Landau-Ginzburg theory, but $\rho \sim i\langle\phi^*\dot{\phi}-\dot{\phi}^*\phi\rangle=0$ in the Higgs theory.

[9]M. Kalb and P. Ramond, Phys. Rev. D 9, 2273 (1974). See also E. Cremmer and J. Scherk, Nucl. Phys. B72, 117 (1974).

[10]L. N. Chang and F. Mansouri, Phys. Rev. D 5, 2235 (1972); Goto, Ref. 7; G. Goddard, J. Goldstone, C. Rebbi, and C. B. Thorn, Nucl. Phys. B56, 109 (1973).

[11]A clearcut answer to this problem seems to be lacking. See, however, Nielsen and Olesen, Ref. 1; G. 't Hooft, CERN Report No. TH-1873-CERN, 1974 (unpublished); Y. Nambu, Ref. 2.

PHYSICAL REVIEW D VOLUME 10, NUMBER 12 15 DECEMBER 1974

Towards a field theory of hadron binding*

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(Received 17 July 1974)

A field-theoretic model for hadron binding is described in which free quarks are totally screened. Quarks interact via a dipole vector-gluon field. A second-quantization procedure for the gluon field, which reduces the field to an embodiment of a direct particle interaction, eliminates unitarity problems. A detailed description of perturbation-theory rules is given. In contrast to the results of the pseudoscalar-meson and massive-vector-meson models (without cutoff), scaling occurs in the electroproduction structure functions. Another possible model having some resemblance to the relativistic harmonic-oscillator quark model of Feynman, Kislinger, and Ravndal is also described. It is unitary and has scaling structure functions.

I. INTRODUCTION

The current understanding of hadronic structure allows two apparently contradictory statements to be made: The constituents of the hadron appear to be loosely bound, quasifree particles. The constituents of the hadron are not produced and do not occur outside of hadrons. Several attempts have been made to resolve this paradoxical situation. They may be divided into two categories: "conventional" field-theoretic approaches,[1,2] and *ad hoc* approaches which postulate manifestly non-field-theoretic structures for confinement, e.g., the "bag" model.[3] In the first approach Casher, Kogut, and Susskind[1] and Wilson[2] showed that quarks could be totally screened and not observed. However, a four-dimensional, Lorentz-invariant field-theoretic model of hadron binding with its attendant conceptual and computational advantages appears to be lacking. We shall discuss a possible candidate, the dipole gluon model, in detail. In addition, another possibility is briefly described in Appendix B which bears some comparison with the quark model of Feynman *et al.*[4] The dipole gluon model has two major qualitative features in common with the bag model[3] and the two-dimensional quantum-electrodynamic model[1]: (1) The dipole gluon field has no independent degrees of freedom; neither does a bag or the two-dimensional electromagnetic field. (2) The "Coulomb" potential between quarks is proportional to the distance between them in all three models. In a sense the bag model may be regarded as a phenomenological approximation to the dipole model, and the dipole model as a generalization of the two-dimensional model to four dimensions.

In Sec. II we describe a quantization procedure which avoids the introduction of indefinite-metric in or out states and thus leads to a unitary S matrix. In Sec. III we describe the properties of the "free" gluon Lagrangian model. In Sec. IV we describe the perturbation-theory rules of the dipole model. Section V contains a discussion of unitarity, causality, quark confinement, and scaling properties of the electroproduction structure functions. For simplicity we shall ignore all but the dipole quark interaction and do not introduce internal quark quantum numbers.

II. SECOND-QUANTIZATION PROCEDURE FOR THE GLUON FIELD

We shall not quantize the gluon field in the conventional manner for three reasons: (1) to be consistent with experiment where no such particle has been identified, (2) to avoid unitarity problems in the S matrix, and (3) to avoid infrared problems in perturbation theory. We attribute no dynamical degrees of freedom to the gluon field. Instead we regard the field as the embodiment of a direct quark-quark interaction. The gluon field can thus be removed from the Lagrangian in favor of a nonlocal current-current interaction. However, it will be of no small technical advantage to keep the gluon field in the Lagrangian. In order to do this we shall second-quantize the field following the normal prescription and then, instead of introducing a Fock space for free gluons, reduce q-number expressions in the gluon field to c-number expressions via suitable operator boundary conditions.

To illustrate this procedure we consider a scalar boson field, ϕ, with Lagrangian L. We second-quantize the in field, ϕ_{in}, with equal-time commutators

$$[\phi_{\text{in}}(x), \phi_{\text{in}}(y)] = 0 , \tag{1}$$

$$[\Pi_{\text{in}}(x), \Pi_{\text{in}}(y)] = 0 , \tag{2}$$

$$[\phi_{\text{in}}(x), \Pi_{\text{in}}(y)] = -i\delta^3(\vec{x}-\vec{y}) , \tag{3}$$

where

$$\Pi_{\text{in}}(x) = \frac{\delta L}{\delta \dot{\phi}_{\text{in}}(x)} \tag{4}$$

and L_F is the free Lagrangian part of L. The usual operator expressions and identities are established. In particular the formal expansion of the S matrix in terms of time-ordered products of in fields can be made. (We are using only in fields for convenience—our remarks apply to out fields also.)

The unequal-time commutator, $[\phi_{\text{in}}(x), \phi_{\text{in}}(y)]$, is a c-number expression which is completely determined if we require that it be consistent with the equations of motion, that it be consistent with Eqs. (1)–(3) in the limit of equal times, and that it vanish at spacelike distances. Consequently all terms with an even number of factors of $\phi(x)$ reduce to sums of c numbers times products of anticommutators $\{\phi(x), \phi(y)\}$. Terms with an odd number of factors have one factor, $\phi(x)$, times sums of c numbers times products of anticommutators. At this point we could introduce a Fock space of states to complete the reduction of q-number expressions to c number expressions. For reasons stated above we do not. In analogy to Dirac's theory[5] of Hamiltonian constraints we impose operator boundary conditions which complete the specification of the dynamics of the system. We choose

$$\Pi_{\text{in}}(x) \approx 0 \approx \phi_{\text{in}}(x) \tag{5}$$

for all x, where $\approx$ means weakly equal in the sense of Dirac, i.e., evaluate all commutators before imposing the constraints. This eliminates ϕ's degrees of freedom. The free Hamiltonian,

$$H_F = \int \Pi\dot{\phi} - L_F , \tag{6}$$

is now arbitrary to the extent that H_F may be replaced by

$$H_T = H_F + \int A\phi_{\text{in}} + \int b\Pi_{\text{in}} , \tag{7}$$

where A and b will be completely determined by requiring Eq. (5) be true for all time:

$$[\Pi_{\text{in}}, H_T] \approx 0 \tag{8}$$

and

$$[\phi_{\text{in}}, H_T] \approx 0 . \tag{9}$$

Thus

$$A = -\frac{\delta H_F}{\delta \phi_{\text{in}}} \tag{10}$$

and

$$b = -\frac{\delta H_F}{\delta \Pi_{\text{in}}} . \tag{11}$$

To see the effects of this procedure more concretely let

$$L_F = \int (\tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{2}m^2\phi^2) ; \tag{12}$$

then (suppressing the subscript "in" for notational convenience)

$$i\Delta(x-y) = [\phi(x), \phi(y)] \tag{13}$$

$$= \int \frac{d^4k}{(2\pi)^3}\epsilon(k_0)\delta(k^2-m^2)e^{-ik\cdot(x-y)} \tag{14}$$

and the time-ordered product becomes

$$i\tilde{\Delta}(x-y) \equiv T(\phi(x)\phi(y)) = \tfrac{1}{2}i\epsilon(x_0-y_0)\Delta(x-y) , \tag{15}$$

with $\epsilon(x) = \pm 1$ for $x \gtrless 0$. More generally, for even N

$$T(\phi(1)\phi(2)\cdots\phi(N)) \equiv \sum_{\text{permutations}} i^{N/2}\tilde{\Delta}(x_1-x_2)\tilde{\Delta}(x_3-x_4)\cdots\tilde{\Delta}(x_{N-1}-x_N) , \tag{16}$$

where $\phi(i) = \phi(x_i)$. The natural correspondence to the Wick expansion

$$\langle 0|T(\psi(x_1)\cdots\psi(x_N))|0\rangle = \sum_{\text{permutations}} i^{N/2}\Delta_F(x_1-x_2)\cdots\Delta_F(x_{N-1}-x_N) \tag{17}$$

(where Δ_F is the Feynman propagator corresponding to the field ψ) allows us to use conventional perturbation-theory rules, except that diagrams with incoming or outgoing ϕ lines do not contribute to the S matrix and the Feynman propagator

$$\Delta_F(k)=\frac{1}{k^2-m^2+i\epsilon} \tag{18}$$

is to be replaced with

$$\tilde{\Delta}(k)=\mathrm{P}\,\frac{1}{k^2-m^2+i\epsilon}\equiv\frac{1}{2}\left(\frac{1}{k^2-m^2+i\epsilon}+\frac{1}{k^2-m^2-i\epsilon}\right) \tag{19}$$

for internal ϕ lines. In configuration space the Green's function corresponding to Eq. (19) is half the sum of the advanced and retarded Green's functions.

If we follow the above procedure in second-quantizing the electromagnetic field the resulting model quantum electrodynamics corresponds to the classical action-at-a-distance electrodynamics of Schwarzschild, Tetrode, and Fokker.[6] The fact that photon production does not occur in the model QED corresponds to the absence of radiation reaction in the classical theory. In Sec. V this will be shown to be the key to maintaining the unitarity of the S matrix in the dipole gluon model.

III. THE DIPOLE GLUON MODEL

We now consider a model[4] for hadron binding which has several major qualitative features in agreement with experimental results: large-transverse-momenta damping, scaling electroproduction structure functions, and complete screening of free quarks. The Lagrangian is

$$\mathcal{L}=-\tfrac{1}{2}F^1_{\mu\nu}F^{2\mu\nu}-\tfrac{1}{2}\lambda^2A^2_\mu A^{2\mu}+\bar{\psi}(i\not\nabla-g\not{A}^1-m)\psi\ , \tag{20}$$

where A^1_μ and A^2_μ are massless gluon fields, $F^i_{\mu\nu}=\partial_\nu A^i_\mu-\partial_\mu A^i_\nu$ for $i=1,2$, ψ is the quark field, and g is a dimensionless and λ a dimensional coupling constant. The equations of motion are

$$\partial^\mu F^1_{\mu\nu}+\lambda^2A^2_\nu=0\ , \tag{21}$$

$$\partial^\mu F^2_{\mu\nu}+gJ_\nu=0\ , \tag{22}$$

$$(i\not\nabla-g\not{A}^1-m)\psi=0\ , \tag{23}$$

with J_μ the quark current. Equation (21) implies $\partial^\mu A^2_\mu=0$ while A^1_μ is a gauge-invariant field. As a result we have

$$\Box\,\partial^\mu F^1_{\mu\nu}+g\lambda^2J_\nu=0\ . \tag{24}$$

We now consider the "free" gluon case whose Lagrangian is the first two terms on the right-hand side of Eq. (20). The canonical momentum conjugate to A^1_μ is

$$\Pi^1_\mu=F^2_{0\mu} \tag{25}$$

and that conjugate to A^2_μ is

$$\Pi^2_\mu=F^1_{0\mu}\ . \tag{26}$$

Since $\Pi^1_0=\Pi^2_0=0$ we find that A^1_0 and A^2_0 are c numbers and thus $\vec{\nabla}\cdot\vec{A}^2$ is also a c number with the possible exception of the zero-frequency mode. If we choose the Coulomb gauge for A^1_μ

$$\vec{\nabla}\cdot\vec{A}^1=0 \tag{27}$$

then we obtain the equal-time commutation relations

$$[\Pi^a_i(x),A^b_j(y)]=i\delta^{ab}\int\frac{d^3k}{(2\pi)^3}\,e^{i\vec{k}\cdot(\vec{x}-\vec{y})}\left(\delta_{ij}-\frac{k_ik_j}{|\vec{k}|^2}\right) \tag{28}$$

for $i,j=1,2,3$, in analogy to similar expressions in quantum electrodynamics. All other equal-time commutators are zero. We can define an electric field, $\vec{E}$, and magnetic field, $\vec{B}$, by

$$\vec{E}=-\vec{\nabla}A^{10}-\frac{\partial}{\partial t}\vec{A}^1 \tag{29}$$

and

$$\vec{B}=\vec{\nabla}\times\vec{A}^1\ , \tag{30}$$

which imply

$$\vec{\nabla}\times\vec{E}=\frac{\partial\vec{B}}{\partial t} \tag{31}$$

and

$$\vec{\nabla}\cdot\vec{B}=0\ . \tag{32}$$

In the Coulomb gauge Eq. (24) can be restated as

$$\Box\vec{\nabla}\cdot\vec{E}=g\lambda^2J^0\ , \tag{33}$$

$$\Box\left(\vec{\nabla}\times\vec{B}-\frac{\partial\vec{E}}{\partial t}\right)=g\lambda^2\vec{J}\ , \tag{34}$$

where J^μ is the quark current. Equations (29) and (33) give our analog to the differential equation for the instantaneous Coulomb potential of QED,

$$\Box\nabla^2A^{10}=-g\lambda^2J^0\ , \tag{35}$$

while the equivalent vector-potential differential equation is

$$\Box(\Box\vec{A}^1+\vec{\nabla}\dot{A}^{10})=g\lambda^2\vec{J}\ . \tag{36}$$

The free gluon unequal-time commutators may be determined from Eqs. (21), (22), (28), (35), and (36) (with the current, of course, set to zero):

$$i\Delta_{ij}^{11}(x-y) \equiv [A_i^1(x), A_j^1(y)] = -i\lambda^2(\delta_{ij} - \nabla_i\nabla_j\nabla^{-2})\left[\frac{\partial}{\partial\mu^2}\,\Delta(x-y,\mu)\right]_{\mu=0}, \tag{37}$$

$$i\Delta_{ij}^{12}(x-y) \equiv [A_i^1(x), A_j^2(y)] = i(\delta_{ij} - \nabla_i\nabla_j\nabla^{-2})\Delta(x-y,0), \tag{38}$$

$$i\Delta_{ij}^{22}(x-y) \equiv [A_i^2(x), A_j^2(y)] = 0, \tag{39}$$

with $i, j = 1, 2, 3$ and

$$i\Delta(x-y,\mu) = \int \frac{d^4k}{(2\pi)^3}\epsilon(k_0)\delta(k^2-\mu^2)\,e^{-ik\cdot(x-y)}. \tag{40}$$

The commutators, Δ^{11} and Δ^{12}, are zero at spacelike separations due to the form of Δ. They are consistent with the equal-time commutation relations in that limit and they are also consistent with the equations of motion due to the identity

$$\Box\left[\frac{\partial}{\partial\mu^2}\,\Delta(x-y,\mu)\right]_{\mu=0} = -\Delta(x-y,0). \tag{41}$$

Assuming that we have established all operator expressions we are now in a position to apply operator boundary conditions to the gluon field. The key quantities so far as the perturbation theory we will consider in the next section is concerned are the time-ordered propagators of the gluon field

$$T(A_i^a(x)A_j^b(y)) \equiv \tfrac{1}{2}\epsilon(x_0-y_0)[A_i^a(x), A_j^b(y)] \tag{42}$$

$$= \tfrac{1}{2}i\epsilon(x_0-y_0)\Delta_{ij}^{ab}(x-y), \tag{43}$$

where we have suppressed the "in" subscript on the field operator. We can take advantage of the gauge invariance of A_μ^1 to express $T(A_\mu^1 A_\nu^1)$ in the Feynman gauge,

$$T(A_\mu^1(x)A_\nu^1(y)) = i\lambda^2 g_{\mu\nu}\int\frac{d^4k}{(2\pi)^4}\left(\mathrm{P}\,\frac{1}{k^4}\right)e^{-ik\cdot(x-y)} \tag{44}$$

with

$$\mathrm{P}\,\frac{1}{k^4} \equiv \frac{1}{2}\left[\frac{1}{(k^2+i\epsilon)^2} + \frac{1}{(k^2-i\epsilon)^2}\right]. \tag{45}$$

In coordinate space

$$T(A_\mu^1(x)A_\nu^1(y)) = ig_{\mu\nu}\lambda^2\theta((x-y)^2)/16\pi. \tag{46}$$

The equations of motion of the dipole model display a close analogy to those of quantum electrodynamics. The main difference (with important physical consequences) is the increased degree of the differential equation for A_μ^1 vis-à-vis the corresponding QED equations. The result is a dipole propagator rather than a monopole propagator in momentum space. One could have second-quantized the dipole field in a manner which leads to dipole Feynman propagators. In that case the S matrix would not be unitary in perturbation theory.

IV. PERTURBATION THEORY

The rules for forming the integral corresponding to a Feynman diagram in the dipole model are identical with those of quantum electrodynamics[7] except that we use

$$iB_{\mu\nu}(q) = ig_{\mu\nu}\lambda^2\mathrm{P}\,\frac{1}{q^4} \tag{47}$$

rather than the Feynman photon propagator

$$iD_{F\mu\nu}(q) = -\frac{ig_{\mu\nu}}{q^2+i\epsilon}. \tag{48}$$

The choice of a principal-value propagator has substantial effects in perturbation theory. For example we shall show that consistency with unitarity requires no diagrams with in or out gluon lines contribute to the S matrix. In addition, there are novelties in the type of divergences in diagrams and the analytic structure of the S matrix. It also appears that conclusions based on summing only a finite number of graphs contributing to an S-matrix element may be misleading. This follows from the fact (to be shown in Sec. V) that free quarks do not exist upon summation of all orders of perturbation theory [Eq. (65)], though this is not seen in a summation to any finite order. The physical states are neutral bound states and thus it appears that the best methods of exploring the physics embodied in this model will involve Bethe-Salpeter equations[8] or eikonal summations. They are currently under study.

We now describe the modifications necessary to compute diagrams in perturbation theory. The propagator of Eq. (47) may be exponentiated through the use of the identity

$$\mathrm{P}\,\frac{1}{k^4} = -\tfrac{1}{2}\int_{-\infty}^{\infty} d\alpha\,\alpha\,\epsilon(\alpha)\exp(i\alpha k^2), \tag{49}$$

where $\epsilon(\alpha) = \pm 1$ for $\alpha \gtrless 0$. Since Feynman parameters are not necessarily positive the following identity will be useful in evaluating loop integrations:

$$\int d^4k\,\exp[iC(\alpha)k^2] = i\pi^2\epsilon(C)/C^2. \tag{50}$$

As a result the Feynman parameter representation of a diagram will have the form

$$I=\int_{-\infty}^{\infty}\prod_{j=1}^{p}\alpha_j\,d\alpha_j\int_0^{\infty}\frac{d\beta_1\cdots d\beta_q}{C^2}\,\epsilon(\alpha_1\alpha_2\cdots\alpha_p C)Ne^{iD/C}\,. \tag{51}$$

where α_i corresponds to an internal gluon line and β_i to an internal fermion line, N symbolizes numerator terms, and C and D are determinantal functions.[9] If we had given the gluons dipole Feynman propagators we would have obtained

$$\int_0^{\infty}\frac{\prod_{j=1}^{p}\alpha_j\,d\alpha_j\,d\beta_1 d\beta_2\cdots d\beta_q N\exp(iD/C)}{C^2}\,, \tag{52}$$

in comparison to Eq. (51). If we now scale all Feynman parameters with u and use the identity

$$\int_0^{\infty}\frac{du}{u}\,\delta\left(1-\frac{|\alpha_1+\alpha_2+\cdots+\alpha_p+\beta_1+\beta_2+\cdots+\beta_q|}{u}\right)=1 \tag{53}$$

(where | | indicates absolute value) to introduce an integration over u in Eq. (51), we obtain

$$I=\Gamma(q+2p-2l)\int_{-\infty}^{\infty}\prod_{j=1}^{p}\alpha_j\,d\alpha_j\int_0^{\infty}\frac{d\beta_1\cdots d\beta_q\,\epsilon(\alpha_1\cdots\alpha_p C)\tilde{N}}{C^2(-iD/C)^{q+2p-2l}}\,\delta\left(1-\left|\sum_i\alpha_i+\sum_j\beta_j\right|\right) \tag{54}$$

where l = the number of loop integrations in the original diagram and $\tilde{N}$ is obtained from N. An example of this procedure is given in Appendix A. As an alternative to the above method one can introduce light-cone coordinates and evaluate pole terms by contour integrations with Eq. (45) specifying the location of the poles relative to the contour.

The divergences occurring in this model are somewhat novel. As one would expect, with a dipole propagator the ultraviolet divergences are restricted to some lower-order diagrams and are logarithmic in nature (see Fig. 1). The dipole propagator, because it is in principal value, does not induce infrared divergences in loop integrations. However, a third type of divergence, which may be called a light-cone divergence,[10] does occur and is connected with a divergence in a loop integration, $\int d^4k$, associated with the region where $k^2=k_0^2-k_3^2-\vec{k}_\perp^{\,2}\approx 0$ and $k_0, k_3\to\infty$. In the Feynman parameter representation of Eq. (54) the divergence will appear at the $\pm\infty$ limits of Feynman parameter integrals. The worst divergence is quadratic and associated with one-loop diagrams with one internal gluon line (Fig. 2). These divergences can be managed through the use of Pauli-Villars regularization. Some diagrams containing light-cone divergences are given in Fig. 2. It should be noted that they are necessarily one-loop diagrams. We can demonstrate this by an examination of the overall degree of light-cone divergence of a graph in the representation of Eq. (54). Let us scale all Feynman parameters in Eq. (54) with Λ and determine the leading behavior as $\Lambda\to\infty$. We find, for $l\equiv$ number of loops >1,

$$\tilde{N}\sim\Lambda^0\,, \tag{55}$$

$$C\sim\Lambda^l\,, \tag{56}$$

$$D\sim\Lambda^{l+1}\,, \tag{57}$$

and as a result

$$I\sim\Lambda^{2p+q-1-2l-(q+2p-2l)}=\Lambda^{-1}\,, \tag{58}$$

or convergence of the integral as a whole. However, for one-loop diagrams ($l=1$)

$$C\sim\Lambda^0 \tag{59}$$

and consequently

$$\tilde{N}\sim\Lambda^q\,, \tag{60}$$

$$I\sim\Lambda^{3-2p}\,. \tag{61}$$

For example, the diagram of Fig. 2(a) has $p=1$ and diverges quadratically.

Light-cone divergences stem directly from the use of principal-value propagators for the gluon. As such they reflect the nontrivial nature of Wick rotation in this model and they lead to divergences in the vertex renormalization constant, the wave-function renormalization constant, and the quark self-mass which prevent this model from being a superrenormalizable theory of the conventional variety.

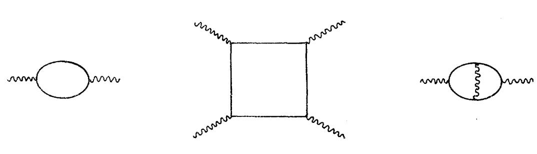

FIG. 1. Some ultraviolet-divergent diagrams.

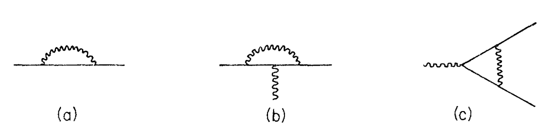

FIG. 2. Some light-cone divergent diagrams.

The Schwinger-Dyson equations for dipole electrodynamics are quite similar to those of quantum electrodynamics, with the exception of the gluon Green's functions, which we now discuss. The proper gluon self-energy, $\Pi_{\mu\nu}(q)$, which couples only to the A_μ^1 channel due to the form of our Lagrangian, satisfies

$$\Pi_{\mu\nu}(q) = i\int \frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\,\gamma_\mu S_F'(k)\Gamma_\nu(k, k+q)S_F'(k+q) \tag{62}$$

$$= (q_\mu q_\nu - g_{\mu\nu}q^2)\Pi(q^2)\,, \tag{63}$$

where S_F' is the quark propagator and Γ_ν is the proper vertex function (see Fig. 3). The gluon self-energy is related to the complete gluon propagator, $B_{\mu\nu}'$, by

$$iB_{\mu\nu}' = iB_{\mu\nu} + ig^2 iB_{\mu\lambda}\Pi^{\lambda\sigma} iB_{\sigma\nu}'\,, \tag{64}$$

with $B_{\mu\nu}$ given by Eq. (47). Using Eq. (63) we find

$$B_{\mu\nu}'(q) = \frac{\lambda^2 g_{\mu\nu}}{q^4 + g^2\lambda^2 q^2\Pi(q)} - \frac{q_\mu q_\nu g^2\lambda^4\Pi(q)}{q^8 + g^2\lambda^2 q^6\Pi(q)}\,. \tag{65}$$

The Green's function for the gluon field, A_μ^2, which is zero within the context of the free gluon Lagrangian [cf. Eq. (39)], is nonzero in the interacting theory due to vacuum-polarization effects. It is related to $B_{\mu\nu}'$ by

$$B_{\mu\nu}^2(q) = \frac{(q^4/\lambda^2)B_{\mu\nu}'(q) - g_{\mu\nu}}{\lambda^2} \tag{66}$$

$$= \frac{-g^2\Pi(q)g_{\mu\nu}}{q^2 + g^2\lambda^2\Pi(q)} \tag{67}$$

up to terms proportional to $q_\mu q_\nu$. Equation (65) will play an instrumental role in the demonstration of free-quark screening in the next section.

V. SOME GENERAL PROPERTIES

In this section we will first consider the screening mechanism for quarks and then discuss unitarity, causality, and scaling properties of the lowest-order contributions to the electroproduction structure functions.

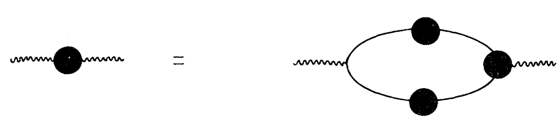

FIG. 3. Representation of the Schwinger-Dyson equation for the gluon self-energy.

In Ref. 1 attention was drawn to the screening of free quarks due to vacuum polarization. Our mechanism is a variation of the Schwinger mechanism[11] but differs from it in an important respect: It is manifest in low order and thus not a matter of conjecture—an important consideration for insoluble field theories. Even in lowest order (Fig. 1), where $\Pi(q)$ is a constant up to a logarithmic term, we find manifest screening.

Let us consider a system containing free quarks in some bounded region. We choose to work in the Coulomb gauge. Because of Eq. (35) the total charge is proportional to

$$Q \propto \int d^3x\,\nabla^2\Box A_0^1 \tag{68}$$

$$= \int d\vec{S}\cdot\vec{\nabla}\Box A_0^1\,. \tag{69}$$

However, an examination of Eq. (65) shows that important vacuum-polarization effects occur at large distances. The potential corresponding to a static free quark located at the origin is

$$A_0^1 = \frac{-\lambda^2 g|\vec{x}|}{8\pi} \tag{70}$$

(if we ignore vacuum-polarization effects) and a finite contribution to Q would result if substituted in Eq. (69). At large distances A_0^1 is substantially modified from the expression in Eq. (70). From Eq. (65) we see that the large-distance behavior of A_0^1 is controlled by

$$\frac{1}{g^2q^2\Pi(q)}\,, \tag{71}$$

and since $\Pi(q)$ is a constant up to logarithms in lowest order and not proportional to a positive power of q^2 in any finite order of perturbation theory we find A_0^1 to be proportional to at most an inverse power of $|\vec{x}|$ at large distances. Substituting an inverse power of $|\vec{x}|$ for A_0^1 in Eq. (69) and letting the surface of integration go to infinity shows $Q = 0$. Thus isolated free quarks do not exist in this model. Only neutral bound states occur.

We have chosen a propagator for the gluon which allows the S matrix to be unitarity. Our gluons are dipole ghosts, and, having indefinite metric, they would normally destroy the unitarity of the S matrix. But the quantization procedure eliminates their appearance in in or out states and their principal-value propagator precludes states containing gluons from contributing to the absorptive part of any Feynman diagram.[12] This is required if the S matrix is to be unitary. But as a result the S matrix is not analytic. The nonanalyticity is closely associated with advanced noncausal effects. Our procedure forces unitarity to be valid at the expense of noncausality. Tradeoffs of this type have recently been discussed by Coleman.[13] We return to the question of causality later.

We have verified that unitarity is maintained in perturbation theory by an explicit calculation of the lowest-order quark self-energy [Fig. 2(a)], which is

$$\Sigma(q)=\frac{-\lambda^2g^2}{8\pi^2}\,\mathrm{P}\sum_i \frac{c_i\Lambda_i^{\,2}\ln(\Lambda_i^{\,2}/q^2)}{q^2(q^2-\Lambda_i^{\,2})}\left(\frac{\Lambda_i^{\,2}}{q^2}\not{q}-2m\right), \tag{72}$$

where $c_1=1$, $\Lambda_1=m$, the regulator identities $\sum_i c_i=\sum_i c_i\Lambda_i^{\,2}=0$ hold, and P signifies $\Sigma(q^2+i\epsilon)=\Sigma(q^2-i\epsilon)$ as is demonstrated in detail in Appendix A. The fact that the singularities in Eq. (72) occur in principal value implies Σ has no absorptive part. This is to be contrasted with the corresponding quantity in QED which has an absorptive part reflecting the physically allowed decay of an off-shell electron into an electron and a photon. No similar possibility exists in our model.

We now will show that the absorptive part (in the physical region) of a Feynman diagram with one internal gluon line only receives contributions from intermediate states (obtained by appropriately "cutting" internal lines in all possible ways) which do not contain the gluon. The generalization to diagrams with many gluon lines is immediate. First we note that a principal-value propagator may be decomposed:

$$\mathrm{P}\,\frac{1}{k^2-m^2}=\frac{1}{k^2-m^2+i\epsilon}+i\pi\delta(k^2-m^2)\,. \tag{73}$$

For the sake of simplicity we shall write the integral corresponding to our hypothetical diagram as

$$I=\int d^4k\,\tilde{I}\,\mathrm{P}\,\frac{1}{k^4} \tag{74}$$

$$=\frac{\partial}{\partial\mu^2}\int d^4k\,\tilde{I}\,\mathrm{P}\,\frac{1}{k^2-\mu^2}\bigg|_{\mu^2=0}\,, \tag{75}$$

where I and $\tilde{I}$ have indices and momenta appropriate to the diagram in question and the limits we have introduced engender no infrared difficulties due to the choice of a principal-value propagator. Substituting Eq. (73) into Eq. (75) we can decompose I into three Feynman integrals (actually their derivative with respect to mass, etc.),

$$I=\frac{\partial}{\partial\mu^2}\int d^4k\left[\frac{\tilde{I}}{k^2-\mu^2+i\epsilon}+i\pi\theta(k_0)\delta(k^2-\mu^2)+i\pi\theta(-k_0)\delta(k^2-\mu^2)\right]_{\mu=0}, \tag{76}$$

in each of which only Feynman propagators are used. The last two terms correspond to opening up the loop containing the gluon. Their Feynman diagrams have in and out gluon lines of momentum k, which is summed over. Let us now restrict ourselves to the physical region[14] of our diagram so that we can take the absorptive part of I in the following way:

$$\begin{aligned}\mathrm{Abs}(I)&=i\pi\frac{\partial}{\partial\mu^2}\int d^4k[-\tilde{I}'\theta(k_0)\delta(k^2-\mu^2)+\tilde{I}'\theta(k_0)\delta(k^2-\mu^2)+\tilde{I}'\theta(-k_0)\delta(k^2-\mu^2)]+R\\&=R\,.\end{aligned} \tag{77}$$

The term in square brackets contains all contributions from intermediate states containing a gluon, while R contains the remainder of the absorptive part. The first two terms cancel, while the third term is zero in the physical region. Thus we have shown that the absorptive part receives no contributions from states containing the gluon. Consequently only states containing quarks contribute to unitarity sums for absorptive parts and diagrams containing external gluons do not contribute to the S matrix.

The principal-value gluon propagator has introduced noncausal effects into our model in the sense that the corresponding configuration-space Green's function is half-advanced and half-retarded. However, because we have maintained the commutativity of field operators at spacelike distances the principle of microscopic causality is not violated. Although advanced effects are not observed in everyday life they do not lead to internal inconsistency or paradoxes.[6] On the microscopic level, for example, within the confines of a hadron, advanced effects are not necessarily ruled out on physical grounds. From the earlier discussion of vacuum-polarization effects it is clear that noncausalities must be limited to very short distances. It thus appears that the only significant question involving causality is whether the nonanalyticity of the S matrix for low-order quark-quark scattering will be reflected in the scattering amplitudes of bound states in a manner which is in substantial disagreement with our understanding of S-matrix analyticity for physical particle scattering. The answer to this question is not known.

As an application of the dipole model we shall study the deep-inelastic electroproduction structure functions in low-order perturbation theory. Previous calculations[15] of the structure functions in pseudoscalar-meson or neutral-vector-meson field-theoretic models (without transverse-momentum cutoffs) contained logarithmic deviations from scaling in apparent conflict with experimental results. The dipole model has strong transverse-momentum damping and as a result one obtains scaling structure functions—in fact, the only asymptotically leading contribution appears to be the diagram of Fig. 4(a). Higher-order diagrams do not scale by powers of q^2, the photon mass squared. For example the diagrams of lowest order in q^2 [Figs. 4(b)–4(d)] contributing to νW_2 are of $O(q^{-4})$. Thus the dipole model establishes a parton picture of the deep-inelastic structure functions since quarks appear to be pointlike particles in the scaling region. The choice of a principal-value propagator for the gluon has the effect of suppressing corrections to the scaling part of νW_2 which would have been of $O(q^{-2})$, such as the contribution of the diagram of Fig. 5. The absorptive part of that diagram is zero due to principal-value gluon propagator. Thus the precocious nature of scaling could be connected with the properties of the principal-value gluon propagator in electroproduction. On the other hand, the principal-value propagator will not play such an important role (at least in low order) in suppressing nonscaling contributions to the absorptive part of the amplitude associated with $e^+e^- \to$ hadrons. Thus low-order calculations are suggestive so far as scaling phenomena are concerned.

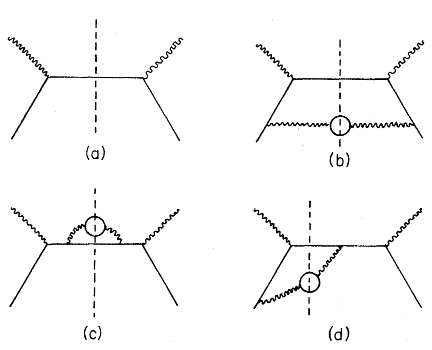

FIG. 4. Lowest-order diagrams contributing to the inelastic electroproduction structure functions. The dashed lines indicate the only contributions to the electroproduction structure functions of the absorptive part of the forward virtual Compton scattering diagram. External "wiggly" lines represent photons, while internal "wiggly" lines represent dipole gluons.

VI. CONCLUSION

The dipole electrodynamics model which we have discussed in the preceding sections is a prototype for a field theory of hadron binding. It has a number of desirable qualitative features such as quark confinement and scaling electroproduction structure functions. The physical content of this model is in the bound-states sector. This sector is currently under study using Bethe-Salpeter and eikonal techniques.

In a more realistic version of this model charge will be replaced by color in such a way that only zero-triality states are physical. The fields A_μ^1 and A_μ^2 will then become Yang-Mills fields. In that case the use of principal-value propagators appears to substantially simplify the model since closed loops of Yang-Mills fields are necessarily zero.[16]

ACKNOWLEDGMENT

I am grateful to the members of the Newman Laboratory of Nuclear Studies for helpful discussions and particularly to Dr. J. Kogut, Dr. D. K. Sinclair, and Dr. L. Susskind.

APPENDIX A

As an example of the modifications in perturbation-theory calculations resulting from principal-value propagators we evaluate the self-energy contribution of Fig. 2(a) and verify Eq. (72):

$$\Sigma(q) = \frac{i g^2 \lambda^2}{(2\pi)^4} \int \frac{d^4k}{(q+k)^2 - m^2 + i\epsilon} \left(\mathrm{P}\,\frac{1}{k^4}\right) \gamma_\nu (\not{q} + \not{k} + m) \gamma^\nu \tag{A1}$$

in the Feynman gauge. Feynman parameters can be introduced, and using Eqs. (49) and (50) we obtain

$$\Sigma(q) = \frac{i g^2 \lambda^2}{32\pi^2} \int_{-\infty}^{\infty} d\alpha\, \epsilon(\alpha)\alpha \int_0^{\infty} \frac{d\beta}{C^2} \left(\frac{-2\alpha}{C}\,\not{q} + 4m\right) \times \epsilon(C) e^{iD/C}, \tag{A2}$$

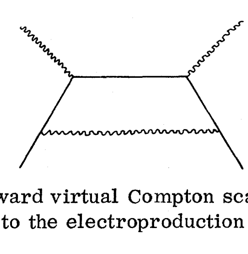

FIG. 5. A forward virtual Compton scattering diagram *not* contributing to the electroproduction structure functions.

where

$$C=\alpha+\beta \tag{A3}$$

and

$$D=\alpha\beta q^2-\beta C m^2 . \tag{A4}$$

Scaling α and β and using Eq. (53) converts $\Sigma(q)$ to the form

$$\Sigma(q)=\frac{-g^2\lambda^2}{32\pi^2}\int_{-\infty}^{\infty}d\alpha\,\alpha\,\epsilon(\alpha)\int_0^{\infty}\frac{d\beta}{D}\left[-2\alpha\epsilon(C)\not{q}+4m\right]\times\delta(1-|\alpha+\beta|) . \tag{A5}$$

Of the two "points" contributing to the integral, $\alpha+\beta=\pm1$, the contribution of the term $\alpha+\beta=-1$ can be included in the other term by an extension of the β integration domain:

$$\Sigma(q)=\frac{-g^2\lambda^2}{32\pi^2}\int_{-\infty}^{\infty}d\alpha\,\epsilon(\alpha)\alpha\int_{-\infty}^{\infty}d\beta\,\frac{\delta(1-\alpha-\beta)(-2\alpha\not{q}+4m)}{\alpha\beta q^2-\beta m^2} . \tag{A6}$$

This may be shown by letting $\alpha\to-\alpha$ and $\beta\to-\beta$ in the term in question. Equation (A6) has divergences at $\alpha=\pm\infty$ after the β integration. These may be handled by introducing Pauli-Villars regulators of mass Λ_i satisfying

$$\sum_i c_i=0 , \tag{A7}$$

$$\sum_i c_i\Lambda_i{}^2=0 , \tag{A8}$$

$$c_1=1 , \tag{A9}$$

$$\Lambda_1=m . \tag{A10}$$

Equation (A6) then becomes

$$\Sigma(q)=\frac{-g^2\lambda^2}{32\pi^2}\sum_i c_i\int_{-\infty}^{\infty}\frac{d\alpha\,\alpha\,\epsilon(\alpha)(-2\alpha\not{q}+4m)}{(1-\alpha)\{\alpha[q^2+i\epsilon(\alpha)\delta]-\Lambda_i{}^2\}} . \tag{A11}$$

which may be shown to give Eq. (72) by elementary integrations. Apparent singularities in the denominator of the integrand of Eq. (A11) do not lead to difficulties if we take account of the $i\epsilon$'s which we have suppressed. The $i\epsilon(\alpha)\delta$ term (δ is infinitesimal) shows $\Sigma(q)$ to be in principal value $[\Sigma(q^2+i\delta)=\Sigma(q^2-i\delta)]$. It originates in the exponentiation of the principal-value propagator using Eq. (49).

APPENDIX B

We will briefly describe another possible model for hadron binding. Like the dipole model it is a member of a class of null-metric gluon theories with multipole Green's functions. The physical motivation for considering this model is a gross similarity to a quark model of Feynman *et al.*[4] which posited a relativistic harmonic oscillator potential, $x^\mu x_\mu$, between quarks and obtained quite successful agreement with experiment. If we neglect factors due to its vectorial nature (and also vacuum polarization effects) the interaction between quarks in our model is $x^\mu x_\mu\theta(x_\mu x^\mu)$ (note that $x_\mu x^\mu=x_0{}^2-\vec{x}^2$). The $\theta(x^\mu x_\mu)$ factor, which is necessary for unitarity to be maintained, seems to imply that only "timelike" excitations are physical, and as a result the analysis of Feynman *et al.* cannot be directly appropriated for our use.

We shall introduce three vector-gluon fields, $A_\mu^i(x)$ ($i=1, 2, 3$), of which only one will interact directly with the prototype spin-$\frac{1}{2}$ quark field, $\psi(x)$:

$$\mathcal{L}=-\tfrac{1}{2}F^1_{\mu\nu}F^{3\mu\nu}+\tfrac{1}{4}F^2_{\mu\nu}F^{2\mu\nu}-\lambda^2A^2_\mu A^{3\mu}+\bar{\psi}(i\nabla\!\!\!/-g A\!\!\!/^1-m)\psi , \tag{B1}$$

with $F^i_{\mu\nu}=\partial_\nu A^i_\mu-\partial_\mu A^i_\nu$, and λ and g coupling constants. Following the canonical procedure we obtain the equations of motion

$$\partial^\mu F^1_{\mu\nu}+\lambda^2A^2_\nu=0 , \tag{B2}$$

$$\partial^\mu F^2_{\mu\nu}-\lambda^2A^3_\nu=0 , \tag{B3}$$

$$\partial^\mu F^3_{\mu\nu}+gJ_\nu=0 , \tag{B4}$$

$$(i\nabla\!\!\!/-gA\!\!\!/^1-m)\psi=0 , \tag{B5}$$

with J^ν the quark current. The equations of motior reveal the Lagrangian to be invariant under local gauge transformations of A^1_μ and ψ while

$$\partial^\mu A^2_\mu=\partial^\mu A^3_\mu=0 . \tag{B6}$$

Furthermore, Eqs. (B2) and (B3) imply

$$\Box F^1_{\mu\nu}-\lambda^2F^2_{\mu\nu}=0 , \tag{B7}$$

$$\Box F^2_{\mu\nu}+\lambda^2F^3_{\mu\nu}=0 , \tag{B8}$$

and as a result

$$\Box^2\partial^\mu F^1_{\mu\nu}=g\lambda^4J_\nu \tag{B9}$$

irrespective of the gauge choice for A^1_μ.

Following the conventional procedure we find that the canonical equal-time commutation relations in the radiation gauge ($\vec{\nabla}\cdot\vec{A}^1=0$) are

$$[F^a_{0i}(x), A^b_j(y)]=ih^{ab}\int\frac{d^3k}{(2\pi)^3}e^{i\vec{k}\cdot(\vec{x}-\vec{y})}\left(\delta_{ij}-\frac{k_ik_j}{|\vec{k}|^2}\right) , \tag{B10}$$

with $h^{13}=h^{31}=-h^{22}=1$ and all other $h^{ab}=0$. We can now choose to quantize the theory as described in the text. Again we may use the perturbation-theory rules of QED if the photon propagator is replaced with the gluon propagator in the following manner:

$$iD_{F\mu\nu} \to iG_{\mu\nu} = i\lambda^4 g_{\mu\nu} \mathrm{P}\frac{1}{k^6} \tag{B11}$$

in the Feynman gauge, with

$$\mathrm{P}\frac{1}{k^6} = \frac{1}{2}\,\frac{1}{(k^2+i\epsilon)^3} + \frac{1}{2}\,\frac{1}{(k^2-i\epsilon)^3}\,. \tag{B12}$$

In coordinate space the gluon propagator is

$$T(A^1_\mu(x)A^1_\nu(y)) = i\lambda^4 g_{\mu\nu}\int d^4k\left(\mathrm{P}\frac{1}{k^6}\right)e^{-ik\cdot(x-y)} \tag{B13}$$

$$= \frac{i}{64\pi}\lambda^4 g_{\mu\nu}(x-y)^2\theta((x-y)^2) \tag{B14}$$

in the Feynman gauge, which suggests a relationship between our model and that of Feynman, Kislinger, and Ravndal[4] as stated previously.

The discussions of unitarity, causality, and quark confinement given in the text apply to this model with only superficial changes. The light-cone divergences encountered in the dipole model are not so extreme here. For example, the overall degree of light-cone divergence for one-loop diagrams is 3-3p [where p is the number of internal gluon lines; cf. Eq. (61)] and thus the lowest-order quark self-energy (Fig. 2) is only logarithmically divergent,

$$\Sigma(q) = \frac{\lambda^4 g^2}{16\pi^2}\,\mathrm{P}\frac{1}{q^4}\left\{ \not{q}\left[\ln\left(\frac{\Lambda^2}{m^2}\right) - \frac{2q^2m^2}{(q^2-m^2)^2} + \frac{3q^4m^2-q^6}{(q^2-m^2)^3}\ln\left(\frac{q^2}{m^2}\right)\right] + 2m\left[\frac{q^2(q^2+m^2)}{(q^2-m^2)^2} - \frac{2q^4m^2}{(q^2-m^2)^3}\ln\left(\frac{q^2}{m^2}\right)\right]\right\}, \tag{B15}$$

where q is the quark four-momentum, m the quark mass, Λ^2 is a regulator mass, and P signifies that all singularities are to be taken in principal value.

Finally, we would like to note again that the deep-inelastic structure functions scale in this model with leading nonscaling corrections of $O(1/q^6)$, where q is the virtual-photon four-momentum. These corrections come from the diagrams of Fig. 4.

*Work supported in part by the National Science Foundation.

[1]A. Casher, J. Kogut, and L. Susskind, Phys. Rev. Lett. 31, 792 (1973).

[2]K. Wilson, Phys. Rev. D 10, 2445 (1974).

[3]A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, and V. F. Weisskopf, Phys. Rev. D 9, 3471 (1974).

[4]R. Feynman, M. Kislinger, and F. Ravndal, Phys. Rev. D 3, 2706 (1971).

[5]P. A. M. Dirac, *Lectures on Quantum Mechanics* (Yeshiva Univ., New York, 1964).

[6]K. Schwarzschild, Göttinger Nachrichten 128, 132 (1903); H. Tetrode, Z. Phys. 10, 317 (1922); A. D. Fokker, *ibid.* 58, 386 (1929); Physica (The Hague) 9, 33 (1929); 12, 145 (1932); J. Wheeler and R. P. Feynman, Rev. Mod. Phys. 17, 157 (1945); 21, 425 (1949).

[7]J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965), p. 382.

[8]E. Salpeter and H. Bethe, Phys. Rev. 84, 1232 (1951).

[9]R. J. Eden, P. V. Landshoff, D. I. Olive, and J. C. Polkinghorne, *The Analytic S-Matrix* (Cambridge Univ. Press, Cambridge, 1966), p. 34.

[10]Suggested by Dr. D. K. Sinclair.

[11]J. Schwinger, Phys. Rev. 128, 2425 (1962).

[12]After this work was completed Dr. K. Subbarao pointed out that Professor E. C. G. Sudarshan has considered the use of action-at-a-distance interactions to remedy unitarity problems in indefinite-metric theories: E. C. G. Sudarshan, Fields and Quanta 2, 175 (1972).

[13]S. Coleman, in *Subnuclear Phenomena*, edited by A. Zichichi (Academic, New York, 1970), Part A, p. 282.

[14]R. J. Eden, P. V. Landshoff, D. I. Olive, and J. C. Polkinghorne, *The Analytic S-Matrix* (Ref. 9), p. 112.

[15]R. Jackiw and G. Preparata, Phys. Rev. Lett. 22, 975 (1969); S. Adler and W. Tung, *ibid.* 22, 978 (1969); S. Blaha, Phys. Rev. D 3, 510 (1971).

[16]This may be proved using the representation of Eq. (54) with $q=0$ (only principal-value propagators) and $l=1$. If we let $\alpha_i \to -\alpha_i$ for $i=1,2,\ldots,p$ then we can show that $I=-I$ and thus $I=0$. $\tilde{N}$ is arbitrary except that it can be written as a sum of terms which are homogeneous in the Feynman parameters. This condition can always be satisfied in perturbation theory.