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Calculability and naturalness in gauge theories*

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Calculability conditions are discussed for local gauge theories with Higgs-type symmetry breaking. We focus on the naturalness of μe universality, the naturalness of the Cabibbo angle θ, the naturalness of CP-violating phases, and the naturalness of the nonleptonic $\Delta I = \frac{1}{2}$ rule. In this context we examine many published gauge models and construct others to illuminate the questions at hand. We note that naturalness of μe universality for charged currents does not necessarily imply universality for neutral currents (natural "restricted" universality), and we emphasize the need for ν_e-beam experiments. For SU(2)×U(1) and SU(2)×U(1)×U(1) we give first examples of how a nontrivial natural θ can appear. Models with CP violation are classified as to whether their CP-violating phases are natural or not. For O(4)×U(1) we give a first example in which all the above naturalness criteria can be implemented. Here the natural μe universality is necessarily restricted. The principal tool used in these investigations is the strict renormalizability relative to a gauge group enlarged by discrete symmetries, and the union of representations reducible under the gauge group to irreducible ones under the enlarged group. To implement this program, it is sometimes necessary to introduce Higgs couplings involving right-handed neutrinos; here the zero neutrino mass is associated with a discrete symmetry which remains unbroken upon spontaneous breakdown. We also find that strict renormalizability can lead to mass relations between fermions. In O(4)×U(1) models, such mass relations as well as right-handed neutrinos are necessary ingredients. Furthermore, for these models the spontaneity of CP violation acquired an operational significance, namely, as a discrete symmetry necessary (but not sufficient) to give a CP-violating phase a natural value (90°). While the models we discuss are rather cumbersome, particularly due to the complexity of the symmetry-breaking mechanism, we expect that the tools we have developed may well have wider applicability.

I. INTRODUCTION

Many gauge models of weak and electromagnetic interactions have been devised in the last few years. The basic strategy for their construction consists in a reconciliation of field-theoretical and phenomenological requirements. From the side of field theory one insists on the renormalizability of the scheme as the principal predictive theoretical tool. From the side of phenomenology one attempts to incorporate all the known regularities of the weak interactions. What is known here almost entirely concerns the rather low-energy and low-momentum-transfer domain. Indeed, it is our ignorance of high-energy weak phenomena which allows, at this stage, for so much play in model building. Thus, experiments have not even confirmed the actual existence of massive vector bosons, the key ingredient in all models. In addition, there are many other questions which, when answered, will sharply delimit the present freedom of theoretical speculation, such as: Are there other weak currents[1] than the one customary pair of charge-carrying currents? Are there heavy leptons? Are there charmed hadronic states and, if so, what is the scheme which combines charm with known hadronic symmetries?

While, therefore, the future lies almost entirely in new experimental information, there is nevertheless much room for further theoretical study at this time. Beyond the construction of further models, there exist already a number of problems of principle which in some way or other have to do with the question: To what extent is some given model phenomenological?

This question has already been much discussed in the context both of specific problems related to some particular gauge model (μe universality,[2-4] strong isospin invariance as a natural versus an artificial symmetry,[5,6] and others), and of broader considerations on the presence and role of counterterms[7] and (related thereto) of "zeroth-order relations."[8] Thus it is known that, in order to answer our question, one must first of all exhibit the Lagrangian $\mathcal{L}$ of a given scheme in its strictly renormalizable[9] form. This means in particular that all necessary counterterms are included in $\mathcal{L}$. Then one phenomenological parameter can be associated with each independent counterterm (except wave-function renormalization counterterms). All observable quantities in the theory are then expressible in terms of these parameters. For example, in spin-$\frac{1}{2}$ electrodynamics, charge and mass need renormalization so they can be chosen to be the phenomenological parameters. We call a quantity "calculable" if no corresponding counterterm need be introduced. In ordinary theories, calculability is determined simply by power counting. For example, in spin-$\frac{1}{2}$ electrodynamics, the anomalous magnetic moment is calculable because the corresponding counterterm is not renormalizable.

In a theory with spontaneously broken symmetry, the situation is more complicated. The counterterms needed for renormalizability have the symmetries of the Lagrangian before spontaneous breakdown. In such a case, there may be nontrivial relations among the counterterms. If so, the masses and coupling constants appearing in the Lagrangian will not be independent phenomenological parameters. Rather there will be "zeroth-order relations" among these quantities, the corrections to which will be calculable higher-order effects.[7,8] We will call such relations "natural." For phenomenological reasons it is sometimes assumed that there are relations among masses and/or coupling constants which are not zeroth-order relations. The "corrections" to such relations are uncontrollable. Relations of this kind are called[6] "artificial."

As an example of a zeroth-order relation, consider the Weinberg[10,11] $SU(2)\times U(1)$ model in its original form. It contains a pair of charged vector mesons $W^{\pm}$ with mass M_W and a neutral vector meson with mass M_Z. Another parameter in the model is the mixing angle θ_W ($\tan\theta_W$ is a ratio of gauge coupling constants). Associated with the three parameters M_W, M_Z, and θ_W, there are only two independent counterterms, so only two of the parameters are phenomenological. There is a natural, zeroth-order relation among them: $\cos^2\theta_W = M_W^2/M_Z^2$. Thus $M_Z^2\cos^2\theta_W - M_W^2$ is calculable and, since all couplings in the theory are relatively weak, it is small. To leading order, the finiteness of this expression has been verified explicitly.[3,4]

If the Weinberg model is correct, then one combination of parameters, $\sin^2\theta_W M_W^2$, is already known because it is related to e^2/G. A measurement of M_W, for instance, would then yield a determination of all three parameters and make specific predictions about, say, neutral-current effects in ν_μ-electron scattering.

The naturalness of this relation between M_W, M_Z, θ_W depends on the details of the Higgs-meson structure of the model; in particular, on the assumption that the only scalar-meson multiplet in the model is the one doublet needed to give mass to the fermions. This choice has the virtue of simplicity, but, on the other hand, it is possible to enlarge the Higgs system without significantly changing the low-energy predictions. The only important new feature of such a modified $SU(2)\times U(1)$ model (aside from the additional Higgs mesons themselves) is that M_W, M_Z, and θ_W become independent phenomenological parameters, and their natural relation is lost. In such theories ν_μ-electron scattering would not be completely predicted, but instead would serve to determine the additional parameter.

This discussion illustrates how natural relations serve to delimit the number of measurements necessary to reach the predictive level of a gauge theory. Closely related to this kind of problem are questions whether (approximate) regularities already observed can be translated, in the context of gauge theory, into "natural" relations; in other words, whether these regularities are a necessary theoretical consequence of the choice of gauge model, rather than just an *ad hoc* phenomenological input. An illustrative example is μe universality. If a gauge group and its adopted representation content are such that the equality of the $\bar{\nu}_e e$ and $\bar{\nu}_\mu \mu$ couplings in the charged current is dictated by the structure of the strictly renormalizable Lagrangian before symmetry breakdown, then the unit value of the coupling-constant ratio will be a zeroth-order relation, the corrections to which will be calculable higher-order effects. This calculability is obviously a sensible theoretical constraint to be imposed on the choice of gauge model. This question will be discussed in more detail in Secs. II A and III B.

It has become customary to choose the set of scalar fields by a tacit criterion of minimality; namely, by introducing just such fields sufficient to attain mass where mass is needed. It is the main point of the present study that it may be worthwhile to replace this criterion by the alternative one to attain as much naturalness or predictive power as possible. In the example just discussed of the Weinberg model, these criteria [with respect both to the naturalness of the (M_W, M_Z, θ_W) relation and to μe universality] are equivalent. But, as we will see, this is not always the case.

In this paper we consider in detail some questions of calculability and naturalness in specific models with the aim of developing insight and theoretical tools which may be generally useful in model building. We focus on four topics, all essentially concerned with low-energy parameters:

(a) naturalness of μe universality,
(b) zeroth-order relations involving the Cabibbo angle θ,
(c) naturalness of *CP*-violating phases,
(d) naturalness of the nonleptonic $\Delta I = \frac{1}{2}$ rule.

We limit ourselves to the context of local gauge theories with a spontaneous breakdown mechanism induced by the presence of scalar fields, some of which acquire nonzero vacuum expectation values. It is sometimes conjectured that this Higgs mechanism itself is of a largely phenomenological character and that the actual symmetry breakdown mechanism is of a more fundamental nature. Since we have nothing to contribute to this question, we will stick to the Higgs mechanism. In fact, for the purpose of the present study we shall take the details of the Higgs-meson couplings very seriously.

Of course, naturalness is a notion valid to all orders in perturbation theory. The order of radiative correction in which lack of naturalness first becomes manifest is often characterized by parameters $\ll \alpha = \frac{1}{137}$ (for examples, see Secs. II B and III B). Hence caution is needed in the study of naturalness questions by graph methods.

Since we do not explicitly consider strong-interaction effects in this paper, it may be asked if we do not push things too far on too narrow a front. It would appear that there is no such objection if strong interactions are sufficiently damped at high virtual frequencies, as is, for example, the case if they enjoy asymptotic freedom. However, it may be well to bear such reservations in mind until we understand better the union with strong interactions.

The next two sections are organized as follows. Section II is devoted to gauge groups in which only a single pair of charged vector bosons appear. These comprise of course $SU(2)\times U(1)$ and $O(3)$, but we shall also find it instructive to consider $SU(2)\times U(1)\times U(1)$, which contains two massive neutral vector bosons. In Sec. III we discuss instances where two pairs of charged vector bosons enter; the groups discussed are $O(4)$ and $O(4)\times U(1)$.

We start with the analysis of natural universality in Sec. II A and briefly review the published models, two of which are natural,[11,12] the others artificial in this regard. We then raise a quite general question: What is the physical meaning of universality?

As is well known, all physical information bearing on universality stems from observations of semileptonic charge-changing processes. In the construction of gauge models, it has almost invariably been assumed tacitly that this universality property extends to all currents. From here on, "universality" shall refer to this situation which is met (whether naturally or artificially) in all $SU(2)\times U(1)$ and $O(3)$ models which have an equivalent representation content for muon-type and for electron-type leptons. In the absence of information to the contrary, we are led to ask the following question: Is it possible to construct models such that (a) the universality in the $|\Delta Q_{\text{hadronic}}| = 1$ processes is natural, while (b) in neutral-current processes this universality does not apply? We shall refer to such a situation as natural *restricted* universality. The formal meaning of natural universality is therefore that the substitutional invariance $\nu_e \leftrightarrow \nu_\mu$, $e \leftrightarrow \mu$ is natural for all currents, while it does not apply to some currents in the restricted case.

The physical meaning of the restricted situation is that it is no longer true that (up to lepton mass corrections) the cross sections for $\nu_\mu + e \to \nu_\mu + e$ and for $\nu_e + \mu \to \nu_e + \mu$ are equal. Nor (more importantly in practice) is it true that the reactions

$$\nu_\mu + \text{nucleon} \to \nu_\mu + X\,, \tag{1.1}$$

$$\nu_e + \text{nucleon} \to \nu_e + X \qquad (1.2)$$

have equal cross sections. Nevertheless, as we shall show by examples, the cross-section ratio for the reactions (1.1) and (1.2) may have simple calculability properties. In the context of a gauge theory, we can evidently have restricted universality if and only if inequivalent representations are involved for muon- and for electron-type leptons, that is, if heavy leptons exist and/or if there is more than one neutral current. We give examples of this in Secs. II A and III B. Since a breakdown of full universality has been a subject of much theoretical speculation through the years (especially in connection with lepton mass problems), we can only hope that experimentation with e-neutrino beams will not be too far off.

In constructing examples of restricted universality, we shall introduce a tool to be used repeatedly in the sequel, namely the extension of a local gauge group $\mathcal{G}$ by a discrete group S, such that in the limit of unbroken symmetry we deal with the full invariance under the group $\mathcal{G}\times S$. The demand of strict renormalizability relative to $\mathcal{G}$ is to be extended to strict renormalizability relative to $\mathcal{G}\times S$. It is a further and crucial feature that we shall need representations which are irreducible under $\mathcal{G}\times S$, though reducible under $\mathcal{G}$ alone. This same situation, reducibility relative to $\mathcal{G}$, irreducibility relative to $\mathcal{G}\times S$ will occur time and again in this paper. Indeed it is by this same device that we shall demonstrate how to construct certain natural values for the Cabibbo angle; and for CP-violating phases. As the patient reader will see, the distinct problems discussed in this paper have in fact many technical traits in common.

In the examination of universality we came upon some features novel to model building. As we shall see [cf. Eqs. (2.2)–(2.6) below], it is *necessary* in one example to introduce explicitly right-handed e neutrinos in the Higgs couplings. The necessity arises from the structure of S. Nevertheless, this neutrino remains massless. The reason is that *one discrete* element of S remains as an invariant operation even after spontaneous symmetry breakdown. In any event the answer to the question "Why is the neutrino massless (if indeed it is)?" may well contain a clue to the structure of gauge theories. In this context, the old answer applies: γ_5 invariance cannot tell the whole story since the neutrino is not singled out by this invariance in the symmetry limit.

In another example [cf. Eqs. (2.9)–(2.13) below] we find that implementation of strict renormalizability leads to a quadratic mass relation between fermions. This relation is natural, in the technical sense, and it is "type one" in a recently given classification.[8]

In Sec. II B we turn to the question of the Cabibbo angle θ and first show that θ is a phenomenological parameter (a renormalization constant) in all published models that fall under the heading of Sec. II. We report here on two models in which θ has a natural value. In the first example, the zeroth-order value of θ is a pure number, namely 45° [hence $\tan\theta = 1 + O(\alpha)$], a case of methodological though hardly of physical interest. The second example is furnished within the context of SU(2) $\times$ U(1) $\times$ U(1). Here a model is constructed in which θ is a natural function of the (four) renormalized quark masses.

The final part (C) of Sec. II is devoted to a brief discussion of gauge models with a single pair of charged vector bosons in which CP violation is incorporated. The inclusion of these effects means that, in some ways or other, a CP-violating phase (or phases) enter in the gauge model. We note that in these models the phases, the fermion masses, and the Cabibbo angle are independent phenomenological parameters.

In summary, in Sec. II the following new points emerge: (a) We learn how to implement μe universality in a restricted way. (b) The necessity may arise for having right-handed neutrinos appear in Yukawa couplings to Higgs fields. (c) Natural fermion mass relations may arise as a concomitant to the implementation of naturalness. All these will reappear as necessary ingredients for the class of models discussed in Sec. III. Since the very design of these models is based on the notion that μe universality and the origins of θ and of CP violation are inseparably intertwined, it is no longer possible, as in Sec. II, to treat these problems one by one. Let us briefly recapitulate the main idea.

The phenomenological starting point of these models[13-15] is the assumption that there are two pairs instead of the usual one pair of charged currents, coupled to pairs $W_1^\pm$, $W_2^\pm$ of charged vector bosons as follows:

$$ig[\bar{\mathcal{P}}\gamma_\mu(1+\gamma_5)\mathfrak{N} + \alpha\bar{\nu}_e\gamma_\mu(1+\gamma_5)e + \beta\bar{\nu}_\mu\gamma_\mu(1+\gamma_5)\mu + \cdots]W_1$$
$$+ ig[\bar{\mathcal{P}}\gamma_\mu(1+\gamma_5)\lambda + \gamma\bar{\nu}_e\gamma_\mu(1+\gamma_5)e + \delta\bar{\nu}_\mu\gamma_\mu(1+\gamma_5)\mu + \cdots]W_2 + \text{H.c.}\,, \qquad (1.3)$$

where the dots denote other terms as they may (and indeed will) arise. $\alpha, \beta, \gamma, \delta$ are phases: $|\alpha| = |\beta| = |\gamma| = |\delta| = 1$. This condition ensures μe universality (always on a phenomenological level). The imposition of Cabibbo universality between μ decay and $\mathcal{P}$ and λ β decay implies that these phases cannot all be real. In fact the latter condition implies that

$$\mathrm{Re}\,\alpha^*\beta\gamma\delta^* = 0 . \tag{1.4}$$

Now any three of these four phases may be eliminated in favor of a single phase by choosing appropriate conventions. Example: We can put $\alpha = \beta = 1$ by redefining e and μ. And we can effectively set $\gamma = 1$ by redefining $W_2 \to \gamma^* W_2$, $\lambda \to \gamma\lambda$. By this convention only δ survives (and cannot be eliminated) and Eq. (1.4) implies that (up to an unimportant sign) $\delta = i$. Hence Cabibbo universality is arrived at via the route of *CP* violation. The one single surviving phase reflects on a property of the lepton terms as a set relative to the hadron terms in the currents, rather than on a property of an individual lepton term.

From the point of view of naturalness of parameters the following problems now arise if Eq. (1.3) is to be implemented via a gauge model:

(1) Clearly the Cabibbo angle is to be defined by $\tan\theta = M_1{}^2/M_2{}^2$ where M_1, M_2 are the respective masses of W_1, W_2. Question: Is this a natural (i.e., zeroth-order) relation? If so, we shall have, more precisely

$$\tan\theta = M_1{}^2/M_2{}^2 + O(\alpha) . \tag{1.5}$$

If realizable this then becomes one of the predictive features of such models: To calculable corrections of order α, there should be two charged vector mesons with mass ratio $(\tan\theta)^{1/2} \approx \frac{1}{2}$. [*Note.* Speaking futuristically, even the discovery of a single charged vector meson might shed light on whether Eq. (1.3) makes any sense, since in models of the present kind a W *cannot* decay both in $\Delta S = 0$ and $\Delta S = 1$ hadronic (charm conserving) channels with relative rates $\sim\tan^2\theta$ as in the single-W models.]

(2) It follows from Eq. (1.4) that, whatever phase convention we adopt, we *cannot* have both $\alpha = \beta$ and $\gamma = \delta$. Therefore, a certain dissymmetry has to appear in the electron-type versus the muon-type leptons. It was therefore clear from the outset[13] that μe universality would be an issue. We are now in a position to state the problem more precisely than was done hithertofore. Question: Is such dissymmetry compatible with natural μe universality, if not fully, then at least in the restricted sense?

(3) Continuing with the above example of conventions, set $\delta = e^{i\psi} = i$ so that $\psi = \pi/2$. Question: Is this a natural value for ψ? If so, we shall have, more precisely,

$$\sin\psi = 1 + O(\alpha) . \tag{1.6}$$

If realizable, *CP* violation is then characterized by a calculable *CP*-violating phase. In obvious language, one may then further call the *CP* violation "maximal." The impact of this maximal *CP* violation is of the "superweak" kind.[16]

It is shown in Sec. III how these questions can all be answered affirmatively. In Sec. III A we give a short review of the models involved and of earlier comments on their calculability properties. There we also refer to the question of the naturalness of the nonleptonic $\Delta I = \frac{1}{2}$ rule. Section III B is devoted to a systematic discussion of the four questions raised above. Once again, discrete symmetries are the key to the arguments presented. Here we discover that one of the discrete symmetries needed to implement the naturalness of Eq. (1.5) is that the theory be *CP*-invariant prior to the onset of spontaneous symmetry breaking (of course all gauge theories are *CPT*-invariant). *CP* noninvariance is then "spontaneous." The esthetic appeal of this particular mode of *CP* invariance breaking was first underlined by T. D. Lee.[17]

It may be useful at this point to state concisely in what way spontaneity of *CP* violation is pertinent to calculability properties of *CP*-violating effects in gauge models. First, there is the question of the imaginary part in K^1-K^2 mass mixing (the superweak mechanism). In the present limited state of the art this effect is associated[18] with the S-matrix element for the quark transition $\bar{\mathfrak{N}}\lambda \to \bar{\lambda}\mathfrak{N}$. Since there cannot be a counterterm for this transition (it would be a four-Fermi interaction) this transition is finite in any event, and the same is true for on-shell *CP*-violating transition elements. Second, consideration has been given to the electric dipole moment of fermions in gauge theories with *CP* violation. Again there cannot be a counterterm for such moments.[19] Thus these two effects are calculable quite independently of the way *CP* violation is implemented in gauge models. But now there are two possibilities: (1) If the *CP*-violating phase is phenomenological, then at least one of these effects serves to determine its renormalized value. (2) If the *CP*-violating phase is natural, then its value is a separate prediction of the theory to which these effects have to conform. It is this second case with which we are dealing here in the realization of Eq. (1.5).

Thus we are led to classify gauge theories with *CP* violation as follows:

(I) The *CP*-violating phase(s) are phenomenological. *CP* violation may or may not be spontaneous. An example of each of these two instances is mentioned in Sec. II C.

(II) The CP-violating phase(s) are natural. This is the case in Sec. III B, where CP violation is spontaneous. We have no example where the phase is natural and CP violation is nonspontaneous.

CP invariance is only one of several discrete symmetries which we shall need in the present context, the more so because we are simultaneously concerned also with the naturalness of θ, of μe universality, and of the $\Delta I = \frac{1}{2}$ rule. We now record our findings for the group O(4)× U(1).

(a) All f^L 4-vectors, all f^R 4-scalars [$f^{L,R}$ *= left- (right-) handed fermions*]. Only for the unphysical zeroth-order value $\theta = 45°$ can all naturalness conditions be met. For other θ values one cannot prevent a lack of naturalness which (under optimal conditions) becomes manifest only to order

$$\alpha^2[(m_0{}^2 m_{ch}{}^4)/(m_H{}^2 m_W{}^4)] \tag{1.7}$$

(m_0, m_{ch} are a typical neutral and charged lepton mass, respectively, m_H = typical Higgs-meson mass, m_W = typical vector-meson mass).

(b) All f^L 4-spinors, all f^R 4-scalars. Again we could push the lack of naturalness at best to the order of Eq. (1.7).

(c) O(4) × U(1) × U(1), same fermion content as under (b). Here full naturalness can be met strictly.

(d) Back to O(4) × U(1), left-handed quarks and electrons (or muons) 4-spinors, left-handed muons (or electrons) in the adjoint representation of O(4). This is the simplest model we have found so far in which simultaneously the Cabibbo angle satisfies a natural zeroth-order relation and is nontrivial; CP violation is natural and maximal; μe universality is natural and restricted; and the $\Delta I = \frac{1}{2}$ rule is natural (to the extent that the quark states used can be integrated in a theory which includes strong interactions). After some general comments on the cases (a)–(c) in Sec. III A we analyze case (d) in detail in Sec. III B. We stress that we have pushed this investigation rather ruthlessly to the present level in order to show by at least one example that the conditions studied here can actually all be met. We regard the complexity of the model, especially of the Higgs system, as a clear indication that these matters are far from closed.

In Sec. IV we make a final comment on what we believe we have learned and on what we are sure we do not understand.

Finally, the following conclusions may be drawn from this methodological investigation, as we see it.

(1) In gauge model building the following three assumptions are most often tacitly made: (a) Charge-changing weak processes are mediated by one and only one pair of charged W mesons. (b) μe universality is desired to be a property of *all* currents. (c) Any occurrence whatsoever of right-handed neutrinos is tabu. For all we know, none of these (independent) assumptions should be taken for granted.

(2) Theoretical demands of naturalness will constrict the choice of gauge group and content in approaches to an electromagnetic weak synthesis. As we tried to make clear, severe demands of this kind already arise from the consideration of low-energy phenomena. The criteria discussed in this paper would seem reasonable, but we are in no position to claim that they are imperatives. Also, there are other constraints which deserve at least as serious consideration, notably the naturalness of hadronic symmetries and of the μ/e mass ratio.

(3) The reader who will have followed this technical discourse on naturalness and artificiality may wonder, along with the authors, whatever has happened to good old-fashioned simplicity. Perhaps the gauge-theory approach is wrong, but this we doubt. Perhaps some essential theoretical ingredients are lacking, in particular in regard to symmetry-breaking mechanisms. Perhaps also what we now consider simplicity may turn out to be deceptive, as experiment progresses; it would not be the first time in particle physics. A linear combination of the last two alternatives is our own best guess.

II. GAUGE THEORIES WITH A SINGLE CHARGED $W^{\pm}$ PAIR

A. μe universality

1. *Models with natural universality*

There are two of these. In the first one, the Weinberg model,[10,11] the left-handed electron and ν_e fields and the muon and ν_μ fields transform according to two equivalent irreducible representations of the gauge group. The renormalizable couplings of the charged intermediate vector boson to leptons is characterized by one parameter, the gauge coupling constant associated with the SU(2) factor of the group, and is the same for electrons and muons. Clearly muon-electron universality is natural. Naturalness here is a direct consequence of the gauge structure of the theory. This is a simple translation into the language of renormalizable field theory of the old idea that universality should have something to do with conserved currents, that is, the transformation properties of the weakly interacting system under some continuous group.

The Weinberg model is unique in that it involves only observed lepton states. (It is possible to change the Abelian gauge structure of the theory—see below.) Almost all other unified models of weak and electromagnetic interactions involve unobserved "heavy leptons." The number of possible theories of this kind is very large. A second published model with universality properties similar to the Weinberg model is the Lee-Prentki-Zumino (LPZ) model.[12] Here the left-handed lepton fields are assigned to gauge SU(2) triplets as follows: $(E^+, \nu_e, e^-)_L$ and $(M^+, \nu_\mu, \mu^-)_L$, where E^+ and M^+ are heavy lepton fields. As in the Weinberg model, the muon and electron fields have identical properties under the gauge group, determined by their assignment to equivalent irreducible representations.

2. *Two examples of natural restricted universality*

If a large number of heavy lepton states is postulated, there is a great deal of flexibility in model building. Consider, for instance, the following problem: Can we write down a model which predicts $\nu_\mu e$ scattering with typical weak-interaction strength, but in which $\nu_e \mu$ scattering is suppressed? The answer is yes, of course, by assigning left-handed leptons to triplets as follows: $(\nu_\mu, \mu^-, M^{--})_L$ and $(E^+, \nu_e, e^-)_L$. In this model, the muon and electron fields have different gauge properties. They have different U(1) gauge quantum numbers. Nevertheless, universality is still natural for charge-changing processes. The point is that when the left-handed leptons are assigned to irreducible representations of the gauge groups, the couplings of the intermediate vector boson to the *charged* muon and electron currents are determined simply by the relevant Clebsch-Gordan coefficients. If these coefficients for the muon and electron currents are equal, universality is natural for the charged currents; if they are unequal, the model does not have universality.

However, our example shows that natural universality for the charged currents does not necessarily extend to neutral-current couplings. In fact, this current does contain $\bar{\nu}_\mu \nu_\mu$—but no $\bar{\nu}_e \nu_e$ terms. Let us now imagine that we complete the model with a quark structure that satisfies all the usual constraints, including Cabibbo universality. (For this purpose one can take over the LPZ-quark representations.) Then the amplitudes for the processes Eq. (1.1) are $O(G)$ and those for Eq. (1.2) are $O(G\alpha)$.

The Clebsch-Gordan coefficients for the charged currents may also be equal for other reasons. As a fanciful example, imagine assigning the left-handed lepton fields to gauge multiplets as follows: the electron in a 4 (gauge isospin $\frac{3}{2}$), $(E^+, \nu_e, e^-, E^{--})_L$, and the muon in a 5 (gauge isospin 2), $(\nu_\mu, \mu^-, M^{--}, M^{---}, M^{----})_L$. Now universality is natural even though the representations are very different, just because the relevant Clebsch-Gordan coefficients happen to be equal. In this example the neutral current does contain both $\bar{\nu}_\mu \nu_\mu$ and $\bar{\nu}_e \nu_e$ terms but with different weight. Again the universality, while natural, is only of the restricted type and the ratio of the amplitudes of the semileptonic reactions (1.1) and (1.2) is as 4:1.

3. *Reducible representations, artificial universality*

We have still not considered all possible models with a single pair of charged vector bosons. It is possible to relax the condition that the left-handed electron (or muon) and ν_e (ν_μ) field belong to an irreducible representation of the gauge group. A well-known example of a model involving reducible representations is the O(3) model.[20] Here the left-handed lepton fields are assigned to singlets and triplets as follows: Triplets are $(E^+, E^0\cos\beta + \nu_e \sin\beta, e^-)_L$ and $(M^+, M^0\cos\beta + \nu_\mu \sin\beta, \mu^-)_L$; singlets are $(\nu_e \cos\beta - E^0 \sin\beta)_L$ and $(\nu_\mu \cos\beta - M^0 \sin\beta)_L$. In this example, it is the neutrino fields which transform like a mixture of singlet and triplet, not like a single irreducible representation.

As written, this model has muon-electron universality, but here it is *not* natural. The reason is that the angle β depends on the details of the Higgs-meson couplings, the bare-mass terms, and the spontaneous-symmetry breaking and there is no reason for it to be the same for electron and muon multiplets. In fact, the angles for ν_e and ν_μ must be renormalized with independent counterterms. So while the O(3) model can describe muon-electron universality, it cannot predict it.

4. *Right-handed neutrinos*

It may be tempting at this point to conclude that irreducibility in the sense described above is a necessary condition for naturalness of universality in a gauge model, but such a conclusion would be premature. Consider, for example, the problem posed earlier in this section: to construct a model in which $\nu_\mu e$ scattering is present but $\nu_e \mu$ scattering is suppressed. One such model was given above, but it is also possible to use reducible representations. Consider the following assignment of the left-handed leptons fields: muon in a doublet $(\nu_\mu, \mu^-)_L$; electrons in two triplets,

$$\psi_1 = \begin{pmatrix} E^+ \\ \frac{\nu_e + N^0}{\sqrt{2}} \\ e^- \end{pmatrix}_L , \quad \psi_2 = \begin{pmatrix} F^+ \\ \frac{N^0 - \nu_e}{\sqrt{2}} \\ f^- \end{pmatrix}_L . \tag{2.1}$$

The first two representations give the usual charged-current structure, while the third representation leads to no presently observable experimental effects. The physics would be the same if $(N^0 - \nu_e)_L/\sqrt{2}$ were assigned as a singlet, at least until the appropriate heavy leptons are observed. But then universality would not be natural. For the assignment given above, on the other hand, it is possible to implement naturalness. To see this, we must analyze this theory in some detail.

The strategy is as follows. In Eq. (2.1), a 45° mixing angle appears between ν_e and the heavy lepton N^0. If this angle is natural, then μe universality will be natural for the charged currents. In turn, the 45° mixing will be natural if the Higgs coupling needed to give mass to the electron-type leptons *forces* us to have mixing at 45°. This can happen when the symmetry group of the Lagrangian is not just $SU(2)\times U(1)$, but $SU(2)\times U(1)\times S$, where S is a group of discrete symmetries. We now explicitly exhibit Yukawa couplings invariant under such an enlarged symmetry group. For the muon system everything is as in the Weinberg model. For the electron system we introduce two real triplets of Higgs mesons, H_1 and H_2 ($t=1$, $Y=0$) and two complex triplets, K_1 and K_2 with $t=1$, $Y=1$ (we write the electric charge as $Q=t_3+Y$). All right-handed fermions are taken to have $t=0$ and the appropriate weak hypercharge. Consider now the following set of interactions:

$$a_1[(\bar\psi_1+\bar\psi_2)N_R^0 H_1 + (\bar\psi_1-\bar\psi_2)\nu_{eR}H_2] + a_2(\bar\psi_1 E_R^+ + \bar\psi_2 F_R^+)K_1 + a_3(\bar\psi_1 E_R^+ - \bar\psi_2 F_R^+)K_2 + a_4(\bar\psi_1 e_R + \bar\psi_2 f_R^-)\bar K_1 + a_5(\bar\psi_1 e_R - \bar\psi_2 f_R^-)\bar K_2 + \text{H.c.} \tag{2.2}$$

Observe that in the first line the right-handed neutrino ν_{eR} appears. The couplings of Eq. (2.2) are invariant under the following discrete operations (AEU = all else unchanged; $\bar K_1$ and $\bar K_2$ are the complex conjugates of K_1, K_2, respectively):

$$\psi_1 \leftrightarrow \psi_2,\ \nu_{eR} \to -\nu_{eR},\ E_R^+ \leftrightarrow F_R^+$$
$$e_R \leftrightarrow f_R^-,\ K_2 \to -K_2,\ \text{AEU}; \tag{2.3}$$
$$\psi_2 \to -\psi_2,\ N_R^0 \leftrightarrow \nu_{eR},\ H_1 \leftrightarrow H_2$$
$$F_R^+ \to -F_R^+,\ f_R^- \to -f_R^-,\ \text{AEU}; \tag{2.4}$$
$$H_2 \to -H_2,\ \nu_{eR} \to -\nu_{eR},\ \text{AEU}. \tag{2.5}$$

In addition, the remainder of the Lagrangian is invariant as well under these discrete operations. Note that because of Eqs. (2.3) and (2.4) the two triplets have become an irreducible representation of the enlarged group. These transformations have the following important properties:

(a) Not only are the couplings in Eq. (2.2) invariant under the transformations Eqs. (2.3), (2.4), and (2.5), but furthermore, *these transformations determine these Higgs couplings uniquely* (up to the values of the coefficients $a_1, \ldots, a_5$, of course). An obvious way to verify this is to write down first the most general set of trilinear couplings between $\psi_{1,2}$, $H_{1,2}$, $K_{1,2}$, and the right-handed fermions which is compatible with the invariance under the continuous group $SU(2)\times U(1)$. Then by imposition of the discrete invariances (2.3)–(2.5) one arrives at Eq. (2.2).

(b) If the vacuum expectation values of the Higgs multiplets are as follows:

$$\langle H_1\rangle = \begin{pmatrix} 0 \\ h_1 \\ 0 \end{pmatrix}, \quad \langle H_2\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad K_1 = \begin{pmatrix} k_1 \\ 0 \\ 0 \end{pmatrix}, \quad K_2 = \begin{pmatrix} k_2 \\ 0 \\ 0 \end{pmatrix}, \tag{2.6}$$

then we will have achieved the proper mass diagonalization *including a null mass for the e neutrino* (h_1, k_1, and k_2 shall be nonzero). Now the Higgs meson self-couplings do allow vacuum expectation values with the properties given by Eq. (2.6) for some region with nonzero measure in the space of renormalized parameters. In particular, $\langle H_2\rangle \equiv 0$ is allowed because Eq. (2.5) tells us that the Higgs potential cannot contain terms linear in H_2.

The reader may well be confused at this point about the reason for introducing ν_{eR} and H_2 in the first place, since the nonintroduction of ν_R, customary in all gauge models proposed so far, is in itself a sufficient ground for having a vanishing neutrino mass. The idea is that ν_{eR} is necessary for naturalness of the form Eq. (2.1) with the gauge group $SU(2)\times U(1)$. We will show below that, without ν_{eR}, naturalness can only be achieved by enlarging the gauge group.

As was stated in the Introduction, the strict masslessness of the neutrino, in schemes like these, is associated with a discrete symmetry which remains valid even after the spontaneous breakdown of symmetry. In the present case, this is the symmetry given by Eq. (2.5).

As a last example in this section of natural restricted universality, we shall show how this can come about via the extension of $SU(2)\times U(1)$ not only by discrete symmetries but also by continuous ones. Here it will not be necessary to introduce ν_R.

5. *The gauge group SU(2)× U(1) × U(1)*

Let us again start with $(\nu_\mu, \mu^-)_L$ as a doublet and with the triplets ψ_1, ψ_2 given in Eq. (2.1). However, we are now going to consider these multiplets as representations of the gauge group SU(2) × U(1) × U(1). This group has the covariant derivative

$$D_\mu = \partial_\mu - i(g\vec{A}_\mu \cdot \vec{t} + g_S B_\mu S + g_R C_\mu R) . \qquad (2.7)$$

We choose the charge operator to be

$$Q = t_3 + S + R , \qquad (2.8)$$

so that the electromagnetic field is given by $e^{-1}A_\mu = g^{-1}A^3_\mu + g_S{}^{-1}B_\mu + g_R{}^{-1}C_\mu$ with $e^2 = (g^{-2} + g_S{}^{-2} + g_R{}^{-2})^{-1}$. The representations may be labeled as $(t)^{S,R}$.

We now make the detailed assignments as follows: $(\nu_\mu, \mu^-)_L$ is $(\frac{1}{2})^{(-1/2,0)}$, ψ_1 and ψ_2 are $(1)^{(0,0)}$. All right-handed leptons shall be SU(2) singlets. For μ_R we take $Q=S$ $(=-1)$, while for the electronic lepton fields E^+_R, F^+_R, e^-_R, and f^-_R we take $Q=R$.

We shall need two real triplets H_1 and H_2 of Higgs mesons of the type $(1)^{(0,0)}$ and two more, K_1 and K_2 which are $(1)^{(0,1)}$. These shall enter in the $\psi_{1,2}$ couplings. [For the muon doublet we will have a Higgs doublet $(\frac{1}{2})^{(1/2,0)}$, as in the familiar SU(2) × U(1) case.] We assume that the Lagrangian is invariant under the following discrete symmetries:

$$[1]\quad \psi_1 \leftrightarrow \psi_2,\ E^+_R \leftrightarrow F^+_R,\ e_R \leftrightarrow f^-_R$$
$$K_2 \to -K_2,\ H_2 \to -H_2,\ \text{AEU}\,; \qquad (2.9)$$
$$[2]\quad \psi_2 \to -\psi_2,\ H_1 \leftrightarrow H_2\ ,$$
$$F^+_R \to -F^+_R,\ f^-_R \to -f^-_R,\ \text{AEU}\ . \qquad (2.10)$$

We further assume that the following symmetry is broken only by mass terms:

$$[3]\quad E^+_R \to e^-_R \to -E^+_R,\ F^+_R \to -f^-_R \to -F^+_R\ ,$$
$$K_1 \to \tilde{K}_2,\ K_2 \to -\tilde{K}_1,\ C_\mu \to -C_\mu,\ \text{AEU}\ , \qquad (2.11)$$

where C_μ is the gauge field defined in Eq. (2.7). Now the most general Yukawa couplings consistent with these symmetries and with the gauge symmetry are

$$a_1[(\overline{E}^+_R\psi_1 + \overline{F}^+_R\psi_2)K_1 + (e_R\psi_1 - f^-_R\psi_2)\tilde{K}_2]$$
$$+ a_2[(\overline{E}^+_R\psi_1 - F^+_R\psi_2)K_2 - (e_R\psi_1 + f^-_R\psi_2)\tilde{K}_1]$$
$$+ a_3\overline{N}^0_R[(\psi_1+\psi_2)H_1 + (\psi_1 - \psi_2)H_2] + \text{H.c.} \qquad (2.12)$$

The Higgs-meson self-couplings allow vacuum expectation values such that $\langle H_2\rangle = 0$, $\langle H_1\rangle$, $\langle K_1\rangle$, and $\langle K_2\rangle$ nonzero; and $\langle K_1\rangle \neq \langle K_2\rangle$. These vacuum expectation values with the Yukawa coupling written above give the theory we want, with the additional constraint that the fermion masses satisfy a quadratic zeroth-order relation:

$$M^2(e) + M^2(E^+) = M^2(f^-) + M^2(F^+)\ . \qquad (2.13)$$

Observe that this is a *natural* mass relation since it is dictated by the symmetry of the system and by the vacuum expectation values for the H and K fields stated above.

The reader will note similarities between Eqs. (2.3) and (2.4) as compared with Eqs. (2.9) and (2.10). On the other hand, Eq. (2.11) is quite a different thing from Eq. (2.5). Let us enlarge on the role of Eq. (2.11). The symmetries Eqs. (2.9) and (2.10) allow Higgs-meson self-couplings of the form $\alpha(K_1^\dagger K_2)(H_1H_2)$ + H.c. Now if $\langle K_1\rangle$, $\langle K_2\rangle$, $\langle H_1\rangle$ are all nonzero, this term gives a direct tadpole contribution to $\langle H_2\rangle$ which spoils the naturalness of the condition $\langle H_2\rangle = 0$. The symmetry [3] is specifically designed to forbid this term. But this symmetry is not consistent with SU(2) × U(1) structure. It is at this point that the need for the extension by another U(1) factor becomes manifest (always as an alternative to the extension discussed previously.) Symmetry [3] cannot be an exact symmetry of the Lagrangian because one can show that in zeroth order it implies $|\langle K_1\rangle| = |\langle K_2\rangle|$ and therefore $m(e^-) = m(F^+)$, so it must be broken by mass terms. That is, we must include in the Lagrangian terms of dimension less than four which break the symmetry. The renormalization of the dimension-four terms can still be done with a symmetric counterterm, as is obvious by power counting. Thus we can forbid the term $(K_1^\dagger K_2)(H_1H_2)$ but still include terms like $K_1^\dagger K_1 - K_2^\dagger K_2$ which break the symmetry[21] between K_1 and K_2. These considerations determine the form of the Lagrangian.

One can again use the catchword irreducibility to describe naturalness of universality in this model. We reiterate, however, that here one means irreducibility under the full symmetry of the Lagrangian which may contain a complicated discrete group in addition to the gauge symmetry.

Finally we note that the Higgs system described above is such that two massive neutral vector bosons appear. We shall not be interested in the details of the necessary diagonalization process, except for one qualitative observation about the two neutral currents coupled to these vector bosons. It is clear from the quantum number assignments given above that a $\overline{\nu}_\mu\nu_\mu$ term will generally appear in both these currents, while $\overline{\nu}_e\nu_e$ terms will not appear in either current. Thus we have another example of restricted universality with different orders of magnitude for the processes (1.1) and (1.2).

B. The Cabibbo angle

1. *Remarks on SU(2)× U(1)*

We begin with a brief description of the way θ appears in the four-quark version of $SU(2)\times U(1)$, for two reasons: first, in order to show that θ is not calculable in this model; second, in order to give some indication of what it may take to promote θ from a renormalization parameter to a calculable quantity.

The model in question has two quark doublets $N=(\mathcal{P},\mathfrak{N}_c)_L$, $N'=(\mathcal{P}',\lambda_c)_L$, $\mathfrak{N}_c=\mathfrak{N}\cos\theta+\lambda\sin\theta$, $\lambda_c=-\mathfrak{N}\sin\theta+\lambda\cos\theta$, $(a)_L=(1+\gamma_5)a/2$. Further there are four singlets $\mathcal{P}_R$, $\mathfrak{N}_R$, λ_R, $\mathcal{P}'_R$, $(a)_R=(1-\gamma_5)a/2$. (We leave aside the lepton structure.) The charge-carrying current contains the terms $i(g_1\overline{\mathcal{P}}_L\gamma_\mu\mathfrak{N}_L+g_2\overline{\mathcal{P}}_L\gamma_\mu\lambda_L+\cdots)$, where $g_1=g\cos\theta$, $g_2=g\sin\theta$. g is one of the coupling constants of the group and is subject to renormalization. If g_1 and g_2 suffer independent renormalization, then θ is not calculable. Note the distinct ways in which g and θ make their appearance: g enters via the group structure; θ enters via the details of mass diagonalization.

The reason that θ is not calculable in this model is that this quantity does not enter the theory in any other way than the one just indicated, and cannot enter into any natural zeroth-order relation. In order to see this in detail, we must examine the other quark interactions in the model, namely their couplings to a scalar field doublet H. These couplings can be written as

$$(f_1\overline{N}\mathcal{P}_R+f_2\overline{N}'\mathcal{P}'_R+f_3\overline{N}\mathcal{P}'_R+f_4\overline{N}'\mathcal{P}_R)H$$
$$+(f_5\overline{N}\mathfrak{N}_R+f_6\overline{N}\lambda_R+f_7\overline{N}'\mathfrak{N}_R+f_8\overline{N}'\lambda_R)\tilde{H}+\text{H.c.}\,,$$

where $\tilde{H}=i\tau_2H^*$. The eight f_i are a new set of coupling constants each of which is subject to independent renormalization. Up to a common factor e/M_W, $f_5,\ldots,f_8$ can be written as

$$f_5=m_\mathfrak{N}\cos\theta',\; f_6=m_\lambda\sin\theta'',\qquad f_7=-m_\mathfrak{N}\sin\theta',\; f_8=m_\lambda\cos\theta''\,, \tag{2.14}$$

where $m_\mathfrak{N}$, m_λ, θ', θ'' suffer independent renormalizations. The phenomenological introduction of θ in the model is of course based on the notion that $\mathcal{P}$, $\mathcal{P}'$, $\mathfrak{N}$, and λ shall be zeroth-order mass eigenstates. This last condition implies that

$$f_3=f_4=0\,,\qquad \theta=\theta'=\theta''\,, \tag{2.15}$$

which are examples of artificial relations in the sense explained in the Introduction. [A simple argument shows that the lack of naturalness of Eq. (2.15) becomes manifest at first in order $G(m_\lambda-m_\mathfrak{N})(m_{\mathcal{P}'}-m\ \ )$.] Therefore we learn two things:

(a) θ is a purely phenomenological parameter.

(b) Eq. (2.15) indicates that a way to seek for a calculable θ is to ask for shared constraints which apply to the couplings of quarks to vector mesons as well as scalar mesons. In this section we again explore the existence of additional discrete symmetries as a means to implement such shared constraints.

As a first example, consider an $SU(2)\times U(1)$ model in which θ is calculable for a special zeroth-order value, namely $\theta=45°$. Of course, the relation $\tan\theta=1$ is hardly of any practical interest. Here it will merely serve as a first instance of a natural relation in which θ enters, namely

$$\tan\theta=1+O(\alpha)\,. \tag{2.16}$$

The model in question has the same quark content as the LPZ model,[12] namely, two left-handed triplets, with representation $(1)^0$. [We may label the representations by $(t)^Y$, t = weak isospin, Y = weak hypercharge, and $Q=t_3+Y$.] The right-handed quarks are singlets, $(0)^Q$. If one employs a minimal set of scalar multiplets, namely[12] one real scalar triplet $(1)^0$ and one complex triplet $(1)^1$ in order to give mass to all quark states, then θ is noncalculable for the model. By the same argument as given above, one arrives at Eq. (2.15). However, if one uses a pair of $(1)^0$ and a pair of $(1)^1$ multiplets, then θ can be calculable if its zeroth-order value is $\theta=\pi/4$.

The argument goes as follows: Take the two L-quark triplets to be

$$Q^1=\begin{pmatrix}\mathcal{P}\\(\mathfrak{N}+\lambda)/\sqrt{2}\\q^-\end{pmatrix}_L,\quad Q^2=\begin{pmatrix}\mathcal{P}'\\(\mathfrak{N}-\lambda)/\sqrt{2}\\q'^-\end{pmatrix}_L, \tag{2.17}$$

corresponding to $\theta=\pi/4$. Both the charged and the neutral vector currents are invariant under the transformation

$$Q^1\leftrightarrow Q^2\,, \tag{2.18}$$

all else unchanged. If we are able to extend this invariance to the full Lagrangian, then we shall have derived Eq. (2.16).

Introduce the following scalar multiplets: H^1,H^2 which are both $(1)^0$ and K^1, K^2 which are both $(1)^1$. Consider the following set of Higgs couplings ($a_1,\ldots,a_6$ are constants):

$$a_1[(\bar{Q}^1+\bar{Q}^2)\mathfrak{N}_R+(\bar{Q}^1-\bar{Q}^2)\lambda_R]H^1+a_2[(\bar{Q}^1+\bar{Q}^2)\mathfrak{N}_R-(\bar{Q}^1-\bar{Q}^2)\lambda_R]H^2+a_3[\bar{Q}^1\mathcal{P}_R+\bar{Q}^2\mathcal{P}'_R]K^1$$
$$+a_4[\bar{Q}^1\mathcal{P}_R-\bar{Q}^2\mathcal{P}'_R]K^2+a_5[\bar{Q}^1q_R+\bar{Q}^2q'_R]\tilde{K}^1+a_6[\bar{Q}^1q_R-\bar{Q}^2q'_R]\tilde{K}^2+\text{H.c.}\,, \quad (2.19)$$

which have the following three properties:

(1) They are invariant under Eq. (2.18) provided we extend the transformation to

$$Q^1\leftrightarrow Q^2,\ \lambda_R\to-\lambda_R,\ \mathcal{P}_R\leftrightarrow\mathcal{P}'_R,$$
$$q_R\leftrightarrow q'_R,\ K^2\to-K^2\,,\ \text{AEU}\,. \quad (2.20)$$

(2) They are also invariant under the discrete symmetry

$$Q^2\to-Q^2,\ H^2\to-H^2,\ \mathfrak{N}_R\leftrightarrow\lambda_R\,,$$
$$\mathcal{P}'_R\to-\mathcal{P}'_R\,,\ q'_R\to-q'_R\,,\ \text{AEU}\,. \quad (2.21)$$

Together with the gauge invariance and the symmetry (2.20), this symmetry forces the Yukawa couplings to have the form (2.19).

(3) The set of Higgs couplings Eq. (2.19) and the symmetries (2.20) and (2.21) do not imply any unwanted mass degeneracies. In this connection note that the vacuum expectation values $\langle K^1\rangle=(0,0,\lambda^1)$, $\langle K^2\rangle=(0,0,\lambda^2)$ are such that the invariances Eqs. (2.20) and (2.21) do *not* imply any connection between λ^1 and λ^2; similarly for H^1 and H^2.

We have now derived Eq. (2.3) but for one point. It should be ascertained that also the lepton-Higgs couplings are compatible with the discrete symmetry under consideration. This is easily done as follows: (1) Use the same lepton representations as in LPZ.[12] (2) Use lepton couplings to K^1 to generate mass for the charged leptons. (3) Let all lepton states be invariant under the discrete transformations Eqs. (2.20) and (2.21).

The same value $\theta=\pi/4$ can also be obtained in the Weinberg doublet model, provided a third discrete symmetry is introduced. This is simply seen by omitting q and q' from Eqs. (2.17) and (2.19)–(2.21). Indeed it may seem that this is all that is needed. However, it is now necessary to invoke the additional symmetry

$$H^1\to-H^1,\ H^2\to-H^2,\ \mathfrak{N}_R\to-\mathfrak{N}_R,\ \lambda_R\to-\lambda_R,\ \text{AEU}\,.$$

The price paid here is the introduction of four Higgs doublets instead of the usual single one.

The above is an example of a zeroth-order value for θ which is a Clebsch-Gordan coefficient. Our next example is of a quite different kind.

2. Extension to $SU(2)\times U(1)\times U(1)$

The covariant derivative for this group was given in Eq. (2.7). We also define Q as in Eq. (2.8) and will continue to label representations as $(t)^{(S,R)}$. However, here we shall operate with different representations as compared with Sec. II A.

We introduce a four-quark model via the following representation content: There are two L-quark doublets

$$\Psi=\begin{pmatrix}P\\N\end{pmatrix}_L,\ \Psi'=\begin{pmatrix}P'\\N'\end{pmatrix}_L:\quad (\tfrac{1}{2})^{(1/6,0)}\,. \quad (2.22)$$

Here P_L, P'_L each are linear combinations of the physical states $\mathcal{P}_L, \mathcal{P}'_L$ $(Q=\frac{2}{3})$ encountered in Sec. II B1, and likewise for N_L, N'_L in regard to $\mathfrak{N}_L, \lambda_L$ $(Q=-\frac{1}{3})$. The precise choice of these combinations will occupy us shortly. There are four R-quark singlets:

$$P_R,\ P'_R:\ (0)^{(1/6,1/2)}\,;\quad N_R,\ N'_R:\ (0)^{(1/6,-1/2)}\,. \quad (2.23)$$

The leptons are assigned as follows:

$$\begin{pmatrix}\nu_e\\e\end{pmatrix}_L=(\tfrac{1}{2})^{(-1/2,0)}\,,\ e_R=(0)^{(-1,0)}\,, \quad (2.24)$$

and similarly for muonic leptons.

Evidently the scalar multiplets needed to generate lepton mass are distinct from those which yield quark masses. For the former purpose one $(\frac{1}{2})^{(1/2,0)}$ suffices. For the latter, we introduce three doublets called ϕ, χ, and η each of which are $(\frac{1}{2})^{(0,-1/2)}$. Obviously these Higgs fields give mass to the charged and to the two neutral vector mesons. For the present purpose the precise nature of the neutral vector normal modes does not concern us. In any event the usual constraints on gauge models imposed by the bound on strangeness-changing effects can be met.

Just as for the case considered previously we now seek for a natural symmetry shared by the vector-meson and the scalar-meson interactions. The vector-meson couplings are invariant under

$$P_R\to N_R\to-P_R\,;\quad P'_R\to N'_R\to-P'_R\,;\quad C_\mu\to-C_\mu\,, \quad (2.25)$$

all else unchanged. This invariance applies also to the following Higgs coupling:

$$a(\bar{P}_R\phi+\bar{N}_R\tilde{\phi})\Psi_L+b(\bar{P}'_R\phi+\bar{N}'_R\tilde{\phi})\Psi_L$$
$$+c(\bar{P}_R\chi+\bar{N}_R\tilde{\eta})\Psi'_L+d(\bar{P}'_R\chi+\bar{N}'_R\tilde{\eta})\Psi'_L+\text{H.c.}\,, \quad (2.26)$$

provided we extend (2.25) to

$$P_R \to N_R \to -P_R,\quad P_R' \to N_R' \to -P_R',\quad C_\mu \to -C_\mu\,, \tag{2.27}$$

$$\phi \to \tilde{\phi},\quad \chi \to \tilde{\eta},\quad \eta \to \tilde{\chi}\,.$$

Equation (2.27) does not yet force the Yukawa couplings to have the form (2.26), but we can impose a second discrete invariance

$$\chi \to i\chi,\quad \eta \to -i\eta,\quad \Psi_L' \to -i\Psi_L'\,,\ \text{AEU}\,. \tag{2.28}$$

Now the form (2.26) is unique. Note that (2.27) and (2.28) do not affect the leptons and their Higgs doublet, so for what follows we can ignore the entire lepton sector.

When the Higgs mesons develop vacuum expectation values, the quark mass matrix becomes

$$(\bar{P}_R\ \ \bar{P}_R')\begin{pmatrix} A & C \\ B & D \end{pmatrix}\begin{pmatrix} P_L \\ P_L' \end{pmatrix} + (\bar{N}_R\ \ \bar{N}_R')\begin{pmatrix} A & \alpha C \\ B & \alpha D \end{pmatrix}\begin{pmatrix} N_L \\ N_L' \end{pmatrix} + \text{H.c.}\,, \tag{2.29}$$

which involves five parameters which we take to be real for simplicity (this can be done naturally by imposition of a CP invariance). In terms of these five parameters, we can express the eight physical quantities, four masses and four angles which describe the zeroth-order mass eigenstates. Therefore, there are three zeroth-order relations among these eight quantities. One of these involves only the Cabibbo angle and the quark masses. It is

$$\tfrac{1}{2}\sin 2\theta = \frac{[-(m_{\mathcal{P}'}{}^2 - m_\lambda{}^2)(m_{\mathcal{P}'}{}^2 - m_{\mathfrak{N}}{}^2)(m_{\mathcal{P}'}{}^2 - m_\lambda{}^2)(m_{\mathcal{P}'}{}^2 - m_{\mathfrak{N}}{}^2)]^{1/2}}{(m_{\mathcal{P}'}{}^2 - m_{\mathcal{P}}{}^2)(m_\lambda{}^2 - m_{\mathfrak{N}}{}^2)}\ \frac{(m_{\mathcal{P}'}m_\lambda + m_{\mathcal{P}}m_{\mathfrak{N}})(m_{\mathcal{P}'}m_{\mathfrak{N}} + m_{\mathcal{P}}m_\lambda)}{(m_{\mathcal{P}'}m_{\mathcal{P}} + m_\lambda m_{\mathfrak{N}})^2}\,. \tag{2.30}$$

This result has its physical limitations. Its consistency demands (among other things) that $m_{\mathcal{P}}{}^2 > m_{\mathfrak{N}}{}^2$, contrary to naive-quark-model expectations. Nevertheless, we believe it is of some interest to display two distinct categories in which θ attains a natural value, whether a nice one or not: one in which θ is a "pure number" as in Eq. (2.16), and one in which θ is a natural function of particle masses as in Eq. (2.30).

C. Comments on *CP* violation

As is well known, important constraints on gauge models follow from the requirements that $|\Delta S| = 1$ and $|\Delta S| = 2$ effects shall be sufficiently suppressed. Thus it is customarily assumed that $\bar{\lambda}\mathfrak{N}$ and $\bar{\mathfrak{N}}\lambda$ terms shall be entirely absent in neutral currents. Beyond that, additional suppression is needed even for $|\Delta S| = 1, 2$ effects mediated by two virtual vector bosons, and by real Higgs scalars. In standard $SU(2)\times U(1)$ models (except one to which we shall come presently) this is achieved as follows: The $\mathcal{P}, \mathfrak{N}, \lambda$ quarks appear in the following two equivalent representations (again, $\mathfrak{N}_c = \mathfrak{N}\cos\theta + \lambda\sin\theta$, $\lambda_c = -\mathfrak{N}\sin\theta + \lambda\cos\theta$):

$$(\ldots\mathcal{P}, \mathfrak{N}_c, \ldots)_L,\quad (\ldots\mathcal{P}', \lambda_c, \ldots)_L\,.$$

Here the $\mathcal{P}'$ is an additional quark which (in some sense or other) is charmed and ... denotes the (possible) presence of other particles. In addition, $\mathfrak{N}_R$ and λ_R are assigned in such a way that they do not contribute to the effects in question. Then the $\Delta S = 2$ transition $\bar{\lambda}\mathfrak{N} \to \bar{\mathfrak{N}}\lambda$ due to exchange of a virtual W^+, W^- pair is proportional to

$$\alpha^2 \sin^2\theta \left[\frac{m_{\mathcal{P}}{}^2 - m_{\mathcal{P}'}{}^2}{M^2}\right]^2\,, \tag{2.31}$$

where $\alpha = \frac{1}{137}$. The mass ratio suppression is due to the action of the Glashow-Iliopoulos-Maiani mechanism.[22] It is this need for some such additional suppression which has led to a proliferation of quark states typical for all gauge models in their present state of development. (Contributions due to virtual Higgs exchange are most often ignored on the ground that the Higgs-scalar masses may be assumed to be sufficiently heavy.)

For our present discussion, the occurrence of the $\sin^2\theta$ factor is of interest. It shows that the Cabibbo angle plays the role of the real (CP-conserving) "mass mixing" parameter in the K-$\bar{K}$ system. Thus this mixing is phenomenological to the extent that θ is phenomenological. In all gauge models with CP violation proposed so far, the imaginary (CP-violating) mass mixing enters via the introduction of one or more new and additional angles which appear in phase factors. We shall briefly indicate here that in those gauge models which fall under the heading of this section, these additional angles are also phenomenological parameters, much like the Cabibbo angle.

Consider for example a variant of the $SU(2)\times U(1)$ variety[23] with representation content: $(\mathcal{P}, \mathfrak{N}_c)_L$, $(\mathcal{P}', \lambda_c)_L$, $(\mathcal{P}', \mathfrak{N}\cos\phi + i\lambda\sin\phi)_R$, all doublets; all other quark states singlets. An upper bound on ϕ follows from physical constraints on $|\Delta S|=1$ neutral current effects. On the other hand, this typically "on-shell" model also has a lower bound on ϕ such as to give the right order of magnitude for CP-violating effects.

(For this as for any on-shell model, the nonleptonic $|\Delta I|=\frac{1}{2}$ rule for CP-violating effects remains unexplained.[24]) A value for $\phi\sim 10^{-4}$ appears acceptable at this stage.

The most general Higgs system which couples to the quark states can contain the weak isospin representations: singlet (mass terms), doublet, and triplet. Such a general system was used in Ref. 22. It is clear that under such circumstances there is no possibility for natural mass relations. As a result, the parameters ϕ, θ, and the fermion masses are an independent phenomenological set. What happens is the occurrence of a new and relatively imaginary coupling constant $ig\sin\phi$ which is subject to separate renormalization.

The question arises whether it is possible to restrict the Higgs system in such a way that constraints appear which involve quark masses as well as parameters θ, ϕ, the constraints being due to diagonalization conditions. This is possible by restricting the Higgs content in such a way that triplets are not introduced. The ensuing constraint relations[25] have not encouraged us to pursue further the question whether an appropriate Higgs system would guarantee that θ, ϕ are natural.

The gauge model briefly reviewed here is of the "small parameter" variety, in the sense that an additional parameter (ϕ in this case) is introduced for the explicit purpose of generating CP-violating effects. The smallness of these effects is associated with the smallness of the parameter in the scale set by θ, the "real" $K\bar{K}$ mixing parameter. The main point we wished to bring out is that, in general, one must be prepared for the fact that such a parameter is phenomenological.

This also applies to a recently studied variant of the $O(3)$ variety where CP violation is spontaneous.[26] Here the neutral current effects enter differently and the phases can be introduced in such a way that they are unconstrained by $\Delta S=1$ effects (and may therefore be large). The number of Yukawa couplings between Higgs mesons and fermions, allowed by strict renormalizability, is too large to permit the phases to appear in natural zeroth-order relations involving only fermion mass and θ, so that these phases remain phenomenological. Their sines enter as proportionality factors in all CP-violating effects, the scale of which is set by the magnitude of Higgs-meson masses and Higgs-field vacuum expectation values.[17,26]

III. GAUGE THEORIES OF THE O(4) × U(1) TYPE

A. Some general features

The covariant derivative for this group is given by

$$D_\mu=\partial_\mu - ig(\vec{A}_\mu\cdot\vec{t}+\vec{C}_\mu\cdot\vec{\rho})-ig'B_\mu Y\ , \qquad (3.1)$$

with $\vec{t}\times\vec{t}=i\vec{t}$, $\vec{\rho}\times\vec{\rho}=i\vec{\rho}$. $\vec{t}$ and $\vec{\rho}$ commute and so does the weak hypercharge Y with both. The electric charge operator is $Q=t_3+\rho_3+Y$, so that

$$g=e\sqrt{2}/\sin\gamma,\quad g'=e/\cos\gamma,\quad e=gg'(g^2+2g'^2)^{-1/2}\ , \qquad (3.2)$$

and γ is the mixing angle of the theory. The reflection operation R with respect to O(4) is $R:\vec{t}\rightarrow\vec{\rho}$. Introduce the following orthonormal set of gauge fields (which are all orthogonal to the electromagnetic field):

$$\begin{aligned}
W^1&=\tfrac{1}{2}[A^1-C^1-i(A^2-C^2)]\ ,\\
W^2&=\tfrac{1}{2}[A^1+C^1-i(A^2+C^2)]\ ,\\
Z&=\frac{1}{\sqrt{2}}[A^3-C^3]\ ,\\
V&=\frac{1}{\sqrt{2}}[(A^3+C^3)\cos\gamma-B\sqrt{2}\sin\gamma]\ .
\end{aligned} \qquad (3.3)$$

$W^{1,2}$ and their conjugates represent singly charged vector fields (we suppress their μ index); Z, V are neutral. Moreover, all these fields are eigenstates of R: W^1 and Z are R-odd, the others (and the electromagnetic field) are R-even.

Equation (3.3) is a trivial rearrangement in the symmetry limit where all vector bosons are massless. However, we shall wish to retain $W^{1,2}$, Z, and V as zeroth-order normal modes upon spontaneous symmetry breakdown. This major constraint has the following implications:

(1) The vacuum expectation values of the Higgs mesons must satisfy R invariance in order to guarantee that the zeroth-order vector-meson mass matrix be R-invariant.

(2) In turn, the Higgs surface must be constrained in a natural way so as to force the R invariance of the vacuum expectation values.

(3) In turn, the Higgs-fermion Yukawa couplings must be naturally compatible with all symmetry conditions implied by (1) and (2) *and* must properly diagonalize the fermion mass matrix in the tree approximation. This strongly delimits the choice of fermion representations. All these points will be explicitly demonstrated in the example discussed in Sec. III B.

Let us suppose that this is achieved and that, moreover, after spontaneous symmetry breaking the charged-current couplings given by Eq. (1.3) and the condition Eq. (1.4) are natural.[27] Then

μe universality will be natural for charge-carrying currents, the CP-violating phases will be natural, and the Cabibbo angle will satisfy the relation Eq. (1.5). Furthermore, in a model of this kind, the charged vector bosons do not mediate strangeness-changing (and charm conserving) nonleptonic weak interactions. Instead, these interactions are mediated by the neutral Z boson, and it is possible to implement a natural $\Delta I = \frac{1}{2}$ rule.

In all published models, the R invariance of the Higgs system is unnatural.

(a) O(4)× U(1). The first model of this kind to be proposed was based on the group O(4), a special case of O(4)×U(1). Left-handed fermions (f^L) were taken to be 4-vectors in O(4). However, no representation for the right-handed fermions (f^R) was found[28] which was consistent with R invariance of the vacuum expectation values of the Higgs mesons. As a result, an $O(\alpha)$ logarithmic divergence in the CP-violating part of the W^1-W^2 mixing was generated by single virtual lepton loops.[29] Indeed, it was for this reason that the study of O(4)× U(1) was initiated.

(b) O(4)× U(1), vector model.[30] Here the f^L are again 4-vectors ($Y=0$), but the f^R are O(4) scalars with $Y=Q$, the electric charge. Higgs mesons are also 4-vectors (with either $Y=0$ or $Y=\pm1$). In this model, the vacuum expectation values of the Higgs mesons could be chosen to be R-invariant, but the choice was unnatural: The logarithmic divergence mentioned above persisted. It was then noted that this divergence could be eliminated by requiring the equality of two neutral lepton masses.[31]

At this point, the present authors took up the problem and began by inquiring whether this model with the lepton mass relation could have a natural R invariance of the Higgs system. We discovered that it did not. However, by enlarging both the Higgs and the lepton system we could "push back" the lack of naturalness so that the leading order in which the logarithmic W^1-W^2 mixing divergence appears is given by Eq. (1.7). We were able to show, furthermore, that this result cannot be improved further, except for the uninteresting case $\theta = 45°$ (where all naturalness conditions can be met). We were not content with the argument that the coefficient of this logarithmic divergence is quite small. Considerations such as these led us to consider the whole question of the Higgs system in more detail and stimulated the investigations of this paper.

(c) O(4)×U(1), spinor representations. Meanwhile, it was noted[15] that most of the same weak interaction properties could be incorporated in an O(4)×U(1) model in which the f^L transform as 4-spinors. In this kind of model, there are necessarily "elastic" neutral currents (though not for both ν_e and ν_μ). For pure spinor models we were again unable to ensure naturalness. Although, as noted in Ref. 15, the CP-violating mixing is finite here to $O(\alpha)$ without any lepton mass constraints, the strict implementation of the needed R invariance causes trouble, once again to an order which cannot be improved beyond Eq. (1.7). However, by enlarging the gauge group to O(4)×U(1)×U(1) and enlarging the lepton content, we were able to implement naturalness in pure spinor models.

In the next subsection, we will describe in detail an O(4)×U(1) model which can be made natural. It employs the quark spinor structure of Ref. 15, but is hybrid as far as leptons are concerned. For the latter, the left muon- (or electron-) type states transform as spinors, while the left electron (or muon) states transform like the adjoint representation of O(4). We shall discover a number of natural mass relations between the fermions. Amongst these there appear zeroth-order mass degeneracies of neutral lepton pairs. This is reminiscent of what was tried[14] for the vector model. But now these degeneracies are truly natural.

Before turning to this, we make a brief comment in regard to the naturalness of the nonleptonic $\Delta I = \frac{1}{2}$ rule, since the appearance of this rule is one of the themes for all the gauge models considered in this section. To the extent that the isospin assignments of the quark states used in these models can eventually be part of a sensible strong-interaction picture, the $\Delta I = \frac{1}{2}$ rule is natural to these models, in the sense that no constraints are involved on coupling constants and/or masses to implement the argument.[32] One may ask the same question for alternative schemes to arrive at this rule. There are two main ideas here: (1) octet dominance, where the $\Delta I = \frac{1}{2}$ rule emerges due to strong-interaction enhancement effects (from the point of view of weak-electromagnetic gauge theories, the question of naturalness is moot here), and (2) the schizon scheme[33] where the $\Delta I = \frac{1}{2}$ rule comes about via the $\Delta I = \frac{3}{2}$ cancellation between a neutral vector-meson coupling (constant g_0, vector mass M_0) and a charged coupling (constant g, vector mass M). It is readily seen that (possibly up to a known Clebsch-Gordan coefficient) the schizon scheme demands the validity of the relation

$$\frac{g_0^2}{M_0^2} = \frac{g^2}{M^2} \sin\theta \cos\theta . \tag{3.4}$$

In the context of a gauge theory this introduces new demands of naturalness (since in general g_0, g, M_0, M will suffer independent renormalizations). This was emphasized by Bég, who recently obtained a realization of the schizon scheme in the context of a gauge model.[34]

We conclude this subsection with two remarks on the order of magnitude estimates for electric dipole moments given elsewhere.[19] First, these estimates for the O(4)× U(1) vector model remain unaffected but, as said, the neutral heavy lepton degeneracy employed there is not natural. Second, these qualitative estimates apply as well to the natural model to be discussed next.

B. A detailed example

In this section we discuss in detail the simplest model we know of with the following properties: μe universality (restricted) is natural; *CP* violation is natural and maximal, and *CP*-violating effects depend in leading order only on fermion and vector-meson masses and the gauge coupling constants; the Cabibbo angle satisfies Eq. (1.5), and there is a natural $\Delta I = \frac{1}{2}$ rule for nonleptonic strangeness-changing processes. As mentioned earlier, this model is based on the gauge group O(4)×U(1) with the right-handed fermions O(4) singlets and the left-handed fermions transforming as 4-spinors and six component tensors (in the adjoint representation).

In the present stage of development, any model in this class always has a counterpart in which the representation content of electronic and muonic leptons are interchanged. Physically, one case differs from the other, for example, in the way ν_e and ν_μ enter in the neutral currents. The case to be described next allows to $O(G)$ for $\nu_\mu + N \to \nu_\mu +$ zero charm hadron system, while the corresponding ν_e reaction is forbidden to this order. The alternative solution allows for "elastic" ν_e but not for ν_μ processes. Now to the example.

A 4-spinor is a pair of doublets (u, v) where u transforms as a doublet under the SU(2) subgroup generated by $\vec{t}$ and v transforms as a doublet under the SU(2) subgroup generated by $\vec{\rho}$. Under the R operation, u and v are interchanged.

The left-handed muonic leptons transform like a pair of 4-spinors with $Y = -\frac{1}{2}$:

$$u_1 = \begin{pmatrix} \nu_\mu \\ (\mu^- + M^-)/\sqrt{2} \end{pmatrix}_L, \quad v_1 = \begin{pmatrix} M^0 \\ (\mu^- - M^-)/\sqrt{2} \end{pmatrix}_L,$$

$$u_2 = \begin{pmatrix} O^0 \\ (o^- + O^-)/\sqrt{2} \end{pmatrix}_L, \quad v_2 = \begin{pmatrix} \nu_0 \\ (o^- - O^-)/\sqrt{2} \end{pmatrix}_L, \tag{3.5a}$$

where ν_0 is supposed to be a second massless neutrino.

The left-handed quarks transform like a pair of 4-spinors with $Y = \frac{1}{2}$:

$$u_3\sqrt{2} = \begin{pmatrix} \mathcal{P} + \mathcal{P}' \\ (\mathfrak{N} + \lambda)/\sqrt{2} + q^0 \end{pmatrix}_L, \quad v_3\sqrt{2} = \begin{pmatrix} \mathcal{P} - \mathcal{P}' \\ (-\mathfrak{N} + \lambda)/\sqrt{2} - r^0 \end{pmatrix}_L, \tag{3.5b}$$

$$u_4\sqrt{2} = \begin{pmatrix} q + r \\ (\mathfrak{N} - \lambda)/\sqrt{2} - r^0 \end{pmatrix}_L, \quad v_4\sqrt{2} = \begin{pmatrix} q - r \\ -(\mathfrak{N} + \lambda)/\sqrt{2} + q^0 \end{pmatrix}_L.$$

q^0, r^0 are neutral. $\mathcal{P}'$, q, and r are positive.

The six-component tensor representation is a pair of triplets (U, V), where U transforms like a 3-vector under $\vec{t}$ and V transforms like a 3-vector under $\vec{\rho}$. In other words, (U, V) transforms like $(\vec{t}, \vec{\rho})$, so this is the adjoint representation. Again, the R operation interchanges U and V. The electronic leptons transform like a pair of these representations with $Y = 0$:

$$U_1\sqrt{2} = \begin{pmatrix} g^+ + h^+ \\ (\nu_e + N^0)/\sqrt{2} + x^0 \\ e^- + f^- \end{pmatrix}_L, \quad V_1\sqrt{2} = \begin{pmatrix} g^+ - h^+ \\ i[-(\nu_e + N^0)/\sqrt{2} + x^0] \\ e^- - f^- \end{pmatrix},$$

$$U_2\sqrt{2} = \begin{pmatrix} G^+ + H^+ \\ (\nu_e - N^0)/\sqrt{2} - X^0 \\ E^- + F^- \end{pmatrix}_L, \quad V_2\sqrt{2} = \begin{pmatrix} G^+ - H^+ \\ i[(\nu_e - N^0)/\sqrt{2} + X^0] \\ E^- - F^- \end{pmatrix}. \tag{3.5c}$$

All right-handed fermion fields are O(4) singlets with $Y = Q$. We will have to include right-handed neutrino fields, for naturalness, even though the neutrinos are massless. However, before we discuss the Higgs-meson system and the Yukawa couplings in detail, a few comments are in order.

(1) The charged currents do have the form Eq. (1.3) where all additional terms involve heavy fermions so far unobserved. This is easily verified from Ref. 15, Eqs. (6)–(8) which hold for any representation content of O(4)×U(1). Indeed since the quark structure Eq. (3.5b) is as in that

paper, the detailed hadronic contributions to all currents are as given explicitly in Ref. 15.

(2) The required form Eq. (1.3) could also have been achieved with a much simpler f^L system (for example, either four 4-vectors in all[14] or four 4-spinors in all[15]) if it were not that we are concerned here about naturalness.

(3) It is also this concern which leads us to introduce the spinor (u_2, v_2) which, as the alert reader will have noticed, only involves unobserved fermions.

(4) The detailed discussion of the currents in this model is not our present concern, except for the remark that the natural μe universality here is restricted to the pairs of currents coupled to W^1 and W^2. Indeed, the neutral vector meson Z_μ [see Eq. (3.3)] is coupled to the operator $t_3 - \rho_3$, from which it follows that the amplitude for $Z \to \overline{\nu}_\mu + \nu_\mu$ is $O(G)$, while the amplitudes for $Z_\mu \to \mu + \overline{\mu}$, or $e + \overline{e}$, or $\nu_e + \overline{\nu}_e$ are each $O(G\alpha)$. The neutral vector meson V_μ is coupled to $t_3 + \rho_3 - Q\sin^2\gamma$ [γ as in Eq. (3.2)]. Thus the associated current contains $\overline{\nu}_\mu \nu_\mu$; $\overline{\mu}\mu$; $\overline{e}e$ but no $\overline{\nu}_e \nu_e$ terms, so that the (calculable) ratio for rates of the processes Eqs. (1.2) and (1.1) is proportional to α^2. (We repeat that lack of universality for certain currents does not imply lack of calculability for models of this kind.)

(5) The model is free of anomalies. This is still true, even if we replace the eight integrally charged quarks by eight "color" triplets of fractionally charged quarks, where the color SU(3) commutes with the weak and electromagnetic gauge group.

(6) If we were not concerned about naturalness, we could give arbitrary masses to the fermions with only three representations of Higgs mesons: one 4-spinor with $Y = \frac{1}{2}$, one 6-tensor with $Y = 0$, and one 6-tensor with $Y = 1$. Instead we will need ten 4-spinors with $Y = \frac{1}{2}$, three 6-tensors with $Y = 0$, and two 6-tensors with $Y = 1$ and the fermion masses will satisfy various mass relations.

The basic strategy in constructing the model is to write down a set of Yukawa couplings such that when the Higgs mesons develop R-invariant vacuum expectation values, the fermion masses are generated consistent with Eq. (3.5). The Yukawa couplings should have enough discrete symmetries to ensure their uniqueness and furthermore, these symmetries must prevent the appearance of Higgs-meson self-couplings which would spoil the naturalness of the R-invariant vacuum expectation values. The list of discrete symmetries will include CP and an R invariance, which in general will be different from the R invariance of the vacuum expectation values. These two symmetries act nontrivially on all the fields. There will also be symmetries which act, for instance, only on the muonic lepton fields. Because of these latter symmetries, the strongest constraints on the system are always obtained by considering only a piece of the model, either the muon system, the quark system, or the electron system at any one time. So in what follows, we will discuss the three subsystems separately.

First consider the muon system. The Higgs mesons needed to generate the fermion masses are 4-spinors with $Y = \frac{1}{2}$, which are pairs of doublets (α, β). To ensure R invariance of the zeroth-order vector meson mass matrix, we must require that for each such pair the vacuum expectation values satisfy $\langle\alpha\rangle = \langle\beta\rangle$. The muon system requires four such spinors. The Yukawa couplings are

$$A[\overline{u}_1\tilde{\alpha}_1\nu_{\mu R} + \overline{v}_2\tilde{\beta}_1\nu_{0R} + \overline{u}_2\tilde{\alpha}_2 O_R^0 + \overline{v}_1\tilde{\beta}_2 M_R^0] + B[(\overline{u}_1\alpha_3 + \overline{v}_1\beta_3)\mu_R^- + (\overline{u}_2\alpha_3 + \overline{v}_2\beta_3)o_R^- + (\overline{u}_1\alpha_4 - \overline{v}_1\beta_4)M_R^- + (\overline{u}_2\alpha_4 - \overline{v}_2\beta_4)O_R^-]$$
$$+ C[(\overline{u}_1\alpha_4 + \overline{v}_1\beta_4)\mu_R^- - (\overline{u}_2\alpha_4 + \overline{v}_2\beta_4)o_R^- - (\overline{u}_1\alpha_3 - \overline{v}_1\beta_3)M_R^- + (\overline{u}_2\alpha_3 - \overline{v}_2\beta_3)O_R^-] + \text{H.c.} \quad (3.6)$$

Here $(\tilde{\alpha}, \tilde{\beta}) = i(\tau_2\alpha^*, \tau_2\beta^*)$ is a 4-spinor with $Y = \frac{1}{2}$. The constants A, B, and C are real, so CP is a good symmetry. Suppressing space-time variables, the Higgs mesons transform as follows under CP: $\alpha_i \to \alpha_i^\dagger$ and $\beta_i \to \beta_i^\dagger$ for $i = 1$ to 4. Under the R symmetry, the fields in Eq. (3.6) transform as follows: $\alpha_i \leftrightarrow \beta_i$ for $i = 1$ to 3, $\alpha_4 \leftrightarrow -\beta_4$, $u_1 \leftrightarrow v_2$, $u_2 \leftrightarrow v_1$, $\nu_{\mu R} \leftrightarrow \nu_{0R}$, $M_R^0 \leftrightarrow O_R^0$, $\mu_R^- \leftrightarrow o_R^-$, $M_R^- \leftrightarrow O_R^-$. This is not the R invariance of the vacuum expectation values, because of the transformation $\alpha_4 \leftrightarrow -\beta_4$ instead of $\alpha_4 \leftrightarrow \beta_4$.

In addition to these two symmetries which act nontrivially on the electronic leptons and the quarks as well as on the muons, we introduce the following symmetries in order to force the Yukawa couplings to have the form Eq. (3.6).

Separate conservation of muon number and o number:

$$\alpha_1 \to -\alpha_1, \ \nu_{\mu R} \to -\nu_{\mu R}, \quad (3.7a)$$

$$\beta_1 \to -\beta_1, \ \nu_{0R} \to -\nu_{0R}, \quad (3.7b)$$

$$\alpha_2 \to -\alpha_2, \ O_R^0 \to -O_R^0, \quad (3.7c)$$

$$\beta_2 \to -\beta_2, \ M_R^0 \to -M_R^0, \quad (3.7d)$$

$$\begin{aligned}&\alpha_1 \leftrightarrow \alpha_2,\ \beta_1 \leftrightarrow \beta_2,\ \alpha_4 \leftrightarrow -\alpha_4,\ \beta_4 \leftrightarrow -\beta_4\ ,\\ &u_1 \leftrightarrow u_2,\ v_1 \leftrightarrow v_2,\ \nu_{\mu R} \leftrightarrow O_R^0, &(3.7\text{e})\\ &\nu_{0R} \leftrightarrow M_R^0,\ \mu_R^- \leftrightarrow o_R^-,\ M_R \leftrightarrow O_R^-\ ,\end{aligned}$$

$$\begin{aligned}&\alpha_3 \to \alpha_4 \to -\alpha_3,\ \beta_3 \to \beta_4 \to -\beta_3,\\ &v_1 \to -v_1,\ v_2 \to -v_2,\ \nu_{0R} \to -\nu_{0R}\ , &(3.7\text{f})\\ &M_R^0 \to -M_R^0,\ \mu_R^- \to M_R^- \to -\mu_R^-,\ o_R^- \to O_R^- \to -o_R^-\ .\end{aligned}$$

In each of these, if a field does not appear, it is unchanged by the transformation. With the exception of Eq. (3.7f) (which is to be broken by quadratic Higgs terms), these transformations are required to be exact symmetries of the Lagrangian.

If the Higgs mesons develop the vacuum expectation values $\langle\alpha_1\rangle = \langle\beta_1\rangle = 0$, $\langle\alpha_i\rangle = \langle\beta_i\rangle \neq 0$ for $i = 2$ to 4, and $\langle\alpha_3\rangle \neq \langle\alpha_4\rangle$, then the fermion mass eigenstates are as shown in Eq. (3.5a), with the mass relations

$$\begin{aligned}&m_{\nu_\mu} = m_{\nu_0} = 0,\ m_{M^0} = m_{O^0}\ ,\\ &m_{\mu^-}{}^{2} + m_{M^-}{}^{2} = m_{o^-}{}^{2} + m_{O^-}{}^{2}\ ,\end{aligned} \qquad (3.8)$$

so that of the eight muon-system masses, only four are independent. Our task now is to show that these vacuum expectation values are natural.

First consider the condition $\langle\alpha_1\rangle = \langle\beta_1\rangle = 0$. Because of the symmetries (3.7a) and (3.7b), these vacuum expectation values are necessarily extremal, because α_1 and β_1 must appear quadratically. For some range of the parameters in the scalar-meson potential, $\langle\alpha_1\rangle = \langle\beta_1\rangle = 0$ will minimize the action. Similarly, the condition $\langle\alpha_i\rangle = \langle\beta_i\rangle$ will be extremal (and minimal for some range of parameters) if the Higgs-meson self-interactions are invariant under the interchange $\alpha_i \leftrightarrow \beta_i$. This is a sufficient, not a necessary condition, but for the muon subsystem it is satisfied.

The only terms which could spoil this invariance are those in which an odd number of α_4 or β_4 fields appear, because, if there are an even number, the true R invariance of the Lagrangian, which involves $\alpha_4 \leftrightarrow -\beta_4$, has the same effect as the R invariance we want, $\alpha_4 \leftrightarrow \beta_4$. The quadratic self-couplings are obviously invariant, as are the quartic terms which involve only α_1, β_1, α_2, and β_2. The quartic terms involving only α_3, β_3, α_4, and β_4 are also invariant because of the symmetry (3.7e). So the only possible problems are terms like $(\alpha_1^\dagger\alpha_3)(\alpha_1^\dagger\alpha_4)$ or $(\alpha_1^\dagger\alpha_1)(\alpha_3^\dagger\alpha_4)$ or others of this type. The first term is forbidden by Eq. (3.7f), while the second is forbidden by Eq. (3.7f) and CP. Finally, Eq. (3.7f) cannot be an exact symmetry of the Lagrangian because it would imply $\langle\alpha_3\rangle = \pm\langle\alpha_4\rangle$ which would yield unwanted mass degeneracies among the charged leptons. So, as said above, this symmetry must be broken by mass terms.

This concludes the discussion of the muon sector. Before going on to a similar study of the other sectors, we should emphasize that the tricks we have used here to enforce naturalness are not special to this model. Indeed, features like the appearance of right-handed neutrino fields, discrete symmetries broken by mass terms, and the quadratic mass relation have already been encountered in the discussion of simpler models. Here they are all necessary, along with the existence of a pair of neutral lepton fields degenerate in zeroth order. We expect that some of these features will be necessary in any model in which naturalness depends on discrete symmetry structure.

For the quark sector, we need six more 4-spinors with $Y = \frac{1}{2}\ (\alpha_i, \beta_i)$ for $i = 5$ to 10. The Yukawa couplings are

$$\begin{aligned}&D\{(\bar u_3\alpha_5 + \bar v_4\beta_5)q_R^0 + (\bar u_4\alpha_5 + \bar v_3\beta_5)r_R^0 + (\bar u_3\alpha_6 + \bar v_4\beta_6)q_R^0 - (\bar u_4\alpha_6 + \bar v_3\beta_6)r_R^0\}\\ &\quad + E\{[(\bar u_3 - \bar u_4)\alpha_7 + (\bar v_3 - \bar v_4)\beta_7]\lambda_R + [(\bar u_3 + \bar u_4)\alpha_7 - (\bar v_3 + \bar v_4)\beta_7]\mathfrak{N}_R\}\\ &\quad + F\{[(\bar u_3 - \bar u_4)\alpha_8 + (\bar v_3 - \bar v_4)\beta_8]\lambda_R + [-(\bar u_3 + \bar u)\alpha_8 + (\bar v_3 + \bar v_4)\beta_8]\mathfrak{N}_R\}\\ &\quad + G\{(\bar u_3\tilde\alpha_9 + \bar v_3\tilde\beta_9)\mathcal{P}_R + (\bar u_4\tilde\alpha_9 + \bar v_4\tilde\beta_9)q_R + (\bar u_3\tilde\alpha_{10} - \bar v_3\tilde\beta_{10})\mathcal{P}'_R + (\bar u_4\tilde\alpha_{10} - \bar v_4\tilde\beta_{10})r_R\}\\ &\quad + H\{(\bar u_3\tilde\alpha_{10} + \bar v_3\tilde\beta_{10})\mathcal{P}_R - (\bar u_4\tilde\alpha_{10} + \bar v_4\tilde\beta_{10})q_R - (\bar u_3\tilde\alpha_9 - \bar v_3\tilde\beta_9)\mathcal{P}'_R + (\bar u_4\tilde\alpha_9 - \bar v_4\tilde\beta_9)r_R\} + \text{H.c.}\end{aligned} \qquad (3.9)$$

Again we want $\langle\alpha_i\rangle = \langle\beta_i\rangle$ for all i, and $\langle\alpha_5\rangle \neq \langle\alpha_6\rangle$ and $\langle\alpha_9\rangle \neq \langle\alpha_{10}\rangle$. The constants $D - H$ are real, so CP is a good symmetry, and again takes $\alpha_i \to \alpha_i^\dagger$ and $\beta_i \to \beta_i^\dagger$.

The R invariance is $\alpha_i \leftrightarrow \beta_i$ for $i = 5$, and 7 to 10, $\alpha_6 \leftrightarrow -\beta_6$, $u_3 \leftrightarrow v_3$, $u_4 \leftrightarrow v_4$, $q_R^0 \leftrightarrow r_R^0$, $\mathfrak{N}_R \leftrightarrow -\mathfrak{N}_R$, $\mathcal{P}'_R \leftrightarrow -\mathcal{P}'_R$, $r_R \leftrightarrow -r_R$.

The invariance peculiar to the quark sector are quark number conservation and the following:

$$\alpha_5 \to -\alpha_5,\ \beta_5 \to -\beta_5,\ \alpha_6 \to -\alpha_6,\ \beta_6 \to -\beta_6,\ q_R^0 \to -q_R^0,\ r_R^0 \to -r_R^0\ , \tag{3.10a}$$

$$\alpha_7 \to -\alpha_7,\ \beta_7 \to -\beta_7,\ \alpha_8 \to -\alpha_8,\ \beta_8 \to -\beta_8,\ \mathfrak{N}_R \to -\mathfrak{N}_R,\ \lambda_R \to -\lambda_R\ , \tag{3.10b}$$

$$\alpha_9 \to -\alpha_9,\ \beta_9 \to -\beta_9,\ \alpha_{10} \to -\alpha_{10},\ \beta_{10} \to -\beta_{10},\ \mathcal{P}_R \to -\mathcal{P}_R,\ \mathcal{P}'_R \to -\mathcal{P}'_R,\ q_R \to -q_R,\ r_R \to -r_R\ , \tag{3.10c}$$

$$\alpha_6 \to -\alpha_6,\ \beta_6 \to -\beta_6,\ \alpha_{10} \to -\alpha_{10},\ \beta_{10} \to -\beta_{10},\ u_3 \leftrightarrow u_4,\ v_3 \leftrightarrow v_4,\ \mathcal{P}_R \leftrightarrow q_R,\ \mathcal{P}'_R \leftrightarrow -r_R,\ q_R^0 \leftrightarrow r_R^0,\ \lambda_R \to -\lambda_R\ , \tag{3.10d}$$

$$\alpha_8 \to -\alpha_8,\ \beta_8 \to -\beta_8,\ \alpha_9 \to -\alpha_9,\ \alpha_{10} \to -\alpha_{10},\ u_4 \to -u_4,\ v_3 \to -v_3,\ \lambda_R \leftrightarrow \mathfrak{N}_R,\ r_R^0 \to -r_R^0,\ \mathcal{P}_R \leftrightarrow -\mathcal{P}_R,\ \mathcal{P}'_R \leftrightarrow -\mathcal{P}'_R\ . \tag{3.10e}$$

$$\alpha_5 \leftrightarrow \alpha_6,\ \beta_5 \leftrightarrow \beta_6,\ r_R^0 \to -r_R^0\ , \tag{3.10f}$$

(to be broken by quadratic Higgs terms).

$$\begin{aligned} &\alpha_9 \to \alpha_{10} \to -\alpha_9,\ \beta_9 \to -\beta_{10} \to -\beta_9, \\ &\mathcal{P}_R \to \mathcal{P}'_R \to -\mathcal{P}_R,\ q_R \to r_R \to -q_R \end{aligned} \tag{3.10g}$$

(to be broken by quadratic Higgs terms). These symmetries force the Yukawa couplings to have the form (3.9) and also ensure that $\langle\alpha_i\rangle = \langle\beta_i\rangle$ is extremal by forcing the Higgs-meson self-couplings to be invariant under the interchange $\alpha_i \leftrightarrow \beta_i$. The quark masses satisfy the quadratic mass relation

$$m_{\mathcal{P}}^2 + m_{\mathcal{P}'}^2 = m_q^2 + m_r^2\ . \tag{3.11}$$

Equation (3.10f) is to be broken as indicated in order to prevent $\langle\alpha_5\rangle = \langle\alpha_6\rangle$ which would suppress strangeness-changing nonleptonic decays. Equation (3.10g) is to be broken in order to prevent unwanted mass degeneracies.

For the electronic lepton system, we need three tensor Higgs-meson representations with $Y=0$, (ϕ_i, χ_i) for $i=1$ to 3 and two with $Y=1$, (ψ_i, ω_i) for $i=1$ or 2. The Yukawa couplings are

$$\begin{aligned} I\{(\overline{U}_1\phi_1 + i\overline{V}_1\chi_1)x_R^0 - (\overline{U}_2\phi_1 - i\overline{V}_2\chi_1)X_R^0\} \\ + J\{[(\overline{U}_1 - \overline{U}_2)\phi_2 - i(\overline{V}_1 + \overline{V}_2)\chi_2]N_R^0 \\ + [(\overline{U}_1 + \overline{U}_2)\phi_3 - i(\overline{V}_1 - \overline{V}_2)\chi_3]\nu_{eR}\} \\ + K\{(\overline{U}_1\tilde{\psi}_1 + \overline{V}_1\tilde{\omega}_1)g_R^+ + (\overline{U}_2\tilde{\psi}_1 + \overline{V}_2\tilde{\omega}_1)G_R^+\} \\ + L\{(\overline{U}_1\tilde{\psi}_2 + \overline{V}_1\tilde{\omega}_2)g_R^+ - (\overline{U}_2\tilde{\psi}_2 + \overline{V}_2\tilde{\omega}_2)G_R^+\} \\ + M\{(\overline{U}_1\tilde{\psi}_1 - \overline{V}_1\tilde{\omega}_1)h_R^+ + (\overline{U}_2\tilde{\psi}_1 - \overline{V}_2\tilde{\omega}_1)H_R^+\} \\ + N\{(\overline{U}_1\tilde{\psi}_2 - \overline{V}_1\tilde{\omega}_2)h_R^+ - (\overline{U}_2\tilde{\psi}_2 - \overline{V}_2\tilde{\omega}_2)H_R^+\} \\ + O\{(\overline{U}_1\psi_1 + \overline{V}_1\omega_1)e_R^- + (\overline{U}_2\psi_1 + \overline{V}_2\omega_1)E_R^-\} \\ + P\{(\overline{U}_1\psi_2 + \overline{V}_1\omega_2)e_R^- + (\overline{U}_2\psi_2 + \overline{V}_2\omega_2)E_R^-\} \\ + Q\{(\overline{U}_1\psi_1 - \overline{V}_1\omega_1)f_R^- + (\overline{U}_2\psi_1 + \overline{V}_2\omega_1)F_R^-\} \\ + R\{(\overline{U}_1\psi_2 - \overline{V}_1\omega_2)f_R^- - (\overline{U}_2\psi_2 - \overline{V}_2\omega_2)F_R^-\} + \text{H.c.} \end{aligned} \tag{3.12}$$

Here $(\tilde{\psi}_i, \tilde{\omega}_i) = (\psi_i^*, \omega_i^*)$ is a tensor with $Y=-1$. The constants I–R are real so the CP invariance involves $\phi_i \to \phi_i^\dagger$, $\chi_i \to -\chi_i^\dagger$, $\psi_i \to \psi_i^\dagger$, $\omega_i \to \omega_i^\dagger$. The R invariance is $\phi_i \to \chi_i \to -\phi_i$, $\psi_i \leftrightarrow \omega_i$, $U_1 \leftrightarrow V_1$, $U_2 \leftrightarrow -V_2$, $x_R^0 \to i x_R^0$, $X_R^0 \to iX_R^0$, $N_R^0 \to -iN_R^0$, $\nu_{eR} \to -i\nu_{eR}$, $h_R^+ \to -h_R^+$, $G_R^+ \to -G_R^+$, $f_R^- \to -f_R^-$, $E_R^- \to -E_R^-$. The other invariances are

$$\phi_1 \to -\phi_1,\ \chi_1 \to -\chi_1,\ x_R^0 \to -x_R^0,\ X_R^0 \to -X_R^0\ , \tag{3.13a}$$

$$\phi_2 \to -\phi_2,\ \chi_2 \to -\chi_2,\ N_R^0 \to -N_R^0\ , \tag{3.13b}$$

$$\phi_3 \to -\phi_3,\ \chi_3 \to -\chi_3,\ \nu_{eR} \to -\nu_{eR}\ , \tag{3.13c}$$

$$\begin{aligned} &\phi_2 \leftrightarrow \phi_3,\ \chi_2 \leftrightarrow \chi_3,\ U_2 \to -U_2,\ V_2 \to -V_2\ , \\ &X_R^0 \to -X_R^0,\ N_R^0 \leftrightarrow \nu_{eR},\ G_R^+ \to -G_R^+\ , \\ &H_R^+ \to -H_R^+,\ E_R^- \to -E_R^-,\ F_R^- \to -F_R^-\ , \end{aligned} \tag{3.13d}$$

$$\begin{aligned} &U_1 \leftrightarrow U_2,\ V_1 \leftrightarrow V_2,\ \phi_1 \to -\phi_1,\ \phi_2 \to -\phi_2,\ \chi_3 \to -\chi_3\ , \\ &\psi_2 \to -\psi_2,\ \omega_2 \to -\omega_2,\ x_R^0 \leftrightarrow X_R^0,\ g_R^+ \leftrightarrow G_R^+,\ h_R^+ \leftrightarrow H_R^+\ , \\ &e_R^- \leftrightarrow E_R^-,\ f_R^- \leftrightarrow F_R^-\ . \end{aligned} \tag{3.13e}$$

These symmetries force the Yukawa couplings to have the form (3.12). Now if the Higgs mesons develop the vacuum expectation values $\langle\phi_3\rangle = \langle\chi_3\rangle = 0$, $\langle\phi_i\rangle = \langle\chi_i\rangle \neq 0$ for $i=1$ or 2 and $\langle\psi_i\rangle = \langle\omega_i\rangle \neq 0$ for $i=1$ or 2, then the fermion mass matrix is consistent with (3.5c) with the mass relation

$$m_{x^0} = m_{X^0}\ . \tag{3.14}$$

All naturalness conditions can be met, as follows. The condition $\langle\phi_3\rangle = \langle\chi_3\rangle = 0$ is extremal because of the γ_5 invariance (3.12c). In this case, we do not need an additional quadratic mass relation to ensure the R invariance of the vacuum expectation values, because terms which involve one of the χ's linearly, such as $(\phi^\dagger\psi)(\omega^\dagger\chi)$, do not affect the vacuum expectation values as long as electromagnetic gauge invariance is not spontaneously broken. So here again, R invariance is natural.

The reader can check that terms which couple Higgs mesons from different subsystems do not spoil naturalness.

We are still not quite finished. All of the Higgs-meson vacuum expectation values we have discussed so far contribute equally to the W^1 and W^2 masses, so we need some additional Higgs structure to avoid the unphysical result, $\tan\theta = 1$. This is easily remedied with a real 4-vector Higgs meson whose vacuum expectation value contributes to the W_2 mass, but not the W_1 mass.

IV. A FINAL COMMENT

We got involved in this investigation by asking what seemed to us at the time to be a rather simple question: Could the desirable properties of the $O(4)\times U(1)$ model be made natural? We have found an answer, but in the process have discovered the very much more important fact that the question itself is far from simple. We could not have foreseen the labyrinth of technical difficulties into which this problem had led us. Possibly, some of the difficulties were self-inflicted, due to our adherence to Higgs-type symmetry breaking; but we know of no other way to ask these detailed questions. We must also admit that we still do not know all the rules of the game we are playing.

The problem is this: There are certain properties which we wish to implement naturally, for instance, 45° angles in the fermion mass matrix or [as in $O(4)\times U(1)$ models] symmetries of the zeroth-order vector meson mass matrix; so the Higgs-meson structure of the theory must be tightly constrained. Not only the Yukawa couplings but also the Higgs vacuum expectation values must be forced to take very specific forms. In some cases, we can satisfy all these constraints by imposing discrete symmetries on the Lagrangian, while in others we can prove that the constraints can never be satisfied. But our results have been obtained at least partially by trial and error. We feel that there must be general strategies for the construction of natural models and further that such general results would be a very important advance in the study of the structure of gauge theories.

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Investigation into dual parafield excitations

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A number of new dual models are presented. They use operators which satisfy parastatistics commutation relations. The properties of these operators are outlined, and the construction of L and G gauge operators from them is described. New amplitudes in which these paraoperators occur instead of, or in addition to, the usual operators are given, and their properties outlined.

I. INTRODUCTION

The concept of a hadron as an extended entity in space has recently received considerable impetus. In a series of investigations[1] it has been shown that the infinitely many independent degrees of freedom associated with the states of a dual model may be interpreted as the transverse harmonic oscillations of a relativistic string. Furthermore, Mandelstam[2] has derived scattering amplitudes for such interacting systems that are closely related to those obtained in the dual resonance model. If the dimensionality of space-time is regarded as a parameter, they can indeed be made to coincide. An attractive feature of the interacting-string approach is the relative ease with which fermions are incorporated into the theory. In common with the operatorial approach this is achieved by introducing extra degrees of freedom in addition to the harmonic excitations which synthesize the motion of the string in time. The resulting Neveu-Schwarz-Ramond (NSR) model[3] has a striking internal consistency which has characterized all dual models with unit-intercept trajectories and an appropriate gauge algebra. At a certain dimensionality of space-time the meson and fermion sectors decouple from negative-norm states and Pomeronic singularities may be interpreted as particles. Even the spectrum of states that appears in the meson sector is intriguingly related to that observed in nature. However, many of the unphysical aspects of dual models are highly correlated and a satisfactory resolution of these difficulties has yet to be found.

In this paper we report on the results of an investigation[4] into the limitations that an alternative method of quantization imposes. In this way we depart from all previous procedures that employ canonical Bose or Fermi commutation relations for the fields that enter into the theory. It must be admitted that the motivation for this approach stems from the physical interpretation of a dual hadron as an extended system that can sustain excitations in addition to orbital and spin degrees of freedom. It is natural to enquire whether additional quarklike modes might not behave differently under quantization, particularly in the light of nondual quark models that require a symmetric three-quark baryon. This raises the question of paraquantization schemes and perhaps the problem of paraparticle scattering. We investigate these problems below in the context of dual physics by systematically exploring the possibilities of replacing the existing dual models (which have orbital and spin excitations) by parafield models. For reasons that will be specified it appears that a satisfactory scheme arises only when additional (paraquarklike) fields are introduced. It is then gratifying to discover that the ensuing model can be made ghost-free in four dimensions if the paraquark fields are rank three.

In Sec. I we review the relevant parts of classical parafield theory that we require. Section II deals with the problem of constructing operators