Finally

$$M_S^{(4)}(q)=\left[\iint M_S(q')M_S(q'')M_S(q-q'-q'')\frac{d^4q'}{(2\pi)^4}\frac{d^4q''}{(2\pi)^4}+\iint M_S(q')M_P(q'')M_P(q-q'-q'')\frac{d^4q'}{(2\pi)^4}\frac{d^4q''}{(2\pi)^4}\right.$$

$$\left.-2\iint M_S(q')M_A^{\nu}(q'')M_{A\nu}(q-q'-q'')\frac{d^4q'}{(2\pi)^4}\frac{d^4q''}{(2\pi)^4}\right]L'. \quad \text{(A10)}$$

Substituting (A5), (A7), and (A10) into (A4) and transforming it back to x space we obtain Eq. (12) of Sec. II. We note that the first term of (A5) is exactly equal to the left-hand side of (A4).

---

*Present address: The Enrico Fermi Institute, University of Chicago, Chicago, Ill. 60637.

†Present address: National Laboratory for High Energy Physics, Oho-machi, Ibaraki-ken, Japan.

[1]Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961).
[2]A. A. Abrikosov, Zh. Eksp. Theor. Fiz. 32, 1442 (1957) [Sov. Phys.—JETP 5, 1174 (1957)].
[3]V. L. Ginzburg and L. D. Landau, Zh. Eksp. Theor. Fiz. 20, 1064 (1950).
[4]For the application of GL equation to particle physics, see H. B. Nielsen and P. Olesen, Nucl. Phys. B61, 45 (1973).
[5]Y. Freundlich and D. Lurié, Nucl. Phys. B19, 557 (1970).
[6]L. P. Gorkov, Zh. Eksp. Theor. Fiz. 36, 1918 (1959) [Sov. Phys.—JETP 9, 1364 (1959)].
[7]A. Chodos, R. L. Jaffe, K. Johnson, and C. B. Thorn, Phys. Rev. D 10, 2599 (1974); T. D. Lee and G. C. Wick, *ibid.* 9, 2291 (1974); Y. Yamaguchi, University of Tokyo Report No. UT 232, 1974 (unpublished); W. A. Bardeen, M. Chanowitz, S. D. Drell, R. Giles, C. K. Lee, M. Weinstein, and T.-M. Yan (unpublished).
[8]Y. Nambu, in *Symmetries and Quark Models*, edited by R. Chand (Gordon and Breach, New York, 1970), p. 269; L. Susskind, Phys. Rev. D 1, 1182 (1970); T. Goto, Prog. Theor. Phys. 46, 1560 (1971).
[9]D. L. Hill and J. A. Wheeler, Phys. Rev. 89, 1102 (1953); R. E. Peierls and J. Yoccoz, Proc. Phys. Soc. A70, 381 (1957); R. E. Peierls and D. J. Thouless, Nucl. Phys. 38, 154 (1962).

PHYSICAL REVIEW D VOLUME 10, NUMBER 12 15 DECEMBER 1974

Strings, monopoles, and gauge fields

Y. Nambu
The Enrico Fermi Institute and The Department of Physics, The University of Chicago, Chicago, Illinois 60637
(Received 9 September 1974)

The Nielsen-Olesen interpretation of dual strings as Abrikosov flux lines is extended to the case of open-ended strings by adapting Dirac's description of magnetic monopoles to a London-type theory. The mathematical formalism turns out to be similar to that of Kalb and Ramond. Translated to hadron physics, it implies that the quarks will act as carriers of magnetic charge, permanently bound in pairs by the string bonds. However, massive axial-vector gluons can be created by hadrons.

I. INTRODUCTION

In a very interesting paper[1] Nielsen and Olesen have pointed out a parallelism between the Higgs model of broken gauge invariance and the Landau-Ginzburg theory of superconductivity on the one hand and the dual string model and the Abrikosov flux lines in type II superconductors on the other. According to their suggestion, a dual string is nothing but a mathematical idealization of a magnetic flux tube in equilibrium against the pressure of the surrounding charged superfluid (Higgs-scalar field) which it displaces. Only strings with no ends (infinite strings or loops) were considered by them. It is known that a closed string could be a candidate for the Pomeron. But what will happen if the string is open-ended? Obviously the magnetic flux will terminate at the end points, thus creating a pair of magnetic charges.[2] In the dual quark model ordinary hadrons are viewed as being made up of quarks bound by dual strings, or, from the string's point of view, as open strings having quarks at their ends. From the Nielsen-Olesen picture it then follows that these quarks will act as a source of magnetic charge. (Here we are using the words electric and magnetic not to refer to actual electromagnetism, but as an analogy in a simplified model of strong interactions.)

At any rate we are led to the following picture. The quarks carry magnetic-type charge g whereas the Higgs field (boson) carries electric-type charge e, although it is a matter of convention to call one magnetic and the other electric. The two charges will be related by the Dirac quantization condition

$$\frac{eg}{4\pi}=\tfrac{1}{2}n\,.$$

The theory also contains two mass parameters m_V and m_S of the vector and Higgs scalar fields A_μ and ϕ, respectively. These are in turn related to two characteristic lengths $\lambda_V=1/m_V$ and $\lambda_S=1/m_S$ which determine the transverse dimensions of the vector-field concentration and of the scalar-field rarification, respectively, around the string. Thus the string is actually two things, a flux of a magnetic field and a hollow vortex line in the Higgs field.

Once we recognize these basic features, it is possible to idealize the situation and formulate the problem without referring to the particular Higgs model. We will do this in the next section by modifying Dirac's description of magnetic monopoles.

II. MODIFIED DIRAC MONOPOLE THEORY

Dirac's extension[3] of the Maxwell equations reads

$$\partial_\mu F_{\mu\nu}=-j_\nu\,, \qquad (1a)$$

$$\partial_\mu F^*_{\mu\nu}=-k_\nu\,, \qquad (1b)$$

where $F^*_{\mu\nu}$ is the dual of $F_{\mu\nu}$; j_ν and k_ν are electric and magnetic currents, respectively.[4] The only new step we take now is to go to the London theory of superconductivity by making the ansatz[5]

$$j_\mu=-m_V{}^2A_\mu \qquad (2)$$

so that Eq. (1a) is replaced by

$$\partial_\mu F_{\mu\nu}-m_V{}^2A_\nu=0\,. \qquad (1a')$$

A classical solution to Eqs. (1b) and (1a′) can be obtained with the aid of the Dirac string. Let us consider a pair of point magnetic charges $\pm g$ joined by a string. Then define

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu-G^*_{\mu\nu}\,,$$
$$G_{\mu\nu}(x)=g\iint dv\,\delta^4(x-y)[y_\mu,y_\nu]\,, \qquad (3)$$
$$[y_\mu,y_\nu]\equiv\frac{\partial(y_\mu,y_\nu)}{\partial(\tau,\sigma)}\,,\quad dv=d\tau d\sigma\,.$$

Here $y_\mu(\tau,\sigma)$ represents the position of a point on the world sheet swept out by the string, and the sheet is parametrized by the internal coordinates τ, σ. $G_{\mu\nu}$ is independent of parametrization.[6] However, for definiteness we fix the range of the parameters as $-\infty<\tau<\infty$, $0\le\sigma\le\pi$ so that $y_\mu(\tau,0)=y_\mu^{(1)}$ and $y_\mu(\tau,\pi)=y_\mu^{(2)}$ represent the world lines of the two magnetic charges.

The $F_{\mu\nu}$ as defined in Eq. (3) automatically satisfies Eq. (1b):

$$\begin{aligned}\partial_\mu F^*_{\mu\nu}&=\partial_\mu G_{\mu\nu}\\&=g\iint dv\frac{\partial}{\partial x_\mu}\delta^4(x-y)[y_\mu,y_\nu]\\&=-g\int dv\left(\frac{\partial}{\partial y_\mu}\delta^4(x-y)\right)[y_\mu,y_\nu]\\&=-g\int dv[\delta^4(x-y),y_\nu]\\&=g\int\frac{dy_\nu}{d\tau}\delta^4(x-y)\,d\tau\Big|_{\sigma=0}^{\sigma=\pi}\\&=-\sum_i k_\nu^{(i)}(x)\,,\end{aligned} \qquad (4)$$

$$k_\nu^{(i)}(x)=g^{(i)}\int\frac{dy_\nu^{(i)}}{d\tau}\delta^4(x-y)d\tau\,,$$
$$g^{(1)}=g\,,\quad g^{(2)}=-g\,.$$

One may therefore regard only Eq. (1a′) as an equation of motion. However, one also needs an equation of motion for the string and the magnetic monopoles at its ends.

For this purpose we take a Lagrangian density in space-time

$$L=-\tfrac{1}{4}F_{\mu\nu}F_{\mu\nu}-\tfrac{1}{2}m_V{}^2A_\mu A_\mu-\chi\,,$$
$$\chi(x)=\sum_{i=1,2}M^{(i)}\int\left(\frac{\partial y_\mu^{(i)}}{\partial\tau}\frac{\partial y_\mu^{(i)}}{\partial\tau}\right)^{1/2}d\tau\,\delta^4(y-x)\,. \qquad (5)$$

Here the last term is a contribution from the monopoles carrying mechanical masses $M^{(i)}$. By varying A_μ in the action integral $\int L\,d^4x$, one gets Eq. (1a′), or

$$(\Box-m_V{}^2)A_\nu=\partial_\mu G^*_{\mu\nu} \qquad (6)$$

after substituting Eq. (3) and observing that $\partial_\mu A_\mu=0$.

By varying y_μ at a point (τ,σ) in the interior of the world sheet, we get

$$\frac{\delta}{\delta y_\lambda}\int L\,d^4x=-\frac{1}{2}\int\frac{\delta F_{\mu\nu}}{\delta y_\lambda}F_{\mu\nu}\,d^4x$$
$$=-\frac{1}{2}\int\frac{\delta G_{\mu\nu}}{\delta y_\lambda}F^*_{\mu\nu}\,d^4x$$
$$=-\frac{g}{2}\iint\frac{\delta}{\delta y_\lambda}\{[y_\mu,y_\nu]\delta^4(y-x)\}\,dv\times F^*_{\mu\nu}(x)\,d^4x$$
$$=-\frac{g}{2}\int\frac{\delta}{\delta y_\lambda}\{[y_\mu,y_\nu]F^*_{\mu\nu}(y)\}\,dv$$
$$=0\ , \tag{7}$$

which leads, in view of the definition of $[y_\mu, y_\mu]$, to

$$\frac{\partial}{\partial\tau}\left(\frac{\partial y_\lambda}{\partial\sigma}F^*_{\mu\nu}(y)\right)-\frac{\partial}{\partial\sigma}\left(\frac{\partial y_\lambda}{\partial\tau}F^*_{\mu\nu}(y)\right)-\tfrac{1}{2}[y_\mu,y_\nu]\frac{\partial F^*_{\mu\nu}}{\partial y_\lambda}$$
$$=[F^*_{\lambda\nu},y_\nu]-\tfrac{1}{2}[y_\mu,y_\nu]\frac{F^*_{\mu\nu}}{y_\lambda}$$
$$=\tfrac{1}{2}[y_\mu,y_\nu]\left(\frac{\partial F^*_{\lambda\nu}}{\partial y_\mu}+\frac{\partial F^*_{\nu\mu}}{\partial y_\lambda}+\frac{\partial F^*_{\mu\lambda}}{\partial y_\nu}\right)$$
$$=0\,. \tag{8}$$

This amounts to

$$[y_\mu,y_\nu]^*\frac{\partial F_{\rho\nu}}{\partial y_\rho}=0\ , \tag{9a}$$

or

$$[y_\mu,y_\nu]^*A_\nu(y)=0 \tag{9b}$$

because of Eq. (1a′).

By varying $y^{(i)}_\mu$, the coordinates of the magnetic poles, we get

$$M^{(i)}\frac{d}{d\tau}\left[\frac{\dot y^{(i)}_\mu}{(\dot y^{(i)}_\lambda\dot y^{(i)}_\lambda)^{1/2}}\right]=g^{(i)}F^*_{\mu\nu}(y^{(i)})\dot y^{(i)}_\nu\ ,$$
$$\dot y^{(i)}_\mu\equiv dy^{(i)}_\mu/d\tau \tag{10}$$

where the right-hand side comes from boundary contributions in Eq. (7).

Equations (6), (9), and (10), then, are our basic equations, the first two of which are equivalent to Eqs. (1a′) and (1b). Equation (6) is a differential equation in the real space-time, whereas Eqs. (9) and (10) are differential equations for y_μ on the two-dimensional world sheet and its boundary. A first observation to make is that Eq. (9) is a linear constraint on A_μ at any point on the sheet. For the existence of a nonzero solution, it is then necessary that the coefficient matrix $\sigma_{\mu\nu}\equiv[y_\mu,y_\nu]$ satisfy

$$\det(\sigma^*_{\mu\nu})=\tfrac{1}{16}(\sigma_{\mu\nu}\sigma^*_{\mu\nu})^2=0\,. \tag{11}$$

This is indeed true because the six-tensor $\sigma_{\mu\nu}\,dv$ represents the (oriented) surface element of the sheet embedded in the real space-time, and one can always choose, unless the surface intersects itself, a local Lorentz frame ("normal frame") so that only one component $\sigma_{\lambda\rho}=-\sigma_{\rho\lambda}$ is nonvanishing, where λ and ρ are in the tangential plane of the sheet. Equation (9) then implies that the vector A_μ must lie in the tangential plane. In physical terms, the London current can only flow along the string.

For a further study of the motion of the string, let us solve Eq. (6) for A,

$$A_\nu(x)=\int\Delta(x-y)\partial_\mu G^*_{\mu\nu}(y)\,d^4y\ , \tag{12}$$

and substitute it in Eqs. (5), (9), and (10). Here $\Delta(x)$ is a Green's function for a field of mass m_V. In this way we obtain nonlinear equations for y_μ, and an effective Lagrangian from which these equations will follow. A straightforward calculation shows that the action integral, after discarding total divergences, can be brought to the form

$$\int d^2v\mathcal{L}_{\rm eff}=\tfrac{1}{4}g^2m_V{}^2\iint d^2v\,d^2v'\sigma_{\mu\nu}\Delta(y-y')\sigma'_{\mu\nu}$$
$$+\tfrac{1}{2}\sum_{i,j}\iint g^{(i)}g^{(j)}\dot y^{(i)}_\mu\Delta(y^{(i)}-y^{(j)\prime})\times\dot y^{(j)\prime}_\mu\,d\tau\,d\tau'$$
$$-\sum_i\int M^{(i)}(\dot v^{(i)}_\mu\dot y^{(i)}_\mu)^{1/2}\,d\tau\,. \tag{13}$$

It is now defined entirely in the two-dimensional world, y_μ merely being regarded as fields with four components. The first term represents a short-range Yukawa interaction between two surface elements; the second is another Yukawa interaction between magnetic currents, including self-interaction; the third is, of course, the mechanical mass term. Everything is manifestly independent of the choice of internal coordinates, and therefore can be given a direct geometrical interpretation.

It is interesting to observe that if $m_V=0$, the first term goes out and we recover the familiar result of magnetic charges interacting via the long-range Maxwell field. The string is unphysical in this case since it does not carry energy-momentum. Once $m_V\neq 0$, however, the string acquires physical reality. From dimensional considerations and the short-range nature of $\Delta(x)$, it is clear that the first term of Eq. (13) is proportional to the surface area of the world sheet, as in the dual string model, with a characteristic coefficient $\sim g^2m_V{}^2$.

A somewhat more elaborate derivation of the string Lagrangian from Eq. (13) will be as follows.

At a point (τ,σ) on the sheet we choose, as before, a "normal frame" so that

$$\frac{\partial y_\mu}{\partial_\sigma}=\delta_{\mu 3}\,,\quad \frac{\partial y_\mu}{\partial \tau}=i\delta_{\mu 4}\,,$$

for example. If the curvature of the surface is small over the range $\sim 1/m_V$ of $\Delta(x)$, and the point in question is not within a distance $\sim 1/m_V$ from the boundaries, one can approximate

$$\begin{aligned}\int \Delta(y-y')\sigma_{34}\,dv&=-i\int \Delta(x)\,dx_3\,dx_4\\&=\frac{-i}{(2\pi)^4}\iint \frac{e^{i(k_3x_3+k_4x_4)}}{k^2+m_V{}^2}\,d^4k\,dx_3\,dx_4\\&=\frac{-i}{(2\pi)^2}\int\frac{dk_1\,dk_2}{k_1{}^2+k_2{}^2+m_V{}^2}\\&=\frac{-i}{4\pi}\ln\left(\frac{K^2}{m_V{}^2}+1\right),\end{aligned}\qquad(14)$$

where a momentum cutoff K has been introduced in directions perpendicular to the sheet, corresponding to the finite thickness of the string. From the work of Nielsen and Olesen[1,5] one may equate it with a mass parameter m_S characteristic of the Higgs field.

With Eq. (14) the first term of Eq. (13) becomes a single integral of surface elements. In covariant notation, then, one gets the string Lagrangian (density)[7] in the (τ,σ) space

$$\mathcal{L}_{\text{string}}=-\frac{1}{2\pi\alpha'}\,|\det\sigma_{\mu\nu}|^{1/2}\,,\qquad(15)$$

where

$$\frac{1}{2\pi\alpha'}=\frac{g^2}{8\pi}\,m_V{}^2\ln\left(\frac{m_S{}^2}{m_V{}^2}+1\right).\qquad(16)$$

This relates the Regge trajectory slope α' to the parameters of our theory.

We have thus seen that Eq. (13) contains the string Lagrangian in the local limit. Beside that, however, Eq. (13) has the following additional important features.

(a) The end points of the string behave like particles with mass $M^{(i)}$ and charge $g^{(i)}$, coupled to a massive vector field. This leads to a Yukawa interaction between the end points and their own self-energies. In the static picture, this interaction energy is

$$-\frac{g^2}{4\pi}\,\frac{e^{-m_V l}}{l}\,,\qquad(17)$$

where l is the distance between the end points. The string energy, on the other hand, will be

$$\gtrsim l\,\frac{g^2m_V{}^2}{8\pi}\ln\left(\frac{m_S{}^2}{m_V{}^2}+1\right).\qquad(18)$$

For a sufficiently long string ($l\gtrsim 1/m_V$), the string energy is dominant; for a short string ($l\lesssim 1/m_V$) the singular Yukawa interaction becomes important if the size of the end-point monopoles is even smaller.

(b) The string is oriented, i.e., has an intrinsic sense of polarization, like a magnet.[8] When two portions of the world sheet nearly overlap, there will be a Yukawa interaction between their surface elements. The interaction is attractive when the two string elements line up antiparallel, and repulsive when they are parallel. Such an effect will become most pronounced if the string folds on itself. An example of normal modes of this type is

$$\begin{aligned}x&=\cos n\sigma\,\cos n\tau\,,\\y&=\cos n\sigma\,\sin n\tau\,,\\z&=0\,,\\t&=n\tau\,.\end{aligned}\qquad(19)$$

If we ignore the end-point effects, its energy will be $1/n$ of that in the naive string model if n is odd, and zero if n is even.

(c) It is an easy matter to generalize our considerations to cases with more than one string. The total action integral will now consist of a sum over actions of the type (13) for individual strings, plus similar terms representing interstring contributions to the surface-surface and boundary-boundary interactions. Interestingly, a picture of this kind has been proposed by Kalb and Ramond.[9]

III. QUANTIZATION

Quantization of our theory can be done following Dirac's procedure. Starting from Eq. (7) one first defines the canonical conjugates to A_μ and y_μ, respectively, under displacement of real time t and fictitious time τ. Dirac has shown that the single-valuedness of the wave function for a system containing both electric and magnetic charges requires the quantization condition

$$\frac{eg}{4\pi}=\tfrac{1}{2}n\,,\quad n=0,\pm1,\pm2,\ldots\,.\qquad(20)$$

As is well known, this condition also appears as flux quantization in superconductivity. In the Landau-Ginzburg-Higgs model, e is the electric charge of a scalar field ϕ, and m_V is given by

$$m_V=|e\langle\phi\rangle|\,,\qquad(21)$$

so that the characteristic coefficient in Eq. (13),

$$g^2m_V{}^2=g^2e^2\langle\phi\rangle^2\,,$$

is independent of e, because $\langle\phi\rangle$ is determined by other parameters in the model.

Beyond this, the actual passage to a quantum theory of strings is beset with various well-known difficulties. We will nevertheless indicate how it might be carried out.

The Lagrangian (5) contains three dynamical quantities, the vector field A_μ, the string variable y_μ, and the magnetic source variables $y_\mu^{(i)}$. For a clear separation of the first two, we use the definition (3) to write Eq. (5) as

$$L = -\tfrac{1}{4}F^0_{\mu\nu}F^0_{\mu\nu} + \tfrac{1}{2}F^0_{\mu\nu}G^*_{\mu\nu} + \tfrac{1}{4}G_{\mu\nu}G_{\mu\nu} - \tfrac{1}{2}m_V{}^2A_\mu A_\mu - \chi\ , \qquad (22)$$
$$F^0_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu\ .$$

The second term shows that $G_{\mu\nu}(x)$ acts as a source for the field A, whereas the third term has the nature of a free Lagrangian for the string. From the point of view of the string, it is more convenient to go to a Lagrangian (density) $\mathcal{L}$ in the (τ,σ) space, related to L by

$$L_{\text{string}}\, d^4x = \mathcal{L}_{\text{string}}\, dv\ ,$$
$$L_{\text{string}} = -\tfrac{1}{4}G_{\mu\nu}G_{\mu\nu} + \tfrac{1}{2}F^{0*}_{\mu\nu}G_{\mu\nu}\ . \qquad (23)$$

We find

$$\mathcal{L}_{\text{string}} = \tfrac{1}{4}[y_\mu, y_\nu]\int \delta^4(y-y')[y'_\mu, y'_\nu]\,dv' + \tfrac{1}{2}[y_\mu, y_\nu]\,F^{0*}_{\mu\nu}(y)\ . \qquad (24)$$

The first term of Eq. (24) is very much like the first term of Eq. (13) except that it is quadratically divergent instead of logarithmically. It seems sensible, therefore, to replace this term by $\mathcal{L}_{\text{string}}$ of Eq. (15). The second term is responsible for emission and absorption of vector-field quanta. Thus we get a new Lagrangian

$$\mathcal{L}_{\text{string}} = -\frac{1}{2\pi\alpha'}(\det\sigma_{\mu\nu})^{1/2} + \tfrac{1}{2}\sigma_{\mu\nu}F^0_{\mu\nu}(y)\ , \qquad (25)$$

which can further be reduced to

$$\mathcal{L}_{\text{string}} = \frac{1}{4\pi\alpha'}\left[\left(\frac{\partial y_\mu}{\partial\tau}\right)^2 - \left(\frac{\partial y_\mu}{\partial\sigma}\right)^2\right] + \tfrac{1}{2}\sigma_{\mu\nu}F^{0*}_{\mu\nu}(y) \qquad (26)$$

under the well-known Virasoro gauge condition[10]

$$\left(\frac{\partial y_\mu}{\partial\tau}\right)^2 + \left(\frac{\partial y_\mu}{\partial\sigma}\right)^2 = \frac{\partial y_\mu}{\partial\tau}\frac{\partial y_\mu}{\partial\sigma} = 0\ . \qquad (27)$$

The canonical conjugate to y_μ following from Eq. (26) is

$$\pi_\mu = \frac{1}{2\pi\alpha'}\frac{\partial y_\mu}{\partial\tau} + F^{0*}_{\mu\nu}\frac{\partial y_\nu}{\partial\sigma}\ , \qquad (28)$$

and the Hamiltonian is

$$\overline{\mathcal{H}}_{\text{string}} = \int \mathcal{H}_{\text{string}}\, d\sigma\ ,$$
$$\mathcal{H}_{\text{string}} = \pi\alpha'\left(\pi_\mu - F^0_{\mu\nu}\frac{\partial y_\nu}{\partial\sigma}\right)^2 + \frac{1}{4\pi\alpha'}\left(\frac{\partial y_\mu}{\partial\sigma}\right)^2\ . \qquad (29)$$

The gauge constraint (27) now reads

$$\mathcal{H}_{\text{string}} = 0\ ,\quad \mathcal{P}_{\text{string}} \equiv \pi_\mu\frac{\partial y_\mu}{\partial\sigma} = 0\ . \qquad (30)$$

$\mathcal{H}_{\text{string}}$ and $\mathcal{P}_{\text{string}}$ are nothing but the components $T_{\tau\tau} = T_{\sigma\sigma}$ and $T_{\tau\sigma} = T_{\sigma\tau}$ of the energy-momentum tensor in the (τ,σ) space. Since $\mathcal{L}_{\text{string}}$ does not explicitly depend on (τ,σ), they satisfy the continuity equations

$$\frac{\partial\mathcal{H}}{\partial\tau} - \frac{\partial\mathcal{P}}{\partial\sigma} = \frac{\partial\mathcal{P}}{\partial\tau} - \frac{\partial\mathcal{H}}{\partial\sigma} = 0 \qquad (31)$$

in the interior of the world sheet. Imposing the condition (30) everywhere on the sheet, including the boundaries, is then compatible with the equations of motion. Actually, one must consider the contribution of magnetic poles in Eq. (22). In quantum theory let us assume these poles to be Dirac particles, and postulate the following equations[3]:

$$\gamma_\mu^{(i)}\pi_\mu^{(i)} - iM^{(i)} = 0 \quad (i = 1, 2)\ . \qquad (32)$$

Equations (31) and (32) are to be regarded as constraints on the wave function $\Psi_{\alpha,\beta}(y_\mu(\sigma), A_\nu(\vec{x}))$ which depends on the Dirac spin indices α,β of the poles in addition to the string and field variables.

The real question is, of course, the compatibility of these constraints with each other and with the Hamiltonian as operator relations. Unless this can be shown, the present formalism will remain only a superficial one.

IV. IMPLICATIONS FOR HADRON PHYSICS

The foregoing model theory has many interesting features which are relevant to the actual strong interactions, although some important pieces are missing. For a more realistic model, one would have to seek generalizations to non-Abelian gauge fields.[11] Nevertheless, it will still be instructive to take stock of what the present model already has. First of all, isolated magnetic charges cannot exist because, in contrast with the case for Dirac monopoles, an infinite amount of energy is required to infinitely stretch a string. If these charges are carried by quarks, then single quarks cannot exist. Only quark-antiquark pairs ("mesons") would exist, which have zero total magnetic charge. Unfortunately, there are no "baryons." However, massive axial-vector gluons can also be produced by mesons, as we shall see below.

The forces that bind "quarks" and "antiquarks" are of two kinds. One is the tension of a string; the other is a Yukawa force. For long strings (highly excited mesons) the former will be dominant, leading to the usual linear trajectories of the dual resonance model. However, the finite thickness of the string and the short-range interactions between string elements will distort deep-lying daughter trajectories. If we use Eq. (16) with the known value $\alpha' = 1\ \mathrm{GeV}^{-2}$ and the reasonable ansatz $m_V, m_S \gtrsim 1$ GeV, we find that $g^2/4\pi$ cannot be very large ($\lesssim \frac{1}{2}$). It is the electric coupling $e^2/4\pi$ which is large ($\gtrsim \frac{1}{2}$).

For low-lying states of the meson (short strings) the Yukawa interaction as well as the kinetic energy of the quarks becomes important since both go like 1/length (the quark mass is here ignored). The former is attractive, while the latter acts like a repulsive force. Their effect would lead to a shift in the trajectory intercepts.

Another effect of the Yukawa interaction would be to change the short-distance behavior of the wave function from the Gaussian form of the dual resonance model to a power form. This should be highly desirable in view of what we know about elastic and inelastic form factors of hadrons.

Strings interact with each other via exchange of the gluon field, whose source is distributed over the entire length of a string. This results in joining and splitting of strings not only by end-to-end contact of opposite magnetic charges, but also by antiparallel lineup of two string segments. (See Fig. 1.)

Another straightforward consequence of the theory is emission of a gluon by a string through the interaction term in Eq. (29),

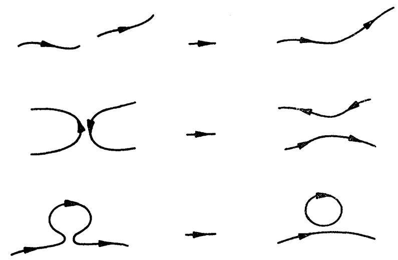

FIG. 1. Examples of joining and breaking of strings.

$$-8\pi\alpha' \int F^{0*}_{\mu\nu}\pi_\mu \frac{\partial y_\nu}{\partial \sigma}\, d\sigma\,. \tag{33}$$

For a gluon of momentum k_μ and polarization ϵ_μ, this is

$$\sim \int k_\mu \epsilon_\nu \pi_\lambda y'_\rho e^{-ik\cdot y} \epsilon_{\mu\nu\lambda\rho}\, d\sigma\,. \tag{34}$$

Obviously the gluon behaves as an axial vector ($J^P = 1^+$) instead of a vector (1^-) particle.[8] A simple example of such a process would involve the transition of a meson string from 0^- to 1^+, as in a hypothetical hadronic reaction

$$\pi \to A_1 + G\,,$$

where G stands for the gluon. It is a curious fact that the axial-vector field nevertheless gives rise to a vector-type interaction between magnetic charges, according to Eq. (13). In any case it appears that unlike the "quarks" the gluons cannot be permanently contained in our Abelian model.

---

*Work supported in part by the National Science Foundation under Grant No. NSF GP 32904 X2.

[1]H. B. Nielsen and P. Olesen, Nucl. Phys. B61, 45 (1973).

[2]This has been independently suggested by G. Parisi, Columbia University Report No. C0-2271-24, 1974 (unpublished). A preliminary account of the present article is given by Y. Nambu, in *Proceedings of the Johns Hopkins Workshop on Current Problems in High Energy Particle Theory, 1974,* edited by G. Domokos *et al.* (Johns Hopkins Univ., Baltimore, 1974).

[3]P. A. M. Dirac, Phys. Rev. 74, 817 (1948).

[4]We use the pseudo-Euclidean metric convention: $g_{\mu\nu} = \delta_{\mu\nu}$; $\mu, \nu = 1, \ldots, 4$. A dual is defined as $F^*_{\mu\nu} = \frac{1}{2} i\, \epsilon_{\mu\nu\lambda\rho} F_{\lambda\rho}$.

[5]As is well known, this is a local approximation to a nonlocal (Pippard) relation $j_\mu(x) = \int K_{\mu\nu}(x-x')A_\nu(x')d^4x'$, where the polarization tensor $K_{\mu\nu}$ has a characteristic range $1/m_S$. With this form the cutoff parameter in Eq. (14) will automatically appear.

[6]It is implicitly assumed that the sheet is essentially Minkowski-like, τ and σ playing the roles of timelike and spacelike coordinates, respectively. The Poisson bracket notation for the Jacobian in Eq. (3) naturally carries an implied suggestion for possible abstractions. However, it is not our intention to pursue it here.

[7]Y. Nambu, lectures prepared for the Copenhagen Summer Symposium, 1970 (unpublished); T. Goto, Prog. Theor. Phys. 46, 1560 (1971).

[8]The theory of Abrikosov flux lines in type II superconductors shows that these lines move only in helical modes of one sign, as determined by the sense of the magnetic flux, whereas the present model does not have such a feature. Herein lies a significant difference between real superconductors and corresponding relativistic models. In the former only electrons exist; in the latter, as in the Higgs model, fluids of both charges respond to magnetic fields. This charge symmetry is responsible for the lack of apparent time-reversal violation in relativistic models. Mathematically, the difference can be traced to different assumptions about static charge density: $\rho \sim \langle \phi^* \phi \rangle \neq 0$ in the

Landau-Ginzburg theory, but $\rho \sim i\langle\phi^*\dot{\phi}-\dot{\phi}^*\phi\rangle=0$ in the Higgs theory.

[9]M. Kalb and P. Ramond, Phys. Rev. D 9, 2273 (1974). See also E. Cremmer and J. Scherk, Nucl. Phys. B72, 117 (1974).

[10]L. N. Chang and F. Mansouri, Phys. Rev. D 5, 2235 (1972); Goto, Ref. 7; G. Goddard, J. Goldstone, C. Rebbi, and C. B. Thorn, Nucl. Phys. B56, 109 (1973).

[11]A clearcut answer to this problem seems to be lacking. See, however, Nielsen and Olesen, Ref. 1; G. 't Hooft, CERN Report No. TH-1873-CERN, 1974 (unpublished); Y. Nambu, Ref. 2.

PHYSICAL REVIEW D VOLUME 10, NUMBER 12 15 DECEMBER 1974

Towards a field theory of hadron binding*

Stephen Blaha
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850
(Received 17 July 1974)

A field-theoretic model for hadron binding is described in which free quarks are totally screened. Quarks interact via a dipole vector-gluon field. A second-quantization procedure for the gluon field, which reduces the field to an embodiment of a direct particle interaction, eliminates unitarity problems. A detailed description of perturbation-theory rules is given. In contrast to the results of the pseudoscalar-meson and massive-vector-meson models (without cutoff), scaling occurs in the electroproduction structure functions. Another possible model having some resemblance to the relativistic harmonic-oscillator quark model of Feynman, Kislinger, and Ravndal is also described. It is unitary and has scaling structure functions.

I. INTRODUCTION

The current understanding of hadronic structure allows two apparently contradictory statements to be made: The constituents of the hadron appear to be loosely bound, quasifree particles. The constituents of the hadron are not produced and do not occur outside of hadrons. Several attempts have been made to resolve this paradoxical situation. They may be divided into two categories: "conventional" field-theoretic approaches,[1,2] and *ad hoc* approaches which postulate manifestly non-field-theoretic structures for confinement, e.g., the "bag" model.[3] In the first approach Casher, Kogut, and Susskind[1] and Wilson[2] showed that quarks could be totally screened and not observed. However, a four-dimensional, Lorentz-invariant field-theoretic model of hadron binding with its attendant conceptual and computational advantages appears to be lacking. We shall discuss a possible candidate, the dipole gluon model, in detail. In addition, another possibility is briefly described in Appendix B which bears some comparison with the quark model of Feynman *et al.*[4] The dipole gluon model has two major qualitative features in common with the bag model[3] and the two-dimensional quantum-electrodynamic model[1]: (1) The dipole gluon field has no independent degrees of freedom; neither does a bag or the two-dimensional electromagnetic field. (2) The "Coulomb" potential between quarks is proportional to the distance between them in all three models. In a sense the bag model may be regarded as a phenomenological approximation to the dipole model, and the dipole model as a generalization of the two-dimensional model to four dimensions.

In Sec. II we describe a quantization procedure which avoids the introduction of indefinite-metric in or out states and thus leads to a unitary S matrix. In Sec. III we describe the properties of the "free" gluon Lagrangian model. In Sec. IV we describe the perturbation-theory rules of the dipole model. Section V contains a discussion of unitarity, causality, quark confinement, and scaling properties of the electroproduction structure functions. For simplicity we shall ignore all but the dipole quark interaction and do not introduce internal quark quantum numbers.

II. SECOND-QUANTIZATION PROCEDURE FOR THE GLUON FIELD

We shall not quantize the gluon field in the conventional manner for three reasons: (1) to be consistent with experiment where no such particle has been identified, (2) to avoid unitarity problems in the S matrix, and (3) to avoid infrared problems in perturbation theory. We attribute no dynamical degrees of freedom to the gluon field. Instead we regard the field as the embodiment of a direct