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Perturbative calculations in a unified gauge-field model of strong, weak, and electromagnetic interactions*

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We calculate several corrections to zeroth-order symmetry relations in a unified gauge-field model of strong, weak, and electromagnetic interactions. Among the topics discussed are the proton-neutron mass difference, the pion mass, the pion mass difference, and parity-violation effects. We discuss the perturbation scheme and establish the gauge invariance of the results. The pion mass originates from electromagnetic corrections and we find the value of 37 MeV. The pion mass difference is not affected by the inclusion of weak interactions and the hard-pion corrections to the mass difference are approximately 0.5 MeV.

I. INTRODUCTION

One of the most important aspects of renormalizable gauge-field theories is that, owing to the strong implications of gauge invariance, various physical quantities are calculable and finite. This was first recognized by 't Hooft,[1] who calculated the electromagnetic mass difference for an isotriplet of fermions in a model based on the O(3) gauge group. The origin of this phenomenon is that certain counterterms which are necessary to render the theory finite are prohibited by gauge invariance. This implies that corresponding quantities vanish in lowest order and pick up possible contributions from closed-loop corrections. For this reason this phenomenon was called a "zeroth-order" symmetry.[2-4] The absence of possible counterterms then implies, because of the renormalizability of the theory,[5,6] that these quantities are finite and calculable.

As stressed in particular by Weinberg,[2,4] the reason why this phenomenon is so important is that such theories can provide a natural explanation for the existence of approximate symmetries in nature, such as isospin or chiral $SU(2)\otimes SU(2)$ symmetry. At present, a large variety of gauge-field models have been investigated in which many examples of zeroth-order symmetry relations have been found. The most simple examples concern the electromagnetic mass differences of hadrons[7,8] and the muon-electron mass ratio.[9] Another promising kind of zeroth-order symmetry is related to the so-called pseudo-Goldstone bosons.[2] Such bosons can be present if the interactions among spinless fields have a higher symmetry than the total Lagrangian for all values of the parameters in the Lagrangian. In the tree approximation the pseudo-Goldstone bosons are necessarily massless, and possible contributions to their masses from closed-loop corrections must again be finite due to the renormalizability. Although it is not clear which approximate symmetries in nature are realized in this way, the general analysis of zeroth-order relations, even in unrealistic models, is important in order to determine their characteristic features and to obtain a general estimate for the higher-order corrections. This may also provide us with new limitations for the construction of more realistic models.

Some time ago Weinberg carried out the one-loop corrections to zeroth-order symmetries in a general renormalizable theory.[4] In particular he considered the electromagnetic mass differences of fermions, and the masses of pseudo-Goldstone bosons. He showed that the final results were gauge-independent and finite in the presence of a corresponding zeroth-order symmetry. However, in the case that all vector-boson masses except that of the photon were roughly equal and larger than the fermion masses, the proton-neutron mass difference generally tended to give the wrong sign. Weinberg also made an estimate of the pseudo-Goldstone–boson masses, and found them to be of order eM, with e and M a typical gauge-field coupling constant and mass, respectively. In the case of different vector-boson masses, M is supposed to be the largest mass. If we want to consider the pions as pseudo-Goldstone bosons, this indicates that in the presence of heavy intermediate vector bosons, the pion mass will be several orders of magnitude too large.

In this paper we will calculate the one-loop corrections to several zeroth-order symmetries in a unified gauge-field model of strong, weak, and electromagnetic interactions. The model, which was previously introduced in discussing the current-algebra properties of gauge-field theories,[10] is a natural extension of the σ model[11] to a gauge-field model of strong interactions, combined with the Weinberg-Salam model[12] of weak and electromagnetic interactions. Models of this type were first constructed by Bars, Halpern, and Yoshimura[13] and by de Wit.[14] A similar model for only strong and electromagnetic interactions was proposed some time ago by Bardakci.[15] The strongly interacting particles in our model are two triplets of vector and axial-vector mesons, presumably the ρ and A_1, a triplet of pions, the nucleon doublet, and one pseudoscalar and two scalar neutral mesons. The weak and electromagnetic interactions are mediated by three massive vector bosons and one massless photon, respectively. Apart from leptons there is one additional spinless particle, as in the Weinberg-Salam model, which interacts only weakly.

The calculations are performed in a continuous set of gauges. For pedagogical reasons we will discuss in some detail the Ward-Takahashi identities for the propagators in the tree approximation. We will make extensive use of those identities in the one-loop calculation, and show that they are crucial for the cancellations among the gauge-dependent parts of our results.

One of the calculated corrections to a zeroth-order symmetry relation is the proton-neutron mass difference. As mentioned previously, Weinberg's result indicated the wrong sign in the case that all vector-boson masses except the photon mass are roughly equal and large. The result in this model is even more discouraging. It turns out that the sign is wrong for all possible values of the parameters. This confirms the general picture that in models based on SU(2) gauge groups the proton is always heavier than the neutron in the one-loop approximation.[7]

Another correction to a zeroth-order relation is the pion mass. As was already mentioned in Ref. 10, the pions in this model are pseudo-Goldstone bosons if an additional reflection symmetry is superimposed. In that case the PCAC (partial conservation of axial-vector current) hypothesis was proved to be correct, and the origins of the chiral-symmetry breaking are the weak and electromagnetic interactions. The first problem is that according to Weinberg's estimate the pion mass will be too large because of its proportionality to the weak intermediate-vector-boson masses. But in addition it was found in a particular model by Lee, Rawls, and Yu,[16] and by Lieberman[17] that although the charged pions picked up a mass due to electromagnetic corrections, the neutral pion remained massless in the one-loop approximation. This necessarily implied that the electromagnetic pion mass difference was enormous. In order to resolve this problem it was proposed that all the pseudo-Goldstone pions in a realistic model should probably pick up their mass in the two-loop approximation, thus being of order e^2M.

In our model we have also performed a calculation of the pion mass, and we will discuss these problems extensively. Our main results are that the pion mass is not proportional to the heavy intermediate-vector-boson masses. If we use the experimental values for the ρ and A_1 meson masses, we find that the pion mass is equal to 37 MeV, which is within one order of magnitude of the experimental value. This is certainly an encouraging result if one wants to consider the pions as pseudo-Goldstone bosons. We also find that the neutral pion remains massless in this approximation. However, we will argue that this is a result of the symmetry structure of the model. Owing to the Abelian character of the electromagnetic gauge group the neutral pion is an exact Goldstone boson[18] in the case that the charged pions are pseudo-Goldstone bosons.[19] This observation shows again an undesirable feature of Abelian gauge groups,[20] and provides another restriction for the construction of realistic models.

Finally, we calculate the pion mass difference for the case that the pion is not a pseudo-Goldstone boson. In the soft-pion limit our result is in agreement with the current-algebra calculation of Das, Guralnik, Mathur, Low, and Young.[21] As was also found by Dicus and Mathur,[22] the contributions from the exchange of weak heavy intermediate bosons are negligible. For hard pions the corrections from the weak interactions, which are in principle comparable to the corrections found by Langacker and Pagels[23] in the context of chiral perturbation theory, cancel in the final answer. We also compare our result with that of Gerstein, Lee, Nieh, and Schnitzer,[24] and find that the hard-pion corrections, which are manifestly finite in our case, are somewhat smaller and of the order of 10%.

In Sec. II we introduce our model. The perturbation scheme and the choice of the gauge are discussed. Ward-Takahashi identities are given in Sec. III, where we also calculate the tadpole diagrams. Section IV contains the calculation of the proton-neutron mass difference and a discussion of higher-order parity-violation effects. The pion mass and pion mass difference are calculated in Sec. V. Finally, in Sec. VI we give our conclusions. Some of the technical details we give in Appendixes A–C.

II. A UNIFIED MODEL OF STRONG, WEAK, AND ELECTROMAGNETIC INTERACTIONS; CHOICE OF THE GAUGE

In this section we will first discuss our unified model, which is a natural extension of the σ model[11] to a gauge model of strong interactions, combined with the Weinberg-Salam model.[12] This model was originally introduced in Ref. 10 as an example in the discussion of the current-algebra properties of gauge-field theories. The gauge group of the strong interactions is the chiral $SU(2)\otimes SU(2)$ group, with corresponding gauge fields X_μ^a and Y_μ^a ($a=1,2,3$). The weak and electromagnetic gauge group is $SU(2)\otimes U(1)$ with gauge fields Z_μ^a and Z_μ^0. The transformation properties of all these fields under the total gauge group are as follows:

$$X_\mu(x)\to U(x)X_\mu(x)U^\dagger(x)+ig_X{}^{-1}U(x)\partial_\mu U^\dagger(x)\,,$$
$$Y_\mu(x)\to V(x)Y_\mu(x)V^\dagger(x)+ig_Y{}^{-1}V(x)\partial_\mu V^\dagger(x)\,,$$
$$Z_\mu(x)\to S(x)Z_\mu(x)S^\dagger(x)+ig_W{}^{-1}S(x)\partial_\mu S^\dagger(x)\,,$$
$$Z_\mu^0(x)\to Z_\mu^0(x)+q^{-1}\partial_\mu\Lambda^0(x)\,.$$

We have used the notation $X_\mu\equiv\frac{1}{2}X_\mu^a\tau_a$, $Y_\mu\equiv\frac{1}{2}Y_\mu^a\tau_a$, $Z_\mu\equiv\frac{1}{2}Z_\mu^a\tau_a$. The corresponding coupling constants are denoted by g_X, g_Y, g_W, and q, and U, V, and S are local SU(2) matrices.

In addition, the model contains a number of spinless, complex doublet fields, K_X, K_Y, K_Σ, and K_Z. These fields, which are represented as 2×2 matrices, have the following transformation properties under the combined $SU(2)\otimes SU(2)\otimes SU(2)\otimes U(1)$ gauge group:

$$K_X(x)\to U(x)K_X(x)S^\dagger(x)\,,$$
$$K_Y(x)\to V(x)K_Y(x)T^\dagger(x)\,,$$
$$K_\Sigma(x)\to U(x)K_\Sigma(x)V^\dagger(x)$$
$$K_Z(x)\to S(x)K_Z(x)T^\dagger(x)\,,$$

with $T(x)=\exp[\frac{1}{2}i\Lambda^0(x)\tau_3]$.

Finally, we will consider the nucleon doublet $N\equiv(p,n)$ and the electron-neutrino doublet $l\equiv(\nu_e,e)$, transforming according to[25,26]

$$N(x)\to\tfrac{1}{2}\exp[\tfrac{1}{2}i\Lambda^0(x)]\times[(1+\gamma_5)U(x)N(x)+(1-\gamma_5)V(x)N(x)]\,,$$
$$l(x)\to\tfrac{1}{2}(1+\gamma_5)\exp[-\tfrac{1}{2}i\Lambda^0(x)]S(x)l(x)+\tfrac{1}{2}(1-\gamma_5)\exp[-\tfrac{1}{2}i\Lambda^0(x)(1-\tau_3)]l(x)\,.$$

The fields Z_μ^a, Z_μ^0, K_Z, and l were already contained in the Weinberg-Salam model and have only weak and electromagnetic interactions. The remaining fields have weak, electromagnetic, and strong interactions.

The most general Lagrangian of dimension less than or equal to four, which is invariant under the combined strong, weak, and electromagnetic gauge transformations, can easily be written down. We divide it into five parts:

$$\mathcal{L}_{\rm INV}=\mathcal{L}_S+\mathcal{L}_{\rm WEM}+\mathcal{L}_\lambda+\mathcal{L}_b+\mathcal{L}_{\rm p.v.}\,. \tag{1}$$

The first term, $\mathcal{L}_S$, contains only the strongly interacting fields together with their interactions with the weak and electromagnetic gauge fields:

$$\begin{aligned}\mathcal{L}_S=&-\tfrac{1}{2}\mathrm{Tr}\{G^X_{\mu\nu}G^X_{\mu\nu}+G^Y_{\mu\nu}G^Y_{\mu\nu}+D_\mu K_X^\dagger D_\mu K_X+D_\mu K_Y^\dagger D_\mu K_Y+D_\mu K_\Sigma^\dagger D_\mu K_\Sigma\}\\&-\overline{N}\gamma_\mu D_\mu N-\tfrac{1}{2}\sqrt{2}\,G_N[\overline{N}(x)K_\Sigma(x)(1-\gamma_5)N(x)+\mathrm{H.c.}]\\&+\mu_1(|K_X|^2+|K_Y|^2)+\mu_2|K_\Sigma|^2+g^2\mu_3(|K_X|^4+|K_Y|^4)+g^2\mu_4|K_X|^2|K_Y|^2+g^2\mu_5|K_\Sigma|^4\\&+g^2\mu_6|K_\Sigma|^2(|K_X|^2+|K_Y|^2)\,.\end{aligned}\tag{2a}$$

The fields that have only weak and electromagnetic interactions are contained in $\mathcal{L}_{\rm WEM}$:

$$\begin{aligned}\mathcal{L}_{\rm WEM}=&-\tfrac{1}{4}G^0_{\mu\nu}G^0_{\mu\nu}-\tfrac{1}{2}\mathrm{Tr}\{G^Z_{\mu\nu}G^Z_{\mu\nu}+D_\mu K_Z^\dagger D_\mu K_Z\}-\bar{l}\gamma_\mu D_\mu l-\tfrac{1}{4}\sqrt{2}\,G_l[\bar{l}K_Z(1-\gamma_5)(1-\tau_3)l+\mathrm{H.c.}]\\&+\rho_1|K_Z|^2+g_W{}^2\rho_2|K_Z|^4\,.\end{aligned}\tag{2b}$$

The remaining terms are given by

$$\mathcal{L}_\lambda=g_W{}^2[\lambda_1(|K_X|^2+|K_Y|^2)+\lambda_2|K_\Sigma|^2]|K_Z|^2\,,\tag{2c}$$

$$\mathcal{L}_b=gg_Wb\,\mathrm{Tr}\{K_Z^\dagger K_X^\dagger K_\Sigma K_Y\}\,,\tag{2d}$$

$$\mathcal{L}_{\rm p.v.}=(|K_X|^2-|K_Y|^2)[\delta_1'+g_W{}^2\delta_2(|K_X|^2+|K_Y{}^2|)+g_W{}^2\delta_3|K_\Sigma|^2+g_W{}^2\delta_4|K_Z|^2]\,.\tag{2e}$$

We have used the following definitions:

$$G^0_{\mu\nu}=\partial_\mu Z^0_\nu-\partial_\nu Z^0_\mu\,,$$

$$G^X_{\mu\nu}=\partial_\mu X_\nu-\partial_\nu X_\mu-ig_X[X_\mu,X_\nu]\,,$$

and similarly for $G^Y_{\mu\nu}$ and $G^Z_{\mu\nu}$. The covariant derivatives are given by

$$\begin{aligned}
D_\mu K_X&=\partial_\mu K_X-ig_XX_\mu K_X+ig_WK_XZ_\mu\,,\\
D_\mu K_Y&=\partial_\mu K_Y-ig_YY_\mu K_Y+\tfrac{1}{2}iqZ^0_\mu K_Y\tau_3\,,\\
D_\mu K_\Sigma&=\partial_\mu K_\Sigma-ig_XX_\mu K_\Sigma+ig_YK_\Sigma Y_\mu\,,\\
D_\mu K_Z&=\partial_\mu K_Z-ig_WZ_\mu K_Z+\tfrac{1}{2}iqZ^0_\mu K_Z\tau_3\,,\\
D_\mu N&=\partial_\mu N-\tfrac{1}{2}ig_XX_\mu(1+\gamma_5)N\\
&\quad-\tfrac{1}{2}ig_YY_\mu(1-\gamma_5)N-\tfrac{1}{2}iqZ^0_\mu N\,,\\
D_\mu l&=\partial_\mu l-\tfrac{1}{2}ig_WZ_\mu(1+\gamma_5)l\\
&\quad+\tfrac{1}{2}iqZ^0_\mu[1-\tfrac{1}{2}\tau_3(1-\gamma_5)]l\,.
\end{aligned}$$

Moreover, we define $|K|^2=\mathrm{Tr}\{K^\dagger K\}$ and $g=\frac{1}{2}(g_X+g_Y)$. We have explicitly extracted the factors of g and g_W in the interaction Lagrangian of the spinless fields so that an expansion in terms of these parameters (and q, G_N, and G_l) corresponds to an expansion in terms of numbers of closed loops.

Although the Lagrangian (1) does not contain explicit mass terms for the gauge fields, these fields can acquire masses by means of the Higgs-Kibble mechanism.[27] This means that the gauge-field masses are generated by the presence of nonzero vacuum expectation values of the spinless fields. In doing so, the local gauge invariance is not disturbed and the renormalizability is preserved.[5,6]

As was argued in Ref. 10, the spinless fields can generally be decomposed as follows:

$$\begin{aligned}
K_X&=\tfrac{1}{2}(2\sqrt{2}\,g^{-1}M_U+\sigma_U+\sigma_V+2i\psi_U+2i\psi_V)\,,\\
K_Y&=\tfrac{1}{2}(2\sqrt{2}\,g^{-1}M_U+\sigma_U-\sigma_V+2i\psi_U-2i\psi_V)\,,\\
K_\Sigma&=\tfrac{1}{2}\sqrt{2}\,(\sqrt{2}\,g^{-1}\epsilon M_U+\sigma_\Sigma+2i\psi_\Sigma)\,,\\
K_Z&=\tfrac{1}{2}\sqrt{2}\,(2g_W{}^{-1}M_Z+\sigma_Z+2i\psi_Z)\,,
\end{aligned}\qquad(3)$$

where we use the notation $\psi\equiv\frac{1}{2}\psi^a\tau_a$.

In these decompositions we have already taken into account the vacuum expectation values of σ_U, σ_Σ, and σ_Z in the tree approximation. In doing so three new parameters, M_U, M_Z, and ϵ, were introduced. Because of their presence the Lagrangian Eq. (1) will contain terms which are linear in the fields $\sigma_U,\sigma_V,\sigma_\Sigma,\sigma_Z$, and M_U,M_Z, and ϵ are determined by the requirement that the terms linear in σ_U, σ_Σ, and σ_Z vanish. Equivalently, we will consider M_U, M_Z, and ϵ as free parameters, instead of μ_1, μ_2, and ρ_1, which are then determined by the previous conditions.[28] In general, σ_V will also have a vacuum expectation value, and the Lagrangian has a term proportional to σ_V. However, we will treat those terms differently for reasons which are explained below. The parameter δ_1' will be replaced by δ_1 such that the coefficient of the term linear in σ_V is equal to $2\sqrt{2}\,g^{-1}M_U\delta_1$.

Owing to the presence of the nonzero vacuum expectation values, all the gauge fields except one will acquire a mass. In order to make this more transparent, let us make the following substitutions:

$$\begin{aligned}
X_\mu&=\tfrac{1}{2}\sqrt{2}\,(U_\mu+V_\mu)+\tfrac{1}{2}eg_X{}^{-1}A_\mu\tau_3\,,\\
Y_\mu&=\tfrac{1}{2}\sqrt{2}(U_\mu-V_\mu)+\tfrac{1}{2}eg_Y{}^{-1}A_\mu\tau_3\,,\\
Z_\mu&=W_\mu+\tfrac{1}{2}eg_W{}^{-1}A_\mu\tau_3\,,\\
Z^0_\mu&=eq^{-1}A_\mu\,,
\end{aligned}$$

with

$$e=g_Xg_Yg_Wq(q^2g_X{}^2g_Y{}^2+q^2g_X{}^2g_W{}^2+q^2g_Y{}^2g_W{}^2+g_X{}^2g_Y{}^2g_W{}^2)^{-1/2}\,.$$

The field A_μ remains massless in all orders of perturbation theory, and is to be identified as the photon field. All the remaining gauge fields will turn out to be massive.

In a gauge-field theory higher-order calculations must be performed in a specific gauge.[27] A convenient way of choosing a gauge is to replace the invariant Lagrangian Eq. (1) by

$$\mathcal{L}=\mathcal{L}_{\mathrm{INV}}-\tfrac{1}{2}C_A{}^2-\mathrm{Tr}\{C_U{}^2+C_V{}^2+C_W{}^2\}\,,\qquad(4a)$$

where the additional terms completely remove the original gauge invariance. We make the following choice for C_U, C_V, C_W, C_A:

$$\begin{aligned}
C_U&=\xi_U\partial_\mu U_\mu-\xi_U{}^{-1}M_U\psi_U\,,\\
C_V&=\xi_V\partial_\mu V_\mu-\xi_V{}^{-1}M_U(\psi_V+\epsilon\psi_\Sigma)\,,\\
C_W&=\xi_W\partial_\mu W_\mu\\
&\quad-\xi_W{}^{-1}M_Z[\psi_Z-\tfrac{1}{2}\sqrt{2}\,g_Wg^{-1}M_UM_Z{}^{-1}(\psi_U+\psi_V)]\,,\\
C_A&=\xi_A\partial_\mu A_\mu\,.
\end{aligned}\qquad(4b)$$

The parameters ξ_U, ξ_V, ξ_W, and ξ_A are arbitrary, and physical quantities should be independent of them. In addition to these gauge-fixing terms we must add the Faddeev-Popov Lagrangian. This Lagrangian follows straightforwardly from the behavior of C_U, C_V, C_W, and C_A under the infinitesimal gauge transformations (see Appendix A):

$$
\begin{aligned}
\mathfrak{L}_{\mathrm{FP}} = & -2\sqrt{2}\,\mathrm{Tr}\{\xi_U \partial_\mu \phi_U^* \partial_\mu^{\mathrm{EM}} \phi_U + \xi_U{}^{-1} M_U{}^2 \phi_U^* \phi_U + \xi_V \partial_\mu \phi_V^* \partial_\mu^{\mathrm{EM}} \phi_V + \xi_V{}^{-1} M_V{}^2 \phi_V^* \phi_V\} \\
& - 2\xi_W\,\mathrm{Tr}\{\partial_\mu \phi_W^* \partial_\mu^{\mathrm{EM}} \phi_W + \xi_W{}^{-2} M_W{}^2 \phi_W^* \phi_W\} \\
& - \xi_A \partial_\mu \phi_A^* \partial_\mu \phi_A + \sqrt{2}\, g_W g^{-1} M_U{}^2\,\mathrm{Tr}\{\xi_U{}^{-1} \phi_U^* \phi_W + \xi_V{}^{-1} \phi_V^* \phi_W + \sqrt{2}\,\xi_W{}^{-1} \phi_W^* (\phi_U + \phi_V)\} \\
& - 2ig\,\mathrm{Tr}\{\xi_U \partial_\mu \phi_U^* ([\phi_U, U_\mu] + [\phi_V, V_\mu] + eg^{-1}[\phi_A \tfrac{1}{2}\tau_3, U_\mu]) + \xi_V \partial_\mu \phi_V^* ([\phi_U, V_\mu] + [\phi_V, U_\mu] + eg^{-1}[\phi_A \tfrac{1}{2}\tau_3, V_\mu])\} \\
& - 2i\xi_W\,\mathrm{Tr}\{\partial_\mu \phi_W^* (g_W[\phi_W, W_\mu] + e[\phi_A \tfrac{1}{2}\tau_3, W_\mu])\} \\
& - gM_U\,\mathrm{Tr}\{\xi_U{}^{-1} \phi_U^* (\phi_U \sigma_U + \phi_V \sigma_V + i[\phi_U, \psi_U] + i[\phi_V, \psi_V]) \\
& \qquad\qquad + \xi_V{}^{-1} \phi_V^* (\phi_U \sigma_V + \phi_V(\sigma_U + 2\epsilon\sigma_\Sigma) + i[\phi_U, \psi_V + 2\epsilon\psi_\Sigma] + i[\phi_V, \psi_U])\} \\
& - g_W M_Z \xi_W{}^{-1}\,\mathrm{Tr}\{\phi_W^* \phi_W (\sigma_Z + \tfrac{1}{2}\sqrt{2}\, g_W g^{-1} M_U M_Z{}^{-1} (\sigma_U + \sigma_V)) + i\phi_W^* [\phi_W, \psi_Z] + 2ieg_W{}^{-1} \phi_W^* [\phi_A \tfrac{1}{2}\tau_3, \psi_Z]\} \\
& + \tfrac{1}{2} g_W M_U\,\mathrm{Tr}\{(\xi_U{}^{-1} \phi_U^* + \xi_V{}^{-1} \phi_V^*)(\phi_W(\sigma_U + \sigma_V) - i[\phi_W, \psi_U + \psi_V])\} \\
& - 2ieM_U\,\mathrm{Tr}\{\xi_U{}^{-1} \phi_U^* [\phi_A \tfrac{1}{2}\tau_3, \psi_U] + \xi_V{}^{-1} \phi_V^* [\phi_A \tfrac{1}{2}\tau_3, \psi_V + \epsilon\psi_\Sigma]\} \\
& + \tfrac{1}{2}\sqrt{2}\, g_W M_U \xi_W{}^{-1}\,\mathrm{Tr}\{\phi_W^* (\phi_U + \phi_V)(\sigma_U + \sigma_V) + i\phi_W^* [\phi_U + \phi_V, \psi_U + \psi_V] \\
& \qquad\qquad + 2ieg^{-1} \phi_W^* [\phi_A \tfrac{1}{2}\tau_3, \psi_U + \psi_V] + ig_W g^{-1} \phi_W^* [\phi_W, \psi_U + \psi_V]\}.
\end{aligned}
\tag{5}
$$

The fields ϕ_A, ϕ_U^a, ϕ_V^a, and ϕ_W^a are the unphysical Faddeev-Popov ghost fields, which obey Fermi-Dirac statistics. We have used the definitions

$$
\begin{aligned}
& \phi_{U,V,W} = \tfrac{1}{2}\phi_{U,V,W}^a \tau_a, \quad \phi_{U,V,W}^* = \tfrac{1}{2}\phi_{U,V,W}^{*a} \tau_a, \quad \partial_\mu^{\mathrm{EM}}\phi = \partial_\mu \phi - \tfrac{1}{2}ieA_\mu[\tau_3, \phi], \\
& M_W{}^2 = M_Z{}^2 + g_W{}^2 g^{-2} M_U{}^2, \quad \text{and } M_V{}^2 = M_U{}^2(1+\epsilon^2).
\end{aligned}
\tag{6}
$$

Subsequently, we consider the effects of the vacuum expectation values in the free part of the Lagrangian, which is given by the terms linear or quadratic in the fields:

$$
\begin{aligned}
\mathfrak{L}_0 = & -\tfrac{1}{2}[(\partial_\mu A_\nu)^2 - (1-\xi_A{}^2)(\partial_\mu A_\mu)^2] \\
& - \mathrm{Tr}\{(\partial_\mu U_\nu)^2 - (1-\xi_U{}^2)(\partial_\mu U_\mu)^2 + M_U{}^2 U_\mu{}^2 + (\partial_\mu V_\nu)^2 - (1-\xi_V{}^2)(\partial_\mu V_\mu)^2 + M_V{}^2 V_\mu{}^2 + (\partial_\mu W_\nu)^2 - (1-\xi_W{}^2)(\partial_\mu W_\mu)^2 \\
& \qquad + M_W{}^2 W_\mu{}^2\} \\
& - e(\partial_\mu A_\nu - \partial_\nu A_\mu)\,\mathrm{Tr}\{g^{-1}\sqrt{2}\,\partial_\mu U_\nu \tau_3 + g_W{}^{-1}\partial_\mu W_\nu \tau_3\} \\
& + g_W g^{-1}\sqrt{2}\, M_U{}^2\,\mathrm{Tr}\{W_\mu(U_\mu + V_\mu)\} \\
& - \mathrm{Tr}\{(\partial_\mu\psi_U)^2 + (\partial_\mu\psi_V)^2 + (\partial_\mu\psi_\Sigma)^2 + (\partial_\mu\psi_Z)^2 \\
& \qquad + (\xi_U{}^{-2} + \tfrac{1}{2} g_W{}^2 g^{-2} \xi_W{}^{-2}) M_U{}^2 \psi_U^2 + (\xi_V{}^{-2} + \tfrac{1}{2} g_W{}^2 g^{-2} \xi_W{}^{-2} - 8\sqrt{2}\,\epsilon b M_Z M_U{}^{-1}) M_U{}^2 \psi_V{}^2 \\
& \qquad + (\epsilon^2 \xi_V{}^{-2} M_U{}^2 - 8\sqrt{2}\,\epsilon^{-1} b M_U M_Z)\psi_\Sigma{}^2 + (\xi_W{}^{-2} M_Z{}^2 - 4\sqrt{2}\,\epsilon b g_W{}^2 g^{-2} M_U{}^3 M_Z{}^{-1})\psi_Z{}^2 \\
& \qquad - 2(\epsilon\xi_V{}^{-2} M_U - 8\sqrt{2}\, b M_Z) M_U \psi_V \psi_\Sigma + g_W{}^2 g^{-2} \xi_W{}^{-2} M_U{}^2 \psi_U \psi_V \\
& \qquad - \sqrt{2}\, g_W g^{-1} \xi_W{}^{-2} M_U M_Z \psi_Z (\psi_U + \psi_V) + 16 b g_W g^{-1} M_U{}^2 \psi_Z (\epsilon\psi_V + \psi_\Sigma)\} \\
& - \tfrac{1}{2}[(\partial_\mu\sigma_U)^2 + (\partial_\mu\sigma_V)^2 + (\partial_\mu\sigma_\Sigma)^2 + (\partial_\mu\sigma_Z)^2] \\
& + 8[4\mu_3 M_U{}^2 \sigma_U{}^2 + (\mu_4 M_U + \sqrt{2}\,\epsilon b M_Z) M_U \sigma_V{}^2 + (\epsilon^2 \mu_5 M_U + \sqrt{2}\,\epsilon^{-1} b M_Z) M_U \sigma_\Sigma{}^2 \\
& \qquad + (2\rho_2 + \tfrac{1}{2}\sqrt{2}\,\epsilon g_W{}^2 g^{-2} M_U{}^3 M_Z{}^{-3}) M_Z{}^2 \sigma_Z{}^2 \\
& \qquad + (2\epsilon\mu_6 M_U - 2\sqrt{2}\, b M_Z) M_U \sigma_U \sigma_\Sigma + \sqrt{2}\, g_W g^{-1} M_U M_Z (2\lambda_1 \sigma_U + \lambda_2 \sigma_\Sigma)\sigma_Z \\
& \qquad + g_W g^{-1} M_U (2 g_W g^{-1} M_U \delta_2 \sigma_U + \epsilon g_W g^{-1} M_U \delta_3 \sigma_\Sigma + \sqrt{2}\, M_Z \delta_4 \sigma_Z)\sigma_V - 2 b g_W g^{-1} M_U{}^2 (\epsilon\sigma_U + \sigma_\Sigma)\sigma_Z] \\
& + \delta_1 (2\sqrt{2}\, g^{-1} M_U \sigma_V + \sigma_U \sigma_V + \psi_U \psi_V) - \overline{N}(\gamma_\mu \partial_\mu + m)N - \overline{l}[\gamma_\mu \partial_\mu + \tfrac{1}{2} m_l (1-\tau_3)]l.
\end{aligned}
\tag{7}
$$

In this expression we neglected terms of order $g_X - g_Y$. The nucleon and lepton masses were given by

$$m = \sqrt{2}\,\epsilon g^{-1} G_N M_U\,, \qquad m_l = 2 g_W{}^{-1} G_l M_Z\,. \tag{8}$$

As follows from Eq. (7) the previously defined quantities M_U, M_V, and M_W correspond to the masses of the strongly interacting gauge fields U_μ and V_μ and of the weak intermediate vector bosons W_μ (up to electromagnetic corrections). The calculation of the propagators in lowest order is now straightforward. Notice that due to the choice of the gauge we have eliminated the transitions between gauge fields and spinless fields. The propagators of the vector bosons are decomposed into two parts:

$$D_{\mu\nu}(q) = D_T(q^2)(\delta_{\mu\nu} - q_\mu q_\nu q^{-2}) + D_L(q^2) q_\mu q_\nu q^{-2}\,.$$

It turns out that only $D_T(q^2)$ depends on the charge of the vector field. Therefore, the first term of the neutral vector propagators will be denoted by $\tilde{D}_T$. Appendix B gives the expressions for D_T, $\tilde{D}_T$, D_L, the propagators of the Faddeev-Popov fields D_{FP}, and the propagators of the spinless triplet fields D_ψ.

The reason why we neglected terms of order $g_X - g_Y$ in the Lagrangian Eq. (7) and why we treated the vacuum expectation value of σ_V differently from those of σ_U, σ_Σ, and σ_Z is that those terms will contribute to the violation of parity. As is well known, the total Lagrangian does not necessarily lead to hadronic parity violations in higher orders which are of the size of G_F, the Fermi coupling constant of the weak interactions. Even if we neglect δ_i, $g_X - g_Y$, and the vacuum expectation value of σ_V in the tree approximation, this will not guarantee that parity violations in higher orders are of the size of G_F. In general, δ_i and $g_X - g_Y$ have to be adjusted in higher orders so that these parity violations are canceled. This is particularly the case when we need δ_i and $g_X - g_Y$ as counterterms in the Lagrangian. We will make some more comments on parity violation in these kinds of models in Sec. IV.

The intermediate vector bosons of the weak interactions are very massive, because the vacuum expectation value of σ_Z is supposed to be large. Therefore the Fermi coupling constant is given by $G_F = \frac{1}{8}\sqrt{2}\, g_W{}^2 M_Z{}^{-2}$. This, however, implies that the coupling of σ_Z with the hadrons must be small, because otherwise the vacuum expectation value of σ_Z would induce effects which would be too large. Hence λ_1 and λ_2 are supposed to be of order G_F, whereas b must be of order $G_F{}^{1/2}$.

III. WARD-TAKAHASHI IDENTITIES; DEFINITION OF THE PION FIELD; CALCULATION OF THE VACUUM EXPECTATION VALUES

Before starting any higher-order calculation, we will analyze the Ward-Takahashi identities for the propagators in lowest order. They give us a check on the consistency of the lowest-order calculations, and provide us with simple relations among various propagators which will turn out to be crucial in order to establish the gauge independence of physical quantities.[27]

Let us first generally give the Ward-Takahashi identities in the diagrammatic formulation of 't Hooft and Veltman.[6] The behavior of the fields under the infinitesimal gauge transformations can be written as

$$A_i(x) \to A_i(x) + t_i^\alpha \Lambda^\alpha(x) + g s_{ij}^\alpha A_j(x) \Lambda^\alpha(x)\,.$$

For our model these transformation properties are given in Appendix A. Here the fields are denoted by A_i, and t_i^α is either a constant or a derivative. The quantities s_{ij}^α are simple constants, which may depend on the coupling constants. With these definitions the generalized Ward-Takahashi identities for the propagators can be graphically represented as in Fig. 1. A solid line with index i belongs to the field A_i, and C_α denotes one of the linear combinations of the fields that are given by C_A, C_U^a, C_V^a, or C_W^a, which were defined in Eqs. (4). A dashed line with index α denotes one of the Faddeev-Popov ghost fields ϕ_A, ϕ_U^a, ϕ_V^a, or ϕ_W^a. The vertices $t_i^\beta \phi^\beta$ and $g s_{ij}^\beta A_j \phi^\beta$, which do not occur in the S matrix, are defined by the infinitesimal transformation properties of the fields A_i.

In the tree approximation the last term of the first identity will not contribute. The identities then have a simple form, especially since, for our choice of the gauge, there are no transitions between gauge fields and spinless meson fields. Using the quantities t_i^α, which are given in Appendix A, we find the following relations for the propagators:

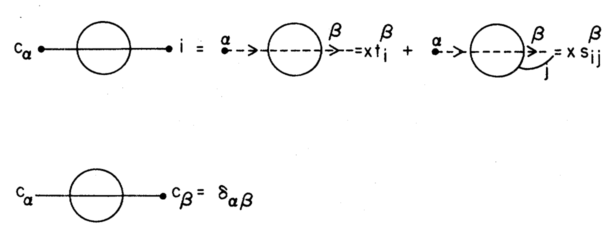

FIG. 1. The Ward-Takahashi identities for the propagators.

$$
\begin{aligned}
&D_L^{AA}=\xi_A{}^{-2}q^{-2}\,,\quad D_L^{Ai}=0\ \text{for}\ i=U,V,W\,,\quad D_L^{UU}=\sqrt{2}\,\xi_U{}^{-1}D_{\rm FP}^{UU}\,,\\
&D_L^{VV}=\sqrt{2}\,\xi_V{}^{-1}D_{\rm FP}^{VV}\,,\quad D_L^{WW}=\xi_W{}^{-1}D_{\rm FP}{}^{WW}\,,\quad D_L^{UV}=D_L^{VU}=\sqrt{2}\,\xi_U{}^{-1}D_{\rm FP}^{UV}=\sqrt{2}\,\xi_V{}^{-1}D_{\rm FP}^{VU}\,,\\
&D_L^{UW}=D_L^{WU}=\xi_U{}^{-1}D_{\rm FP}^{UW}=\sqrt{2}\,\xi_W{}^{-1}D_{\rm FP}^{WU}\,,\quad D_L^{VW}=D_L^{WV}=\xi_V{}^{-1}D_{\rm FP}^{VW}=\sqrt{2}\,\xi_W{}^{-1}D_{\rm FP}^{WV}\,,\\
&D_\psi^{UU}=\sqrt{2}\,\xi_U D_{\rm FP}^{UU}-\tfrac{1}{2}\sqrt{2}\,\xi_U g_W g^{-1}D_{\rm FP}^{UW}\,,\quad D_\psi^{UV}=\sqrt{2}\,\xi_U D_{\rm FP}^{UV}-\tfrac{1}{2}\sqrt{2}\,\xi_U g_W g^{-1}D_{\rm FP}^{UW}\,,\\
&D_\psi^{U\Sigma}=\sqrt{2}\epsilon\,\xi_U D_{\rm FP}^{UV}\,,\quad D_\psi^{UZ}=\xi_U M_Z M_U{}^{-1}D_{\rm FP}^{UW}\,,\quad D_\psi^{VU}+\epsilon D_\psi^{\Sigma U}=\sqrt{2}\,\xi_V D_{\rm FP}^{VU}-\tfrac{1}{2}\sqrt{2}\,\xi_V g_W g^{-1}D_{\rm FP}^{VW}\,,\\
&D_\psi^{VV}+\epsilon D_\psi^{\Sigma V}=\sqrt{2}\,\xi_V D_{\rm FP}^{VV}-\tfrac{1}{2}\sqrt{2}\,\xi_V g_W g^{-1}D_{\rm FP}^{VW}\,,\quad D_\psi^{V\Sigma}+\epsilon D_\psi^{\Sigma\Sigma}=\sqrt{2}\,\epsilon\,\xi_V D_{\rm FP}^{VV}\,,\quad D_\psi^{VZ}+\epsilon D_\psi^{\Sigma Z}=\xi_V M_Z M_U{}^{-1}D_{\rm FP}^{VW}\,;\\
&M_Z D_\psi^{ZU}-\tfrac{1}{2}\sqrt{2}\,g_W g^{-1}M_U(D_\psi^{UU}+D_\psi^{VU})=\sqrt{2}\,\xi_W M_U D_{\rm FP}^{WU}-\tfrac{1}{2}\sqrt{2}\,\xi_W g_W g^{-1}M_U D_{\rm FP}^{WW}\,,\\
&M_Z D_\psi^{ZV}-\tfrac{1}{2}\sqrt{2}\,g_W g^{-1}M_U(D_\psi^{UV}+D_\psi^{VV})=\sqrt{2}\,\xi_W M_U D_{\rm FP}^{WV}-\tfrac{1}{2}\sqrt{2}\,\xi_W g_W g^{-1}M_U D_{\rm FP}^{WW}\,,\\
&M_Z D_\psi^{Z\Sigma}-\tfrac{1}{2}\sqrt{2}\,g_W g^{-1}M_U(D_\psi^{U\Sigma}+D_\psi^{V\Sigma})=\sqrt{2}\,\epsilon\,\xi_W M_U D_{\rm FP}^{WV}\,,\quad M_Z D_\psi^{ZZ}-\tfrac{1}{2}\sqrt{2}\,g_W g^{-1}M_U(D_\psi^{UZ}+D_\psi^{VZ})=\xi_W M_Z D_{\rm FP}^{WW}\,;\\
&\xi_U{}^2q^2D_L^{UU}+\xi_U{}^{-2}M_U{}^2D_\psi^{UU}=1\,,\quad \xi_V{}^2q^2D_L^{VV}+\xi_V{}^{-2}M_U{}^2(D_\psi^{VV}+\epsilon^2D_\psi^{\Sigma\Sigma}+2\epsilon D_\psi^{V\Sigma})=1\,,\\
&\xi_W{}^2q^2D_L^{WW}+\xi_W{}^{-2}[M_Z{}^2D_\psi^{ZZ}-\sqrt{2}\,g_W g^{-1}M_U M_Z(D_\psi^{UZ}+D_\psi^{VZ})+\tfrac{1}{2}g_W{}^2g^{-2}M_U{}^2(D_\psi^{UU}+D_\psi^{VV}+2D_\psi^{UV})]=1\,,\\
&\xi_U{}^2\xi_V{}^2q^2D_L^{UV}+M_U{}^2(D_\psi^{UV}+\epsilon D_\psi^{U\Sigma})=0\,,\quad \xi_U{}^2\xi_W{}^2q^2D_L^{UW}+M_U M_Z D_\psi^{UZ}-\tfrac{1}{2}\sqrt{2}\,g_W g^{-1}M_U{}^2(D_\psi^{UU}+D_\psi^{UV})=0\,,\\
&\xi_V{}^2\xi_W{}^2q^2D_L^{VW}+M_U M_Z D_\psi^{VZ}+\epsilon M_U M_Z D_\psi^{\Sigma Z}-\tfrac{1}{2}\sqrt{2}\,g_W g^{-1}M_U{}^2(D_\psi^{UV}+D_\psi^{VV}+\epsilon D_\psi^{\Sigma U}+\epsilon D_\psi^{\Sigma V})=0\,.
\end{aligned}
\tag{9}
$$

Although the ψ propagators are in general ξ-dependent, there are certain combinations which do not depend on the gauge. This follows from the fact that there is one linear combination of the fields ψ,

$$
\pi=(1+\epsilon^2+\tfrac{1}{2}\epsilon^2g_W{}^2g^{-2}M_U{}^2M_Z{}^{-2})^{-1/2}\times(-\epsilon\psi_V+\psi_\Sigma-\tfrac{1}{2}\sqrt{2}\,\epsilon g_W g^{-1}M_U M_Z{}^{-1}\psi_Z)\,, \tag{10}
$$

which, in this order of perturbation theory, has a gauge-independent mass μ^2. This field is the physical pion field and is the only physical spinless triplet field in the model. Its mass is given by

$$
\mu^2=8\sqrt{2}\,\epsilon M_U M_Z b(1+\epsilon^{-2}+\tfrac{1}{2}g_W{}^2g^{-2}M_U{}^2M_Z{}^{-2})\,. \tag{11}
$$

Using the fact that the pion field is an eigenstate of the ψ propagator, we have the following useful relations:

$$
\begin{aligned}
&\epsilon D_\psi^{UV}-D_\psi^{U\Sigma}+\tfrac{1}{2}\sqrt{2}\,\epsilon g_W g^{-1}M_U M_Z{}^{-1}D_\psi^{UZ}=0\,,\\
&\epsilon D_\psi^{VV}-D_\psi^{V\Sigma}+\tfrac{1}{2}\sqrt{2}\,\epsilon g_W g^{-1}M_U M_Z{}^{-1}D_\psi^{VZ}=\epsilon(q^2+\mu^2)^{-1}\,,\\
&\epsilon D_\psi^{\Sigma V}-D_\psi^{\Sigma\Sigma}+\tfrac{1}{2}\sqrt{2}\,\epsilon g_W g^{-1}M_U M_Z{}^{-1}D_\psi^{\Sigma Z}=-(q^2+\mu^2)^{-1}\,,\\
&\epsilon D_\psi^{ZV}-D_\psi^{Z\Sigma}+\tfrac{1}{2}\sqrt{2}\,\epsilon g_W g^{-1}M_U M_Z{}^{-1}D_\psi^{ZZ}=\tfrac{1}{2}\sqrt{2}\,\epsilon g_W g^{-1}M_U M_Z{}^{-1}(q^2+\mu^2)^{-1}\,.
\end{aligned}
\tag{12}
$$

Because $\mathcal{L}_b$ is the only term in the original Lagrangian (1) which was linear in each of the fields K_X, K_Y, K_Σ, and K_Z, we can consider $b=0$ consistently in all orders of perturbation theory. In that case the pion is a pseudo-Goldstone boson in this model, as was already obvious from Eq. (11), because the pion mass in tree approximation is proportional to b. If $b=0$, the contributions to the pion mass must originate from closed-loop corrections, and those contributions should be finite because there is no corresponding mass counterterm in the Lagrangian.

Finally, we turn to the calculation of the vacuum expectation values of the fields σ_U, σ_V, σ_Σ, and σ_Z in the one-loop approximation. The diagrams which contribute are depicted in Fig. 2. Notice that we have an additional contribution to the vacuum expectation values coming from the term $2\sqrt{2}g^{-1}M_U\delta_1\sigma_V$ in the Lagrangian. This was discussed in Sec. II.

Making use of the Ward-Takahashi identities Eqs. (9), it turns out that the contribution of the Faddeev-Popov ghost loops and the D_L part of the gauge-field loops cancel each other. If we then use the relations between certain coupling constants and the inverse propagators of the σ fields, as was first proposed by Weinberg,[4] we can write the result in the following form:

$$
T_i^{\rm tot}=D_\sigma{}^{-1}(0)^{ij}t_j+T_i\,,\quad i,j=U,V,\Sigma,Z \tag{13a}
$$

where $T_i^{\rm tot}$ is the sum of all one-loop tadpole graphs with a σ_i line vanishing into the vacuum, and D_σ^{ij} is the propagator of the σ fields. Making use of Eq. (12) we find that the quantities T_i are ξ-independent, as they should be. The results are given by

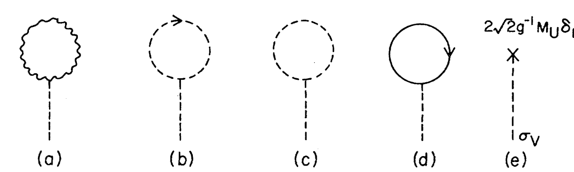

FIG. 2. The diagrams which contribute to the vacuum expectation values of σ_U, σ_V, σ_Σ, and σ_Z in the one-loop approximation: (a) vector-boson tadpole; (b) Faddeev-Popov tadpole; (c) scalar-boson tadpole; (d) fermion tadpole; (e) contribution from the linear term in σ_V in the Lagrangian.

$$t_U = -\tfrac{1}{8}\sqrt{2}\,gM_U{}^{-1}\int d^nq[3D_\psi^{UU}(q)+3D_\psi^{VV}(q)+D_\sigma^{UU}(q)+D_\sigma^{VV}(q)],$$

$$t_V = -\tfrac{1}{4}\sqrt{2}\,gM_U{}^{-1}\int d^nq[3D_\psi^{UV}(q)+D_\sigma^{UV}(q)],$$

$$t_\Sigma = -\tfrac{1}{4}\sqrt{2}\,gM_U{}^{-1}\epsilon^{-1}\int d^nq[3D_\psi^{\Sigma\Sigma}(q)+D_\sigma^{\Sigma\Sigma}(q)],$$

$$t_Z = -\tfrac{1}{4}g_WM_Z{}^{-1}\int d^nq[3D_\psi^{ZZ}(q)+D_\sigma^{ZZ}(q)],$$

$$\begin{aligned}T_U = {}& -\tfrac{1}{4}\sqrt{2}\,g^{-1}M_U(n-1)\int d^nq[g^2(2D_T^{UU}+\tilde{D}_T^{UU}+2D_T^{VV}+\tilde{D}_T^{VV})\\&\qquad -\sqrt{2}\,gg_W(2D_T^{UW}+\tilde{D}_T^{UW}+2D_T^{VW}+\tilde{D}_T^{VW})+g_W{}^2(2D_T^{WW}+\tilde{D}_T^{WW})]\\&+\tfrac{1}{4}\sqrt{2}\,gM_U{}^{-1}\int d^nq\,q^2(D_\sigma^{UU}+D_\sigma^{VV})\\&+4gM_Zb\int d^nq[\epsilon D_\sigma^{VV}-\epsilon^{-1}D_\sigma^{\Sigma\Sigma}+\sqrt{2}\,g_Wg^{-1}M_UM_Z{}^{-1}D_\sigma^{\Sigma Z}-\tfrac{1}{2}\epsilon g_W{}^2g^{-2}M_U{}^2M_Z{}^{-2}D_\sigma^{ZZ}\\&\qquad -3\epsilon(\epsilon^{-2}+\tfrac{1}{2}g_W{}^2g^{-2}M_U{}^2M_Z{}^{-2})(q^2+\mu^2)^{-1}],\end{aligned}$$

$$\begin{aligned}T_V = {}& 2\sqrt{2}\,i(2\pi)^4g^{-1}M_U\delta_1-\tfrac{1}{2}\sqrt{2}\,g^{-1}M_U(n-1)\int d^nq[g^2(2D_T^{UV}+\tilde{D}_T^{UV})-\tfrac{1}{2}\sqrt{2}\,gg_W(2D_T^{UW}+\tilde{D}_T^{UW}+2D_T^{VW}+\tilde{D}_T^{VW})\\&\qquad +\tfrac{1}{2}g_W{}^2(2D_T^{WW}+\tilde{D}_T^{WW})]\\&+\tfrac{1}{2}\sqrt{2}\,gM_U{}^{-1}\int d^nq\,q^2D_\sigma^{UV}+4gM_Zb\int d^nq(2\epsilon D_\sigma^{UV}-D_\sigma^{V\Sigma}-\sqrt{2}\,\epsilon g_Wg^{-1}M_UM_Z{}^{-1}D_\sigma^{VZ}),\end{aligned}$$

$$\begin{aligned}T_\Sigma = {}& -\tfrac{1}{2}\sqrt{2}\,\epsilon g^{-1}M_U\int d^nq[(n-1)g^2(2D_T^{VV}+\tilde{D}_T^{VV})+16G_N{}^2(q^2+m^2)^{-1}-g^2M_U{}^{-2}\epsilon^{-2}q^2D_\sigma^{\Sigma\Sigma}]\\&+2gM_Zb\int d^nq[-4\epsilon^{-1}D_\sigma^{U\Sigma}+6\epsilon^{-2}D_\sigma^{\Sigma\Sigma}-2D_\sigma^{VV}+2\sqrt{2}\,g_Wg^{-1}M_UM_Z{}^{-1}(D_\sigma^{UZ}-\epsilon^{-1}D_\sigma^{\Sigma Z})-g_W{}^2g^{-2}M_U{}^2M_Z{}^{-2}D_\sigma^{ZZ}\\&\qquad -6(1-\epsilon^{-2}+\tfrac{1}{2}g_W{}^2g^{-2}M_U{}^2M_Z{}^{-2})(q^2+\mu^2)^{-1}],\end{aligned}\tag{13b}$$

$$\begin{aligned}T_Z = {}& -\tfrac{1}{2}g_W{}^{-1}M_Z\int d^nq[(n-1)g_W{}^2(2D_T^{WW}+\tilde{D}_T^{WW})+16G_l{}^2(q^2+m_l{}^2)^{-1}-g_W{}^2M_Z{}^{-2}q^2D_\sigma^{ZZ}]\\&+2\sqrt{2}\,g_WM_Ub\int d^nq[2D_\sigma^{U\Sigma}-\epsilon D_\sigma^{VV}-\epsilon^{-1}D_\sigma^{\Sigma\Sigma}-\sqrt{2}\,\epsilon g_Wg^{-1}M_UM_Z{}^{-1}(D_\sigma^{UZ}+\epsilon^{-1}D_\sigma^{\Sigma Z})\\&\qquad +\tfrac{3}{2}\epsilon g_W{}^2g^{-2}M_U{}^2M_Z{}^{-2}D_\sigma^{ZZ}-3\epsilon(1+\epsilon^{-2}-\tfrac{1}{2}g_W{}^2g^{-2}M_U{}^2M_Z{}^{-2})(q^2+\mu^2)^{-1}].\end{aligned}$$

We calculated this result in the context of the n-dimensional regularization method,[29] and the argument of the various propagators is the n-dimensional integration variable q. An important observation, which was originally made by Weinberg,[4] is that T_U, T_V, T_Σ, and T_Z are gauge-independent.[28] This must be the case, because a ξ-dependent term in T_i, in general, can never be canceled by other closed-loop contributions to physical quantities.

IV. THE NUCLEON PROPAGATOR AND THE PROTON-NEUTRON MASS DIFFERENCE

In this section we will discuss a number of aspects which are related to the higher-order corrections to the nucleon propagator. The diagrams which contribute in the one closed-loop approximation are depicted in Fig. 3. After a straightforward calculation we find the following result for the inverse nucleon propagator:

$$S_N{}^{-1}(p) = \not{p} - im - (2\pi)^{-4}G_N[t_\Sigma + D_\sigma^{\Sigma i}(0)T_i] - i(2\pi)^{-4}\int d^nq\,\frac{\Sigma(p,q)}{(p-q)^2+m^2},\tag{14a}$$

with

$$
\begin{aligned}
\Sigma(p,q) = {} & G_N{}^2[D_\sigma^{\Sigma\Sigma}(q^2)(\not{p}-\not{q}+im)+3D_\psi^{\Sigma\Sigma}(q^2)(\not{p}-\not{q}-im)] \\
& -4g^2[D_T^{UU}(q^2)+\tfrac{1}{2}\tilde{D}_T^{UU}(q^2)+D_T^{VV}(q^2)+\tfrac{1}{2}\tilde{D}_T^{VV}(q^2)-2D_T^{UV}(q^2)\gamma_5-\tilde{D}_T^{UV}(q^2)\gamma_5][(3-n)\not{p}+(n-2)\not{q}-(p^2+m^2)q^{-2}\not{q}] \\
& -\tfrac{1}{8}g^2 im(n-1)[2D_T^{UU}(q^2)+\tilde{D}_T^{UU}(q^2)-2D_T^{VV}(q^2)-\tilde{D}_T^{VV}(q^2)] \\
& +\tfrac{3}{8}g^2[D_L^{UU}(q^2)+D_L^{VV}(q^2)+2D_L^{UV}(q^2)\gamma_5][\not{p}-im-(p^2+m^2)q^{-2}\not{q}]+\tfrac{3}{4}img^2[D_L^{VV}(q^2)+D_L^{UV}(q^2)\gamma_5] \\
& -\tfrac{1}{4}\sqrt{2}\,eg(1+\tau_3)\{[\tilde{D}_T^{UA}(q^2)-\tilde{D}_T^{VA}(q^2)\gamma_5][(3-n)\not{p}+(n-2)\not{q}-(p^2+m^2)q^{-2}\not{q}]+im(n-1)\tilde{D}_T^{UA}(q^2)\} \\
& -\tfrac{1}{2}e^2(1+\tau_3)\{\tilde{D}_T^{AA}(q^2)[(3-n)\not{p}+(n-2)\not{q}-(p^2+m^2)q^{-2}\not{q}+im(n-1)]+\xi_A{}^{-2}q^{-4}[(p^2+m^2)\not{q}-q^2(\not{p}-im)]\}\,.
\end{aligned}
\tag{14b}
$$

In deriving this result, we have made use of symmetric integration (which is allowed in the n-dimensional regularization method) and replaced $2(pq)\not{q}$ by $(p^2+m^2+q^2)\not{q}$ in $\Sigma(p,q)$.

We will first establish the gauge independence of the nucleon masses in the one-loop approximation. The gauge-dependent contribution to the masses is given by

$$
-(2\pi)^{-4}G_N t_\Sigma + 3i(2\pi)^{-4}\int\frac{d^nq}{(p-q)^2+m^2}\times[G_N{}^2D_\psi^{\Sigma\Sigma}(q^2)\not{q}-\tfrac{1}{4}img^2D_L^{VV}(q^2)]\,.
$$

Notice that in this order of perturbation theory terms proportional to γ_5 do not contribute to the masses. Making use of Lorentz covariance and symmetrical integration enables us to write the integrand as

$$
[(p-q)^2+m^2]^{-1}\times[+\tfrac{1}{2}G_N{}^2D_\psi^{\Sigma\Sigma}(q^2)(p^2+m^2+q^2)p^{-2}\not{p}+\tfrac{1}{4}img^2D_L^{VV}(q^2)] - \tfrac{1}{2}G_N{}^2D_\psi^{\Sigma\Sigma}(q^2)p^{-2}\not{p}\,.
$$

On the mass shell, the last term will cancel the gauge-dependent terms in t_Σ, whereas the first term turns out to be gauge-independent by virtue of the relation

$$
q^2D_\psi^{\Sigma\Sigma}(q^2)+\epsilon^2M_U{}^2D_L^{VV}(q^2) = 1-8\sqrt{2}\,\epsilon^{-1}M_UM_Z b(q^2+\mu^2)^{-1}\,,
$$

which can either be derived from the Ward-Takahashi identities Eqs. (9) together with Eqs. (12), or found from the propagators listed in Appendix B. Hence we have established the gauge independence of the masses, which follows from a delicate cancellation between the various diagrams.

The expression for the proton-neutron mass difference can easily be found from the nucleon propagator (14). The result is

$$
\frac{m_p-m_n}{m} = -i(2\pi)^{-4}e^2\int\frac{d^nq}{q^2-2p\cdot q}[2-(n-2)(p\cdot q)m^{-2}][\tilde{D}_T^{AA}(q^2)+\tfrac{1}{2}\sqrt{2}\,ge^{-1}\tilde{D}_T^{UA}(q^2)]\,,
\tag{15}
$$

where we have again used Lorentz covariance and symmetric integration. Inserting the results for the propagators $\tilde{D}_T$ from Appendix B, we find

$$
\tilde{D}_T^{AA}(q^2)+\tfrac{1}{2}\sqrt{2}\,ge^{-1}\tilde{D}_T^{UA}(q^2) = M_U{}^2q^{-2}\tilde{D}^{-1}(q^2)[\tfrac{1}{2}q^4+(\tfrac{1}{2}M_V{}^2+M_W{}^2-\tfrac{1}{2}g_W{}^2g^{-2}M_U{}^2)q^2+M_V{}^2M_W{}^2-\tfrac{1}{2}g_W{}^2g^{-2}M_U{}^2(M_U{}^2+M_V{}^2)]\,,
$$

where $\tilde{D}(q^2)$ is also given in Appendix B.

Now one can easily verify that the mass difference is finite, as it should be, since the gauge invariance does not allow a counterterm for the mass difference in the Lagrangian. Unfortuantely, the sign turns out to be positive for all allowed values of the parameters. To show this, we write

$$
\frac{m_p-m_n}{m} = \frac{e^2}{8\pi^2}\left[\tfrac{1}{2}M_U{}^2\left(1-\frac{e^2}{g_W{}^2}-\frac{2e^2}{g^2}\right)^{-1}I_1+M_U{}^2\left(M_W{}^2+\tfrac{1}{2}M_V{}^2-\frac{g_W{}^2}{2g^2}M_U{}^2\right)\left(1-\frac{e^2}{g_W{}^2}-\frac{2e^2}{g^2}\right)^{-1}I_2+I_3\right],
\tag{16a}
$$

with

$$
\begin{aligned}
I_1 &= \int_0^1\int_0^x\int_0^y dx\,dy\,dz\left\{\frac{5-3x}{[(\Lambda_W{}^2-\Lambda_U{}^2)z+(\Lambda_U{}^2-\Lambda_V{}^2)y+\Lambda_V{}^2x+m^2(1-x)^2]} - \frac{m^2(1-x)^2(2-x)}{[(\Lambda_W{}^2-\Lambda_U{}^2)z+(\Lambda_U{}^2-\Lambda_V{}^2)y+\Lambda_V{}^2x+m^2(1-x)^2]^2}\right\}, \\
I_2 &= \int_0^1\int_0^x\int_0^y dx\,dy\,dz\,\frac{(2-x)}{[(\Lambda_W{}^2-\Lambda_U{}^2)z+(\Lambda_U{}^2-\Lambda_V{}^2)y+\Lambda_V{}^2x+m^2(1-x)^2]^2}, \\
I_3 &= \int_0^1\int_0^x\int_0^y\int_0^z dx\,dy\,dz\,dw\,\frac{2(2-x)\Lambda_U{}^2\Lambda_V{}^2\Lambda_W{}^2}{[(\Lambda_W{}^2-\Lambda_U{}^2)w+(\Lambda_U{}^2-\Lambda_V{}^2)z+\Lambda_V{}^2y+m^2(1-x)^2]^3},
\end{aligned}
\tag{16b}
$$

where $\Lambda_U^2, \Lambda_V^2, \Lambda_W^2$ are defined by

$$\tilde{D}(q^2) = \left(1 - \frac{e^2}{g_W^2} - 2\frac{e^2}{g^2}\right) \times (q^2 + \Lambda_U^2)(q^2 + \Lambda_V^2)(q^2 + \Lambda_W^2) . \tag{16c}$$

I_1, I_2, and I_3 are always positive, because Λ_U^2, Λ_V^2, and Λ_W^2 must be positive. Finally, the coefficients of these integrals are always positive, as follows from the original definition of e and M_W in Sec. II so that the mass difference is positive-definite. This confirms the general picture that in models based on SU(2) gauge groups the proton is always heavier than the neutron in the one-loop approximation.[7] As argued by Lieberman,[17] it is, in general, possible to change the sign by adding additional spinless fields, something which will, however, drastically change the model.

Finally, let us discuss the parity-breaking effects in the nucleon propagator (14). The terms proportional to γ_5 turn out to be finite, and moreover these terms are damped by a factor M_W^{-2}, which means that all parity violations are of order G_F, rather than g_W^2 or e^2. The reason for this suppression of parity violation is clear. Owing to the fact that the weak interactions with hadrons occur mainly through the exchange of the strongly interacting gauge fields U_μ^a and V_μ^a, most of the weak radiative corrections to hadronic amplitudes are convergent, and, as is well known, finite corrections from heavy intermediate bosons are always of order G_F. The only diagrams which can give infinite contributions to weak hadronic corrections must necessarily involve the spinless fields σ_U, σ_V, ψ_U^a, or ψ_V^a, because it is only to those hadron fields that the intermediate bosons W_μ^a are coupled directly. An example of such diagrams will be calculated in the next section, and one of our results given in Eq. (18) indicates that, especially when $b=0$, the parity-violating terms are much weaker than one would generally expect.[30]

V. THE PION MASS AND PION MASS DIFFERENCE

We argued in Sec. III that the $\mathfrak{L}_b$ term in the Lagrangian is not needed for the theory to be renormalizable. If we take $b=0$, the mass of the pion will be finite and calculable in all orders of perturbation theory. The possibility of this so-called pseudo-Goldstone character of the pion was suggested some time ago by Weinberg,[2] who also presented some arguments about the typical order of magnitude of its mass.[4] In the one-loop approximation, he found that the masses of pseudo-Goldstone bosons are of the order of eM, with e and M a typical gauge-field coupling constant and mass, respectively. However, when the gauge fields have different masses, M is defined to be the largest mass. If this is indeed the case, it would imply that weak or electromagnetic corrections to hadronic amplitudes are not necessarily small, because they can be proportional to the large intermediate vector-boson masses of the weak interactions.

In order to still have a reasonable mass for a pseudo-Goldstone pion, one could assume that the pseudo-symmetry (or "accidental" symmetry: the symmetry that is connected with the pseudo-Goldstone mechanism), is broken by weakly interacting vector bosons with masses around 1 or 2 GeV. A strong interaction model of this type was proposed some time ago by Bars and Lane,[31] and a reasonable answer for the pion mass was obtained. Although this model ignored the weak interactions completely, the actual equation for the pion mass indicated that it remains finite when one of the vector-boson masses becomes infinitely large. This would contradict Weinberg's estimate that a pseudo-Goldstone mass is proportional to the largest vector-boson mass.[32]

Making explicit use of Weinberg's result, the pion mass was also calculated by Lee, Rawls, and Yu,[16] and Lieberman[17] in a model where all the vector-boson masses (except the photon mass) are large. In this model the origin of the pion mass is electromagnetic and it was found that the charged-pion mass was of order eM, where e is the electromagnetic charge and M is the vector-boson mass. However, it turned out that the neutral-pion mass remained zero in the one-loop approximation, which means that the electromagnetic pion mass difference was unusually large.[33] In order to resolve this problem, it was conjectured that the mass of the pion possibly originates purely from the two-loop contributions, thus being of order e^2M. Hence, one should try to find a realistic model where this is the case.

However, a more careful analysis shows that the neutral pion is an exact Goldstone boson in the case that the charged pions are pseudo-Goldstone bosons which pick up their masses from higher-

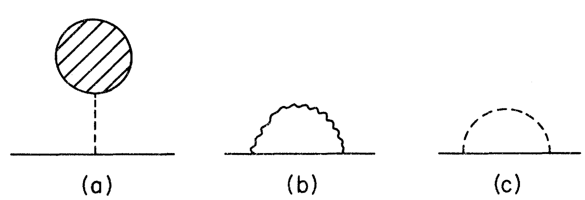

FIG. 3. The diagrams which contribute to the nucleon propagators in the one-loop approximation: (a) σ_Σ tadpole; (b) vector-boson exchange; (c) scalar-boson exchange.

order electromagnetic contributions. This implies that the neutral pion will remain massless in all orders of perturbation theory. The reason for this phenomenon can be traced back to the fact that a subgroup of the pseudo-symmetry, which is still sufficient to prove that the pions are massless in tree approximation, can be extended to a symmetry acting on all the fields which is broken only by electromagnetic interactions. However, the interactions with the Abelian photon field do not completely break this extended symmetry. There is a surviving subgroup which has the neutral pion as its real Goldstone boson. If one wants to have pseudo-Goldstone bosons, then it is clear that this phenomenon can be an important constraint for the construction of realistic models with an Abelian gauge field.[19]

In our model the photon is also related to an Abelian gauge group. In order to show somewhat more explicitly that we will have the same phenomenon, consider the following subgroup $\hat{G}$ of the pseudo-symmetry group defined by

$$K_X \to W_1 K_X W_3^\dagger, \quad K_Y \to W_2 K_Y W_4^\dagger, \qquad K_\Sigma \to W_1 K_\Sigma W_2^\dagger, \quad K_Z \to W_3 K_Z W_5^\dagger, \tag{17}$$

where W_1–W_4 are independent global SU(2) transformations and $W_5 = \exp(i\Lambda_5 \tau_3)$. As stated previously, we can consistently take b equal to zero. In that case the scalar potential is invariant under $\hat{G}$, and making use of the Goldstone theorem[18] we can show that this invariance is sufficient to prove the pseudo-Goldstone character of the pions. However, $\hat{G}$ can be extended to a group which acts on all fields in Lagrangian such that (if $b=0$) the only breaking of this extended group comes from the interactions with the photon field (provided we have chosen a gauge which is also symmetric under these transformations). One can then explicitly show that due to the Abelian character of the photon field, the U(1)⊗ U(1)⊗ U(1)⊗ U(1) subgroup of $\hat{G}$ defined by

$$W_n = \exp(i\Lambda_n \tau_3), \quad n = 1, \ldots, 5$$

can similarly be extended to an exact invariance group of the total Lagrangian. This invariance is sufficient to show that all neutral members of the spinless isotriplets ψ, and thus the neutral pion, are exact Goldstone bosons.

This conclusion holds in all orders of perturbation theory, and our one-loop calculation confirmed that if $b=0$, the neutral pion picks up no mass. Hence, the charged-pion mass will be equal to the electromagnetic mass difference. However, if the weak interactions are ignored, we expect the mass difference to be given by Bardakci's result,[15] which was in agreement with the current-algebra prediction.[21] One of the questions we will consider in this section is whether the addition of weak interactions through heavy intermediate vector bosons will change this result by large terms of order eM_W. If it changes, in agreement with Weinberg's estimate, this would imply that something essential is missing in the current understanding of electromagnetic radiative corrections to hadronic amplitudes.

Let us now turn to the calculations in our unified model. We have determined the full propagator of the spinless triplets ψ_U, ψ_V, ψ_Σ, and ψ_Z again under the previously mentioned assumptions that $g_X = g_Y = g$ and $\delta_1 = 0$ in the tree approximation. The diagrams that contribute to the ψ propagator in the one-loop approximation are depicted in Fig. 4. In the calculations we made extensive use of the Ward-Takahashi identities Eqs. (9). Moreover, we often rewrote terms by making use of the relations between certain coupling constants and inverse propagators. After rather involved calculations, we found that only the parts of diagrams (a), (c), and (d) of Fig. 4 containing the transversal part D_T of the vector-meson propagators contribute to the pion mass in agreement with Weinberg.[4] We then calculated the form for the inverse propagator of the spinless fields $\psi_U, \psi_V, \psi_\Sigma, \psi_Z$. At zero momentum and in the limit $b=0$ this inverse propagator is

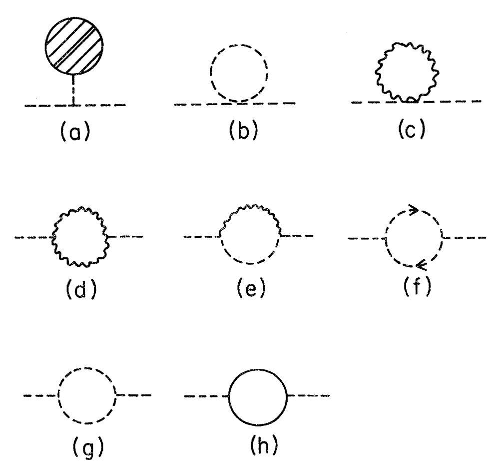

FIG. 4. The diagrams which contribute to the ψ propagators in the one-loop approximation: (a) sum of all tadpoles; (b) scalar-boson seagull diagram; (c) vector-boson seagull diagram; (d) vector-boson–vector-boson exchange; (e) vector-boson–scalar-boson exchange; (f) Faddeev-Popov exchange; (g) scalar-boson–scalar-boson exchange; (h) fermion exchange.

$$D_\psi{}^{-1}(0)+(1-\delta_{a3})e^2\gamma^2\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & \epsilon^2 & -\epsilon & \frac{1}{2}\sqrt{2}\,\epsilon^2 g_W g^{-1}M_U M_Z{}^{-1}\\ 0 & -\epsilon & 1 & -\frac{1}{2}\sqrt{2}\,\epsilon g_W g^{-1}M_U M_Z{}^{-1}\\ 0 & \frac{1}{2}\sqrt{2}\,\epsilon^2 g_W g^{-1}M_U M_Z{}^{-1} & -\frac{1}{2}\sqrt{2}\,\epsilon g_W g^{-1}M_U M_Z{}^{-1} & \frac{1}{2}\epsilon^2 g_W{}^2 g^{-2}M_U{}^2 M_Z{}^{-2}\end{bmatrix}. \tag{18}$$

D_ψ is the zeroth-order propagator as given in Appendix B, with $b=0$, a denotes the isospin component, and γ^2 is defined by the equation

$$\gamma^2=-6i(2\pi)^{-4}g^2 g_W{}^{-2}M_Z{}^2\int d^4q\,\tilde{D}_T^{AA}(q)D_T^{UV}(q)\,.$$

Making use of the expressions listed in Appendix B, we can rewrite the previous equation as

$$\begin{aligned}\gamma^2&=-3i(2\pi)^{-4}M_U{}^4M_Z{}^2\int d^4q[q^2\tilde{D}(q^2)]^{-1}\\ &=\frac{3}{16\pi^2}M_U{}^4M_Z{}^2\left[\left(1-\frac{e^2}{g_W{}^2}-2\frac{e^2}{g^2}\right)(\Lambda_W{}^2-\Lambda_U{}^2)(\Lambda_W{}^2-\Lambda_V{}^2)(\Lambda_U{}^2-\Lambda_V{}^2)\right]^{-1}\left(\Lambda_U{}^2\ln\frac{\Lambda_V{}^2}{\Lambda_W{}^2}+\Lambda_V{}^2\ln\frac{\Lambda_W{}^2}{\Lambda_U{}^2}+\Lambda_W{}^2\ln\frac{\Lambda_U{}^2}{\Lambda_V{}^2}\right).\end{aligned} \tag{19}$$

where $\Lambda_U,\Lambda_V,\Lambda_W$ were defined by Eq. (16c).

This result is finite and gauge-independent, as it should be. It turns out that the one-loop corrections do not contribute to the propagator of the neutral fields ($a=3$), so that the neutral pion remains massless as conjectured previously. In spite of the corrections, the physical pion field in lowest approximation (10) is still an eigenstate. Its corresponding eigenvalue, which is the charged-pion mass in this approximation, is given by

$$M_{\pi^\pm}{}^2=e^2\gamma^2(1+\epsilon^2+\tfrac{1}{2}\epsilon^2 g_W{}^2g^{-2}M_U{}^2M_Z{}^{-2})\,.$$

Evaluating this equation for physical values of the parameters ($M_Z\to\infty$, $e/g\ll 1$, $g_W/g\ll 1$), we find the following result:

$$M_{\pi^\pm}{}^2=\frac{3e^2}{16\pi^2}\frac{M_U{}^2M_V{}^2}{M_V{}^2-M_U{}^2}\ln\frac{M_V{}^2}{M_U{}^2}. \tag{20}$$

If we interpret this result as the electromagnetic mass difference $M_{\pi^\pm}{}^2-M_{\pi^0}{}^2$, it is in agreement with previous calculations.[15,21] The important point is that we find no substantial corrections from the weak interactions, in contradiction to Weinberg's general estimate. If we identify U_μ and V_μ as the ρ and A_1 vector mesons, we find that the value for the charged-pion mass is

$$M_{\pi^\pm}\simeq 37\text{ MeV}\,.$$

Of course, a one-loop result is not very appropriate, and will be affected by strong interaction corrections. Moreover, due to the specific symmetry structure of the model, the neutral pion remains massless. However, we find it very encouraging that the experimental pion mass can be obtained within one order of magnitude for pseudo-Goldstone pions which receive their mass from electromagnetic interactions.

In order to demonstrate more explicitly how the cancellations among gauge-dependent and divergent parts occur, we have also calculated the pion mass difference in the case where $b\neq 0$. This calculation, which is presented in some detail in Appendix C, yields the following result for the mass difference:

$$\begin{aligned}\delta\mu^2&=M_{\pi^\pm}{}^2-M_{\pi^0}{}^2\\ &=\frac{i}{(2\pi)^4}\left(1+\epsilon^{-2}+\frac{1}{2}\frac{g_W{}^2}{g^2}\frac{M_U{}^2}{M_Z{}^2}\right)^{-1}\\ &\quad\times[\Pi_1(-\mu^2)+\Pi_2(-\mu^2)+\Pi_3(-\mu^2)\\ &\quad+\Pi_4(-\mu^2)+\Pi_5(-\mu^2)]\,.\end{aligned}$$

The functions Π_1–Π_5 are given in Appendix C and are finite and gauge-independent. If we evaluate the previous equation, neglecting terms of order G_F and μ^4, we find

$$\begin{aligned}\delta\mu^2=&\frac{3e^2}{16\pi^2}\frac{1+\epsilon^2}{\epsilon^2}M_U{}^2\ln(1+\epsilon^2)\\ &+\frac{3e^2}{16\pi^2}\mu^2\left\{\ln\frac{M_U{}^2}{\mu^2}+\frac{1}{4}\frac{\epsilon^2}{1+\epsilon^2}\frac{x}{1-x}\left(\frac{x}{1-x}\ln x+1\right)+\left[3\epsilon^{-6}(1+\epsilon^2)+\frac{5}{4}\epsilon^{-2}+\frac{2+\epsilon^2}{1+\epsilon^2}\right]\ln(1+\epsilon^2)\right.\\ &\left.-2\epsilon^{-2}-\frac{3\epsilon^{-4}}{1+\epsilon^2}-\frac{5}{2}\frac{\epsilon^{-2}}{1+\epsilon^2}-\frac{3}{4}\frac{1}{1+\epsilon^2}-\frac{1}{2}\right\},\end{aligned} \tag{21}$$

where we made the following substitution for the propagator of the field σ_V:

$$D_\sigma^{VV}(q^2)=\frac{1}{q^2+xM_U{}^2}\,.$$

The pion mass in lowest order, μ^2, was defined previously in Eq. (11), and $\epsilon^2=M_V{}^2/M_U{}^2-1$.

The first term of Eq. (21), which is independent of μ^2, is exactly the current-algebra result found by Das, Guralnik, Mathur, Low, and Young[21] for soft pions. More recently this answer was found by Bardakci[15] in a gauge-field model for strong and electromagnetic interactions. Although it has been shown[10] that the current algebra and PCAC assumptions are valid in the soft-pion limit $\mu^2=0$, this agreement is somewhat surprising because we do not find additional contributions from the exchange of the heavy vector bosons of the weak interactions, which were ignored in Refs. 15 and 21. A similar result was also found by Dicus and Mathur,[22] who repeated the current-algebra calculation for soft pions, taking into account the additional terms from the weak currents. In their model, which was based on SU(4), the corresponding contributions proportional to $\ln M_W{}^2$ were negligible if the ρ and A_1 leptonic decay constants g_ρ and g_A were equal. In our case we have terms proportional to $\mu^2\ln M_W{}^2$ in both Π_3 and Π_4 which, however, cancel exactly in the final answer (21).

The term proportional to $\mu^2\ln\mu^2$ in Eq. (21) was found by Langacker and Pagels[23] by using chiral perturbation theory. However, we wish to point out that in applications of chiral perturbation theory to amplitudes which are of order e^2, the previously mentioned terms proportional to $\mu^2\ln M_W{}^2$ are of the same size as the $\mu^2\ln\mu^2$ terms found from chiral perturbation theory. Although these $\mu^2\ln M_W{}^2$ terms are absent in our result for the pion mass difference, they may occur in two-loop contributions, as well as in other amplitudes like, for example, the $\eta\to 3\pi$ decay amplitude.

Finally we compare Eq. (21) with the hard-pion calculation of Gerstein, Lee, Nieh, and Schnitzer.[24] The main difference is that our result is finite. If we identify U_μ and V_μ as the ρ and A_1 vector mesons, we find

$$\delta\mu^2=\frac{3e^2}{16\pi^2}M_\rho{}^2\left\{2\ln 2+\frac{\mu^2}{M_\rho{}^2}\left[-\frac{45}{8}+\ln\frac{M_\rho{}^2}{\mu^2}+\frac{35}{4}\ln 2+\frac{1}{8}\frac{x}{1-x}\left(\frac{x}{1-x}\ln x+1\right)\right]\right\}.$$

Numerically, we have for the mass difference

$$\delta\mu=5\left[1+0.09+0.003\frac{x}{1-x}\left(\frac{x}{1-x}\ln x+1\right)\right].$$

For reasonable values of x, the correction is approximately 10%, which is smaller than the result found by Gerstein *et al.*

VI. CONCLUSIONS

We have calculated several one-loop corrections to zeroth-order symmetry relations in a unified gauge-field model of strong, weak, and electromagnetic interactions. We explicitly established the finiteness and the gauge invariance of our results by making use of the Ward-Takahashi identities for the propagators. The proton-neutron mass difference turns out to have the wrong sign for all possible values of the parameters. This result has been found in a large class of models based on the SU(2) group, and in order to find the correct sign one probably has to choose a higher symmetry group like SU(3).[8]

In our final results, the parity violations from weak radiative corrections are found to be of the order of the Fermi coupling constant G_F.

In the case that the pions are pseudo-Goldstone bosons we calculated their mass. The neutral pion remains massless in all orders, due to the fact that the electromagnetic gauge group is Abelian in our model. This problem can easily be resolved in principle if the pseudo-Goldstone boson masses are also due to interactions other than with an Abelian gauge field. The mass of the charged pions is 37 MeV.

We also calculated the pion mass difference for the case that the pions are not pseudo-Goldstone bosons. It is remarkable that in the final result for both the pion mass and mass difference the contributions from the weak interactions are negligible. This is not necessarily true in higher orders or in other amplitudes, and it may be that weak interactions play a more important role in the calculation of, for example, kaon mass differences or the $\eta\to 3\pi$ amplitude.

The hard-pion corrections to the pion mass difference were compared with previous calculations, and found to be approximately 0.5 MeV.

APPENDIX A: THE TRANSFORMATION PROPERTIES OF THE FIELDS

In this appendix we will list the behavior of the fields under the infinitesimal local transformations of the total SU(2)⊗ SU(2)⊗ SU(2)⊗ U(1) gauge group of strong, weak, and electromagnetic interactions. We use the following parameterization of the gauge transformations:

$$U(x)\approx 1+ig[\Lambda_U(x)+\Lambda_V(x)]+\tfrac{1}{2}ie\Lambda_A(x)\tau_3\,,$$

$$V(x)\approx 1+ig[\Lambda_U(x)-\Lambda_V(x)]+\tfrac{1}{2}ie\Lambda_A(x)\tau_3\,,$$

$$S(x) \approx 1 + ig_W \Lambda_W(x) + \tfrac{1}{2} ie \Lambda_A(x) \tau_3 ,$$

$$T(x) \approx 1 + \tfrac{1}{2} ie \Lambda_A(x) \tau_3 ,$$

where the transformations $U(x)$, $V(x)$, $S(x)$, and $T(x)$ were introduced in Sec. II. We use the notation $\Lambda_{U,V,W}(x) \equiv \tfrac{1}{2}\Lambda^a_{U,V,W}(x)\tau_a$. A straightforward calculation gives the following transformation properties for the various fields.

Vector fields.

$$U_\mu \to U_\mu + \sqrt{2}\,\partial_\mu \Lambda_U + ig[\Lambda_U, U_\mu] + ig[\Lambda_V, V_\mu] + ie[\Lambda_A \tfrac{1}{2}\tau_3, U_\mu] + ie\sqrt{2}\,[\Lambda_U, A_\mu \tfrac{1}{2}\tau_3],$$

$$V_\mu \to V_\mu + \sqrt{2}\,\partial_\mu \Lambda_V + ig[\Lambda_U, V_\mu] + ig[\Lambda_V, U_\mu] + ie[\Lambda_A \tfrac{1}{2}\tau_3, V_\mu] + ie\sqrt{2}\,[\Lambda_V, A_\mu \tfrac{1}{2}\tau_3],$$

$$W_\mu \to W_\mu + \partial_\mu \Lambda_W + ig_W[\Lambda_W, W_\mu] + ie[\Lambda_A \tfrac{1}{2}\tau_3, W_\mu] + ie[\Lambda_W, A_\mu \tfrac{1}{2}\tau_3],$$

$$A_\mu \to A_\mu + \partial_\mu \Lambda_A .$$

Spinless triplet fields.

$$\psi_U \to \psi_U + \sqrt{2}\,M_U \Lambda_U - \tfrac{1}{2}\sqrt{2}\,g_W g^{-1} M_U \Lambda_W + \tfrac{1}{2} g \Lambda_U \sigma_U + \tfrac{1}{2} g \Lambda_V \sigma_V - \tfrac{1}{4} g_W \Lambda_W(\sigma_U + \sigma_V) + \tfrac{1}{2} ig[\Lambda_U, \psi_U] + \tfrac{1}{2} ig[\Lambda_V, \psi_V] + ie[\Lambda_A \tfrac{1}{2}\tau_3, \psi_U] + \tfrac{1}{4} ig_W[\Lambda_W, \psi_U + \psi_V],$$

$$\psi_V \to \psi_V + \sqrt{2}\,M_U \Lambda_V - \tfrac{1}{2}\sqrt{2}\,g_W g^{-1} M_U \Lambda_W + \tfrac{1}{2} g \Lambda_U \sigma_V + \tfrac{1}{2} g \Lambda_V \sigma_U - \tfrac{1}{4} g_W \Lambda_W(\sigma_U + \sigma_V) + \tfrac{1}{2} ig[\Lambda_U, \psi_V] + \tfrac{1}{2} ig[\Lambda_V, \psi_U] + ie[\Lambda_A \tfrac{1}{2}\tau_3, \psi_V] + \tfrac{1}{4} ig_W[\Lambda_W, \psi_U + \psi_V],$$

$$\psi_\Sigma \to \psi_\Sigma + \epsilon\sqrt{2}\,M_U \Lambda_V + g \Lambda_V \sigma_\Sigma + ig[\Lambda_U, \psi_\Sigma] + ie[\Lambda_A \tfrac{1}{2}\tau_3, \psi_\Sigma],$$

$$\psi_Z \to \psi_Z + M_Z \Lambda_W + \tfrac{1}{2} g_W \Lambda_W \sigma_Z + \tfrac{1}{2} ig_W[\Lambda_W, \psi_Z] + ie[\Lambda_A \tfrac{1}{2}\tau_3, \psi_Z].$$

Spinless singlet fields.

$$\sigma_U \to \sigma_U - \tfrac{1}{2} g \Lambda^a_U \psi^a_U - \tfrac{1}{2} g \Lambda^a_V \psi^a_V + \tfrac{1}{4} g_W \Lambda^a_W(\psi^a_U + \psi^a_V),$$

$$\sigma_V \to \sigma_V - \tfrac{1}{2} g \Lambda^a_U \psi^a_V - \tfrac{1}{2} g \Lambda^a_V \psi^a_U + \tfrac{1}{4} g_W \Lambda^a_W(\psi^a_U + \psi^a_V),$$

$$\sigma_\Sigma \to \sigma_\Sigma - g \Lambda^a_V \psi^a_\Sigma ,$$

$$\sigma_Z \to \sigma_Z - \tfrac{1}{2} g_W \Lambda^a_W \psi^a_Z .$$

Nucleon fields.

$$N \to N + ig\Lambda_U N + ig\Lambda_V \gamma_5 N + ie\Lambda_A \tfrac{1}{2}(1 + \tau_3) N .$$

Lepton fields.

$$l \to l + ig_W \Lambda_W \tfrac{1}{2}(1 + \gamma_5) l - ie\Lambda_A \tfrac{1}{2}(1 - \tau_3) l .$$

APPENDIX B: THE PROPAGATORS OF THE FIELDS

The propagator of the vector fields U^a_μ, V^a_μ, W^a_μ, and A_μ is decomposed as follows:

$$D_{\mu\nu}(q) = D_T(q^2)\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + D_L(q^2)\frac{q_\mu q_\nu}{q^2} .$$

For the charged vector fields, we find the following expressions for D_T:

$$D^{UU}_T(q^2) = [(q^2 + M_V{}^2)(q^2 + M_W{}^2) - \tfrac{1}{2} g_W{}^2 g^{-2} M_U{}^4] \times D^{-1}(q^2),$$

$$D^{VV}_T(q^2) = [(q^2 + M_U{}^2)(q^2 + M_W{}^2) - \tfrac{1}{2} g_W{}^2 g^{-2} M_U{}^4] \times D^{-1}(q^2),$$

$$D^{WW}_T(q^2) = (q^2 + M_U{}^2)(q^2 + M_V{}^2) D^{-1}(q^2),$$

$$D^{UV}_T(q^2) = D^{VU}_T(q^2) = \tfrac{1}{2} g_W{}^2 g^{-2} M_U{}^4 D^{-1}(q^2),$$

$$D^{UW}_T(q^2) = D^{WU}_T(q^2) = \tfrac{1}{2}\sqrt{2}\,g_W g^{-1} M_U{}^2 (q^2 + M_V{}^2) D^{-1}(q^2),$$

$$D^{VW}_T(q^2) = D^{WV}_T(q^2) = \tfrac{1}{2}\sqrt{2}\,g_W g^{-1} M_U{}^2 (q^2 + M_U{}^2) D^{-1}(q^2),$$

with $D(q^2)$ defined by

$$D(q^2) = (q^2 + M_U{}^2)(q^2 + M_V{}^2)(q^2 + M_W{}^2) - \tfrac{1}{2} g_W{}^2 g^{-2} M_U{}^4 (2q^2 + M_U{}^2 + M_V{}^2) .$$

We distinguish the neutral vector propagators from the charged vector propagators by a tilde. The exact formulas are

$$\tilde{D}^{UU}_T(q^2) = [(q^2 + M_V{}^2)(q^2 + M_W{}^2) - \tfrac{1}{2} g_W{}^2 g^{-2} M_U{}^4 - e^2 g_W^{-2} q^2 (q^2 + M_V{}^2)]\tilde{D}^{-1}(q^2),$$

$$\tilde{D}^{VV}_T(q^2) = [(q^2 + M_U{}^2)(q^2 + M_W{}^2) - \tfrac{1}{2} g_W{}^2 g^{-2} M_U{}^4 - e^2 g_W{}^{-2} q^2 (q^2 + M_U{}^2) - 2e^2 g^{-2} q^2 (q^2 + M_W{}^2 + M_U{}^2)]\tilde{D}^{-1}(q^2),$$

$$\tilde{D}^{WW}_T(q^2) = [(q^2 + M_U{}^2)(q^2 + M_V{}^2) - 2e^2 g^{-2} q^2 (q^2 + M_V{}^2)]\tilde{D}^{-1}(q^2),$$

$$\tilde{D}^{AA}_T(q^2) = q^{-2} D(q^2) \tilde{D}^{-1}(q^2),$$

$$\tilde{D}^{UV}_T(q^2) = \tilde{D}^{VU}_T(q^2) = (\tfrac{1}{2} g_W{}^2 g^{-2} M_U{}^4 + e^2 g^{-2} M_U{}^2 q^2)\tilde{D}^{-1}(q^2),$$

$$\tilde{D}^{UW}_T(q^2) = \tilde{D}^{WU}_T(q^2) = [\tfrac{1}{2}\sqrt{2}\,g_W g^{-1} M_U{}^2 (q^2 + M_V{}^2) + \sqrt{2}\,e^2 g^{-1} g_W{}^{-1} q^2 (q^2 + M_V{}^2)]\tilde{D}^{-1}(q^2),$$

$$\tilde{D}^{UA}_T(q^2) = \tilde{D}^{AU}_T(q^2) = -\tfrac{1}{2}\sqrt{2}\,e g^{-1}[M_U{}^2 (q^2 + M_V{}^2) + 2(q^2 + M_V{}^2)(q^2 + M_W{}^2) - g_W{}^2 g^{-2} M_U{}^4]\tilde{D}^{-1}(q^2),$$

$$\tilde{D}^{VW}_T(q^2) = \tilde{D}^{WV}_T(q^2) = [\tfrac{1}{2}\sqrt{2}\,g_W g^{-1} M_U{}^2 (q^2 + M_U{}^2) - \sqrt{2}\,e^2 g_W g^{-3} M_U{}^2 q^2]\tilde{D}^{-1}(q^2),$$

$$\tilde{D}^{VA}_T(q^2) = \tilde{D}^{AV}_T(q^2) = -\tfrac{1}{2}\sqrt{2}\,e g^{-1}[M_U{}^2 (q^2 + M_U{}^2) + g_W{}^2 g^{-2} M_U{}^4]\tilde{D}^{-1}(q^2),$$

$$\tilde{D}_T^{WA}(q^2)=\tilde{D}_T^{AW}(q^2)=-eg_W{}^{-1}[(q^2+M_U{}^2)(q^2+M_V{}^2)+g_W{}^2g^{-2}M_U{}^2(q^2+M_V{}^2)]\tilde{D}^{-1}(q^2)\,,$$

with

$$\tilde{D}(q^2)=D(q^2)-e^2g_W{}^{-2}q^2(q^2+M_U{}^2)(q^2+M_V{}^2)-2e^2g^{-2}q^2(q^2+M_V{}^2)(q^2+M_W{}^2)-2e^2g^{-2}M_U{}^2q^2(q^2+M_V{}^2)$$
$$+e^2g_W{}^2g^{-4}M_U{}^4q^2\,.$$

The second part of the gauge-field propagators, D_L, does not depend on the charge in this approximation. It is given by

$$D_L^{UU}(q^2)=[(\xi_V{}^2q^2+M_V{}^2)(\xi_W{}^2q^2+M_W{}^2)-\tfrac{1}{2}g_W{}^2g^{-2}M_U{}^4]\overline{D}^{-1}(q^2)\,,$$
$$D_L^{VV}(q^2)=[(\xi_U{}^2q^2+M_U{}^2)(\xi_W{}^2q^2+M_W{}^2)-\tfrac{1}{2}g_W{}^2g^{-2}M_U{}^4]\overline{D}^{-1}(q^2)\,,$$
$$D_L^{WW}(q^2)=(\xi_U{}^2q^2+M_U{}^2)(\xi_V{}^2q^2+M_V{}^2)\overline{D}^{-1}(q^2)\,,$$
$$D_L^{AA}(q^2)=\xi_A{}^{-2}q^{-2}\,,$$
$$D_L^{UV}(q^2)=D_L^{VU}(q^2)=\tfrac{1}{2}g_W{}^2g^{-2}M_U{}^4\overline{D}^{-1}(q^2)\,,$$
$$D_L^{UW}(q^2)=D_L^{WU}(q^2)=\tfrac{1}{2}\sqrt{2}\,g_Wg^{-1}M_U{}^2(\xi_V{}^2q^2+M_V{}^2)\overline{D}^{-1}(q^2)\,,$$
$$D_L^{VW}(q^2)=D_L^{WV}(q^2)=\tfrac{1}{2}\sqrt{2}\,g_Wg^{-1}M_U{}^2(\xi_U{}^2q^2+M_U{}^2)\overline{D}^{-1}(q^2)\,,$$
$$D_L^{UA}(q^2)=D_L^{AU}(q^2)=0\,,$$
$$D_L^{AV}(q^2)=D_L^{VA}(q^2)=0\,,$$
$$D_L^{AW}(q^2)=D_L^{WA}(q^2)=0\,,$$

with

$$\overline{D}(q^2)=(\xi_U{}^2q^2+M_U{}^2)(\xi_V{}^2q^2+M_V{}^2)(\xi_W{}^2q^2+M_W{}^2)-\tfrac{1}{2}g_W{}^2g^{-2}M_U{}^4(\xi_U{}^2q^2+M_U{}^2+\xi_V{}^2q^2+M_V{}^2)\,.$$

The propagators of the Faddeev-Popov fields are related to the gauge field propagators D_L through the Ward-Takahashi identities. We have explicitly verified those relations. The Faddeev-Popov propagators can easily be read off from the identities in Eq. (9) using the explicit forms for D_L. Notice that these propagators are not symmetric.

Finally, we give the propagators of the spinless triplet fields ψ_U, ψ_V, ψ_Σ, and ψ_Z, which are charge-independent in this approximation. It turns out that they can be expressed in the following form:

$$D_\psi^{UU}(q^2)=[1-M_U{}^2D_L^{UU}(q^2)+\sqrt{2}\,g_Wg^{-1}M_U{}^2D_L^{UW}(q^2)-\tfrac{1}{2}g_W{}^2g^{-2}M_U{}^2D_L^{WW}(q^2)]q^{-2}\,,$$
$$D_\psi^{VV}(q^2)=[1-M_U{}^2D_L^{VV}(q^2)+\sqrt{2}\,g_Wg^{-1}M_U{}^2D_L^{VW}(q^2)-\tfrac{1}{2}g_W{}^2g^{-2}M_U{}^2D_L^{WW}(q^2)-8\sqrt{2}\,\epsilon M_UM_Zb(q^2+\mu^2)^{-1}]q^{-2}\,,$$
$$D_\psi^{\Sigma\Sigma}(q^2)=[1-\epsilon^2M_U{}^2D_L^{VV}(q^2)-8\sqrt{2}\,\epsilon^{-1}M_UM_Zb(q^2+\mu^2)^{-1}]q^{-2}\,,$$
$$D_\psi^{ZZ}(q^2)=[1-M_Z{}^2D_L^{WW}(q^2)-4\sqrt{2}\,\epsilon g_W{}^2g^{-2}M_U{}^3M_Z{}^{-1}b(q^2+\mu^2)^{-1}]q^{-2}\,,$$
$$D_\psi^{UV}(q^2)=D_\psi^{VU}(q^2)=\{-M_U{}^2D_L^{UV}(q^2)+\tfrac{1}{2}\sqrt{2}\,g_Wg^{-1}M_U{}^2[D_L^{UW}(q^2)+D_L^{VW}(q^2)]-\tfrac{1}{2}g_W{}^2g^{-2}M_U{}^2D_L^{WW}(q^2)\}q^{-2}\,,$$
$$D_\psi^{U\Sigma}(q^2)=D_\psi^{\Sigma U}(q^2)=\epsilon[-M_U{}^2D_L^{UV}(q^2)+\tfrac{1}{2}\sqrt{2}\,g_Wg^{-1}M_U{}^2D_L^{VW}(q^2)]q^{-2}\,,$$
$$D_\psi^{UZ}(q^2)=D_\psi^{ZU}(q^2)=M_UM_Z[-D_L^{UW}(q^2)+\tfrac{1}{2}\sqrt{2}\,g_Wg^{-1}D_L^{WW}(q^2)]q^{-2}\,,$$
$$D_\psi^{V\Sigma}(q^2)=D_\psi^{\Sigma V}(q^2)=[-\epsilon M_U{}^2D_L^{VV}(q^2)+\tfrac{1}{2}\sqrt{2}\,\epsilon g_Wg^{-1}M_U{}^2D_L^{VW}(q^2)+8\sqrt{2}M_UM_Zb(q^2+\mu^2)^{-1}]q^{-2}\,,$$
$$D_\psi^{VZ}(q^2)=D_\psi^{ZV}(q^2)=[-M_UM_ZD_L^{VW}(q^2)+\tfrac{1}{2}\sqrt{2}\,g_Wg^{-1}M_UM_ZD_L^{WW}(q^2)-8\epsilon g_Wg^{-1}M_U{}^2b(q^2+\mu^2)^{-1}]q^{-2}\,,$$
$$D_\psi^{\Sigma Z}(q^2)=D_\psi^{Z\Sigma}(q^2)=[-\epsilon M_UM_ZD_L^{VW}(q^2)+8g_Wg^{-1}M_U{}^2b(q^2+\mu^2)^{-1}]q^{-2}\,.$$

μ^2 was defined in Eq. (11) as the pion mass in the tree approximation. These decompositions of the ψ propagators turn out to be very convenient in order to find the cancellations necessary for the gauge independence of various quantities. They satisfy the Ward-Takahashi identities Eqs. (9) and the relations Eq. (12) and could in fact be derived from them.

The propagator for the spinless meson fields D_σ^{ij} can be found by inverting the quadratic terms in the Lagrangian. We have not required the explicit expressions for these propagators so there is no need to list them. In Sec. V we assume that D_σ^{VV} has a pole at $q^2=-xM_U{}^2$, and give our result for the hard-pion mass difference in terms of the parameter x.

APPENDIX C: THE PION MASS DIFFERENCE

We will show in some detail that the pion mass difference is finite in our model, and show how the gauge-dependent parts cancel such that the final answer is gauge-independent. The diagrams that contribute to the pion mass difference are the diagrams (c), (d), and (e) depicted in Fig. 4. The pion mass difference also receives contributions from the interactions with leptons, cf. diagram 4(h). However, these contributions are of order $m_l^4 G_F^2$ so they can be neglected. A straightforward calculation of the remaining diagrams gives the following results, where the vector-boson propagators are decomposed as in Appendix B.

Diagram 4(c):

$$
\begin{aligned}
&-e^2 D_L^{AA} + (n-1)(1+\epsilon^2+\tfrac{1}{2}\epsilon^2 g_W^2 g^{-2} M_U^2 M_Z^{-2})^{-1} \\
&\quad\times[-e^2\tilde{D}_T^{AA}(1+\epsilon^2+\tfrac{1}{2}\epsilon^2 g_W^2 g^{-2}M_U^2 M_Z^{-2}) - \tfrac{1}{2}\sqrt{2}\,eg(2+\epsilon^2)\tilde{D}_T^{UA} - \tfrac{1}{2}\epsilon^2 e g_W M_W^2 M_Z^{-2}\tilde{D}_T^{WA} \\
&\quad+\tfrac{1}{4}\sqrt{2}\,\epsilon^2 g g_W(\Delta^{WU}+\Delta^{WV}) + \tfrac{1}{2}g^2(\Delta^{UU}-\Delta^{VV})]\,.
\end{aligned}
$$

Diagram 4(d):

$$
\begin{aligned}
&(1+\epsilon^{-2}+\tfrac{1}{2}g_W^2 g^{-2}M_U^2 M_Z^{-2})^{-1}\left(n-2+\frac{(p\cdot q)^2}{p^2q^2}\right) \\
&\quad\times\{-\tfrac{1}{2}g^2 M_U^2(\Delta^{UU}D_T^{VV}+\Delta^{VV}D_T^{UU}-2\Delta^{UV}D_T^{UV}) \\
&\quad-\tfrac{1}{4}g_W^2 M_U^2[\Delta^{WW}(D_T^{UU}+D_T^{VV}+2D_T^{UV})+(\Delta^{UU}+\Delta^{VV}+2\Delta^{UV})D_T^{WW}-2(\Delta^{UW}+\Delta^{VW})(D_T^{UW}+D_T^{VW})] \\
&\quad-\tfrac{1}{2}\sqrt{2}\,g g_W M_U^2[\Delta^{VW}(D_T^{UU}+D_T^{UV})-\Delta^{UW}(D_T^{VV}+D_T^{UV})+\Delta^{UU}D_T^{VW}-\Delta^{VV}D_T^{UW}+\Delta^{UV}(D_T^{VW}-D_T^{UW})]\} \\
&+(1+\epsilon^{-2}+\tfrac{1}{2}g_W^2 g^{-2}M_U^2 M_Z^{-2})^{-1}\left(1-\frac{(p\cdot q)^2}{p^2q^2}\right) \\
&\quad\times\{-\tfrac{1}{2}g^2 M_U^2(\Delta^{UU}D_L^{VV}+\Delta^{VV}D_L^{UU}-2\Delta^{UV}D_L^{UV}) \\
&\quad-\tfrac{1}{4}g_W^2 M_U^2[\Delta^{WW}(D_L^{UU}+D_L^{VV}+2D_L^{UV})+(\Delta^{UU}+\Delta^{VV}+2\Delta^{UV})D_L^{WW}-2(\Delta^{UW}+\Delta^{VW})(D_L^{UW}+D_L^{VW})] \\
&\quad-\tfrac{1}{2}\sqrt{2}\,g g_W M_U^2[\Delta^{VW}(D_L^{UU}+D_L^{UV})-\Delta^{UW}(D_L^{VV}+D_L^{UV})+\Delta^{UU}D_L^{VW}-\Delta^{VV}D_L^{UW}+\Delta^{UV}(D_L^{VW}-D_L^{UW})]\}\,.
\end{aligned}
$$

Diagram 4(e):

$$
\begin{aligned}
&(k^2-p^2)^2 q^{-2} e^2 D_L^{AA}[\epsilon^2 D_\psi^{VV}+D_\psi^{\Sigma\Sigma}-2\epsilon D_\psi^{V\Sigma}+\sqrt{2}\,\epsilon g_W g^{-1}M_U M_Z^{-1}(\epsilon D_\psi^{VZ}-D_\psi^{\Sigma Z})+\tfrac{1}{2}\epsilon^2 g_W^2 g^{-2}M_U^2 M_Z^{-2}D_\psi^{ZZ}] \\
&\quad+4(1+\epsilon^2+\tfrac{1}{2}\epsilon^2 g_W^2 g^{-2}M_U^2 M_Z^{-2})^{-1}\left(1-\frac{(p\cdot q)^2}{p^2q^2}\right)p^2 \\
&\quad\times\{e^2\tilde{D}_T^{AA}[\epsilon^2 D_\psi^{VV}+D_\psi^{\Sigma\Sigma}-2\epsilon D_\psi^{V\Sigma}+\sqrt{2}\,\epsilon g_W g^{-1}M_U M_Z^{-1}(\epsilon D_\psi^{VZ}-D_\psi^{\Sigma Z})+\tfrac{1}{2}\epsilon^2 g_W^2 g^{-2}M_U^2 M_Z^{-2}D_\psi^{ZZ}] \\
&\quad+\tfrac{1}{2}\sqrt{2}\,eg\tilde{D}_T^{UA}[\epsilon^2 D_\psi^{VV}+2D_\psi^{\Sigma\Sigma}-3\epsilon D_\psi^{V\Sigma}+\tfrac{1}{2}\sqrt{2}\,\epsilon g_W g^{-1}M_U M_Z^{-1}(\epsilon D_\psi^{VZ}+\epsilon D_\psi^{UZ}-D_\psi^{\Sigma Z})] \\
&\quad+\tfrac{1}{2}\sqrt{2}\,\epsilon eg\tilde{D}_T^{VA}(\epsilon D_\psi^{UV}-D_\psi^{U\Sigma}+\tfrac{1}{2}\sqrt{2}\,\epsilon g_W g^{-1}M_U M_Z^{-1}D_\psi^{UZ}) \\
&\quad+\tfrac{1}{2}\epsilon e g_W\tilde{D}_T^{WA}[\epsilon D_\psi^{VV}+\epsilon D_\psi^{UV}-D_\psi^{V\Sigma}-D_\psi^{U\Sigma}+\tfrac{1}{2}\sqrt{2}\,g_W g^{-1}M_U M_Z^{-1}(\epsilon D_\psi^{UZ}+\epsilon D_\psi^{VZ}-2D_\psi^{\Sigma Z})] \\
&\quad-\tfrac{1}{2}g^2\Delta^{UU}(D_\psi^{\Sigma\Sigma}+\tfrac{1}{4}\epsilon^2 D_\psi^{VV})-\tfrac{1}{8}\epsilon^2 g^2\Delta^{VV}D_\psi^{UU}-\tfrac{1}{4}\epsilon^2 g^2\Delta^{UV}D_\psi^{UV} \\
&\quad-\tfrac{1}{16}\epsilon^2 g_W^2\Delta^{WW}[D_\psi^{UU}+D_\psi^{VV}+2D_\psi^{UV}+2\sqrt{2}\,g_W g^{-1}M_U M_Z^{-1}(D_\psi^{VZ}+D_\psi^{UZ})+2g_W^2 g^{-2}M_U^2 M_Z^{-2}D_\psi^{ZZ}] \\
&\quad-\tfrac{1}{8}\sqrt{2}\,\epsilon g g_W\Delta^{UW}[\epsilon D_\psi^{VV}+\epsilon D_\psi^{UV}-2D_\psi^{V\Sigma}-2D_\psi^{U\Sigma}+\sqrt{2}\,g_W g^{-1}M_U M_Z^{-1}(\epsilon D_\psi^{VZ}-2D_\psi^{\Sigma Z})] \\
&\quad-\tfrac{1}{8}\sqrt{2}\,\epsilon^2 g g_W\Delta^{VW}(D_\psi^{UU}+D_\psi^{UV}+\sqrt{2}\,g_W g^{-1}M_U M_Z^{-1}D_\psi^{UZ})\} \\
&\quad+(1+\epsilon^{-2}+\tfrac{1}{2}g_W^2 g^{-2}M_U^2 M_Z^{-2})^{-1}\left(1-\frac{(q\cdot p)^2}{p^2q^2}\right)p^2 \\
&\quad\times\{\tfrac{1}{2}g^2\Delta^{UU}D_\sigma^{VV}+\tfrac{1}{2}g^2\Delta^{VV}(D_\sigma^{UU}+4\epsilon^{-2}D_\sigma^{\Sigma\Sigma}-4D_\sigma^{U\Sigma})+g^2\Delta^{UV}(D_\sigma^{UV}-2D_\sigma^{VZ}) \\
&\quad+\tfrac{1}{2}g_W^2\Delta^{WW}[\tfrac{1}{2}D_\sigma^{UU}+\tfrac{1}{2}D_\sigma^{VV}+D_\sigma^{UV}-\sqrt{2}\,g_W g^{-1}M_U M_Z^{-1}(D_\sigma^{UZ}+D_\sigma^{VZ})+g_W^2 g^{-2}M_U^2 M_Z^{-2}D_\sigma^{ZZ}] \\
&\quad-\tfrac{1}{2}\sqrt{2}\,g g_W\Delta^{UW}[(D_\sigma^{VV}+D_\sigma^{UV})-\sqrt{2}\,g_W g^{-1}M_U M_Z^{-1}(D_\sigma^{VZ}+D_\sigma^{UZ})] \\
&\quad-\tfrac{1}{2}\sqrt{2}\,g g_W\Delta^{VW}[D_\sigma^{UU}+D_\sigma^{UV}-2\epsilon^{-2}(D_\sigma^{U\Sigma}+D_\sigma^{V\Sigma})-2\sqrt{2}\,\epsilon^{-1}g_W g^{-1}M_U M_Z^{-1}D_\sigma^{\Sigma Z}]\}\,.
\end{aligned}
$$

We have suppressed a factor $i(2\pi)^{-4}$ and an integration over q, the momentum of the propagators $\tilde{D}_T$ or Δ. The argument of a second propagator is denoted by p, which is defined by $p=k-q$, where k is the external momentum. We used the definition $\Delta(q^2)=D_T(q^2)-\tilde{D}_T(q^2)$.

If we now substitute the expressions for D_ψ as given in Appendix B, we find that the gauge-dependent terms involving D_L cancel straightforwardly against the gauge-dependent terms from the diagram 4(d) except for the term

$$eD_L^{AA}(q^2)(k^2-p^2)^2q^{-2}(p^2+\mu^2)^{-1}\,.$$

This expression together with the term $-e^2D_L^{AA}$ from diagram 4(c) are the only gauge-dependent terms which are left. After symmetric integration we can write these terms as

$$(k^2+\mu^2)e^2D_L^{AA}(q^2)(k^2-p^2)q^{-2}(p^2+\mu^2)^{-1}\,.$$

After some tedious algebra,[34] rewriting certain coupling constants in terms of the elements of D_T^{-1}, we can write the difference of the self-energy graphs for charged and neutral pions as

$$\begin{aligned}\Pi^\pm(k^2)-\Pi^0(k^2)={}&i(2\pi)^{-4}e^2\xi_A{}^{-2}(k^2+\mu^2)\int d^nq\,(q^2-2k\cdot q)q^{-4}[(k-q)^2+\mu^2]^{-1}\\&+i(2\pi)^{-4}(1+\epsilon^{-2}+\tfrac12 g_W{}^2g^{-2}M_U{}^2M_Z{}^{-2})^{-1}[\Pi_1(k^2)+\Pi_2(k^2)+\Pi_3(k^2)+\Pi_4(k^2)+\Pi_5(k^2)]\,,\end{aligned}$$

with

$$\begin{aligned}\Pi_1(k^2)={}&(n-1)\int d^nq\{-\epsilon^{-2}(e^2\tilde{D}_T^{AA}+\tfrac12\sqrt2\,eg\tilde{D}_T^{UA})(M_U{}^2D_T^{UU}-M_V{}^2D_T^{UV})\\&+\tfrac12\sqrt2\,\epsilon^{-2}eg[\tilde{D}_T^{UA}(M_U{}^2D_T^{UU}-M_V{}^2D_T^{VV})-M_V{}^2\tilde{D}_T^{VA}(D_T^{UU}-D_T^{UV})]\\&+eg^2g_W{}^{-1}q^2(\tilde{D}_T^{UA}D_T^{VW}-\tilde{D}_T^{VA}D_T^{UW})-M_W{}^2(e^2\tilde{D}_T^{AA}+\tfrac12\sqrt2\,eg\tilde{D}_T^{UA})D_T^{WW}\\&+\tfrac14\sqrt2\,g_Wg^{-1}M_U{}^2(e^2\tilde{D}_T^{AA}+\sqrt2\,eg\tilde{D}_T^{UA})(D_T^{UW}+D_T^{VW})-\tfrac12\epsilon^2e^2M_U{}^2M_W{}^2M_Z{}^{-2}\tilde{D}_T^{AA}D_T^{UV}\\&+\tfrac14\sqrt2\,eg(M_U{}^2\tilde{D}_T^{UA}-M_V{}^2\tilde{D}_T^{VA})D_T^{WW}-\tfrac12\sqrt2\,egq^2\tilde{D}_T^{WA}(D_T^{UW}+D_T^{VW})\}\\&+(n-1)\int d^nq\,(k^2-2k\cdot q)\{\tfrac12\epsilon^{-2}g^2[\Delta^{UV}(D_T^{UU}-D_T^{VV})+(\Delta^{UU}-\sqrt2\,eg^{-1}\tilde{D}_T^{UA})(D_T^{VV}-D_T^{UV})\\&\qquad-(\Delta^{VV}+2e^2g^{-2}\tilde{D}_T^{AA}+\sqrt2\,eg^{-1}\tilde{D}_T^{UA})(D_T^{UU}-D_T^{UV})]\\&\qquad-g^2(\Delta^{VW}D_T^{UW}-\Delta^{UW}D_T^{VW})-\tfrac12[e^2\tilde{D}_T^{AA}+\sqrt2\,eg\tilde{D}_T^{UA}-\tfrac12\sqrt2\,gg_W(\Delta^{UW}+\Delta^{VW})]D_T^{WW}\\&\qquad-\tfrac12M_W{}^2M_Z{}^{-2}[(e^2\tilde{D}_T^{AA}+eg_W\tilde{D}_T^{WA})D_T^{WW}+eg_W\tilde{D}_T^{VA}D_T^{VW}\\&\qquad+\sqrt2\,g_Wg^{-1}(e^2\tilde{D}_T^{AA}+\tfrac12\sqrt2\,eg\tilde{D}_T^{UA})D_T^{UW}]\}\,,\end{aligned}$$

$$\begin{aligned}\Pi_2(k^2)={}&\tfrac12g^2M_U{}^2\int d^nq\,\frac{k^2q^2-(k\cdot q)^2}{q^2(k-q)^2}\\&\times\{\Delta^{UU}(D_T^{VV}+\sqrt2\,g_Wg^{-1}D_T^{VW}+\tfrac12g_W{}^2g^{-2}D_T^{WW})+\Delta^{VV}(D_T^{UU}-\sqrt2\,g_Wg^{-1}D_T^{UW}+\tfrac12g_W{}^2g^{-2}D_T^{WW})\\&-\Delta^{UV}[2D_T^{UV}-\sqrt2\,g_Wg^{-1}(D_T^{UW}+D_T^{VW})-g_W{}^2g^{-2}D_T^{WW}]+\tfrac12g_W{}^2g^{-2}\Delta^{WW}(D_T^{UU}+D_T^{VV}+2D_T^{UV})\\&+\sqrt2\,g_Wg^{-1}\Delta^{VW}[D_T^{UU}+D_T^{UV}-\tfrac12\sqrt2\,g_Wg^{-1}(D_T^{UW}+D_T^{VW})]\\&-\sqrt2\,g_Wg^{-1}\Delta^{UW}[D_T^{VV}+D_T^{UV}+\tfrac12\sqrt2\,g_Wg^{-1}(D_T^{UW}+D_T^{VW})]\}\,,\end{aligned}$$

$$\begin{aligned}\Pi_3(k^2)={}&\tfrac12g^2\int d^nq\,\frac{k^2q^2-(k\cdot q)^2}{q^2}\\&\times\{\Delta^{UU}D_\sigma^{VV}+\Delta^{VV}(D_\sigma^{UU}-4D_\sigma^{U\Sigma}+4\epsilon^{-2}D_\sigma^{\Sigma\Sigma})+2\Delta^{UV}(D_\sigma^{UV}-2D_\sigma^{V\Sigma})\\&+g_W{}^2g^{-2}\Delta^{WW}[\tfrac12D_\sigma^{UU}+\tfrac12D_\sigma^{VV}+D_\sigma^{UV}-\sqrt2\,g_Wg^{-1}M_UM_Z{}^{-1}(D_\sigma^{UZ}+D_\sigma^{VZ})\\&\qquad+g_W{}^2g^{-2}M_U{}^2M_Z{}^{-2}D_\sigma^{ZZ}-M_W{}^2M_Z{}^{-2}(k-q)^{-2}]\\&-\sqrt2\,g_Wg^{-1}\Delta^{UW}[D_\sigma^{VV}+D_\sigma^{UV}-\sqrt2\,g_Wg^{-1}M_UM_Z{}^{-1}(D_\sigma^{UZ}+D_\sigma^{VZ})]\\&-\sqrt2\,g_Wg^{-1}\Delta^{VW}(D_\sigma^{UU}+D_\sigma^{UV}-2\epsilon^{-2}D_\sigma^{U\Sigma}-2\epsilon^{-2}D_\sigma^{V\Sigma}+2\sqrt2\,\epsilon^{-1}g_Wg^{-1}M_UM_Z{}^{-1}D_\sigma^{\Sigma Z})\}\,,\end{aligned}$$

$$\begin{aligned}\Pi_4(k^2)={}&\int d^nq\,\frac{k^2q^2-(k\cdot q)^2}{q^2(k-q)^2}\{2M_W{}^2M_Z{}^{-2}(e^2\tilde{D}_T^{AA}+eg_W\tilde{D}_T^{WA})+2(1+2\epsilon^{-2})(e^2\tilde{D}_T^{AA}+\sqrt2\,eg\tilde{D}_T^{UA})\\&-\tfrac12g^2(1+4\epsilon^{-2})\Delta^{UU}-\tfrac12g^2\Delta^{VV}-\tfrac12\sqrt2\,g_Wg(\Delta^{UW}+\tfrac12\Delta^{VW})\}\,,\end{aligned}$$

$$\Pi_5(k^2)=8\sqrt{2}\,\epsilon^{-1}M_UM_Zb\int d^nq\,\frac{k^2q^2-(k\cdot q)^2}{q^2(k-q)^2[(k-q)^2+\mu^2]}$$
$$\times\{-2\epsilon^2(1+\epsilon^{-2}+\tfrac{1}{2}g_W{}^2g^{-2}M_U{}^2M_Z{}^{-2})[M_W{}^2M_Z{}^{-2}(e^2\tilde{D}_T^{AA}+eg_W\tilde{D}_T^{WA})$$
$$+(1+2\epsilon^{-2})(e^2\tilde{D}_T^{AA}+\sqrt{2}\,eg\tilde{D}_T^{UA})]$$
$$-g^2(2+2\epsilon^{-2}+\tfrac{1}{2}\epsilon^2)\Delta^{UU}+\tfrac{1}{2}\sqrt{2}\,gg_W(2+\epsilon^2)M_W{}^2M_Z{}^{-2}\Delta^{UW}-\tfrac{1}{4}\epsilon^2g_W{}^2M_W{}^4M_Z{}^{-4}\Delta^{WW}\}.$$

It is obvious that on the mass shell where $k^2+\mu^2=0$, the answer is gauge-independent. Moreover, it follows straightforwardly from the explicit expressions for the various propagators that all the functions $\Pi_1-\Pi_5$ are finite.

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