# Comments and Addenda

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## Relativistic motion in a uniform magnetic field. II*

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The solution to the Lorentz-Dirac equation for a particle moving in a uniform magnetic field is developed with the coordinate time as the independent variable. This perturbation solution, which is exact to fourth order in the interaction constant $\lambda = 2e^3H/3m^2c^4$, is compared with other solutions.

### I. INTRODUCTION

A general solution to the Lorentz-Dirac equation, describing the relativistic motion of a charged particle in a uniform magnetic field, has been given in Ref. 1. Therein, the particle proper time ($\tau$) was used as the independent variable. It is our purpose in this second paper to express our previous results in terms of the coordinate time ($t$). This is desirable for the sake of completeness. In addition, the results may answer some recent questions[2-5] regarding the correctness of other solutions which are expressed as functions of the coordinate time.

### II. COORDINATE TIME

In paper I we have represented the ratio of the particle total energy at any proper time to its initial total energy by Eq. I(5a). (The prefix I refers to the equations of paper I.) This can be written as

$$\gamma(\tau)/\gamma(0) = [\zeta_0 - (\zeta_0 - 1)\exp(-2\lambda\phi)]^{-1/2}\ , \tag{1}$$

with $\zeta_0 = \gamma^2(0)/\gamma_L{}^2$. Integrating this equation with respect to $\phi$ from $\phi = 0$ to $\phi$ yields, after some manipulation,

$$\exp(\lambda\phi) = \frac{1}{2\zeta_0{}^{1/2}}\{(\zeta_0{}^{1/2}+1)\exp[Z(t)] + (\zeta_0{}^{1/2}-1)\exp[-Z(t)]\}\ , \tag{2}$$

where

$$Z(t) = \frac{\lambda}{\gamma_L}\int_0^t \dot{\phi}\,dt\ . \tag{3}$$

Since $\dot{\phi}$ is known from Eq. I(15), we readily derive the function $Z(t)$ to fourth order in the interaction constant $\lambda$. Thus,

$$Z(t) = \lambda\frac{\omega t}{\gamma_L} + \lambda^3(1-6\zeta_0)\frac{\omega t}{\gamma_L} + \lambda^4 6\zeta_0(\zeta_0 - 1)\frac{\omega^2 t^2}{\gamma_L \gamma(0)}\ . \tag{4}$$

With the aid of Eq. (2), one now obtains the ratio given by Eq. (1) as a function of $Z(t)$ which only depends on the coordinate time $t$, that is,

$$\frac{\gamma(t)}{\gamma_L} = \frac{(\zeta_0{}^{1/2}+1) + (\zeta_0{}^{1/2}-1)\exp[-2Z(t)]}{(\zeta_0{}^{1/2}+1) - (\zeta_0{}^{1/2}-1)\exp[-2Z(t)]}\ . \tag{5}$$

As a last step in finding the proper time as a function of the coordinate time, we expand the reciprocal of Eq. (5) into a Taylor series in the interaction constant $\lambda$ and integrate the terms with respect to $t$. It then follows that

$$\tau = \frac{t}{\gamma(0)} \{1 + \tfrac{1}{2}\lambda(\zeta_0 - 1)T - \tfrac{1}{3}\lambda^2(\zeta_0 - 1)T^2 + \tfrac{1}{3}\lambda^3(\zeta_0 - 1)[\tfrac{3}{2}(1 - 6\zeta_0)T - \tfrac{1}{4}(\zeta_0 - 3)T^3] + \tfrac{1}{3}\lambda^4(\zeta_0 - 1)[2(3\zeta_0^2 + 3\zeta_0 - 1)T^2 + \tfrac{1}{5}(2\zeta_0 - 3)T^4]\} \ . \tag{6}$$

Here, for the sake of simplicity, we have introduced the variable $T = \omega t/\gamma(0)$.

## III. RESULTS AND CONCLUSIONS

The knowledge of the particle proper time as a function of the coordinate time allows us to readily write the equations that are analogous to Eqs. I(5), I(15), and I(16). Thus, letting $h(t) = \phi(\tau)$ and $f(t) = \theta(\tau)$, we have

$$\gamma(t)/\gamma(0) = \{\zeta_0 - (\zeta_0 - 1)\exp[-2\lambda h(t)]\}^{-1/2} \ , \tag{7a}$$

$$u_1(t) + iu_2(t) = [u_1(0) + iu_2(0)]\exp[-\lambda h(t) - if(t)] \ , \tag{7b}$$

and

$$u_3(t) = u_3(0) \ . \tag{7c}$$

The two functions $h(t)$ and $f(t)$ are given by

$$h(t) = T\{1 + \tfrac{1}{2}\lambda(\zeta_0 - 1)T + \lambda^2(1 - 6\zeta_0) - \tfrac{1}{3}\lambda^2(\zeta_0 - 1)T^2 + \lambda^3(\zeta_0 - 1)[T - \tfrac{1}{12}(\zeta_0 - 3)T^3] + \lambda^4 2(40\zeta_0^2 - 20\zeta_0 + 1) + \lambda^4(\zeta_0 - 1)[(2\zeta_0 - 1)T^2 + \tfrac{1}{15}(2\zeta_0 - 3)T^4]\} \ , \tag{8}$$

and

$$f(t) = T\{1 + \tfrac{1}{2}\lambda(\zeta_0 - 1)T - \lambda^2 2\zeta_0 - \tfrac{1}{3}\lambda^2(\zeta_0 - 1)T^2 + \tfrac{1}{2}\lambda^3(\zeta_0 - 1)[(1 - 4\zeta_0)T - \tfrac{1}{6}(\zeta_0 - 3)T^3] + \lambda^4 2\zeta_0(10\zeta_0 - 3) + \tfrac{1}{3}\lambda^4(\zeta_0 - 1)[2(2\zeta_0^2 + 3\zeta_0 - 1)T^2 + \tfrac{1}{5}(2\zeta_0 - 3)T^4]\} \ . \tag{9}$$

We note that, though the method used in paper I to obtain the basic solution is quite different from that recently developed by Pascual and Mas,[2] the solutions, expressed as functions of the coordinate time, are similar to the same order in the interaction constant. For example, Eq. (10) of Ref. 2 is equivalent to our Eq. (5) when we restrict the function $Z(t)$ to first order in the interaction constant. In addition, consistent with the observation of these authors, we find that our final solution, Eqs. (7)–(9), does not reduce to those of Shen[3–5] for the case of highly relativistic particles $[\gamma(0) \gg 1]$. For the case of nonrelativistic, planar motion $[\gamma(0) \approx 1,\ \gamma_L = 1]$, our general solution does, as expected, approach that of Plass.[6]

In conclusion, we would like to emphasize that, though we have given the perturbation solution as a function of the coordinate time to fourth order in the interaction constant, the basic method presented can be employed to derive the solution to higher order with no fundamental difficulty.

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[1]J. C. Herrera, Phys. Rev. D 7, 1567 (1973), hereafter referred to as paper I.

[2]P. Pascual and L. Mas, Phys. Rev. D 9, 2482 (1974).

[3]C. S. Shen, Phys. Rev. D 6, 2736 (1972).

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[5]C. S. Shen, Phys. Rev. D 6, 3039 (1972).

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