma^0\Gamma_i\psi[z+\tfrac{1}{2}(\eta-\zeta)]\,\psi[z+\tfrac{1}{2}(\zeta-\eta)]\gamma^0\Gamma_i\psi[z-\tfrac{1}{2}(\zeta+\eta)] . \quad (34)$$

The last two terms are the contact terms in the photon and electron channels, respectively, which will be determined in Sec. VI. The region of integration for x and y is determined by the nonvanishing of Δ, that is

$$xy - 4\lambda^2 x - 4\lambda^4 \geq 0 . \quad (35)$$

It might seem that the amplitude corresponding to V is not acceptable because, for large values of x, $h_2 \to -2x$ so that the spectral integral does not converge. However, this is not the case because the vector is antisymmetric. For large values of x, the coupling becomes local in the variable ζ and the resulting quadratic structure of $\psi(z \pm \frac{1}{2}\eta)$ vanishes. Alternatively, in any application of Eq. (33), there will be two terms from $\Delta_+(\zeta, m^2(x+4))$ that have a relative minus sign so that, once again, the large-x behavior cancels. Thus condition (2) stated in Sec. II must always be applied within the context in which the spectral forms are to be used.

IV. PHOTON SINGLE-SPECTRAL FORM

In this section we will consider the causal scattering process which arises from the causal exchange of two photons (see Fig. 2). The external electrons are on-shell and the acts of creation and detection of the intermediate photons are causally related. The corresponding vacuum amplitude is

$$\begin{aligned}\langle 0_+|0_-\rangle = {} & -e^4\int \frac{(dP_1)}{(2\pi)^4}\frac{(dP_{1'})}{(2\pi)^4}\frac{(dP_2)}{(2\pi)^4}\frac{(dP_{2'})}{(2\pi)^4}\,d\omega_k\,d\omega_{k'}(2\pi)^4\delta(P_1+P_{1'}-k-k')(2\pi)^4\delta(P_2+P_{2'}-k-k') \\ & \times \psi_1(-P_1)\gamma^0\gamma^\nu\frac{1}{m-\gamma(k-P_1)}\gamma^\mu\psi_{1'}(-P_{1'})\left[\psi_2(P_2)\gamma^0\gamma_\nu\frac{1}{m-\gamma(P_2-k)}\gamma_\mu\psi_{2'}(P_{2'}) + (P_2 \leftrightarrow P_{2'})\right] .\end{aligned} \quad (36)$$

The kinematics is as given in Eqs. (9)–(11). In order to determine the contact terms, we can consider any arbitrary scattering angle; for simplicity we will choose the case of forward scattering which is characterized by

$$T^2 = 0 .$$

As in Ref. 4, the momentum vectors and spinor functions are kept general and only the various scalar coefficients are calculated in the forward direction.

Following the same procedure as for the double-spectral form, integrating over phase space, and using the integrals given in Appendix B, we find

$$\langle 0_+|0_-\rangle = -\pi\alpha^2 \frac{1}{m^2}\int \frac{(dP_1)}{(2\pi)^4}\frac{(dP_{1'})}{(2\pi)^4}\frac{(dP_2)}{(2\pi)^4}\frac{(dP_{2'})}{(2\pi)^4}\frac{(y-4\lambda^2)^{1/2}}{y^{1/2}}\frac{1}{(y-4)^2}(2\pi)^4\delta(P_1+P_{1'}-P_2-P_{2'}) \times \sum_{i=1}^{5} H_i\Gamma_i(P_1,P_{1'};P_2,P_{2'}), \tag{37}$$

where

$$H_1 = (\tfrac{19}{3}-\lambda^2)y - \tfrac{4}{3} + 2\lambda^2 - \tfrac{10}{3}\lambda^4 + \lambda^6 + \lambda^4(\tfrac{10}{3}-\lambda^2)\frac{(2-\lambda^2)^2}{y-4\lambda^2+\lambda^4} + \frac{4+8\lambda^2-6\lambda^4}{y-2\lambda^2} - 4\lambda^2\frac{(2-\lambda^2)(1-\lambda^2)}{(y-2\lambda^2)^2}$$
$$+\left[-2y-12+8\lambda^2-25\frac{2-\lambda^2}{y-4}-\frac{2+5\lambda^2}{y-2\lambda^2}+\lambda^2\frac{8+2\lambda^2-4\lambda^4}{(y-2\lambda^2)^2}-4\lambda^4\frac{(2-\lambda^2)(1-\lambda^2)}{(y-2\lambda^2)^3}\right]L, \tag{38}$$

$$H_2 = (2+\lambda^2)y - 28 + 18\lambda^2 - 4\lambda^4 - \lambda^6 + (8+\lambda^2)\frac{(2-\lambda^2)^4}{y-4\lambda^2+\lambda^4} - 12\frac{(2-\lambda^2)^2}{y-2\lambda^2}$$
$$+\left[-4y+32-20\lambda^2-20\frac{2-5\lambda^2+2\lambda^4}{y-2\lambda^2}-12\lambda^2\frac{(2-\lambda^2)^2}{(y-2\lambda^2)^2}\right]L, \tag{39}$$

$$H_3 = 8y - 48 + 8\lambda^2 - 8\lambda^4 - 8(2+\lambda^2-\lambda^4)\frac{(2-\lambda^2)^2}{y-4\lambda^2+\lambda^4} + \frac{32}{y} + 48\frac{2-\lambda^2}{y-2\lambda^2}$$
$$+\left[-8\frac{4+\lambda^2}{y}-\frac{24\lambda^2}{y-2\lambda^2}+48\lambda^2\frac{2-\lambda^2}{(y-2\lambda^2)^2}\right]L, \tag{40}$$

$$H_4 = (-2+\lambda^2)y - 4 + 10\lambda^2 - 8\lambda^4 - \lambda^6 - \lambda^2(20-8\lambda^2-\lambda^4)\frac{(2-\lambda^2)^2}{y-4\lambda^2+\lambda^4} + 12\frac{4-\lambda^2}{y-2\lambda^2}$$
$$+\left[16-12\lambda^2-4\frac{6-\lambda^2+2\lambda^4}{y-2\lambda^2}+12\lambda^2\frac{4-\lambda^4}{(y-2\lambda^2)^2}\right]L, \tag{41}$$

$$H_5 = -\tfrac{1}{3}y + \tfrac{40}{3} - 8\lambda^2 + \tfrac{22}{3}\lambda^4 + \lambda^2(16-\tfrac{22}{3}\lambda^2)\frac{(2-\lambda^2)^2}{y-4\lambda^2+\lambda^4} - \frac{4+32\lambda^2-18\lambda^4}{y-2\lambda^2} + 4\lambda^2\frac{(2-\lambda^2)(1-\lambda^2)}{(y-2\lambda^2)^2}$$
$$+\left[2y-12+4\lambda^2-11\frac{2-\lambda^2}{y-4}+\frac{2-3\lambda^2+8\lambda^4}{y-2\lambda^2}-\lambda^2\frac{8+26\lambda^2-16\lambda^4}{(y-2\lambda^2)^2}+4\lambda^4\frac{(2-\lambda^2)(1-\lambda^2)}{(y-2\lambda^2)^3}\right]L, \tag{42}$$

and

$$L = \frac{(y-4)^{1/2}}{(y-4\lambda^2)^{1/2}}\ln\frac{y-2\lambda^2+(y-4)^{1/2}(y-4\lambda^2)^{1/2}}{y-2\lambda^2-(y-4)^{1/2}(y-4\lambda^2)^{1/2}}. \tag{43}$$

The threshold for this channel is $4\lambda^2$; for $y<4$, the logarithm can be written as an arctangent.

V. ELECTRON SINGLE-SPECTRAL FORM

Here we consider the contribution to causal forward scattering resulting from electron exchange (see Fig. 3). The vacuum amplitude is given by the expression

$$\langle 0_+|0_-\rangle = -e^4\int\frac{(dP_1)}{(2\pi)^4}\frac{(dP_{1'})}{(2\pi)^4}\frac{(dP_2)}{(2\pi)^4}\frac{(dP_{2'})}{(2\pi)^4}d\omega_p\,d\omega_q(2\pi)^4\delta(P_1+P_{1'}-P_2-P_{2'})(2\pi)^4\delta(P_1+P_{1'}-p-q)$$
$$\times\frac{1}{(P_1-p)^2+\mu^2}\frac{1}{(P_2-p)^2+\mu^2}\psi_1(-P_1)\gamma^0\gamma^\nu(m-\gamma p)\gamma^\mu\psi_2(P_2)$$
$$\times\psi_{1'}(-P_{1'})\gamma^0\gamma_\nu[m-\gamma(P_2+P_{2'}-p)]\gamma_\mu\psi_{2'}(P_{2'}). \tag{44}$$

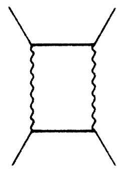

FIG. 2. Causal diagram for the photon channel leading to the photon single-spectral form.

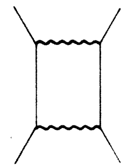

FIG. 3. Causal diagram for a forward electron-exchange process.

The kinematic vectors are as given by Eqs. (9)–(11) with the substitutions

$$P_{1'} \rightarrow -P_2, \quad P_2 \rightarrow -P_{1'} .$$

Therefore, in this channel, the energy variable is T, with

$$-T^2 = -(P_2 + P_{2'})^2 = m^2(x+4) \geq 4m^2 , \qquad (45)$$

and forward scattering is characterized by

$$S^2 = 0 . \qquad (46)$$

Again following the procedure of the double-spectral form, integrating over phase space, and using the integrals given in Appendix C, we find

$$\langle 0_+ | 0_- \rangle = -\pi\alpha^2 \frac{1}{m^2} \int \frac{(dP_1)}{(2\pi)^4} \frac{(dP_{1'})}{(2\pi)^4} \frac{(dP_2)}{(2\pi)^4} \frac{(dP_{2'})}{(2\pi)^4} \frac{1}{x^{1/2}(x+4)^{1/2}} (2\pi)^4 \delta(P_1 + P_{1'} - P_2 - P_{2'})$$
$$\times \sum_{i=1}^{5} \mathcal{K}_i \Gamma_i(P_1, -P_2; -P_{1'}, P_{2'}) , \qquad (47)$$

where

$$\mathcal{K}_1 = \frac{4}{x+\lambda^2} - \frac{12}{x} - \frac{4}{x+4} + \left(\frac{2+\lambda^2}{x} + \frac{2-\lambda^2}{x+4} + 12\frac{\lambda^2}{x^2} \right) \ln\Sigma , \qquad (48)$$

$$\mathcal{K}_2 = -\frac{4}{\lambda^2} x - \frac{8}{\lambda^2} - 2 - \frac{4+2\lambda^2}{x+\lambda^2} + \left(4 + \frac{12+4\lambda^2}{x} \right) \ln\Sigma , \qquad (49)$$

$$\mathcal{K}_3 = -\frac{8}{3}\frac{1}{\lambda^2} - \frac{64}{3(x+\lambda^2)} + \frac{16}{x} + \left(\frac{16}{x} - 16\frac{\lambda^2}{x^2} \right) \ln\Sigma , \qquad (50)$$

$$\mathcal{K}_4 = -4 - \frac{12+4\lambda^2}{x+\lambda^2} + \frac{12+8\lambda^2}{x} \ln\Sigma , \qquad (51)$$

$$\mathcal{K}_5 = -6 + \frac{4+2\lambda^2}{x+\lambda^2} + \frac{4}{x+4} - \frac{12}{x} + \left(4 + \frac{2+3\lambda^2}{x} - \frac{2-\lambda^2}{x+4} + 12\frac{\lambda^2}{x^2} \right) \ln\Sigma , \qquad (52)$$

and

$$\Sigma = \frac{x+\lambda^2}{\lambda^2} . \qquad (53)$$

It is to be noted that in the above results some of the coefficients behave as $1/\lambda^2$. In particular, one of these terms is found in the vector amplitude ($\mathcal{K}_2$), which again has a superficially bad behavior for large values of x. In order for the final amplitudes to behave properly in the nonforward direction (that is, no worse than $\ln\lambda^2$), these singular terms must be exactly reproduced by the double-spectral form. We will see this is indeed the case.

Of course, the vacuum amplitude as given in Eq. (44) is not complete. There is an additional term of the same form as above except with $P_2 \leftrightarrow P_{2'}$. Forward scattering here corresponds to $U^2 = 0$ and yields a structure different from Eq. (47) (see Appendix D).

VI. CONTACT TERMS

The double-spectral form with the accompanying single-spectral forms [Eq. (33)] is the general expression for the vacuum amplitude. Therefore, it must correctly describe the causal forward scattering processes considered in Secs. IV and V. Upon applying the appropriate causal and kinematic restrictions to Eq. (33), we find

$$\langle 0_+|0_-\rangle_{\text{causal}} = -4\pi\alpha^2 \frac{1}{m^2}\int \frac{(dP_1)}{(2\pi)^4}\frac{(dP_{1'})}{(2\pi)^4}\frac{(dP_2)}{(2\pi)^4}\frac{(dP_{2'})}{(2\pi)^4}(2\pi)^4\delta(P_1+P_{1'}-P_2-P_{2'})$$
$$\times\sum_{i=1}^{5}\left\{\left[\int_{x_0}^{\infty}\frac{dx}{\sqrt{\Delta}}h_i\left(\frac{1}{x+4}+\frac{\theta_i}{x+y}\right)+\frac{(y-4\lambda^2)^{1/2}}{y^{1/2}}\frac{1}{(y-4)^2}(1+\theta_i)\chi_i\right]\right.$$
$$\times\Gamma_i(P_1,P_{1'};P_2,P_{2'})$$
$$\left.+\left[\int_{y_0}^{\infty}\frac{dy}{\sqrt{\Delta}}h_i\frac{1}{y}+\frac{1}{x^{1/2}(x+4)^{1/2}}\bar{\chi}_i\right]\Gamma_i(P_1,-P_2;-P_{1'},P_{2'})\right\}, \tag{54}$$

where

$$x_0=\frac{4\lambda^4}{y-4\lambda^2},\quad y_0=4\lambda^2\frac{x+\lambda^2}{x}. \tag{55}$$

Here

$$\theta_i=\begin{cases}-1, & i=1,2\\ +1, & i=3,4,5\end{cases} \tag{56}$$

which is determined by the symmetry of the appropriate Fermi invariant. Since the vector and tensor Dirac matrices are antisymmetrical, there can be no corresponding contact terms in the photon channel. Comparison of Eq. (54) with Eq. (37) and Eq. (47) provides the following equations to determine the contact terms:

$$(y-4)^2\frac{y^{1/2}}{(y-4\lambda^2)^{1/2}}\int_{x_0}^{\infty}\frac{dx}{\sqrt{\Delta}}h_i\left(\frac{1}{x+4}+\frac{\theta_i}{x+y}\right)+(1+\theta_i)\chi_i=\tfrac{1}{4}H_i \tag{57}$$

and

$$x^{1/2}(x+4)^{1/2}\int_{y_0}^{\infty}\frac{dy}{\sqrt{\Delta}}h_i\frac{1}{y}+\bar{\chi}_i=\tfrac{1}{4}\mathcal{H}_i. \tag{58}$$

Performing the necessary integrations we find

$$\chi_i=0,\quad i=1,2,3,5 \tag{59}$$

$$\chi_4=-(y-4)+(2-\lambda^2)L, \tag{60}$$

$$\bar{\chi}_i=0,\quad i=3,5 \tag{61}$$

$$\bar{\chi}_1=-\bar{\chi}_2=-\bar{\chi}_4=\frac{1}{2}-\frac{1}{2}\frac{\lambda^2}{x}\ln\Sigma. \tag{62}$$

Once again, the vacuum amplitude given in Eq. (54) is not complete. For the electron channel a term involving $\Gamma_i(P_1,-P_{2'};-P_{1'},P_2)$ should be included. A reasonably simple consistency check on the amplitudes can now be made. An equation similar to Eq. (57) is derived for this case by comparison with the $U^2=0$ results mentioned at the end of Sec. V. In this way we confirm Eqs. (61) and (62). The details are presented in Appendix D.

VII. INSERTIONS

In the preceding sections we have been analyzing the so-called box diagram. There remain the contributions that arise from the remaining processes in the electron channel plus the pole term (single-particle exchange) in the photon channel. These processes are depicted in Figs. 4 and 5. However, they just correspond to vertex and vacuum polarization insertions into the lowest-order interaction, which is given by

$$\langle 0_+|0_-\rangle=i\tfrac{1}{8}e^2\int(dz)(dz')\psi(z)\gamma^0\hat{q}\gamma^\mu\psi(z)\times D_+(z-z')\psi(z')\gamma^0\hat{q}\gamma_\mu\psi(z'), \tag{63}$$

where $\hat{q}$ is the charge matrix. The insertions[12,17] consist of, in momentum space $[M^2=m^2(x+4)]$,

$$\gamma^\mu\to\gamma^\mu\left[1-\frac{k^2}{m^2}\frac{\alpha}{2\pi}\int_{4m^2}^{\infty}\frac{dM^2}{k^2+M^2-i\epsilon}f_1(x)\right]+\frac{1}{2m}\sigma^{\nu\mu}ik_\nu\frac{\alpha}{\pi}\int_{4m^2}^{\infty}\frac{dM^2}{k^2+M^2-i\epsilon}f_2(x), \tag{64}$$

$$f_1(x)=\frac{1}{x^{1/2}(x+4)^{1/2}}\left[\frac{x+2}{x+4}\ln\Sigma-\frac{1}{2}\frac{3x+4}{x+4}\right], \tag{65}$$

$$f_2(x)=\frac{1}{x^{1/2}(x+4)^{1/2}}, \tag{66}$$

and

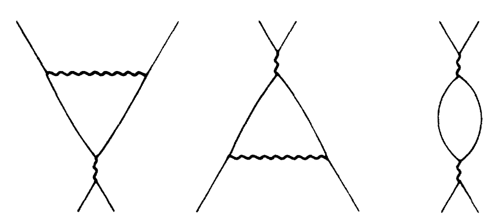

FIG. 4. Causal diagrams for the remaining electron-exchange processes.

$$\frac{1}{k^2} \to \frac{1}{k^2} + \frac{1}{m^2}\frac{\alpha}{\pi}\int_{4m^2}^{\infty} \frac{dM^2}{k^2+M^2-i\epsilon}\, a(x)\,, \tag{67}$$

$$a(x) = \frac{1}{3}\frac{x(x+6)}{(x+4)^2}\frac{1}{x^{1/2}(x+4)^{1/2}}\,. \tag{68}$$

The resulting structure, to order α^2, can be written in terms of the Fermi set except for the charge matrix, $\hat{q}$, which remains between the spinors. This inclusion of $\hat{q}$ will be denoted by a corresponding caret on the G_i, that is, $\hat{G}_i$ [cf. Eq. (34)]. The insertions then yield

$$\langle 0_+|0_-\rangle = i\,\frac{\alpha^2}{2m^2}\int (dz)(d\eta)\,d(m^2x)\,\Delta_+(\eta, m^2(x+4)) \times \sum_{i=1}^{5} D_i(s,t,u)\,\hat{G}_i(z,\eta,\zeta=0)\,, \tag{69}$$

where

$$D_1 = \tfrac{1}{4} f_2(x)\,, \quad D_3 = D_5 = 0\,, \tag{70}$$

$$D_2(s,t,u) = -f_1(x) + a(x) + \left(\frac{1}{s} - \frac{1}{4}\right) f_2(x)\,, \tag{71}$$

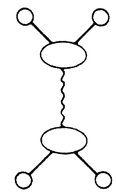

FIG. 5. Single-particle-exchange mechanism.

$$D_4(s,t,u) = \frac{u-t}{4s} f_2(x)\,, \tag{72}$$

$$m^2 s = S^2, \quad m^2 t = T^2, \quad m^2 u = U^2\,. \tag{73}$$

VIII. CONCLUSIONS

The final result for the complete fourth-order description of electron-electron (positron) scattering is given in terms of the vacuum amplitude contributions of Eqs. (33) and (69). The various weight functions are given in Eqs. (25)–(29), (59)–(62), and (70)–(73). From the vacuum amplitude the scattering amplitudes can be obtained in a straightforward way. The invariant amplitude for the electron-positron scattering process is

$$\langle 1_{P_1\sigma_1 q} 1_{P_1'\sigma_1',-q} | 1_{P_2\sigma_2 q} 1_{P_2'\sigma_2',-q}\rangle = 8i\alpha^2(2\pi)^4\,\delta(P_1+P_{1'}-P_2-P_{2'})(d\omega_{P_1}\,d\omega_{P_{1'}}\,d\omega_{P_2}\,d\omega_{P_{2'}})^{1/2} \times \sum_{i=1}^{5}\left(\mathfrak{M}_1^i\, u^*_{P_1\sigma_1}\gamma^0\Gamma_i u^*_{P_{1'}\sigma_{1'}}\, u_{P_2\sigma_2}\gamma^0\Gamma_i u_{P_{2'}\sigma_{2'}} - \mathfrak{M}_2^i\, u^*_{P_1\sigma_1}\gamma^0\Gamma_i u_{P_2\sigma_2} u^*_{P_{1'}\sigma_{1'}}\gamma^0\Gamma_i u_{P_{2'}\sigma_{2'}}\right), \tag{74}$$

where we have defined two probability amplitudes $\mathfrak{M}_1$ and $\mathfrak{M}_2$, which are $[m^2 s = (P_1+P_{1'})^2;\ m^2 t = (P_1 - P_2)^2;\ m^2 u = (P_1 - P_{2'})^2]$

$$\mathfrak{M}_1^i = \int dx\,dy\,\frac{h_i}{\sqrt{\Delta}}\frac{1}{s+y}\left(\frac{1}{t+x+4} + \frac{\theta_i}{u+x+4}\right) + \int dy\,\frac{(y-4\lambda^2)^{1/2}}{y^{1/2}}\frac{\chi_i}{(y-4)^2}\frac{1}{s+y}(1+\theta_i) + \int dx\,\frac{\bar{\chi}_i}{x^{1/2}(x+4)^{1/2}}\left(\frac{1}{t+x+4} + \frac{\theta_i}{u+x+4}\right) - \int dx\,\frac{2D_i(s,t,u)}{s+x+4}\,, \tag{75}$$

$$\mathfrak{M}_2^i = \int dx\,dy\,\frac{h_i}{\sqrt{\Delta}}\frac{1}{t+y}\left(\frac{1}{s+x+4} + \frac{\theta_i}{u+x+4}\right) + \int dy\,\frac{(y-4\lambda^2)^{1/2}}{y^{1/2}}\frac{\chi_i}{(y-4)^2}\frac{1}{t+y}(1+\theta_i) + \int dx\,\frac{\bar{\chi}_i}{x^{1/2}(x+4)^{1/2}}\left(\frac{1}{s+x+4} + \frac{\theta_i}{u+x+4}\right) - \int dx\,\frac{2D_i(t,s,u)}{t+x+4}\,. \tag{76}$$

In a subsequent communication we will apply the above result to calculate the fourth-order helicity amplitudes. From these amplitudes we will rederive the unpolarized cross section.[8] Also considered there will be the contributions due to soft-photon emission.

It should be noted that throughout this work we have treated the photon mass exactly. Except for forward (backward) scattering or near threshold, the factors that are linear or quadratic in λ^2 can, in fact, be ignored. However, there are situations for which the exact dependence on λ^2 is crucial. One illustration of this is the work of Ref. 11, where it is shown that the $\lambda^2 \to 0$ limit is not uniform.

In summary, we have calculated the invariant amplitudes that contain all the polarizations information to order α^2. Our method has been to consider the causal particle exchanges that give rise to the processes of interest. In particular, the causal analysis of the "diamond" process begins with all internal particles on-shell and all external particles as virtual. There are two excitations which are associated with physical exchanges of an electron pair and a photon pair, respectively. The use of the principle of space-time uniformity, which consists of the removal of the causal restrictions and the application to free external particles, leads directly to the double-spectral form. The contact terms that arise as the fields are allowed to overlap are calculated by reference to (on-shell) causal scattering. The remaining particle-exchange processes are also presented as spectral forms. This procedure is to be contrasted with the more conventional approach of operator field theory or the use of the Mandelstam representation in conjunction with analyticity arguments. It has been our experience that our approach is at least as efficient as that of field theory if all that is required is an unpolarized cross section. But for detailed polarization information, the application of spectral analysis is far superior. And being more physically motivated the source-theory derivation of these spectral forms is more straightforward than the S-matrix methodology.

APPENDIX A: DOUBLE-SPECTRAL FORM INTEGRALS

Here we present the averaged value of various powers of the loop variable k^μ that occur in Sec. III. In terms of the vectors S, T, and U the various tensors are of the forms

$$\langle 1\rangle = K, \tag{A1}$$

$$\langle k\rangle = aS + bT + cU, \tag{A2}$$

$$\langle kk\rangle = dSS + eTT + fUU + g(ST+TS) + h(SU+US) + l(TU+UT) + n1. \tag{A3}$$

Then, in terms of the variables of Eq. (21), we have, after mass extrapolation,

$$K=1, \quad a=\tfrac{1}{2}, \quad c=\frac{1}{2}\,\frac{y-2\lambda^2}{x+y}, \tag{A4}$$

$$d=\frac{1}{4}+\frac{1}{y}m^{-2}n, \quad e=\frac{1}{x+4}m^{-2}n, \tag{A5}$$

$$f=c^2-\frac{1}{x+y}m^{-2}n, \quad h=\tfrac{1}{2}c, \tag{A6}$$

$$m^{-2}n=\frac{1}{4}\,\frac{1}{x+y}\left[x(y-4\lambda^2)-4\lambda^4\right], \tag{A7}$$

$$b=g=l=0. \tag{A8}$$

APPENDIX B: PHOTON-CHANNEL INTEGRALS

The following integrals are used to evaluate Eq. (36). As mentioned in Sec. IV, we impose the forward scattering condition

$$T^2=(P_1-P_2)^2=0 \tag{B1}$$

on the coefficients. Defining

$$\langle\cdots\rangle=\frac{16\pi m^4(y-4)^2}{(1-4\lambda^2/y)^{1/2}} \times \int d\omega_k\, d\omega_{k'}(2\pi)^4\,\delta(S-k-k') \times \frac{1}{m^2+(P_1-k)^2}\,\frac{1}{m^2+(P_2-k)^2}(\cdots), \tag{B2}$$

where $\cdots = 1, k^\mu, k^\mu k^\nu$, we expand the tensors just as in Eqs. (A1)–(A3). The various coefficients will be denoted with a prime, that is, $K\to K'$, etc. We then find, in terms of the variable y [see Eq. (21)] and function L [see Eq. (43)],

$$K'=2(y-8+4\lambda^2-\lambda^4)+2\,\frac{(2-\lambda^2)^4}{y-4\lambda^2+\lambda^4}, \tag{B3}$$

$$a'=\tfrac{1}{2}K', \quad b'=0, \tag{B4}$$

$$c'=y-4+2\lambda^2-\lambda^4-\lambda^2\,\frac{(2-\lambda^2)^3}{y-4\lambda^2+\lambda^4}-2L, \tag{B5}$$

$$d'=\tfrac{1}{2}(y-12+4\lambda^2-\lambda^4)+\frac{8}{y}+\frac{1}{2}\,\frac{(2-\lambda^2)^4}{y-4\lambda^2+\lambda^4}+\left(1-\frac{2\lambda^2}{y}\right)L, \tag{B6}$$

$$f'=\tfrac{1}{2}(y+8-\lambda^4)+\tfrac{1}{2}\lambda^4\,\frac{(2-\lambda^2)^2}{y-4\lambda^2+\lambda^4}-3\left(1+\frac{4-2\lambda^2}{y-4}\right)L, \tag{B7}$$

$$e'=\tfrac{1}{3}f', \quad g'=l'=0, \quad h'=\tfrac{1}{2}c', \tag{B8}$$

$$m^{-2}n'=-2(y-4)+(y-2\lambda^2)L. \tag{B9}$$

The above is for the first term of Eq. (36). For the second part ($P_2\leftrightarrow P_{2'}$) we define the expansion coefficients exactly as above except $K'\to\tilde{K}'$, etc. We have

$$\tilde{K}'=4\left(1-2\,\frac{2-\lambda^2}{y-2\lambda^2}\right)L, \tag{B10}$$

$$\tilde{a}'=\tfrac{1}{2}\tilde{K}', \quad \tilde{c}'=0, \tag{B11}$$

$$\tilde{b}'=-2+4\,\frac{2-\lambda^2}{y-2\lambda^2}+\left(2-\frac{4}{y-2\lambda^2}+4\lambda^2\,\frac{2-\lambda^2}{(y-2\lambda^2)^2}\right)L, \tag{B12}$$

$$\tilde{d}'=1-\frac{4}{y}+\left(1-\frac{4-\lambda^2}{y}-\frac{2-\lambda^2}{y-2\lambda^2}\right)L, \tag{B13}$$

$$\tilde{e}'=-2+\frac{2}{y-2\lambda^2}-2\lambda^2\frac{2-\lambda^2}{(y-2\lambda^2)^2}+\left[1+\frac{3}{2}\frac{2-\lambda^2}{y-4}-\frac{1}{2}\frac{2+\lambda^2}{y-2\lambda^2}+\lambda^2\frac{4-\lambda^2}{(y-2\lambda^2)^2}-2\lambda^4\frac{2-\lambda^2}{(y-2\lambda^2)^3}\right]L, \quad (B14)$$

$$\tilde{f}'=-3+\left(1+3\frac{2-\lambda^2}{y-4}+\frac{\lambda^2}{y-2\lambda^2}\right)L, \quad (B15)$$

$$\tilde{g}'=\tfrac{1}{2}\tilde{b}', \quad \tilde{h}'=\tilde{l}'=0, \quad (B16)$$

$$m^{-2}\tilde{n}'=y-4-2\left(1-\lambda^2\frac{2-\lambda^2}{y-2\lambda^2}\right)L. \quad (B17)$$

APPENDIX C: ELECTRON-CHANNEL INTEGRALS

For this channel we require the integrals of various tensors involving the momentum of one of the exchanged electrons, p^μ. Defining

$$\langle\cdots\rangle=16\pi m^4x^{1/2}(x+4)^{1/2}\times\int d\omega_p\,d\omega_q(2\pi)^4\,\delta(T-p-q)\times\frac{1}{(P_1-p)^2+\mu^2}\,\frac{1}{(P_2-p)^2+\mu^2}(\cdots), \quad (C1)$$

where $\cdots=1,p^\mu,p^\mu p^\nu$, we expand the tensors just as in Eqs. (A1)–(A3) except we use $K\to K''$, etc. Using the forward scattering condition [Eq. (46)], the variable x [Eq. (45)], and Σ [Eq. (53)], we find

$$K''=\frac{2}{\lambda^2}-\frac{2}{x+\lambda^2}, \quad a''=0, \quad b''=\tfrac{1}{2}K'', \quad (C2)$$

$$c''=\frac{1}{\lambda^2}+\frac{1}{x+\lambda^2}-\frac{2}{x}\ln\Sigma, \quad d''=\tfrac{1}{3}f'', \quad (C3)$$

$$e''=\frac{1}{2\lambda^2}-\frac{1}{2}\frac{1}{x+\lambda^2}-\frac{2}{x+4}+\frac{1}{2}\left(\frac{2-\lambda^2}{x+4}+\frac{\lambda^2}{x}\right)\ln\Sigma, \quad (C4)$$

$$f''=\frac{6}{x}+\frac{1}{2\lambda^2}-\frac{1}{2}\frac{1}{x+\lambda^2}-3\left(\frac{1}{x}+\frac{2\lambda^2}{x^2}\right)\ln\Sigma, \quad (C5)$$

$$g''=h''=0, \quad l''=\tfrac{1}{2}c'', \quad (C6)$$

$$m^{-2}n''=-2+\left(1+\frac{2\lambda^2}{x}\right)\ln\Sigma. \quad (C7)$$

APPENDIX D: CONSISTENCY CHECK

As mentioned at the end of Sec. V the right-hand side of Eq. (44) should be supplemented by a term of the same form except with $P_2\leftrightarrow P_{2'}$. The necessary integrals are the same as those in Appendix C except now in the definition of $\langle\cdots\rangle$, P_2 is replaced by $P_{2'}$. Denoting the expansion coefficients by $\tilde{K}''$, etc., and using the forward scattering condition $U^2=0$ appropriate for this term, we find

$$\tilde{K}''=\frac{4}{x+2\lambda^2}\ln\Sigma, \quad \tilde{b}''=\tfrac{1}{2}\tilde{K}'', \quad \tilde{c}''=0, \quad (D1)$$

$$\tilde{a}''=-\frac{2}{x+2\lambda^2}+\left[\frac{1}{x}+\frac{x}{(x+2\lambda^2)^2}\right]\ln\Sigma, \quad (D2)$$

$$\tilde{d}''=-\frac{2(x^2+3x\lambda^2+3\lambda^4)}{x(x+2\lambda^2)^2}+\left[\frac{3x+6\lambda^2}{4x^2}+\frac{x^2}{4(x+2\lambda^2)^3}\right]\ln\Sigma, \quad (D3)$$

$$\tilde{e}''=\frac{1}{x+4}+\frac{1}{x+2\lambda^2}\left[1-\frac{2x\lambda^2+2\lambda^4}{x^2+4x}\right]\ln\Sigma, \quad (D4)$$

$$\tilde{f}''=-\frac{3}{x}+\frac{x^2+6x\lambda^2+6\lambda^4}{x^2(x+2\lambda^2)}\ln\Sigma, \quad (D5)$$

$$\tilde{g}''=\tfrac{1}{2}\tilde{a}'', \quad \tilde{h}''=\tilde{l}''=0, \quad (D6)$$

$$m^{-2}\tilde{n}''=1-\frac{2(x\lambda^2+\lambda^4)}{x(x+2\lambda^2)}\ln\Sigma. \quad (D7)$$

The additional contribution to the vacuum amplitude has the same form as Eq. (47) except for the following replacements: $\Gamma_i(P_1,-P_2;-P_{1'},P_{2'})\to\Gamma_i(P_1,-P_{2'};-P_{1'},P_2)$ and $\mathcal{K}_i\to\tilde{\mathcal{K}}_i$, which are found to be

$$\tilde{\mathcal{K}}_1=\frac{3}{x}+1+\frac{-2\lambda^2+2\lambda^4}{(x+2\lambda^2)^2}-\frac{3}{x+2\lambda^2}+\frac{2}{x+4}+\left[-\frac{3\lambda^2}{x^2}+\frac{1-4\lambda^2}{2x}+\frac{-2\lambda^4+2\lambda^6}{(x+2\lambda^2)^3}-\frac{2\lambda^2+\lambda^4}{(x+2\lambda^2)^2}-\frac{1}{2(x+2\lambda^2)}\left(1+\lambda^2+\frac{4\lambda^2}{4-2\lambda^2}\right)+\frac{1}{2(x+4)}\left(\lambda^2+\frac{4\lambda^2}{4-2\lambda^2}\right)\right]\ln\Sigma, \quad (D8)$$

$$\tilde{\mathcal{K}}_2=2+\frac{12-6\lambda^2}{x+2\lambda^2}+\left[\frac{-6+\lambda^2}{x}+\frac{12\lambda^2-6\lambda^4}{(x+2\lambda^2)^2}+\frac{-10+17\lambda^2}{x+2\lambda^2}-8\right]\ln\Sigma, \quad (D9)$$

$$\tilde{\mathcal{K}}_3=-\frac{24}{x}+\frac{24}{x+2\lambda^2}+\left[\frac{24\lambda^2}{x^2}+\frac{24\lambda^2}{(x+2\lambda^2)^2}\right]\ln\Sigma, \quad (D10)$$

$$\tilde{\mathcal{K}}_4 = -2 + \frac{12+6\lambda^2}{x+2\lambda^2} + \left[-\frac{6+\lambda^2}{x} + \frac{12\lambda^2+6\lambda^4}{(x+2\lambda^2)^2} + \frac{6-\lambda^2}{x+2\lambda^2} \right] \ln\Sigma , \tag{D11}$$

$$\tilde{\mathcal{K}}_5 = \frac{3}{x} + 3 + \frac{-2\lambda^2+2\lambda^4}{(x+2\lambda^2)^2} - \frac{3+6\lambda^2}{x+2\lambda^2} - \frac{2}{x+4}$$
$$+ \left[-\frac{3\lambda^2}{x^2} + \frac{1}{2x} + \frac{-2\lambda^4+2\lambda^6}{(x+2\lambda^2)^3} - \frac{2\lambda^2+7\lambda^4}{(x+2\lambda^2)^2} + \frac{1}{2(x+2\lambda^2)} \left(-1+\lambda^2+\frac{4\lambda^2}{4-2\lambda^2} \right) - \frac{1}{2(x+4)} \left(\lambda^2 + \frac{4\lambda^2}{4-2\lambda^2} \right) \right] \ln\Sigma . \tag{D12}$$

Likewise, for the electron channel we must add a term to the right-hand side of Eq. (54), with $\Gamma_i(P_1, -P_2; -P_{1'}, P_{2'}) \to \Gamma(P_1, -P_{2'}; -P_{1'}, P_2)$ and $y^{-1} \to (x+y)^{-1}$ in the double-spectral-form denominator. Since the two terms for the electron channel have different structures, the contact terms $\bar{\chi}_i$ must also satisfy the following equation [cf. Eq. (58)]:

$$x^{1/2}(x+4)^{1/2} \int_{y_0}^{\infty} \frac{dy}{\sqrt{\Delta}} h_i \frac{1}{x+y} + \bar{\chi}_i = \tfrac{1}{4} \tilde{\mathcal{K}}_i . \tag{D13}$$

Indeed Eqs. (61) and (62) are recovered. Since $P_2 \leftrightarrow P_{2'}$ relates backward scattering to forward scattering and vice versa, the above test can be viewed as a consistency check for the amplitudes between the two processes.

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[17]J. Schwinger, Phys. Rev. 158, 1391 (1967).