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Conditions for an $SU_L(3) \otimes U(1)$ gauge theory*†

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We study the possibility of constructing an $SU_L(3) \otimes U(1)$ gauge theory of weak and electromagnetic interactions consistent with experiment and allowing room for the incorporation of renormalizable strong interactions. Greater than usual attention is paid to the stability of the Higgs potential and serious difficulties are encountered. We argue that a satisfactory $SU(3)_L \otimes U(1)$ model based on elementary Higgs fields requires a global or discrete symmetry beyond SU(3) in the Higgs sector.

I. INTRODUCTION

Within the past few years the possibility of constructing a model of weak and electromagnetic interactions based on gauge field theories has been much discussed.[1] Because of hadron spectra and quark statistics it is widely accepted that the elementary hadronic constituents (quarks) appearing in these models have, in addition to the familiar SU(3), a "color" or SU(3)′ degree of freedom. In order to avoid strangeness-changing neutral currents one has in general either resorted to weak currents that are not singlets under the SU(3)′ or postulated the existence of extra "charmed" quarks. The first alternative seems to lead to insuperable difficulties[2] if one ultimately intends to use the color degree of freedom to construct a renormalizable theory of strong interactions. The second alternative, as exemplified in the 4-quartet[3] model, seems more reasonable but requires the eventual discovery of charmed hadrons. If these are not discovered in the near future it becomes interesting to ask whether a renormalizable scheme with room for strong interactions and consistent with the experimental constraints on $\Delta S = 2$ processes and neutral currents can be assembled without resort to charmed quarks. To this end we have investigated $SU_L(3) \otimes U(1)$ gauge theories of weak and electromagnetic interactions in which the basic constituents of matter are the usual three Han-Nambu triplets. Previous theoretical work[4] on this gauge group has avoided contradicting experiment only by postulating ultraheavy vector bosons ($m \approx 10^4$ GeV), which we feel are unsatisfactory,[5] and has not paid adequate attention to the stability problem of the Higgs potential.

The $SU_L(3) \otimes U(1)$ gauge symmetry in principle offers a possibility of explaining the Cabibbo angle, in contrast to $SU(2) \otimes U(1)$ models in which the Cabibbo rotation is put in by hand. This possibility arises from the existence of more than one charged intermediate boson in the $SU(3) \otimes U(1)$ model. The Cabibbo angle is related to the mixing angle and mass eigenvalues of the ρ- and K^*-type gauge bosons in a way that depends only on ratios of vacuum expectation values of Higgs fields. It is possible that with the proper choice of Higgs multiplet(s) these ratios would be predicted, although we have found no such suitable choice.

In Sec. II of this paper we discuss the conditions on the gauge-boson mass matrix that will satisfy all the experimental constraints, and in Sec. III we address ourselves to the problem of constructing a renormalizable Higgs potential that will provide the specified mass matrix. None of the previous papers on $SU(3) \otimes U(1)$ have constructed an adequate potential, for reasons discussed in Sec. III. We also have failed to discover a suitable potential, and we present a theorem stating that a successful implementation of an $SU(3) \otimes U(1)$ gauge model requires global or discrete symmetries beyond SU(3) among the elementary Higgs particles. The possibility exists, of course, that the real Higgs scalars are composite particles instead of elementary fields appearing in a Lagrangian. In this case the obstacles we encounter may not arise and $SU_L(3) \otimes U(1)$ might become a very attractive model for weak and electromagnetic interactions.

II. PROPERTIES OF THE GAUGE-BOSON MASS MATRIX

In one-to-one correspondence with the generators of $SU_L(3) \otimes U(1)$ we naturally postulate a singlet and an octet of gauge bosons W^0 and W^α ($\alpha = 1, \ldots, 8$) whose couplings to the quarks are given by

$$\mathcal{L}_{\text{int}} = g L_\mu^\alpha W_\mu^\alpha + g' Y_\mu^0 W_\mu^0 \; ; \tag{1}$$

g and g' are the gauge coupling constants, and in terms of the elementary quarks

$$L_\mu^\alpha = \sum_{\gamma=1}^{3} \bar{\psi}^\gamma \gamma_\mu (1+\gamma_5) \tfrac{1}{2} \Lambda^\alpha \psi^\gamma \; . \tag{2}$$

The SU(3)′ (color) index γ is summed over to make each current a color singlet. Y_μ^0 is the U(1) current, also a color singlet, whose exact structure need not be specified here but is chosen to secure the correct charge couplings of the photon in the usual way. Given that there are three triplets of quarks each transforming according to the $\underline{3}$ representation of $SU_L(3)$, the absence of Adler anomalies requires that there be an additional three (anti)triplets of fermions each transforming according to the $\underline{3}^*$ representation. We identify these as the leptons and thus arrive at the quark and lepton multiplets of Table I. The lepton couplings to the gauge bosons are as in Eqs. (1) and (2), except that the generators Λ^i for the $\underline{3}^*$ representation are given by

$$\Lambda^{\alpha}(\underline{3}^*) = -\lambda^{\alpha *} = -\lambda^{\alpha T} , \tag{3a}$$

whereas

$$\Lambda^{\alpha}(\underline{3}) = \lambda^{\alpha}, \tag{3b}$$

λ^{α} being the usual Gell-Mann matrices for SU(3). The E^0, M^0, l^-, l^0, and L^0 are as-yet-undiscovered leptons presumed to be very massive.

The W^6 and W^7 mediate strangeness-changing neutral currents; however, these currents L_μ^6 and L_μ^7 always couple to heavy leptons. Hence, there are no strangeness-changing neutral currents observable at low energies. The E^0 and M^0 would, however, eventually be produced in high-energy neutrino reactions by neutral strangeness-changing currents. They can also be produced weakly by high-energy electron and muon beams via the charged intermediate bosons coupling to L_μ^4 and L_μ^5. The E^0 and M^0, once produced, should decay weakly into a strangeness-1 hadronic state and a neutrino or charged lepton. If the $SU(3) \otimes U(1)$ is spontaneously broken down to the electromagnetic U(1) gauge symmetry, all of the nine intermediate bosons will become massive except for one (the photon) coupling to the charge current. In Sec. III we will investigate various Higgs potentials to accomplish this and we search in addition for a mixing between the ρ- and K^*-type gauge bosons so that Eq. (1) rewritten in terms of the gauge boson mass eigenstates becomes

TABLE I. Quark and lepton triplets required for an anomaly-free $SU(3) \otimes U(1)$ gauge theory.

Quarks			Leptons		
$\mathcal{P}_1$	$\mathcal{P}_2$	$\mathcal{P}_3$	e^-	μ^-	l^-
$\mathfrak{N}_1$	$\mathfrak{N}_2$	$\mathfrak{N}_3$	ν_e	ν_μ	l^0
λ_1	λ_2	λ_3	E^0	M^0	L^0

$$\begin{aligned}\mathcal{L}_{\rm int} = {} & \frac{g}{\sqrt{2}} (L_\mu^{1+i2}\cos\gamma + L_\mu^{4+i5}\sin\gamma) W_\mu^- + \text{H.c.} \\ & + \frac{g}{\sqrt{2}} (L_\mu^{4+i5}\cos\gamma - L_\mu^{1+i2}\sin\gamma) V_\mu^- + \text{H.c.} \\ & + g(L_\mu^6 W_\mu^6 + L_\mu^7 W_\mu^7) + g j_\mu^{(Z)} Z_\mu \\ & + g j_\mu^{(B)} B_\mu + e V_\mu^{\rm em} A_\mu ,\end{aligned} \tag{4}$$

where

$$W_\mu^{\pm} = \frac{1}{\sqrt{2}} (W_\mu^{1\pm i2}\cos\gamma + W_\mu^{4\pm i5}\sin\gamma) , \tag{5a}$$

$$V_\mu^{\pm} = \frac{1}{\sqrt{2}} (W_\mu^{4\pm i5}\cos\gamma - W_\mu^{1\pm i2}\sin\gamma) . \tag{5b}$$

The photon is

$$A_\mu = \left(W_\mu^3 + \frac{1}{\sqrt{3}} W_\mu^8 - \lambda W_\mu^0 \right) \left(\tfrac{4}{3} + \lambda^2\right)^{-1/2} , \tag{5c}$$

and B_μ, Z_μ are mixtures of W_μ^3, W_μ^8, and W_μ^0 orthogonal to the photon; $j_\mu^{(B)}$ and $j_\mu^{(Z)}$ are the corresponding strangeness-conserving currents.

To second order in $\mathcal{L}_{\rm int}$ the effective β-decay interaction is then

$$\begin{aligned}\mathcal{L}_{\rm eff} &= g^2 [\bar{e}\gamma_\mu(1+\gamma_5)\nu_e] \left[\frac{L_\mu^{1+i2}\cos\gamma + L_\mu^{4+i5}\sin\gamma}{\sqrt{2}} \frac{1}{m_W^2} \frac{\cos\gamma}{\sqrt{2}} + \frac{L_\mu^{4+i5}\cos\gamma - L_\mu^{1+i2}\sin\gamma}{\sqrt{2}} \frac{1}{m_V^2} \frac{(-\sin\gamma)}{\sqrt{2}} \right] \\ &\equiv \frac{G_F}{\sqrt{2}} (L_\mu^{1+i2}\cos\theta_C + L_\mu^{4+i5}\sin\theta_C) \bar{e}\gamma_\mu(1+\gamma_5)\nu_e .\end{aligned} \tag{6}$$

Thus, the Fermi constant and the Cabibbo angle are given by

$$G_F \frac{\cos\theta_C}{\sqrt{2}} = \frac{g^2}{2} \left(\frac{\cos^2\gamma}{m_W^2} + \frac{\sin^2\gamma}{m_V^2} \right) \tag{7a}$$

and

$$G_F \frac{\sin\theta_C}{\sqrt{2}} = \frac{g^2}{2} \sin\gamma \cos\gamma \left(\frac{1}{m_W^2} - \frac{1}{m_V^2} \right) , \tag{7b}$$

where G_F is the phenomenological Fermi constant and θ_C is the Cabibbo angle. The mixing angle γ and all the vector-boson masses depend, of course, on the representations chosen for the Higgs bosons.

The interaction $g(L_\mu^6 W_\mu^6 + L_\mu^7 W_\mu^7)$ could in general cause $\Delta S = 2$ nonleptonic reactions of order G_F through the effective one-boson exchange interaction

$$\mathcal{L}_{\rm eff} = g^2 \left(L_\mu^6 \frac{1}{m_6^2 + q^2} L_\mu^6 + L_\mu^7 \frac{1}{m_7^2 + q^2} L_\mu^7 \right) . \tag{8}$$

We avoid this disaster by requiring $m_6 = m_7$, whereupon the effective interaction becomes

$$\mathcal{L}_{\text{eff}} = \frac{g^2}{(m_6^{\,2}+q^2)}\,(L_\mu^6 L_\mu^6 + L_\mu^7 L_\mu^7)\ . \tag{9}$$

This combination is purely $\Delta S=0$. Note that we do not have to appeal to ultrahigh masses to suppress $\Delta S=2$ in second order. However, we must ensure that the relation $m_6=m_7$ is not a result of precisely fixing the parameters in the Higgs potential or the minimum point but is maintained over a finite range of those parameters. Otherwise, renormalization corrections to the Higgs parameters would unbalance the relation, resulting in $\Delta S=2$ transitions of order $G_F\alpha$, contrary to experiment.

In this connection we should mention another possible problem in the $SU_L(3)\otimes U(1)$ gauge model. Lee, Primack, and Treiman[6] have estimated the $\Delta S=2$ contribution from the fourth-order graph in $\mathcal{L}_{\text{int}}$ in which two charged bosons are exchanged. Although this contribution is difficult to calculate reliably, they estimate its order of magnitude to be $G_F\alpha$ in models with no charmed quarks. If the $SU(3)\otimes U(1)$ gauge theory is otherwise satisfactory our attitude toward this problem would be that the form factors of the observed hadrons might make these loop graphs so rapidly convergent that they are actually of order $G_F(\alpha/m_W^{\,2})$, i.e., $G_F^{\,2}$ consistent with experiment. For example, in the related process $K_L\to\mu^+\mu^-$ via double charged boson exchange one can use the soft-kaon approximation and the Weinberg spectral-function sum rules to argue that the graph in question is of order $G_F^{\,2}$ instead of the $G_F\alpha$ of the earlier estimates. With additional assumptions involving vacuum saturation of the 4-current matrix element between kaon states, the $K_0,\overline{K}_0$ mixing can also be estimated to be of order $G_F^{\,2}$.

From Eqs. (7a) and (7b) we see that the Cabibbo angle is given by

$$\tan\theta_C = \left|\frac{\sin\gamma\cos\gamma(m_V^{\,2}-m_W^{\,2})}{m_V^{\,2}\cos^2\gamma + m_W^{\,2}\sin^2\gamma}\right|\ . \tag{10}$$

The trigonometric functions of γ as well as the gauge-boson mass ratio $m_W^{\,2}/m_V^{\,2}$ will ultimately be functions of ratios only of the Higgs vacuum expectation values.

To summarize this section, we feel that $SU_L(3)\otimes U(1)$ is a workable gauge group for unified weak and electromagnetic interactions provided that

(i) W_6 and W_7 are physical (diagonalized) gauge bosons and $m_6^{\,2}=m_7^{\,2}$ over a finite range of Higgs potential parameters and vacuum expectation values,

(ii) the physical charged vector mesons are mixtures of $W^{1\pm i2}$ and $W^{4\pm i5}$ and have unequal masses, and

(iii) the model should have only one photon (massless particle) in the combination $A_\mu \sim W_\mu^3 + (1/\sqrt{3})\,W_\mu^8 + \lambda W_\mu^0$.

To have Higgs scalars which after spontaneous symmetry breaking give rise to the above desired features is a totally nontrivial problem.

III. THE HIGGS POTENTIAL

In this section we would like to study the general properties of the Higgs potential needed to satisfy the requirements listed in Sec. II. It is important to note that although one is completely free to choose which representations the Higgs scalars belong to, once they have been chosen one must write the most general (quartic) gauge-invariant potential in order that the theory be renormalizable. Otherwise there will in general be infinities arising in the perturbation expansion that cannot be absorbed into coupling-constant renormalizations. An exception to this rule requiring the most general potential is the case in which one can impose an additional discrete or global symmetry on the total Lagrangian. Such an additional symmetry will be respected by the renormalization program and maintained to all orders.

Similarly, it is essential as mentioned before that the gauge-boson mass requirements eliminating $\Delta S=2$ in second order not be the result of precise relations among the Higgs vacuum expectation values or potential parameters. Such relations will in general be broken in renormalization. Both of the above criteria are overlooked in the most recent paper of Schechter and Singer,[5] which is otherwise somewhat similar to our work. We have searched through all possible combinations of low-lying Higgs representations (i.e., triplets, sextets, octets) and have failed to find a combination capable of guaranteeing criteria (i) and (ii) of Sec. II simultaneously. Rather than explain one by one why each possibility tried by us (or by other authors) fails, we will present below our current group-theoretical understanding of the obstacles. We hope that our analysis will focus attention on the root of the problems involved.

To begin, we note the obvious fact that if the Higgs potential, $V(\phi)$, is invariant under transformations of some group $\overline{G}$, every point in the representation space moves along an equipotential subspace under the action of $\overline{G}$. This can be stated in a slightly stronger theorem:

Theorem 1. If $\overline{G}$ is the *largest* symmetry of the potential (including global symmetries and discrete symmetries) and if λ is a point in the Higgs representation space that minimizes the potential, then any other point λ' is also a minimum point if and only if λ and λ' are related by a transformation in $\overline{G}$. It is clear that if $\lambda'=\overline{G}\lambda$, then $V(\lambda)=V(\lambda')$.

Furthermore, if $\overline{G}$ is the *largest* symmetry, and $\lambda' \neq \overline{G}\lambda$, then $V(\lambda')$ equals $V(\lambda)$ at most for isolated values of λ and λ'. Such an accidental equality, $V(\lambda') = V(\lambda)$, depends on precise relations among the parameters in the Higgs potential, which, as discussed previously, will not be maintained in renormalization. In the context of gauge theories, theorem 1 holds after one dismisses the possibility of accidental equality.

Ignoring for the present the singlet gauge boson, which does not affect our conclusions, we would like to propose the following crucial theorem:

Theorem 2. If the highest symmetry of the potential is SU(3) the gauge boson mass (squared) matrix is diagonalized by an SU(3) transformation.

This theorem is important because if as in the present model we have eight gauge bosons the mass eigenstates could in general be related to the original eight by any unitary 8-by-8 matrix, i.e., by any element of the group U(8). Theorem 2 states that the possible mixing among gauge bosons is severely restricted by the symmetry of the Higgs potential. Before discussing the present consequences of this theorem, let us examine the basis for its truth.

Let us examine the SU(3) gauge-invariant kinetic term in the Higgs Lagrangian

$$\begin{aligned}\mathcal{L}_{H,\,\mathrm{kin}} &= [(\partial_\mu \delta_{jk} - ig\theta^\alpha_{jk} W^\alpha_\mu)\phi_k]^\dagger \\ &\quad\times [(\partial_\mu \delta_{jm} - ig\theta^\beta_{jm} W^\beta_\mu)\phi_m]\ ,\end{aligned} \tag{11}$$

θ^α being a representation of the SU(3) generators in the Higgs representation space, and ϕ_k being the Higgs field. Under a local SU(3) transformation

$$\begin{aligned}\phi_m \to \phi'_m &= T(x)_{mh}\phi_h \\ &= (e^{i\overline{\alpha}(x)\cdot\overline{\theta}})_{mh}\phi_h\ ,\end{aligned} \tag{12}$$

$$W^\beta_\mu \to W'^\beta_\mu = S^{\beta\gamma}(x)W^\gamma_\mu + \frac{1}{g}\partial_\mu\alpha^\beta(x)\ , \tag{13}$$

and local gauge invariance is ensured by the relation

$$(\theta^\beta)_{jm}S^{\beta\gamma}(x) = [T(x)\theta^\gamma T^{-1}(x)]_{jm}\ . \tag{14}$$

S is a real, unitary matrix belonging to the adjoint representation of SU(3). In the spontaneous symmetry breakdown the fields ϕ_m are assigned vacuum expectation values $\langle\phi_m\rangle$ consistent with the potential minimum and, from Eq. (11), the W bosons acquire the mass matrix

$$(\mu^2)_{\alpha\beta} = g^2\langle\phi^{k\dagger}\rangle\{\theta^\alpha, \theta^\beta\}_{km}\langle\phi^m\rangle\ . \tag{15}$$

After symmetry breaking, the other minimum points related to $\langle\phi^k\rangle$ by SU(3) are no longer physically equivalent.[7] Let us suppose, however, that we had chosen other vacuum expectation values $\langle\phi^k_0\rangle$ also consistent with the potential minimum and therefore related to $\langle\phi^k\rangle$ by an SU(3) transformation. Let us further suppose that the mass matrix induced by the choice $\langle\phi^k_0\rangle$ is diagonal:

$$\begin{aligned}(\mu_0{}^2)_{\alpha\beta} &= (\mu_0{}^2)_\alpha\delta_{\alpha\beta} \\ &= \langle\phi^{k\dagger}_0\rangle\{\theta^\alpha, \theta^\beta\}_{km}\langle\phi^m_0\rangle\ .\end{aligned} \tag{16}$$

If as we have assumed $\langle\phi^m_0\rangle = T_{mh}\langle\phi^m\rangle$, since $T^\dagger\theta^\alpha T = \theta^\gamma(S^{-1})^{\gamma\alpha}$,

$$\begin{aligned}(\mu_0{}^2)_{\alpha\beta} &= \langle\phi^{k\dagger}\rangle\{\theta^\gamma, \theta^\lambda\}_{km}\langle\phi^m\rangle(S^{-1})^{\gamma\alpha}(S^{-1})^{\lambda\beta} \\ &= S^{\alpha\gamma}(\mu^2)_{\gamma\lambda}(S^{-1})^{\lambda\beta}\ .\end{aligned}$$

Thus the gauge-boson mass matrix is diagonalized by the SU(3) transformation S. Theorem 2 is thus easily proven, subject only to the assumption that there exists a minimum point $\langle\phi_0\rangle$ for which the gauge-boson mass matrix is diagonal. For all the representations we have studied this condition is met, but it remains a possible loophole through theorem 2.

The relevance of theorem 2 to $SU(3)\otimes U(1)$ gauge theories is as follows. Any SU(3) transformation that mixes the ρ^+ and K^{*+} members of the gauge boson octet will necessarily mix the ρ^0, ω^0, and K^{*0} members also. The transformation that accomplishes criteria (i) and (ii) (mixing ρ^+ and K^{*+} but leaving K^{*0}_L and K^{*0}_S unmixed) is therefore not in SU(3). The addition of the U(1), of course, does not affect the K^* mixings at issue here. Thus, if we wish our diagonalized gauge-boson mass matrix to satisfy criteria (i) and (ii), we must have a larger symmetry than SU(3) (global or discrete) in the Higgs potential and this larger symmetry must be preserved at least to first order in the renormalization.

The natural suggestion at this point would be to assume that the Higgs mesons fall into a single SU(3) octet. The most general quartic potential invariant under SU(3) is then also invariant under O(8), and we then have a suitable pseudosymmetry. Unfortunately, the octet Higgs representation does not constitute a realistic model because of charge constraints on the fields. The integrally charged lepton triplets require the octet part of the photon to be mixed with the singlet W^0_μ, which cannot be accomplished with only octet Higgs representations.

We have also tried unsuccessfully to implement criteria (i) and (ii) by imposing various discrete symmetries on the Higgs potential. It is our opinion therefore that theorem 2 constitutes a serious obstacle to the construction of a satisfactory $SU(3)\otimes U(1)$ gauge model.

The possibility always exists, as mentioned in Sec. I, that the spontaneous symmetry breaking must be implemented in the way of Nambu and Jona-Lasinio by composite scalars rather than by elementary Higgs bosons. The theorems of this section then dissolve in our sea of ignorance about composite Higgs particles. This point of view, however, does not seem to be a practical attitude towards the problems we have encountered.

IV. CONCLUSION

In this paper we have discussed the general features of SU(3)⊗ U(1) gauge models satisfying experimental constraints. We have avoided the tendency of other workers in this gauge group to postulate ultraheavy vector mesons and we have studied in detail the Higgs sector. We have found no combination of low-lying Higgs representations capable of producing the required gauge-boson mass matrix and we have summarized the difficulty in two group-theoretical theorems.

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[1]See for example B. W. Lee, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 2, p. 273.

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[4]J. Schechter and Y. Ueda, Phys. Rev. D 8, 484 (1973); G. G. Ross, Rutherford report, 1973 (unpublished).

[5]However, within the last few weeks we have received two reports on SU(3)⊗U(1) models which avoid these ultraheavy gauge particles in a way similar to our treatment. These are J. Schechter and M. Singer, Phys. Rev. D 9, 1769 (1974) and V. Gupta and H. S. Mani, Phys. Rev. D (to be published). Both of these papers overlook difficulties in the Higgs sector as discussed in the present work.

[6]B. W. Lee, J. Primack, and S. Treiman, Phys. Rev. D 7, 510 (1973).

[7]We assume that the quarks are given differing masses by the spontaneous breakdown so that the transformation from $\langle\phi^k\rangle$ to $\langle\phi_0^k\rangle$ cannot be "rotated away" by redefining quark fields.