

Photon corrections to the graviton propagator

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The one-loop photon contribution to the graviton self-energy is calculated using the technique of dimensional regularization and the resulting amplitude is found to satisfy the appropriate Slavnov-Ward identities. The infinite part can be removed by counterterms of the form $\sqrt{g} (R^2 - 3R_{\mu\nu}R^{\mu\nu})$ and the finite part is compared with the previously derived graviton-loop contribution.

I. INTRODUCTION

Recent work¹ has sought to confirm the belief that the major results of classical general relativity may be obtained from the manifestly covariant quantized theory by limitation to tree diagrams. Genuine quantum effects, of course, necessitate the introduction of closed loops. What is remarkable about quantum gravity is that as soon as these contributions are considered, one is forced, by virtue of the universality of the gravitational interaction, to include not only closed graviton loops, but also closed loops of every other quantized matter field in nature. As an example, we might consider the radiative corrections to the scattering of a body by an external gravitational field. To a given order in the gravitational coupling constant, all such closed-loop diagrams enter on an equal basis into the same matrix element. Indeed, one cannot be sure whether the grave difficulties associated with a theory which confines itself to the self-interaction of the gravitational field alone (in particular the problem of nonrenormalizability) might not be due to the neglect of all the other virtual quantized matter fields with which gravitation inevitably interacts.

The inclusion of every elementary particle in such a calculation, of course, represents an almost impossible task. However, if we first confine our attention to massless particles, then it is instructive to compare the graviton-loop corrections to the graviton propagator with those of the photon (and possibly neutrino) loops.

Historically it was the nongravitational modifications which were considered first,² but a rigorous calculation of the magnitude of these quantum effects was made inaccessible by infrared-divergence problems of a peculiar kind. Recently Capper, Leibbrandt, and Ramón Medrano³⁻⁵ have shown, using dimensional regularization,⁶ how one may extract the finite part of the single-graviton-loop corrections in a manner which admits no violation to the underlying symmetry of the theory

(i.e., consistently with the Slavnov-Ward identities), and in which the infrared problems encountered in Ref. 2 do not arise. This method is equally applicable to the (doubly) gauge-invariant interaction of gravitons and photons, in particular to the photon one-loop corrections to the graviton propagator, and it is to this problem that the present paper is devoted.

In Sec. II we quote the relevant Feynman rules which are employed in Sec. III to calculate the photon-loop contribution to the graviton self-energy. Throughout the calculation we work formally in a Euclidean space of 2ω dimensions (where ω is in general complex) and only continue to a four-dimensional Minkowski space at the end of the calculation. It is verified that this photon loop satisfies the Slavnov-Ward identities^{4,7} which are derived in the Appendix. Furthermore, by expanding the connected graviton Green's function about $\omega=2$, the finite part (both real and imaginary) may be extracted. The infinite part can be canceled by a counterterm in the Lagrangian which is the first term in the expansion of $-\sqrt{g} [30(4\pi)^2 (\omega-2)]^{-1} \times (R^2 - 3R_{\mu\nu}R^{\mu\nu})$. Finally the photon loop [Eqs. (4.10) and (4.11)] is compared with the previously derived³ graviton and fictitious particle loops, quoted in Eqs. (5.1) and (5.2). We employ natural units with $\hbar=c=1$ and $\kappa^2=32\pi G$, where G is Newton's constant. Also we refer to Refs. 3 and 4 as I and II, respectively.

II. FEYNMAN RULES

In this section we summarize the relevant Feynman rules. The Lagrangian density is given by

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_A, \quad (2.1)$$

where

$$\mathcal{L}_G = \frac{2}{\kappa^2} \sqrt{g} g^{\mu\nu} R_{\mu\nu} \quad (2.2)$$

is the familiar Einstein Lagrangian and, by minimal substitution, the generally covariant photon

Lagrangian is

$$\mathcal{L}_A = -\frac{1}{4}\sqrt{g} g^{\mu\nu} g^{\alpha\beta} F_{\mu\alpha} F_{\nu\beta}, \quad (2.3)$$

with

$$F_{\mu\alpha} = A_{\alpha,\mu} - A_{\mu,\alpha}. \quad (2.4)$$

Following I, we define

$$\tilde{g}^{\mu\nu} \equiv \sqrt{g} g^{\mu\nu} \quad (2.5)$$

so that \mathcal{L}_G takes the form given in I, and \mathcal{L}_A becomes

$$\mathcal{L}_A = -\frac{1}{4}(\det \tilde{g}^{\rho\sigma})^{-1/(n-2)} \tilde{g}^{\mu\nu} \tilde{g}^{\alpha\beta} F_{\mu\alpha} F_{\nu\beta}, \quad (2.6)$$

where n is the dimension of the space.

If we define the graviton field $\phi^{\mu\nu}$ by

$$\tilde{g}^{\mu\nu} = \delta^{\mu\nu} + \kappa \phi^{\mu\nu}, \quad (2.7)$$

$\delta^{\mu\nu}$ being the n -dimensional Kronecker delta, then there is no need to distinguish between upper and lower indices on $\phi_{\mu\nu}$. The determinant factor now becomes

$$(\det \tilde{g}^{\mu\nu})^{-1/(n-2)} = 1 - \frac{\kappa}{n-2} \phi_{\lambda\lambda} + O(\kappa^2). \quad (2.8)$$

Writing

$$\mathcal{L}_A = \sum_{j=2}^{\infty} \kappa^{j-2} \mathcal{L}_{A(j)}, \quad (2.9)$$

we find that

$$\mathcal{L}_{A(2)} = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} \quad (2.10)$$

and

$$\begin{aligned} \mathcal{L}_{A(3)} = & -\frac{1}{4} \left(\delta_{\mu\nu} \phi_{\alpha\beta} + \delta_{\alpha\beta} \phi_{\mu\nu} - \frac{1}{(n-2)} \delta_{\mu\nu} \delta_{\alpha\beta} \phi_{\lambda\lambda} \right) \\ & \times F_{\mu\alpha} F_{\nu\beta}. \end{aligned} \quad (2.11)$$

The electromagnetic gauge is fixed by adding to $\mathcal{L}_{A(2)}$ the gauge-breaking term (see Appendix)

$$-\frac{1}{2} (A_{\mu,\mu})^2, \quad (2.12)$$

which results in the free photon propagator

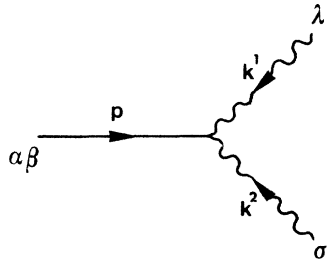


FIG. 1. The graviton-photon-photon vertex (the wavy lines are photons).

$$D_{\mu\nu}(k^2) = \frac{-\delta_{\mu\nu}}{k^2}. \quad (2.13)$$

$\mathcal{L}_{A(3)}$ provides the graviton-photon-photon vertex function, which, with the labeling of Fig. 1, is given by

$$\begin{aligned} V_{\alpha\beta,\lambda,\sigma}(p, k^1, k^2) = & \delta_{\lambda\sigma} k_{(\alpha}^1 k_{\beta)}^2 - \delta_{\sigma(\beta} k_{\alpha)}^1 k_{\lambda)}^2 \\ & - \delta_{\lambda(\alpha} k_{\sigma)}^1 k_{\beta)}^2 + \delta_{\sigma(\beta} \delta_{\alpha)\lambda} k^1 \cdot k^2 \\ & - \frac{1}{n-2} \delta_{\alpha\beta} (\delta_{\lambda\sigma} k^1 k^2 - k_{\sigma}^1 k_{\lambda}^2), \end{aligned} \quad (2.14)$$

where we employ the notation

$$A_{(\alpha} B_{\beta)} = \frac{1}{2} (A_{\alpha} B_{\beta} + A_{\beta} B_{\alpha}). \quad (2.15)$$

III. LOWEST-ORDER PHOTON SELF-ENERGY INSERTIONS

All possible photon self-energy corrections to order κ^2 are depicted in Fig. 2. In fact it is necessary to compute only the contribution from Fig. 2(a), since the massless tadpole diagrams can consistently be set equal to zero within the framework of dimensional regularization.⁸ Hence there is no need to display explicitly the three-graviton and two-photon-two-graviton vertex functions.

From the Feynman rules developed in Sec. II, the contribution to the self-energy from Fig. 2(a) is given by

$$\begin{aligned} T_{\alpha\beta\alpha'\beta'} = & \kappa^2 \int \frac{d^{2\omega}q}{(2\pi)^{2\omega}} V_{\alpha\beta,\lambda,\sigma}(p, -q, -q-p) D_{\lambda\lambda'}(q^2) \\ & \times V_{\alpha'\beta',\lambda,\sigma}(-p, q, p-q) D_{\sigma\sigma'}((p-q)^2), \end{aligned} \quad (3.1)$$

where, following the dimensional-regularization technique, the momentum-space integration is defined over a 2ω -dimensional Euclidean space. The regulating parameter 2ω (in general complex) now replaces the integer n of Sec. II. The evaluation of this integral follows precisely the same lines as those described in I, and here we merely quote the final result:

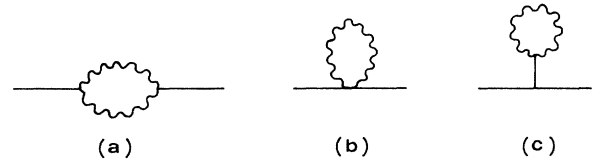


FIG. 2. Lowest-order photon contributions to the graviton self-energy.

$$\begin{aligned}
T_{\alpha\beta\alpha'\beta'} = & \kappa^2 [p_\alpha p_\beta p_{\alpha'} p_{\beta'} T_1(p^2) \\
& + \delta_{\alpha\beta} \delta_{\alpha'\beta'} (p^2)^2 T_2(p^2) \\
& + (\delta_{\alpha\alpha'} \delta_{\beta\beta'} + \delta_{\beta'\alpha} \delta_{\alpha\beta'}) (p^2)^2 T_3(p^2) \\
& + (\delta_{\alpha\beta} p_{\alpha'} p_{\beta'} + \delta_{\alpha'\beta'} p_\alpha p_\beta) p^2 T_4(p^2) \\
& + (\delta_{\alpha\alpha'} p_\beta p_{\beta'} + \delta_{\beta\alpha'} p_\alpha p_{\beta'}) \\
& + \delta_{\alpha\beta} p_{\beta'} p_{\alpha'} + \delta_{\beta\beta'} p_\alpha p_{\alpha'}] p^2 T_5(p^2), \tag{3.2}
\end{aligned}$$

where

$$T_1 = \frac{\omega(\omega-1)^2}{2(4\omega^2-1)} I_1, \tag{3.3}$$

$$T_2 = \frac{-3\omega^2+2\omega+4}{8(4\omega^2-1)(\omega-1)} I_1, \tag{3.4}$$

$$T_3 = \frac{4\omega^2-3\omega-4}{8(4\omega^2-1)} I_1, \tag{3.5}$$

$$T_4 = \frac{\omega(\omega-1)}{4(4\omega^2-1)} I_1, \tag{3.6}$$

$$T_5 = \frac{-4\omega^2+3\omega+4}{8(4\omega^2-1)} I_1 \tag{3.7}$$

and I_1 is the basic integral

$$\begin{aligned}
I_1 &= \int \frac{dq^{2\omega}}{(2\pi)^{2\omega}} \frac{1}{q^2(p-q)^2} \\
&= \frac{1}{(4\pi)^\omega} \frac{\Gamma(2-\omega)\Gamma(\omega-1)\Gamma(\omega-1)}{\Gamma(2\omega-2)} (p^2)^{\omega-2}. \tag{3.8}
\end{aligned}$$

We are now in a position to verify that our regularization technique correctly preserves the underlying gauge invariance of the theory, and that the Slavnov-Ward identities are indeed satisfied. To see this explicitly we first construct the connected Green's function

$$Q_{\nu\sigma,\mu\lambda}(p^2) = D_{\nu\sigma\alpha\beta}(p^2) T_{\alpha\beta\alpha'\beta'}(p^2) D_{\alpha'\beta',\mu\lambda}(p^2), \tag{3.9}$$

where

$$D_{\nu\sigma\alpha\beta}(p^2) = \frac{1}{2p^2} (\delta_{\nu\alpha} \delta_{\sigma\beta} + \delta_{\nu\beta} \delta_{\sigma\alpha} - \delta_{\nu\sigma} \delta_{\alpha\beta}) \tag{3.10}$$

is the free graviton propagator derived in I.

The result is

$$\begin{aligned}
Q_{\nu\sigma\mu\lambda}(p^2) = & \frac{\kappa^2}{4(p^2)^2} \{ a_{1\nu\sigma\mu\lambda} T_1 + (\omega-1)^2 a_{2\nu\sigma\mu\lambda} T_2 \\
& + [a_{3\nu\sigma\mu\lambda} + (\omega-2) a_{2\nu\sigma\mu\lambda}] T_3 \\
& + (\omega-1) a_{4\nu\sigma\mu\lambda} T_4 + a_{5\nu\sigma\mu\lambda} T_5 \}, \tag{3.11}
\end{aligned}$$

with

$$a_{1\nu\sigma\mu\lambda} = 4p_\nu p_\sigma p_\mu p_\lambda - 2\delta_{\mu\nu} p_\nu p_\sigma p^2 - 2p^2 \delta_{\nu\sigma} p_\mu p_\lambda + \delta_{\nu\sigma} \delta_{\mu\lambda} (p^2)^2, \tag{3.12}$$

$$a_{2\nu\sigma\mu\lambda} = 4\delta_{\nu\sigma} \delta_{\mu\lambda} (p^2)^2, \tag{3.13}$$

$$a_{3\nu\sigma\mu\lambda} = 4[\delta_{\mu\nu} \delta_{\sigma\lambda} (p^2)^2 + \delta_{\nu\lambda} \delta_{\sigma\mu} (p^2)^2], \tag{3.14}$$

$$a_{4\nu\sigma\mu\lambda} = 4[\delta_{\nu\sigma} \delta_{\mu\lambda} (p^2)^2 - \delta_{\mu\lambda} p_\nu p_\sigma p^2 - \delta_{\nu\sigma} p_\mu p_\lambda p^2], \tag{3.15}$$

$$\begin{aligned}
a_{5\nu\sigma\mu\lambda} = & 4(\delta_{\mu\nu} p_\sigma p_\lambda + \delta_{\sigma\mu} p_\nu p_\lambda + \delta_{\nu\lambda} p_\sigma p_\mu \\
& + \delta_{\sigma\lambda} p_\nu p_\mu - 2\delta_{\nu\sigma} p_\mu p_\lambda \\
& - 2\delta_{\mu\lambda} p_\nu p_\sigma + p^2 \delta_{\mu\lambda} \delta_{\nu\sigma}) p^2. \tag{3.16}
\end{aligned}$$

The Slavnov-Ward identity derived in the Appendix, Eq. (A15), is given in momentum space by

$$p_\nu Q_{\nu\sigma\mu\lambda} = 0 \tag{3.17}$$

or equivalently

$$T_3 + T_5 = 0, \tag{3.18}$$

$$T_1 - 2(\omega-1) T_4 = 0, \tag{3.19}$$

$$-T_1 + 4(\omega-1)^2 T_2 + 4(\omega-2) T_3 - 4 T_5 = 0. \tag{3.20}$$

These identities are valid for *all* values of the regulating parameter ω (in particular for $\omega=2$), as may be verified directly from Eqs. (3.3) to (3.7).

IV. THE CONNECTED GREEN'S FUNCTION

To obtain the finite part of the connected Green's function $Q_{\nu\sigma\mu\lambda}$ we proceed as in I by expanding the entire right-hand side of Eq. (3.11) about $\omega=2$ and then separating the various pole terms from the real and imaginary parts. Expressing the amplitudes T_j in the form

$$T_j = \Gamma(2-\omega) (p^2)^{\omega-2} f_j(\omega), \quad j=1, \dots, 5 \tag{4.1}$$

we expand each T_j about $\omega=2$. Before expanding, however, one ought to comment on the dimensionality of the coupling constant κ . Whereas in four dimensions κ^2 is simply proportional to Newton's constant, in 2ω dimensions we must have

$$\kappa^2 \propto \frac{G}{(\mu^2)^{\omega-2}}, \tag{4.2}$$

where μ is an arbitrary constant having the dimensions of mass. Bearing this in mind we rewrite Eq. (4.1) in the form

$$T_j = \Gamma(2-\omega) \left(\frac{p^2}{\mu^2} \right)^{\omega-2} f_i(\omega), \tag{4.3}$$

with the understanding that κ^2 in Eq. (3.11) now takes on its traditional four-dimensional value, $(32\pi G)$. We may now expand about $\omega=2$, obtaining

$$T_j = \frac{f_j(2)}{2-\omega} + \left[\psi(1)f_j(2) - f_j(2) \ln \frac{p^2}{\mu^2} - f_j'(2) \right] + O((\omega-2)), \quad (4.4)$$

which, when continued analytically from Euclidean to Minkowski space ($p^2 = p_0^2 - \vec{p}^2$), becomes

$$T_j = \frac{f_j(2)}{2-\omega} + \left[\psi(1)f_j(2) - f_j(2) \ln \left| \frac{p^2}{\mu^2} \right| - f_j'(2) \right] + i\pi f_j(2) + O((\omega-2)). \quad (4.5)$$

Here the prime denotes differentiation with respect to ω and $\psi(\omega) = (d/d\omega) \ln \Gamma(\omega)$. As noted in I it is essential to expand the connected Green's function $Q_{\nu\sigma\mu\lambda}$ rather than the self-energy $T_{\alpha\beta\alpha'\beta'}$. Writing

$$Q_{\nu\sigma\mu\lambda}(p^2) = \frac{\kappa^2}{4(p^2)^2 (4\pi)^2} (Q_{\nu\sigma\mu\lambda}^{\text{pole}} + Q_{\nu\sigma\mu\lambda}^{\text{real}} + i Q_{\nu\sigma\mu\lambda}^{\text{Im}}), \quad (4.6)$$

we have the final results

$$Q_{\nu\sigma\mu\lambda}^{\text{pole}} = \frac{1}{2-\omega} Q_{\nu\sigma\mu\lambda}^1, \quad (4.7)$$

$$Q_{\nu\sigma\mu\lambda}^{\text{real}} = \left(\psi(1) + 2 - \ln \left| \frac{p^2}{(4\pi)\mu^2} \right| \right) Q_{\nu\sigma\mu\lambda}^1 + Q_{\nu\sigma\mu\lambda}^2, \quad (4.8)$$

$$Q_{\nu\sigma\mu\lambda}^{\text{Im}} = \pi Q_{\nu\sigma\mu\lambda}^1, \quad (4.9)$$

where

$$Q_{\nu\sigma\mu\lambda}^1 = \frac{1}{60} (4a_{1\nu\sigma\mu\lambda} - 2a_{2\nu\sigma\mu\lambda} + 3a_{3\nu\sigma\mu\lambda} + 2a_{4\nu\sigma\mu\lambda} - 3a_{5\nu\sigma\mu\lambda}) \quad (4.10)$$

and

$$Q_{\nu\sigma\mu\lambda}^2 = \frac{1}{1800} (-172a_{1\nu\sigma\mu\lambda} + 56a_{2\nu\sigma\mu\lambda} - 99a_{3\nu\sigma\mu\lambda} - 86a_{4\nu\sigma\mu\lambda} + 99a_{5\nu\sigma\mu\lambda}). \quad (4.11)$$

Our major goal has now been achieved, namely the extraction from $Q_{\nu\sigma\mu\lambda}$ of the finite part $Q^{\text{real}} + i Q^{\text{Im}}$ in a manner consistent with the three Slavnov-Ward identities (3.18), (3.19), and (3.20). Moreover, each of the terms Q^{pole} , Q^{real} , and Q^{Im} satisfies these identities *separately*, as the reader may verify.

Having obtained $Q_{\nu\sigma\mu\lambda}^{\text{pole}}$, it is now a straightforward matter to compute what counterterms must be added to our original Lagrangian (2.1) in order to render finite the one-loop photon corrections to the graviton propagator. They will necessarily involve four derivatives of the graviton field. We state without proof that if we construct from the curvature scalar R and the Ricci tensor $R_{\mu\nu}$ the counterterms

$$\Delta \mathcal{L} = \frac{-1}{30(4\pi)^2 (\omega-2)} \sqrt{g} (R^2 - 3R_{\mu\nu} R^{\mu\nu}), \quad (4.12)$$

then the lowest-order contributions correctly cancel the unwanted divergences in $Q_{\nu\sigma\mu\lambda}^{\text{pole}}$.

V. COMPARISON WITH GRAVITON-LOOP CONTRIBUTION

It is now of interest to compare the graviton-loop corrections to the graviton propagator, as calculated in I, with the photon-loop corrections calculated in this paper. In I, the quantity analogous to $Q_{\nu\sigma\mu\lambda}$ of Eq. (4.6) was found to have precisely the same form as that given in Eqs. (4.7), (4.8), and (4.9), except that, for the graviton, instead of Eqs. (4.10) and (4.11) we have

$$Q_{\nu\sigma\mu\lambda}^1 = \frac{1}{4} \frac{1}{60} (328a_{1\nu\sigma\mu\lambda} - 59a_{2\nu\sigma\mu\lambda} + 81a_{3\nu\sigma\mu\lambda} + 104a_{4\nu\sigma\mu\lambda} - 81a_{5\nu\sigma\mu\lambda}) \quad (5.1)$$

and

$$Q_{\nu\sigma\mu\lambda}^2 = \frac{1}{4} \frac{1}{1800} (896a_{1\nu\sigma\mu\lambda} + 1517a_{2\nu\sigma\mu\lambda} - 1143a_{3\nu\sigma\mu\lambda} + 598a_{4\nu\sigma\mu\lambda} + 1143a_{5\nu\sigma\mu\lambda}). \quad (5.2)$$

Thus the coefficients in Q^1 (and therefore also in Q^{pole}) appear with the same sign in both the electromagnetic and gravitational cases, though the former are smaller in magnitude than the latter. No pattern seems to emerge for Q^2 , however.

The major difference between photon corrections and graviton corrections is a qualitative one. $Q_{\nu\sigma\mu\lambda}^{\text{photon}}$ satisfies the "naive" identity

$$p_\nu Q_{\nu\sigma\mu\lambda}^{\text{photon}} = 0 \quad (5.3)$$

which gives rise to the three identities of Eqs. (3.18), (3.19), and (3.20); Q^{graviton} on the other hand is not transverse but obeys a more complicated identity owing to the presence of fictitious-particle contributions. This has been discussed extensively in II. It does, however, satisfy

$$p_\nu p_\mu Q_{\nu\sigma\mu\lambda}^{\text{graviton}} = 0. \quad (5.4)$$

Consequently, the counterterms required to cancel the one-loop divergences in pure gravity are not generally covariant, but rather covariant under a particular nonlinear realization of the general coordinate transformation group. One naturally expects to recover the gauge invariance when physical on-mass-shell S -matrix elements are computed.

VI. CONCLUSIONS

We have succeeded in regularizing the lowest-order photon contributions to the graviton Green's function by employing the method of dimensional regularization. The appropriate Slavnov-Ward identity was derived, and both the pole terms and finite parts were found to be consistent with this identity. The pole terms were removed by adding

generally covariant counterterms to the original Lagrangian, and the finite parts were compared with those arising from the graviton loop.

With regard to the parameter μ having dimensions of mass which entered naturally into the calculation, we remark that in a recent paper by one of us (MJD)⁹ graviton-loop corrections to the Schwarzschild solution were considered and there it was found that the parameter μ disappeared from the final results when the gravitational field was coupled to a conserved source. Photon-loop contributions would not alter the results of that paper, save for a small modification ($\sim 10\%$) of the numerical coefficients.

Finally, the question of renormalizability of the quantum theory of gravitation is still an open one,^{10,11} and there is a corresponding ambiguity in the finite part of the self-energy corrections. It also remains to be seen what are the effects of including a closed neutrino loop, and this is now under consideration.

Note added in proof. The neutrino contribution to the self-energy has now been calculated [D. M. Capper and M. J. Duff, Imperial College, London, Report No. ICTP/73/12 (unpublished)] and yields results similar to those found for the photon case, with $\Delta\mathcal{L}^\nu = \frac{1}{4}\Delta\mathcal{L}^\gamma$ (there is no cancellation). That the photon and the neutrino yield similar counterterms follows from conformal invariance. In both cases, however, conformal anomalies arise [e.g., in (4.10) and (4.11) $Q_{\sigma\sigma\mu\lambda}^1 = 0$ but $Q_{\sigma\sigma\mu\lambda}^2 \neq 0$]. These are discussed in a recent report [D. M. Capper and M. J. Duff, Imperial College, London, Report No. ICTP/73/10. (unpublished)]. A discussion of the divergences arising from graviton-photon and graviton-fermion interactions has also been given by S. Deser and P. van Nieuwenhuizen [Phys. Rev. Lett. **32**, 245 (1974); this issue, Phys. Rev. D **10**, 401 (1974); **10**, 411 (1974)].

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APPENDIX

In this appendix we derive the fictitious-particle contribution for the combined graviton-photon Lagrangian and also derive some of the Slavnov-Ward identities; for details of the techniques involved we refer the reader to papers I and II, respectively. The "naive" generating functional is given by

$$Z[0, 0] = \int d[\bar{g}^{\mu\nu}] d[A_\alpha] \exp\left[i \int dx (\mathcal{L}_g + \mathcal{L}_A)\right], \quad (\text{A1})$$

where \mathcal{L}_g is given in I and \mathcal{L}_A in Eq. (2.3). With this expression for Z there are two problems—both due to gauge invariance. Firstly, the bilinear part of neither \mathcal{L}_g nor \mathcal{L}_A has an inverse and therefore propagators cannot be constructed from the generating functional of Eq. (A1). Secondly, the integration over the function space of $\bar{g}^{\mu\nu}$ contains a infinite factor due to integrating over points related by (non-Abelian) gravitational gauge transformations. These problems can be resolved by the Faddeev-Popov technique.¹² To this end we insert in Eq. (A1) a constant factor given by

$$\Delta[\bar{g}^{\mu\nu}, A_\alpha] \int d\Omega \delta[\partial_\mu \bar{g}^{\mu\nu} - B^\nu] \delta[\partial_\alpha A_\alpha - c] = 1. \quad (\text{A2})$$

$B^\nu(x)$ and $c(x)$ are arbitrary functions and Ω is any transformation in the combined gauge group which leaves (A1) invariant. As in Appendix B of I we make an inverse Ω transformation on the new Z and then discard the now irrelevant integration over Ω , obtaining

$$\begin{aligned} Z[0, 0] &= \int d[\bar{g}^{\mu\nu}] d[A_\alpha] \Delta[\bar{g}^{\mu\nu}, A_\alpha] \\ &\quad \times \delta[\partial_\mu \bar{g}^{\mu\nu} - B^\nu] \delta[\partial_\alpha A_\alpha - c] \\ &\quad \times \exp\left[i \int (\mathcal{L}_g + \mathcal{L}_A) dx\right]. \end{aligned} \quad (\text{A3})$$

Because of the δ functionals in Eq. (A3) we only need to evaluate the integral in (A2) over the hypersurface defined by $\delta[\partial_\mu \bar{g}^{\mu\nu} - B^\nu] \delta[\partial_\alpha A_\alpha - c]$. Consider an electromagnetic gauge transformation given by

$$A_\mu(x) \rightarrow A_\mu(x) + \partial_\mu \Lambda(x) \quad (\text{A4})$$

together with a gravitational gauge transformation given by

$$\begin{aligned} \bar{g}^{\mu\nu}(x) &\rightarrow \bar{g}^{\mu\nu}(x) + [-\xi^\lambda(x) \bar{g}^{\mu\nu},{}_{,\lambda}(x) + \xi^\mu{}_{,\rho}(x) \bar{g}^{\rho\nu}(x) \\ &\quad + \xi^\nu{}_{,\sigma}(x) \bar{g}^{\mu\sigma}(x) - \xi^\alpha{}_{,\alpha}(x) \bar{g}^{\mu\nu}(x)]. \end{aligned} \quad (\text{A5})$$

The total variation in $\partial_\mu A_\mu$ under a combined gravitational and electromagnetic gauge transformation is of the form

$$\delta(A_{\mu,\mu}) = F(\xi_\mu, A_\lambda, \phi_{\alpha\beta}) + \lambda_{,\alpha\alpha}. \quad (\text{A6})$$

Evaluating near $\partial_\alpha A_\alpha = c$, we find

$$\begin{aligned} \delta[\partial_\alpha A_{\alpha\Omega} - c] &= \int d\lambda \delta[\lambda_{,\alpha\alpha} + F(\xi_\mu, A_\lambda, \phi_{\alpha\beta})] \\ &= 1. \end{aligned} \quad (\text{A7})$$

Thus, introducing photons and using the particular type of gauge for the photon propagator implied by (A2), no additional fictitious particles are needed.

It is convenient to integrate out the arbitrary functions $B^\nu(x)$ and $c(x)$ by introducing a suitable weight function such as

$$\rho[B^\nu, c] = \exp \left\{ -i \int \left[\frac{1}{\alpha\kappa^2} B(x)^2 + \frac{1}{2\beta} c(x)^2 \right] dx \right\}. \quad (\text{A8})$$

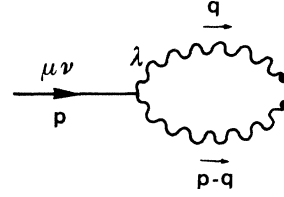


FIG. 3. The lowest-order contribution to the contracted three-point Green's function, which occurs in the Slavnov-Ward identities.

The resultant effect is to add two gauge-breaking terms to the Lagrangian, giving a final generating functional

$$Z[j_{\mu\nu}, J^\alpha] = \int d[\tilde{g}^{\mu\nu}] d[A_\alpha] \Delta[\tilde{g}^{\mu\nu}] \exp \left\{ i \int dx \left[\mathcal{L}_g + \mathcal{L}_A - \frac{1}{\kappa^2 \alpha'} (\partial_\mu \tilde{g}^{\mu\nu})^2 - \frac{1}{2\beta} (\partial_\alpha A_\alpha)^2 + \frac{1}{\kappa} \tilde{g}^{\mu\nu} j_{\mu\nu} + A_\alpha J^\alpha \right] \right\}. \quad (\text{A9})$$

In fact we find it convenient to choose $\alpha' = -1$ and $\beta = 1$, which gives rise to the propagators of Eqs. (2.14) and (3.10).

The derivation of the Slavnov-Ward identities resulting from *gravitational* gauge transformations follows closely that given in II, except there is an extra term from the variation of $\partial_\alpha A_\alpha$. A short calculation shows that (in the notation of II) the Slavnov identities are given by

$$\int dx d[\tilde{g}^{\mu\nu}] d[A_\alpha] \bar{Z}[j_{\mu\nu}, J^\alpha] \left[j_{\mu\nu}(x) A_{\mu\nu\lambda}(x) N_{\lambda\beta}(x, y) - \frac{2}{\alpha} \delta(x-y) \partial_\lambda \phi_{\lambda\beta}(x) + \frac{\kappa}{\beta} \delta(x-y) \partial^\alpha A_\alpha(x) A_\beta(x) \right] = 0 \quad (\text{A10})$$

[only the last term is different from II], where

$$A_{\mu\nu\lambda} = -\phi_{\mu\nu,\lambda} + \phi_{\rho\nu} \delta_{\lambda\mu} \partial_\rho + \phi_{\mu\rho} \delta_{\nu\lambda} \partial_\rho - \phi_{\mu\nu} \partial_\lambda + \kappa^{-1} (\delta_{\mu\lambda} \partial_\nu + \delta_{\nu\lambda} \partial_\mu - \delta_{\mu\nu} \partial_\lambda), \quad (\text{A11})$$

$$(N_{\nu\lambda})^{-1} = -\phi_{\mu\nu,\lambda\mu} + \phi_{\mu\rho} \delta_{\nu\lambda} \partial_\mu \partial_\rho + \phi_{\mu\rho,\mu} \delta_{\nu\lambda} \partial_\rho - \phi_{\mu\nu,\mu} \partial_\lambda + \kappa^{-1} \delta_{\nu\lambda} \partial^2, \quad (\text{A12})$$

$$\bar{Z}[j_{\mu\nu}, J^\alpha] = \Delta \exp \left\{ i \int dx \left[\mathcal{L}_A + \mathcal{L}_g + \frac{1}{\kappa} \tilde{g}^{\mu\nu} j_{\mu\nu} + A_\alpha J^\alpha - \frac{1}{\kappa^2 \alpha} \partial_\mu \tilde{g}^{\mu\nu} \partial_\alpha \tilde{g}^{\alpha\beta} \delta_{\nu\beta} - \frac{1}{2\beta} (\partial_\alpha A_\alpha)^2 \right] \right\}. \quad (\text{A13})$$

We thus obtain the following identity for the totally corrected graviton Green's function:

$$\begin{aligned} \frac{2i}{\alpha} \langle T(\phi_{\mu\nu}(z) \phi_{\lambda\beta,\lambda}(y)) \rangle \\ = \langle T(A_{\mu\nu\lambda}(z) N_{\lambda\beta}(z, y)) \rangle \\ + \frac{\kappa i}{\beta} \langle T(\phi_{\mu\nu}(z) \partial_\lambda A_\lambda(y) A_\beta(y)) \rangle. \end{aligned} \quad (\text{A14})$$

To order κ^2 the additional term in Eq. (A14) gives rise to the contracted three-point Green's function shown in Fig. 3. However, on using the ver-

text given in Eq. (2.15), this diagram is found to be identically zero (at least in the gauges employed in this paper). Hence to *this order* the Slavnov-Ward identities derived in II are unaltered. Since these were satisfied by the graviton corrections alone, they must also be separately satisfied by the photon corrections to the graviton propagator, i.e., to order κ^2 ,

$$\langle T\phi_{\mu\nu}(z) \phi_{\lambda\beta,\lambda}(y) \rangle_{\text{photon corrections}} = 0. \quad (\text{A15})$$

This identity, or equivalently Eq. (3.17), is indeed verified in the text.

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