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[21]Our interest in this paper has been restricted to singularities related to the rotation group. However, we note here that $f(s)$ and $g(s)$ contain singularities in s which are inherited by the helicity amplitude. These singularities are discussed from a variety of viewpoints in the papers cited in Ref. 8.

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Three triplets, paraquarks, and "colored" quarks*

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We investigate the interconnection between various "nine-quark" models of hadrons: paraquarks of order 3, tri-"color" Gell-Mann quarks, our distinguishable three triplets with SU(3)′×SU(3)″ symmetry, and another version with SU(3)′×SO(3) symmetry. A natural framework in which to distinguish one model from another is provided by a recent theorem by Ohnuki and Kamefuchi on the equivalence relation between a para-Fermi field of order p and a Fermi field with a hidden variable which takes on p different values. Owing to the difference in charge assignment that is natural to each model, the e^+e^- annihilation ratio R $[=\sigma(e^+e^-\to\text{hadrons})/\sigma(e^+e^-\to\mu^+\mu^-)]$ is equal to 2, 4, and 8 for para-(color) quarks, our triplets, and (fractionally charged) triplets with SU(3)′×SO(3) symmetry, respectively. The experimental data may be interpreted to favor distinguishable triplet models. We then study the dynamics of superstrong interactions implied by the two versions of distinguishable triplet models, and find the SU(3)″ variety to be favored. Assuming integral charge and baryon-number assignment, we discuss the gross mass spectrum of hadrons. Single quarks will have a mass of ~2 GeV, and decay into ordinary baryons or antibaryons.

I. INTRODUCTION

The three-triplet model[1,2] with double SU(3) symmetry has had several rather successful applications in hadron spectroscopy and reactions. In addition to allowing integral charges, symmetric baryon wave functions, and triality-zero states saturation, the three triplets are compatible with both the $\pi^0\to 2\gamma$ decay rates and the total $e^+e^-\to$ hadrons cross sections. The model has also been widely applied with respect to the quark-parton picture of hadrons as well as various theories of unified gauge models.

In the family of what may be called "nine-quark" models,[3] there are many distinct models with varying degrees of similarities between them. The paraquarks of Greenberg[4] consist of a single SU(3) triplet of parafermions of order 3; if the Green-component fields are to be taken as independent fields, then this model contains nine quarks. The basic features of our three-triplet model were independently proposed by Tavkhelidze[5] (however, without an explicit classification scheme). Miyamoto[6] has considered slightly different integrally charged three triplets within the framework of SU(9) symmetry. Yet another three-triplet model was proposed by Tati[7] in which the quarks are assigned a "spin" of magnitude one with the symmetry group being SU(3) ×SO(3). Recently, Gell-Mann[8] has revived the paraquark model in a version in which the Green index is called color.

In this paper we will discuss similarities and differences between some of these models; in particular, the SU(3)′× SU(3)″ triplets, the paraquarks or "colored" quarks, and the SU(3)′× SO(3) triplets, which are an extension of the original proposal by Tati. Crucial to our discussion is a recent theorem by Ohnuki and Kamefuchi[9] concerning the equivalence relations between a parafermi field of order p and a Fermi field with a hidden variable which takes on p different values. They have shown that different classes of equivalence exist depending on the choice of types of observables, consistent with the locality condition.

First, however, we mention the equivalence between Gell-Mann's "colored" quarks and Greenberg's paraquarks of order 3 for the sake of completeness. As is well known, a para-Fermi field $\psi(x)$ of order p is reducible in terms of its Green components:

$$\psi(x)=\sum_{\alpha=1}^{p}\psi^{(\alpha)}(x)\ , \tag{1}$$

where the set of p Fermi Green components satisfy anomalous commutation relations:

$$\{\psi^{(\alpha)}(x),\ \psi^{(\alpha)\dagger}(y)\}=\delta^{(3)}(x-y)\ ,$$
$$\{\psi^{(\alpha)}(x),\ \psi^{(\alpha)}(y)\}=0\ , \tag{2a}$$
$$[\psi^{(\alpha)}(x),\ \psi^{(\beta)\dagger}(y)]=[\psi^{(\alpha)}(x),\ \psi^{(\beta)}(y)]=0\ \text{ for }\ \alpha\neq\beta. \tag{2b}$$

In the usual para-Fermi field theory, the parafield operator $\psi(x)$ is the one that has a direct physical meaning. The Green component fields $\psi^{(\alpha)}(x)$ are auxiliary mathematical quantities.

The anomalous commutation relations (2b) can be removed by the Klein transformation K_α, which converts $\psi^{(\alpha)}(x)$ into another set $\phi^{(\alpha)}(x)$ satisfying normal anticommutation relations.

Let

$$\psi^{(\alpha)}(x)=K_\alpha\phi^{(\alpha)}(x)\ , \tag{3}$$

where

$$K_\alpha=\exp\left[i\pi\int\sum_{\gamma=\alpha}^{p}\phi^{(\gamma)\dagger}(x)\,\phi^{(\gamma)}(x)\,d^3x\right]\ . \tag{4}$$

The K_α's then satisfy

$$[K_\alpha,\phi^{(\beta)}(x)]=0\quad\text{for }\ \alpha>\beta\ ,$$
$$\{K_\alpha,\phi^{(\beta)}(x)\}=0\quad\text{for }\ \alpha\leqslant\beta\ . \tag{5}$$

In terms of $\phi^{(\alpha)}(x)$, the commutation relations (2) become

$$\{\phi^{(\alpha)}(x),\ \phi^{(\beta)\dagger}(y)\}=\delta_{\alpha\beta}\,\delta^{(3)}(x-y),$$
$$\{\phi^{(\alpha)}(x),\ \phi^{(\beta)}(y)\}=0\ . \tag{6}$$

The $\phi^{(\alpha)}(x)$ fields, therefore, correspond to a set of p ordinary Fermi fields. Gell-Mann's "color"-quark scheme is set up with three fields, q_R, q_W, q_B. These fields are taken to anticommute with one another as well as with themselves,[10] which is tantamount to identifying the Klein-transformed Green component fields $\phi^{(\alpha)}(x)$ as the basic constituent (as well as current) quark fields.

Now we return to the Ohnuki-Kamefuchi theorem. Denoting any observable by $F(V_\text{I})$ defined in a spatial domain V_I, which is a Hermitian operator and a functional of $\psi(x)$ and $\psi^\dagger(x)$ $(x\in V_\text{I})$, these authors use, instead of the usual local commutativity,

$$[F(V_\text{I}),\ F'(V_\text{II})]=0\ \text{ for }\ V_\text{I}\cap V_\text{II}=0\ , \tag{7}$$

a stronger form:

$$[F(V_\text{I}),\ \psi(x)]=0\ \text{ for }\ x\notin V_\text{I}\ . \tag{8}$$

Under the stronger condition, they have shown[11] that $F(V_\text{I})$ must in general be a functional of $[\psi(x),\ \psi(y)]$, $[\psi(x),\ \psi^\dagger(y)]$, and $[\psi^\dagger(x),\ \psi^\dagger(y)]$ $(x,y\in V_\text{I})$.

Within this framework, the question of the equivalence relations between a single para-Fermi field of order p (PF field) and a system of a single Fermi field with a hidden variable which takes on p different values (F field) is investigated. Ohnuki and Kamefuchi define three kinds of equivalence relations:

(a) Strong equivalence. PF field $\rightleftarrows$ F field for all possible observations.

(b) Weak equivalence. PF field $\rightarrow$ F field for all possible observations, but the converse is not true.

(c) Local equivalence. PF field $\rightleftarrows$ F field for observations of any subsystem of PF fields (cluster property).

The Ohnuki-Kamefuchi theorem may now be stated as follows:

(1) If all quantities that satisfy the locality condition (8) are adopted as observables, i.e., functionals of $[\psi,\psi]$, $[\psi^\dagger,\psi^\dagger]$, and $[\psi,\psi^\dagger]$, then the *weak equivalence* holds true. The *local equivalence* holds true provided that the total system is sufficiently large compared to the subsystem in question.

(2) If observables are further restricted to functionals of $[\psi,\psi^\dagger]$ only, then the strong equivalence as well as the local equivalence holds true.[12]

Now an important point here is that the two different choices for the class of observables are directly connected with two different symmetry groups among the (Klein-transformed) Green component fields $\phi^{(\alpha)}(x)$. As pointed out by Ohnuki and Kamefuchi,[9] we have

$$[\psi(x), \psi(y)] = \sum_{\alpha=1}^{p} [\phi^{(\alpha)}(x), \phi^{(\alpha)}(y)] ,$$

$$[\psi^\dagger(x), \psi^\dagger(y)] = \sum_{\alpha=1}^{p} [\phi^{(\alpha)\dagger}(x), \phi^{(\alpha)\dagger}(y)] , \quad (9)$$

$$[\psi(x), \psi^\dagger(y)] = \sum_{\alpha=1}^{p} [\phi^{(\alpha)}(x), \phi^{(\alpha)\dagger}(y)]$$

and these relations are invariant under the gauge transformations SO(p) defined by

$$\phi^{(\alpha)\prime}(x) = \sum_{\beta=1}^{p} g_{\alpha\beta}\phi^{(\beta)}(x), \quad g \in \mathrm{SO}(p) \quad (10)$$

whereas the last relation alone, that is, $[\psi, \psi^\dagger]$, is invariant under SU(p) with $g \in \mathrm{SU}(p)$. Hence all the observables are either SO(p)-invariant or SU(p)-invariant, depending upon the choices of restrictions.

One should keep in mind, of course, that these equivalence relations are between a *single* PF field of order p and the system of a single Fermi field with a p-valued hidden variable. When one considers a system of N interacting PF fields[13] with the physical SU(3) symmetry endowed on them, the Ohnuki-Kamefuchi theorem does not imply *a priori* that the paraquark model of order 3 is automatically equivalent to the SU(3) × SU(3) or the SU(3) × SO(3) scheme. What the theorem tells us, however, is that the symmetry group of gauge transformations among the Green component fields $\phi^{(\alpha)}(x)$ is of the form of either SO(p) or SU(p).

II. NINE-QUARK MODELS

Let us now discuss the various "nine-quark" models. These nine-quark models will have for their respective symmetry groups either SU(3) × SU(3) or SU(3) × SO(3) structure, where the first SU(3) in each case is, of course, the usual one, whereas the second groups [SU(3) or SO(3)] are the ones for the symmetry of hidden variables.

(i) The paraquarks or color quarks.[14] In this case, we have a set of three indistinguishable SU(3) quarks; all their quantum numbers are to be identical. The freedom provided by the hidden-variable symmetry group is not explicitly made use of. This model can be associated with either SU(3) × SU(3) or SU(3) × SO(3) in the sense that such an identification may be irrelevant. Either case simply provides some unknown and/or unspecified three degrees of freedom.[15] The model consists of three indistinguishable sets of SU(3) quarks with the electric-charge assignments $(Q, Q-1, Q-1) = (\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$, where $Q = I_3 + \frac{1}{2}Y$.

Now the $\pi^0 \to 2\gamma$ decay amplitude is given by

$$F^\pi(0) = (-\alpha/2\pi)\,(2S)\,\sqrt{2}\,\mu^2/f_\pi , \quad (11)$$

where

$$S = \tfrac{1}{2}Q^2 - \tfrac{1}{2}(Q-1)^2$$
$$= Q - \tfrac{1}{2} \quad (12)$$

for each quark triplet with charges $(Q, Q-1, Q-1)$. In the asymptotic energy limit, the ratio R defined by $R = \sigma_{\text{total}}(e^+e^- \to \text{hadrons})/\sigma_{\text{total}}(e^+e^- \to \mu^+\mu^-)$ is given by

$$R \to R_\infty = \sum_i (\tfrac{1}{4} + \tfrac{3}{2}s_i)\, Q_i^2 , \quad (13)$$

where s_i is the spin (0 or $\frac{1}{2}$) of the ith field. In the absence of any integer-spin charged partons, this ratio is

$$R_\infty = \sum_i Q_i^2 . \quad (14)$$

As is well known,[8] the paraquark or color-quark model gives values of S and R three times those for the Gell-Mann–Zweig (GMZ) quarks:

$$S = 3S_{\text{GMZ}} = 3(\tfrac{1}{6}) = \tfrac{1}{2},$$
$$R = 3R_{\text{GMZ}} = 3(\tfrac{2}{3}) = 2 . \quad (15)$$

(ii) Three triplets with SU(3)′ × SU(3)″ *symmetry.* This is the case in which the para-Fermi observables are restricted to the form $[\psi, \psi^\dagger]$ only and hence, with the strong-equivalence condition, the SU(3)″ degrees of freedom are equivalent to the para-Fermi degrees of freedom.

As Ohnuki and Kamefuchi have noted, it is an interesting feature of their analysis that the symmetry corresponding to this new SU(3) symmetry can be derived as a consequence of the locality condition. We make, however, a departure from para-Fermi theories by allowing the observables to distinguish between different components; dynamics is thus no longer color-independent. Since ordinary hadrons (low-lying states) are assigned to SU(3)″ singlets only, one may identify the SU(3) symmetry group of strong interactions with either the diagonal subgroup of SU(3)′ × SU(3)″ or just SU(3)′, depending on one's point of view. In the latter case, strong interactions are still color-independent.[16]

As far as electromagnetic interactions are concerned, charge quantum number is defined within the diagonal SU(3) as

$$Q = I_3' + \tfrac{1}{2}Y' + I_3'' + \tfrac{1}{2}Y'' , \quad (16)$$

which gives integral charges (1, 0, 0,), (1, 0, 0,), and (0, −1, −1) for the nine quarks. For the two parameters R and S we find[17, 18]

$$S=+\tfrac{1}{2}, \quad R_\infty=4 . \tag{17}$$

The three triplets t^1, t^2, and t^3 are shown in Fig. 1.

(iii) Three triplets with SU(3)′ × SO(3) *symmetry.* With the gauge group between the $\phi^{(\alpha)}(x)$ fields chosen as SO(3) let us now consider another possible three-triplet scheme under the symmetry group SU(3)′ × SO(3). Such a possibility was first considered by Tati.[7] By assigning a new "spin" of magnitude one to the quarks and assuming that the two-body force is attractive for triplet states and repulsive for singlet and quintet states with respect to this new spin, Tati made the saturation possible only for the $j=\frac{3}{2}$ decouplet and $j=\frac{1}{2}$ octet of baryons.

According to the Ohnuki-Kamefuchi theorem above, this is a rather distinct case in the sense that the equivalence relation is a weak one, i.e., this model or, more precisely, the SO(3) degrees of freedom are not equivalent to the para-Fermi degrees of freedom, even though the converse is true. The class of observables consists of all possible forms $[\psi, \psi]$ and $[\psi^\dagger, \psi^\dagger]$ as well as $[\psi, \psi^\dagger]$. We note, however, that the currents of type $[\psi, \psi]$ and $[\psi^\dagger, \psi^\dagger]$ carry charge and baryon number, and hence would not be allowed to occur in a Hamiltonian unless, for example, they are coupled to certain Bose fields which carry compensating quantum numbers. Otherwise the para-Fermi and colored quarks will be strongly equivalent.

The situation becomes different if we give up indistinguishability. The SO(3) can be characterized by the presence of its generators in the Hamiltonian rather than the currents $[\psi, \psi]$ and $[\psi^\dagger, \psi^\dagger]$. Let us then treat this case in the same spirit as in our original SU(3)″ model. Let $\vec{L}$ be

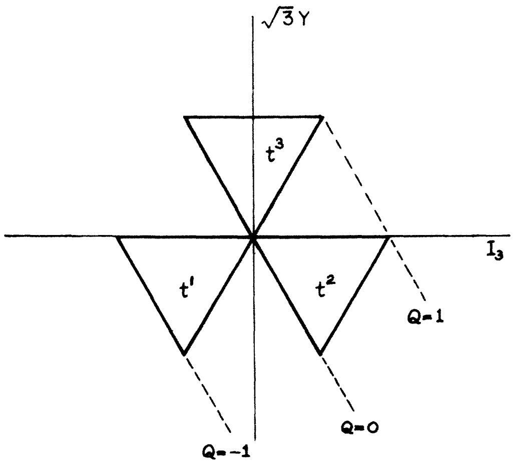

FIG. 1. Quantum numbers of the three triplets in the SU(3)′ × SU(3)″ model.

the three generators of SO(3) acting on the para-Fermi degrees of freedom. The third component L_3 will take eigenvalues 1, 0, −1. Let the electric-charge operator be defined by

$$Q=I_3'+\tfrac{1}{2}Y'+L_3\,, \qquad Y=Y' . \tag{18}$$

In contrast to the SU(3)′ × SU(3)″ triplets, this new variety of triplets have fractional charges; the charges are $(\frac{5}{3}, \frac{2}{3}, \frac{2}{3})$, $(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$, and $(-\frac{1}{3}, -\frac{4}{3}, -\frac{4}{3})$ for t^1, t^2, and t^3, respectively. All known hadrons are assumed to be singlets with respect to the SO(3). The meson and baryon states are given by $t\bar{t}$ and ttt. The SU(3)′ × SO(3) contents of these 81- and 729-plets are

$$\begin{aligned}(3,3)\times(\bar{3},3)&=(8+1,1)+(8+1,3)+(8+1,5)\,,\\ (3,3)\times(3,3)\times(3,3)&\\ &=(1+2(8)+10,1)+3(1+2(8)+10,3)\\ &\quad+2(1+2(8)+10,5)+(1+2(8)+10,7)\,.\end{aligned} \tag{19}$$

For this model, the S and R take on the following values:

$$S=+\tfrac{1}{2} \text{ and } R_\infty=8 . \tag{20}$$

Summarizing, we have

$$Q=(I_3'+\tfrac{1}{2}Y')+\begin{cases}0\\ I_3''+\tfrac{1}{2}Y''\\ L_3\end{cases} \tag{21}$$

and

$$S=\begin{cases}\frac{1}{2},\\ \frac{1}{2},\\ \frac{1}{2},\end{cases} \quad R_\infty=\begin{cases}2,\\ 4,\\ 8,\end{cases} \quad \text{for}\begin{cases}\text{para (color) quarks}\\ \text{three triplets with SU(3)}''\\ \text{three triplets with SO(3)} .\end{cases} \tag{22}$$

At this point it seems appropriate to discuss the experimental situation. As for S, all three models are satisfactory with respect to both magnitude and sign. On the other hand, the experimental value[17] for R appears to be increasing with energy, reaching a value ≃ 5 at the highest available energy of 4 GeV. It may still be increasing. One may question the validity of the supposedly asymptotic formula at these relatively low energies. But if R should remain high at higher energies, distinguishable nine-quark models would be favored over the para-Fermi model. (See Sec. IV for further discussion.)

It is often argued that the ratio of structure functions for electron-neutron and electron-proton inelastic processes also provides a test of various models. Thus this ratio will be $\geq\frac{1}{4}$, $\geq\frac{1}{2}$, $\geq\frac{7}{10}$ for the paraquark, SU(3)″, and SO(3) models, respectively. Experimentally the lower limit seems to be less than $\frac{1}{2}$ but consistent with the paraquark case. However, we should keep in mind that the SU(3)″ and SO(3) results are based on the assumption that the nucleon is a pure singlet. By breaking the symmetry, as we must in any case (see Sec. III), the limit can be lowered. The absolute limit in the SU(3)′ × SU(3)″ scheme is zero. (Another possibility would be that the SLAC energies are still below the color threshold, in which case all models give the same results.[19] See Sec. IV.)

III. BARYON-NUMBER ASSIGNMENT

Let us next discuss the problem of baryon-number assignment for the triplets. In the case of paraquarks or color quarks, the indistinguishability of component fields necessarily requires their baryon numbers to be $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$, and furthermore excludes the case of SO(3) symmetry (weak equivalence), as has already been mentioned. In the case of distinguishable triplets, baryon numbers may be either fractional or integral. If baryon numbers were fractional, the quarks would be stable against decay into hadrons even if charges were integral. Determination of the baryon number of a particle, however, is a much more difficult task than that of charge. (Imagine how one can determine the baryon number of a nucleus in a direct way.) Still, such an assignment goes against one of the original motivations of the three-triplet model. On the other hand, if both charge and baryon number were integral, we would not have difficulty in explaining the nonexistence of free quarks, because, being sufficiently massive, they can decay into hadrons.

We consider here two examples of integral baryon number assignment[1,2,20]:

$$B=(0, 0, 1) \tag{23a}$$

and

$$B=(1, 1, -1) \quad \text{for } (t^1, t^2, t^3) . \tag{23b}$$

Henceforth, we will use the notation t^α, $\bar{t}^\alpha$ to denote the three triplets. Each t consists of three components t_i^α, where i refers to the SU(3)′.[21]

In the case (23a), the quarks with zero baryon number can only decay, if they can decay at all, by emitting an odd number of leptons, which in turn implies that they must carry lepton number.[22] Presumably such an interaction is part of the weak interactions, and the corresponding decay lives will be relatively long (but shorter than those of the strange particles because of larger Q values). At any rate, we should expect a production of at least two leptons per nucleon when a nucleon breaks up into free triplets. In this connection, one may mention a recent proposal by Kalbfleisch and Fowler[23] in which the $B=0$ triplets are actually identified with the electronic and muonic leptons (augmented by new neutrinos to complete two triplets). A difficulty with their model is that the leptons could not be easily contained in a hadron—the same old difficulty in nuclear β decay that was to be overcome by the Fermi theory. Furthermore, they would also have strong interactions just as our triplets do.

The case (23b) looks less drastic, if only slightly. The $B=1$ quarks can decay into baryons, and the $B=-1$ quarks into antibaryons. Such processes can go only through violation of the SU(3)′ × SU(3)″ symmetry. For this purpose, a piece in the Hamiltonian which transforms like $(\bar{3}, 3)$ and $(3, \bar{3})$ of this group will do because the transition is $(3, \bar{3})$ or $(\bar{3}, 3) \to (8, 1)$. A more general alternative is $(3, \bar{3}) \times (3, \bar{3})$ and its conjugate. This latter choice can cause transitions to (10, 1), $(\overline{10}, 1)$ as well.

If the symmetry-breaking Hamiltonian transforms like $(3, \bar{3}) + (\bar{3}, 3)$, it is precisely the Okubo-type $\sim T_3^3$ (octet). With $(3, \bar{3}) \times (3, \bar{3})$ we get also T_{33}^{33} (27-plet). The octet-dominance assumption will, however, lead to the prediction that $t \to 10$ or $\overline{10}$ will be suppressed. The simplest such example is an off-diagonal mass term

$$a(t_1^1 C\gamma_5 t_3^3 + t_2^2 C\gamma_5 t_3^3) + b(t_3^1 C\gamma_5 t_1^3 + t_3^2 C\gamma_5 t_2^3) + \text{H.c.} \tag{24}$$

Here C is the charge conjugation matrix: $tC = \bar{t}^C$. Octet dominance means $a=-b$. With this perturbation, the basic chains of transitions will be

$$\begin{aligned} &t^1 \to t^1 + (t^2 t^3) \to t^1 t^2 t^3 , \\ &t^2 \to t^2 + (t^3 t^1) \to t^2 t^3 t^1 , \\ &t^3 \to (\bar{t}^1 \text{ or } \bar{t}^2) \to \bar{t}^1 \bar{t}^2 \bar{t}^3 . \end{aligned} \tag{25}$$

Thus the decay is of first order for t^1 and t^2, and of second order for t^3, with respect to the symmetry-breaking interaction (24). Of course it must be accompanied by emission of mesons in addition. Equation (24) is constructed in such a way as to be invariant under the diagonal isospin SU(2) of SU(3)′ × SU(3)″. Since baryon number is related to Y'' we cannot have diagonal SU(3). As far as the decay modes are concerned, then, these triplets would look like Λ^*, Σ^*, and Ξ^* resonances in the case of t^1 and t^2, and $\bar{\Lambda}^*$ and $\bar{\Xi}^*$ resonances in the case of t^3. Alternatively, we could take the baryon-number assignment to be (1, −1, 1) or (−1, 1, 1), but the isospin would not commute with baryon number in these cases.[24] In principle there is nothing wrong with it. If, however, Eq. (23b) is adopted, we may also say that the Gell-Mann–Okubo breaking of SU(3) down to SU(2) is a natural concomitant of the baryon-number assignment, and the latter may in fact be the cause of the former, as was suggested before.[1,2]

Another interesting point is that Eq. (24) is an extension of the observables of the type $[\psi, \psi]$ and $[\psi^\dagger, \psi^\dagger]$ in the para-Fermi theory that belong to $SO(3) \simeq SU(2)$. It is therefore not surprising that we can have a diagonal SU(2) but not SU(3).

IV. SYSTEMATICS OF SUPERSTRONG INTERACTIONS

Within the framework of triplets with integral charges and baryon numbers, we will now speculate on the dynamics of superstrong interactions that make the ordinary hadrons possible as bound states of the triplets. Such an attempt was already made by one of us some time ago,[2] so we will first recapitulate the basic results below, during the course of which the merits of the original three-triplet model compared to the other models will become apparent.

As a working hypothesis we assume the superstrong interactions to operate in the SU(3)″ space. For example, they may be mediated by an octet of vector-gluon gauge fields (some of which will carry charge and baryon number). But in order to make things tractable we simplify them drastically, and express the total energy of a system of quarks as the sum of their rest masses and pairwise static interactions, the latter being considered as coordinate-independent. From our experience with the quark and parton models, this may not be as bad an approximation as one might think offhand. At any rate, the energy E is expressed as[2,25]

$$\begin{aligned} E &= N\mu + \tfrac{1}{4} g^2 \sum_{n>m} \sum_{i=1}^{8} \lambda_i^{(n)} \lambda_i^{(m)} \\ &= N\mu_0 + \tfrac{1}{2} g^2 C_2(l_1, l_2), \\ \mu_0 &= \mu - \tfrac{1}{2} g^2 C_2(1,0) \\ &= \mu - \tfrac{2}{3} g^2 . \end{aligned} \tag{26}$$

Here μ is the mass of a free t or $\bar{t}$; N is the total number of t's and $\bar{t}$'s; the λ's are SU(3) matrices in the SU(3)″ space for individual constituents (Gell-Mann's λ_i or $-\lambda_i^*$, depending on whether they refer to $\bar{t}$ or t); g^2 is an effective interaction strength having the dimensions of energy; μ_0 is the bare mass of a quark which also serves as its effective mass inside a hadron; finally, C_2 is the quadratic Casimir operator of SU(3)″ for the entire system, given by the formula

$$C_2(l_1, l_2) = \tfrac{1}{3}(l_1^2 + l_1 l_2 + l_2^2) + (l_1 + l_2) \tag{27}$$

for a representation $D(l_1, l_2)$. Its values are listed in Table I.

A remarkable feature of the formula (26) is that it only depends on N and C_2. For SU(3)″ singlets $D(0,0)$, which we expect to correspond to ordinary hadrons, the total energy is simply equal to the sum of the effective masses of the constituents, whereas nonsinglet SU(3)″ states will have an excitation energy proportional to C_2. The former property is what has been known empirically for a long time. In fact we may set

$$\mu_0 = 0.3 \text{ GeV}$$

since the lowest baryons and mesons (0^- and 1^-) should have average energies equal to $\sim 3\mu_0$ and $2\mu_0$, respectively.

TABLE I. Eigenvalues of $C_2(l_1, l_2)$.

Representation	Dimensionality	C_2
$D(0,0)$	1	0
$D(1,0)$	3	$\frac{4}{3}$
$D(1,1)$	8	3
$D(2,0)$	6	$\frac{10}{3}$
$D(3,0)$	10	6

At this point we will make a digression and ask what will happen if the other models discussed earlier are adopted. First, the para-Fermi or colored-quark model will allow only a neutral SU(3)″ singlet gluon field. The resulting interaction is repulsive between quarks and attractive between quark and antiquark, but it does not have a saturation property, at least in our approximation.

Next take the $SU(3)' \times SO(3)$ version of the three-triplet model. We have an SO(3) not because we allow $[\psi, \psi]$- and $[\psi^\dagger, \psi^\dagger]$-type currents, but because we allow three real gluon fields coupled to $[\psi^{\dagger\alpha}, \psi^\beta]$-type currents. We get a formula similar to Eq. (26), with C_2 being replaced by the one appropriate to SO(3):

$$C_2 = l(l+1), \quad l = 0, 1, \ldots \tag{28}$$

for a representation $D(l)$. This can account for the stability of baryons, as was noted by Tati, but it cannot distinguish between quark-quark and quark-antiquark interactions. For example, tt pairs will be degenerate with $t\bar{t}$ pairs. (The situation is analogous to the π-π interaction through a ρ exchange.) From this point of view the Tati model is inadmissible unless additional mechanisms are invoked.

Let us therefore come back to the SU(3)″ model, and examine the SU(3)″ nonsinglet excitations. The first thing we would like to point out is that the octet $t\bar{t}$ states, which are the first excitations of the ordinary mesons, are already unbound, i.e., unstable against decay into single quarks, by an amount $\frac{1}{6}g^2$. This suggests that in processes such as e^+-e^- annihilation, nonsinglet excitations start taking place around two quark masses. Now the available data indicate that R reaches 4 around $E = 3$–4 GeV. Equating E with 2μ, we thus find

$$\mu = 1.5\text{–}2 \text{ GeV} \quad (g^2 = 1.8\text{–}2.6 \text{ GeV}) , \qquad (29)$$

a rather low value.[26] However, in view of what we have said about the decay characteristics of single quarks, a value of 2 GeV, say, would not necessarily be ruled out. A similar point on exotic hadrons has been made by Freund[27] in a different context. Taking $\mu = 2$ GeV, $\mu_0 = 0.3$ GeV as a tentative choice, we then predict the mass systematics of hadrons as given in Table II.

From Table II we draw the following conclusions:

(1) The lowest SU(3)″ nonsinglet excitations are single quarks, t, $\bar{t}$ (fermions) at 2 GeV, and diquarks (tt), $(\bar{t}\bar{t})$ (bosons) at 2.3 GeV. They can be produced in the e^+-e^- annihilation, for example.

(2) The lowest nonsinglets states in the baryon sector that can be created in hadron-hadron collisions are a (ttt) octet at 4.7 GeV, and a $t + (tt)_3$ (ionization into a quark and a diquark) at 4.3 GeV. According to our assumption (23b), t will have baryon number ± 1, $(tt)_3$ will have 2 or 0. These nonsinglet baryon states are above the present SLAC energies. If this is taken seriously, the present SLAC data on e-p scattering will not be true asymptotic results. An increase in the cross section will be observed as we go above the threshold for nonsinglet excitations.[28]

(3) The "exotic" meson $(tt\bar{t}\bar{t})$ and baryon $(tttt\bar{t})$ configurations begin to appear at 1.2 GeV and 1.5 GeV, respectively. These are SU(3)″ singlets, but contain 27-plets and others in the SU(3)′. So far there is no firm evidence for such exotic states.

TABLE II. Predicted mass levels.

Quark content	SU(3)″ dimensionality	E (GeV)
t, $(\bar{t})$	$\bar{3}$, (3)	2
(tt)	3	2.3
(tt)	$\bar{6}$	4.9
$(t\bar{t})$	8	4.4
$t+t$, $t+\bar{t}$		4
(ttt)	8	4.7
$(tt)_3 + t$		4.3
$t+t+t$		6
$(tt\bar{t}\bar{t})$	1	1.2
$(tt\bar{t}\bar{t})$	8	5.0
$(tttt)$	$\bar{3}$	2.9
$(tttt)$	6	5.5
$(tttt\bar{t})$	1	1.5
$(tttt\bar{t})$	8	5.3
$(ttttt)$	3	3.2
$(ttttt)$	$\bar{6}$	5.7

These figures are based on the simple formula (26). It ignores, for example, the mass splitting among different representations of the diagonal SU(3), the Gell-Mann–Okubo type breaking of SU(3), a dependence on spin and orbital configurations, and possible three-body interactions. The last effect may be represented by adding to Eq. (26) a term proportional to the cubic Casimir operator of SU(3). It goes without saying that these estimates are extremely crude, and should not be taken as more than a qualitative guide.

A few remarks may be in order about charge and baryon number in relation to the superstrong interactions discussed in this section. As far as these interactions are concerned, we are dealing here with the case of *a priori* distinguishable triplets.[29] It is theoretically possible, however, for the electromagnetic (and weak) interactions and baryon number to be incapable of reading the SU(3)″ quantum numbers. Thus the triplets are fractional, and the experimental consequences of Sec. II will be the same as those in the paraquark model. The main problem in this type of model is how to make the quarks unphysical, namely, how not to create single quarks even at very high energies.

On the other hand, the integrally-charged triplets combined with an SU(3)″ octet of elementary vector fields mediating the superstrong interactions cause a new problem in that these latter fields are also charged and will affect the e^+-e^- cross section, the Callan-Gross relation[30] in deep-inelastic scattering, and other phenomena unless they are sufficiently massive so as not to be excited in the present energy range. As a matter of fact, the ratio R will rise linearly with the square of the center-of-mass energy in the case of vector-meson production.[31] It should be interesting to see if R will keep rising above 4,[32] and if there will be a serious deviation from the Callan-Gross relation at higher energies.

V. SUMMARY AND DISCUSSION

In the first few sections we have examined the intrinsic similarities and differences among four versions of the nine-quark model. Each may be characterized by two different criteria: distinguishability of the three triplets and the underlying gauge group [SU(3) or SO(3)]. If the triplets are indistinguishable, the para-Fermi- and color-quark versions are strongly or weakly equivalent with each other depending on whether the group is SU(3) or SO(3), which in turn depends on the nature of admissible observables (Ohnuki-Kamefuchi theorem). If charge- and baryon-number-carrying currents ($\sim[\psi, \psi]$ and $[\psi^\dagger, \psi^\dagger]$) do not occur in the Hamiltonian, the case of SO(3) is excluded, and the para-Fermi and color theories become strongly equivalent.

In the case of distinguishable triplets, the SU(3) and SO(3) versions lead to different natural charge assignments, as well as different superstrong interactions.

Experimental data on e^+-e^- annihilation cross sections tend to favor distinguishable triplet models over indistinguishable ones. When the dynamics of superstrong interactions is taken into account the SU(3) version [$SU(3)'\times SU(3)''$ symmetry] is definitely favored over the SO(3) version [$SU(3)' \times SO(3)$ symmetry] because the former can account for the saturation properties of superstrong interactions.

In later sections we have therefore explored in some detail the consequences of the model based on $SU(3)''$, taking full advantage of the fact that the quarks may have integral charges and baryon numbers, and therefore may freely be produced. The e^+-e^- annihilation data fix the mass of free quarks to be around 2 GeV, whereas the bare mass is 0.3 GeV. A simple mass formula then enables one to predict the gross mass spectrum of hadrons under the superstrong interactions. Carrying integral baryon numbers, these quarks will be able to decay into baryons or antibaryons (or into leptons if zero baryon number is assigned), via a symmetry-breaking interaction. This will make the nine quarks look like ordinary hadrons. In proton-proton collision, the threshold energy for production of single quarks and diquarks will be 5–6 GeV in the center-of-mass frame.

Some of the theoretical problems that have not been discussed in this article are (a) weak interactions, (b) the relationship with duality and the dual resonance model, (c) the relationship with gauge theories, and (d) the possibility of a renormalizable field theory. To cite an example, the dual-quark-model interpretation of strong interactions would have to be modified if single-quark production were allowed.

As for simple tests of our basic assumptions, we would like to remark that the naive interpretation of the parton model, or the concept of quasi-free partons, is more natural in our model than in the case of fractionally charged, unphysical quarks because we do not have to invoke final-state interactions in order to produce hadrons. Thus in deep-inelastic e-p scattering, for example, we should expect to see under proper kinematic conditions jets of nucleons and mesons arising from the decay of the parent constituents which are characterized by a set of quantum numbers and a mass of the order of 2 GeV. According to our assumption about the baryon number, Eq. (23b), there should be roughly comparable amounts of nucleon, antinucleon, K and $\bar{K}$ components, their ratios being calculable from the composition of the proton. Predictions would be, in general, different and more complicated if the quarks were unphysical.[33]

Other tests we have mentioned in earlier sections are such possibilities as color threshold effects in e-p scattering above the SLAC energies, a violation of the Callan-Gross relation, and an increasing R with energy, the latter two being due to charged vector gluons.

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‡Work supported in part by the National Science Foundation under Contract No. NSF GP 333-0177 and in part by the Army Research Office (Durham).

[1]M-Y. Han and Y. Nambu, Phys. Rev. 139, B1006 (1965).

[2]Y. Nambu, in *Preludes in Theoretical Physics*, edited by A. de-Shalit, H. Feshbach, and L. Van Hove (North-Holland, Amsterdam, 1966), p. 133.

[3]We will generically refer to as "nine-quark" models all those models consisting of either nine fundamental constituents or three of them, each with further three degrees of freedom.

[4]O. W. Greenberg, Phys. Rev. Lett. 13, 598 (1964). As mentioned in footnote 15 of this paper, the construction of meson states in terms of paraquarks works for all paraorders, including $p = 1$, the Fermi case. For an arbitrary paraorder p (as far as mesons are concerned), the $\pi^0 \rightarrow 2\gamma$ decay parameter S would have the theoretical value given by $\frac{1}{6}P$. It is amusing to note that $S_{\exp} = \frac{1}{2}$ gives an independent determination for the value $p = 3$.

[5]A. Tavkhelidze, in *Seminar on High Energy Physics and Elementary Particles* (International Atomic Energy Agency, Vienna, Austria, 1965) p. 763. See also P. G. O. Freund, Phys. Lett. 15, 352 (1965).

[6]Y. Miyamoto, Prog. Theor. Phys. Suppl., extra number, 187 (1965).

[7]T. Tati, Prog. Theor. Phys. 35, 126 (1966); 35, 973 (1966).

[8]M. Gell-Mann, Acta Phys. Austriaca Suppl. 9, 733 (1972); W. A. Bardeen, H. Fritzsch, and M. Gell-Mann, in *Scale and Conformal Symmetry in Hadron Physics*, edited by R. Gatto (Wiley, New York, 1973), p. 139; M. Gell-Mann, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 4, p. 333. At present, however, the word "colored quark model" seems to be used by many people, including Gell-Mann

[H. Fritzsch, M. Gell-Mann, and H. Leutwyler, Phys. Lett. 47B, 365 (1973)], to mean a special case of the three-triplet model in which the U(3)″ symmetry is strictly observed by all (strong, electromagnetic, and weak) interactions. Color is then another name for the U(3)″ quantum numbers. An equivalence in this sense between paraquark (after Klein's transformation) and three-triplet models was pointed out as early as 1966 by O. W. Greenberg and D. Zwanziger [Phys. Rev. 150, 1177 (1966), see p. 1180].

[9]Y. Ohnuki and S. Kamefuchi, Prog. Theor. Phys. 50, 258 (1973).

[10]W. A. Bardeen, H. Fritzsch, and M. Gell-Mann, Ref. 8.

[11]Y. Ohnuki and S. Kamefuchi, Phys. Rev. 170, 1279 (1968); Ann. Phys. (N.Y.) 51, 337 (1969); 57, 543 (1970); 65, 19 (1971).

[12]K. Drühl, R. Haag, and J. E. Roberts, Commun. Math. Phys. 18, 204 (1970). These authors have shown that if the observables are restricted to functionals of $[\psi, \psi^\dagger]$ only, then the PF and F fields are equivalent, using, however, the locality condition (7) rather than (8).

[13]In this connection, several authors have investigated the properties of representations of SO(2N + 1), which is isomorphic to the algebra of N para-Fermi fields (or particles), other than the one corresponding to the ordinary para-Fermi representations with a unique vacuum. They are led to a situation of degenerate vacua with U(N) symmetry. See, for example, T. Palev, CERN Report No. TH-1653, 1973 (unpublished); A. J. Bracken and H. S. Green, J. Math. Phys. 14, 1784 (1973); A. B. Govorkov, Zh. Eksp. Teor. Fiz. 54, 1785 (1968) [Sov. Phys.—JETP 27, 960 (1968)]. We do not concern ourselves with this aspect.

[14]Here we discuss these two models together since they both deal with quarks which are indistinguishable, in contradistinction to distinguishable triplet models below. We do not imply that these two models are identical to each other in all their physical aspects. See Ref. 29 for a discussion on the meaning of distinguishability.

[15]To the extent that the currents are of the form $[\psi, \psi^\dagger]$, one may be tempted to say that SU(3) × SU(3) might be more appropriate. On the other hand, unless there is some distinction between "color" and "anticolor," one may be led to considering SU(3) × SO(3) to be more suiting.

[16]This corresponds to the SUB version of the three triplets as proposed by N. Cabibbo, L. Maiani, and G. Preparata, Phys. Lett. 25B, 132 (1967). For a discussion of differences between the SUB version and the original version of Ref. 1, see J. C. Pati and C. H. Woo, Phys. Rev. D 3, 1173 (1971).

[17]The latest published experimental value is reported to be $R = 4.7 \pm 1.1$, at a center-of-mass energy of 4 GeV by A. Litke *et al.*, Phys. Rev. Lett. 30, 1189 (1973).

[18]In addition to the difference in the value for R between the three triplets (R = 4) and para (color) quarks (R = 2), H. Suura *et al*,[Nuovo Cimento Lett. 4, 505 (1972)] show that the two-photon amplitudes also give different values: T = 4 for our triplets (T = 2 in the case of no-charm-particle production) and $T = \frac{2}{3}$ for the para (color) quarks where $T = \sigma_{2\gamma}(e + e \to e + e + \text{hadrons})/\sigma_{2\gamma}(e + e \to e + e + \mu^+ + \mu^-)$.

[19]H. Lipkin, Phys. Rev. Lett. 28, 63 (1972).

[20]Y. Nambu, in *Proceedings of the Second Coral Gables Conference on Symmetry Principles at High Energies, University of Miami, 1965,* edited by B. Kurşunoğlu, A. Perlmutter, and I. Sakmar (W. H. Freeman, San Francisco, 1965), p. 275.

[21]In terms of the generators of U(3)″, Eq. (23) reads (a) $B = \frac{1}{3}A'' + Y''$, (b) $B = \frac{1}{3}A'' - 2Y''$, where A'' is the U(1)″ quantum number ($\equiv \pm 1$ for t and $\bar{t}$, respectively). In contrast, the para- and color-quark models assign $B = \frac{1}{3}A''$.

[22]For example, we could assign a lepton number $L = (1, -1, 0)$ $[= -2I_3'']$ in conjunction with Eq. (23a). The ordinary baryon then will have no lepton number. The leptonlike members t^1, t^2 could decay through a virtual process t^1, $t^2 \to t^3 + \bar{t}^3 + \text{lepton}$; $t^3(\bar{t}^3) \to$ baryon (antibaryon). The possibility of a quark decaying into leptonic channels has been recently discussed by J. C. Pati and A. Salam [Phys. Rev. Lett. 31, 661 (1973)] in connection with the possible baryon- and lepton-number nonconservation arising out of their model for a unified gauge theory of strong, electromagnetic, and weak interactions. We are, however, not contemplating such nonconservation. Unified gauge theories go beyond the scope of this article.

[23]G. R. Kalbfleisch and E. C. Fowler, Nuovo Cimento 19A, 173 (1974).

[24]We may take the attitude that the only requirement for the baryon number B is that it commutes with charge. Since B depends only on the U(3)″ generators, it is possible that B is diagonal with respect to a rotated basis: $(t^2, t^3) \to (t^{2\prime}, t^{3\prime})$, as in Cabibbo theory. However, such a rotation would be physically meaningful only in relation to other interactions and symmetry breakings.

[25]This has recently been discussed by H. J. Lipkin, Phys. Lett. 45B, 267 (1973). He considers only the interaction energy. This amounts to setting $\mu = 0$.

[26]Of course we have to keep in mind that Eq. (14) is only an asymptotic formula. Actually the lowest-order perturbation theory gives (for spin-$\frac{1}{2}$ particles)

$$R = R_\infty(1 + 2\mu^2/E^2)(1 - 4\mu^2/E^2)^{1/2} .$$

This rises from zero, at the threshold, quickly and monotonically toward R_∞. For example, $R = 0.75\, R_\infty$ at $E = 1.2 \times 2\mu$.

[27]P. G. O. Freund, talk given at the Purdue Conference, 1973 (unpublished); Nuovo Cimento 8A, 525 (1972); C. E. Carlson and P. G. O. Freund, Phys. Lett. 39B, 349 (1972).

[28]J. D. Bjorken, SLAC Report No. 1318, 1973 (unpublished).

[29]There is a subtle ambiguity in the meaning of distinguishability. If the quarks are coupled to an octet of gluons and the SU(3)″ is an exact symmetry observed by all interactions, can we call quarks of different color distinguishable? We are inclined to say yes because an octet of currents serve as a source for the gluon fields and therefore may be considered as observables. But are the gluon fields observable? Perhaps a better criterion for distinguishability is obtained by asking, for example, whether the two states $q^{\dagger\alpha}(x)q^{\dagger\beta}(y)|0\rangle$ and $q^{\dagger\alpha}(y)q^{\dagger\beta}(x)|0\rangle$ [or equivalently $(q^{\dagger\alpha}(x)q^{\dagger\beta}(y) \pm q^{\dagger\alpha}(y)q^{\dagger\beta}(x))|0\rangle$], $\alpha \neq \beta$, $x \neq y$, exist and

are distinct. This would be the case if states having different SU(3)″ symmetries (singlets, triplets, etc.) are allowed to occur with finite energy. In this sense the octet-gluon model belongs *a priori* to the distinguishable case, but it will not if all nonsinglet states are somehow forbidden, a situation speculated or desired by some people.

[30]C. G. Callan and D. Gross, Phys. Rev. Lett. 22, 156 (1969).

[31]N. Cabibbo and R. Gatto, Phys. Rev. 124, 1577 (1961). This result may not asymptotically hold in a gauge theory which incorporates electromagnetism.

[32]A recent report from SLAC [B. Richter, in proceedings of the American Physical Society Meeting, Chicago, 1974 (unpublished)] strengthens the suspicion that R may indeed be rising with energy. If this is confirmed, our estimate of μ in Sec. IV will become less meaningful.

[33]See for example, G. R. Farrar and J. L. Rosner, Phys. Rev. D 7, 2747 (1973).

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Electron-electron scattering: Spectral forms for the invariant amplitudes to order e^4*

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The five invariant amplitudes for electron-electron scattering to order e^4 are calculated using the causal methods of source theory. The basis set employed, which is free of kinematic singularities and zeros, consists of the Fermi invariants. The amplitudes are expressed in terms of double-spectral forms with accompanying single-spectral forms which are determined by analysis of causal forward scattering. The radiative corrections required for the analysis of polarization experiments are contained in these amplitudes.

I. INTRODUCTION

Quite a number of fourth-order processes have been calculated in quantum electrodynamics, including the magnetic moment of the electron,[1] photon-photon scattering,[2] and Compton scattering.[3,4] And some aspects of fermion-fermion scattering[5,6] have been considered, among which is an investigation of the hard-photon corrections[7] to electron-electron scattering. Except for the unpolarized differential cross section,[8] the corresponding polarization calculations for electron-electron (positron) scattering have not been previously done. Theoretically, it is of interest to know the invariant amplitudes and to have them available for use in higher-order processes. Besides, more can be learned of the role played by dynamics in the choice of a basis for the invariant amplitudes. Experimentally, with the advent of colliding beams, a more detailed investigation of electron-electron scattering is possible. In particular, the asymmetry parameters[9] derived from the invariant amplitudes can be compared with experiment. Also, the determination of production cross sections for hadronic states requires an accurate knowledge of the purely electrodynamic processes.

Electron-positron scattering has been considered by McEnnan[10] within a context that attempts to move beyond perturbation theory, to include the effects of bound states and to eliminate the dependence on an artificial photon mass. Barbieri *et al.*[11] have used dispersive techniques for the "box diagram" in their recent calculation of the magnetic moment and charge radius of the electron. However, explicit expressions for the invariant amplitudes or helicity cross sections have not been presented.

The purpose of this paper is to calculate the electron-electron scattering to order e^4 and to present the invariant amplitudes in spectral form. An appropriate choice of a complete set of spinor basis for these amplitudes is considered in Sec. II. The general approach that we will use is the causal methodology of source theory.[12] We start the calculation in Sec. III by considering four virtual electron sources that are causally related (see Fig. 1). The removal of the causal restrictions (space-time generalization) and the application to free external particles (mass extrapolation) yields the invariant amplitudes in double-spectral form. To this must be added possible contact terms which are themselves single-spectral forms. The latter are determined by comparison with the causal forward scattering amplitudes for the photon-