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Is the Pomeron a Goldstone boson?

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A Reggeon field theory is proposed using as an input two exchange-degenerate mesonlike trajectories, each of them corresponding to a pair of conjugate poles. A Pomeron and a lower-lying Reggeon are generated by spontaneous symmetry breaking. Both the input and output trajectories have a $\sqrt{t}$ dependence and are complex for $t < 0$, reflecting the conventional treatment of Regge-pole-cut collisions. The gauge symmetry of the model is briefly discussed, and the model is compared with earlier work.

Considerable effort has recently been applied to the study of the Gribov calculus in a field-theoretic formulation.[1] This formulation has the advantage of guaranteeing the correct t-channel discontinuity relations. Furthermore, new insight may be gained by drawing on the vast experience with conventional field theories. Since the Gribov calculus contains many uncertainties and approximations, this program offers a great deal of freedom. There is a large number of field-theoretic models to choose from, and it is not obvious at the starting point which particular model or models may be relevant for describing nature. Guided by conventional Regge theory, one can set out to study a number of different models and test, in the theoretical laboratory, whether they can lead to a consistent description of Regge phenomena.

In this note we would like to incorporate, within the framework of spontaneous symmetry breakdown, two ideas which are familiar from many conventional Regge models. One is the notion that the Pomeron is generated by lower-lying meson trajectories.[2] The second is the idea that the collision between Regge poles and cuts can approximately be described by a complex trajectory.[3]

The Reggeon field theory is generated by the formal identifications

$$E = 1 - J \tag{1}$$

between energy E and angular momentum J, and

$$\vec{q}^{\,2} = -t \tag{2}$$

between momentum $\vec{q}$ and momentum transfer t. Thereby the Reggeon amplitude, originally a function of J and t, becomes a function of E and $\vec{q}$ in a space with one time and two space dimensions.[1] With this identification the trajectory function $J = f(t)$ translates into a relation between E and $\vec{q}$,

$$E = 1 - f(-\vec{q}^{\,2}), \tag{3}$$

which is a generalized mass-shell condition, so to speak. Clearly, the choice of the trajectory function determines to a large extent the structure of the resulting theory. Before choosing a particular trajectory, we note that one of the fundamental problems which the Reggeon calculus attempts to answer is the effect of collisions between poles and cuts. This problem has been studied intensively and one of the most effective techniques for handling it has been the use of complex trajectories.[3] Therefore, this type of trajectory appears to be an excellent starting point for a Reggeon field theory. In the models studied, the pole can be a Pomeron or a meson and it collides with cuts that either are Pomeron generated or arise from normal thresholds. Generally speaking, when the pole hits the cut it splits into a pair of complex-conjugate poles which can move off in the J plane in various ways. We need not go into further detail of this mechanism other than to note that the propagators in such models generally take the form

$$\frac{1}{(J-\alpha_+)(J-\alpha_-)} = \frac{1}{(J-\alpha_R - i\alpha_I)(J-\alpha_R + i\alpha_I)} = \frac{1}{(J-\alpha_R)^2 + \alpha_I{}^2}, \tag{4}$$

where α_I should be nonzero in the region $t<0$.

Let us now choose the pair of square-root trajectories[7] α_+ and α_- given by

$$\alpha_\pm = 1 \pm (\alpha' t - \mu^2)^{1/2}. \tag{5}$$

They lead to a propagator

$$\frac{1}{(J-\alpha_+)(J-\alpha_-)} = \frac{1}{(1-J)^2 - \alpha' t + \mu^2} = \frac{1}{E^2 + \alpha' \vec{q}^{\,2} + \mu^2}, \tag{6}$$

which is just the propagator of a free particle with mass μ^2 in a three-dimensional space with the metric tensor

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \alpha' & 0 \\ 0 & 0 & \alpha' \end{pmatrix}. \tag{7}$$

The free Lagrangian that generates this propagator is

$$L_0 = -\tfrac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - \tfrac{1}{2}\mu^2 \varphi^2 . \tag{8}$$

In conventional Regge models for the Pomeron and mesons it has been generally assumed, at least as a first-order approximation, that the Pomeron is weak in some sense and that it should be generated by lower trajectories.[2] In practice, this usually means that the input trajectories are taken to be only mesons and that the interaction of these trajectories generates a new trajectory with a higher intercept; this higher trajectory is then identified with the Pomeron. Pomeron self-interactions are viewed as "fine tuning." A natural choice for the input, suggested by experiment, is an exchange-degenerate pair of trajectories. Following this pattern, we write down the free Lagrangian

$$L_0 = -\tfrac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - \tfrac{1}{2}\partial_\mu \psi \partial^\mu \psi - \tfrac{1}{2}\mu^2(\varphi^2 + \psi^2), \tag{9}$$

where φ and ψ represent the exchange-degenerate pair.

The selection of an interaction term introduces new arbitrariness. One of the possible candidates for the interaction Lagrangian is[4,5]

$$L_I = -\frac{\lambda^2}{4!}(\varphi^2 + \psi^2)^2 . \tag{10}$$

We must note that this interaction will cause the creation (and destruction) of four Reggeons from (into) the vacuum, which are not graphs that are originally in the Gribov calculus. This is a problem common to "relativisticlike" theories (see, for example, Ref. 10); however, it leads to the intriguing possibility of shifting a trajectory and generating the Pomeron by spontaneous symmetry breakdown. It has already been pointed out that spontaneous symmetry breakdown can play a role in Reggeon field theory, specifically, that it can force the effective trajectory intercept to be less than or equal to one, a result which is usually thought to arise from s-channel unitarity constraints.[5]

We note that the full Lagrangian

$$L = -\tfrac{1}{2}(\partial_\mu \varphi \partial^\mu \varphi + \partial_\mu \psi \partial^\mu \psi) - \tfrac{1}{2}\mu^2(\varphi^2 + \psi^2) - \frac{\lambda^2}{4!}(\varphi^2 + \psi^2)^2 \tag{11}$$

has a gauge symmetry

$$\begin{aligned} \varphi &\to \varphi - i\epsilon\psi , \\ \psi &\to \psi + i\epsilon\varphi , \end{aligned} \tag{12}$$

which implies a conserved current

$$j_\mu \sim i(\psi\partial_\mu\varphi - \varphi\partial_\mu\psi) \tag{13}$$

and a conserved charge. Since this conserved charge reflects the exchange degeneracy which we built into the Lagrangian, we may expect this conservation of "Reggeon charge" to have validity beyond the scope of this particular model.

Following the well-known discussion of spontaneous symmetry breaking in the σ model[6] we look for the minimum of the potential

$$V = \tfrac{1}{2}\mu^2(\varphi^2 + \psi)^2 + \frac{\lambda^2}{4!}(\varphi^2 + \psi^2)^2 \tag{14}$$

by requiring

$$\frac{\partial V}{\partial \varphi} = \varphi\left(\mu^2 + \frac{\lambda^2}{3!}(\varphi^2 + \psi^2)\right) = 0 , \tag{15a}$$

$$\frac{\partial V}{\partial \psi} = \psi\left(\mu^2 + \frac{\lambda^2}{3!}(\varphi^2 + \psi^2)\right) = 0 , \tag{15b}$$

$$\frac{\partial^2 V}{\partial \varphi^2} = \mu^2 + \frac{\lambda^2}{3!}(\varphi^2 + \psi^2) + \frac{2\lambda^2}{3!}\varphi^2 > 0 , \tag{15c}$$

and

$$\frac{\partial^2 V}{\partial \psi^2} = \mu^2 + \frac{\lambda^2}{3!}(\varphi^2 + \psi^2) + \frac{2\lambda^2}{3!}\psi^2 > 0 . \tag{15d}$$

If $\langle\varphi\rangle_0 = \langle\psi\rangle_0 = 0$, then μ^2 must be positive, which in turn implies that the intercepts of the trajectories are

$$\alpha_\pm(0) = 1 \pm i\mu . \tag{16}$$

These of course lead to a completely acceptable theory of a pair of exchange-degenerate trajectories that collide with branch points at $t = \mu^2/\alpha'$ and become complex.

If we now attempt to violate the Froissart bound by taking μ^2 negative so that the intercept of the trajectory with positive slope (physical sheet) is

$$\alpha_+(0) = 1 + (-\mu^2)^{1/2} , \tag{17}$$

then V no longer has a minimum at $\langle\varphi\rangle = \langle\psi\rangle = 0$. This situation is remedied by making a shift in the ψ field and defining a new field ψ',

$$\psi' = \psi - (-3!\,\mu^2/\lambda^2)^{1/2} . \tag{18}$$

In terms of the new field the Lagrangian becomes

$$L = -\tfrac{1}{2}(\partial_\mu\varphi\partial^\mu\varphi + \partial_\mu\psi'\partial^\mu\psi') - \frac{(-2\mu^2)}{2}\psi'^2 - \frac{\lambda^2}{4!}(\varphi^2 + \psi'^2)^2 - (-\mu^2\lambda^2/3!)^{1/2}\psi'(\varphi^2 + \psi'^2) , \tag{19}$$

and the minimum of the potential now occurs at $\langle\varphi\rangle_0 = \langle\psi'\rangle_0 = 0$. The free φ field becomes massless and its mass-shell condition (trajectory function) reads

$$E^2 + \alpha' \vec{q}^2 = 0 , \qquad (20)$$

while the free ψ' field acquires a mass $-2\mu^2$ with a corresponding mass-shell condition

$$E^2 + \alpha' \vec{q}^2 - 2\mu^2 = 0 . \qquad (21)$$

The trajectory of the φ field can now be identified with the Pomeron since at $t=0$ ($\vec{q}^2=0$) it has $J=1$ ($E=0$). The ψ' field generates a mesonlike trajectory with an intercept of $1 \pm i(-2\mu^2)^{1/2}$. Of course, each of the trajectory functions (20) and (21) corresponds to a conjugate pair of trajectories given by a form similar to that of Eq. (5).

As a result of the spontaneous symmetry breaking our Lagrangian develops cubic interactions which are expected to be present in any complete Regge theory. It is interesting to note that no triple-Pomeron coupling appears; the same phenomenon is true for a typical multiperipheral model where the Pomeron is generated from lower trajectories. However, in the same model one would expect a Pomeron-meson-meson coupling,[9] while we find a Pomeron-Pomeron meson coupling instead. It is important to note, however, that the coupling that is generated is real, while the work of Gribov indicates that it should be purely imaginary.[1] This is likely to be a difficulty of any model with negative bare mass, and there are indications that this is an important class of models.[11]

It is instructive to compare our results to those of Ref. 5. Even though much of the algebra involved can be made to look quite similar, the results are quite different from each other. By introducing the fields

$$\phi_+ = \frac{\psi + i\varphi}{\sqrt{2}}, \quad \phi_- = \frac{\psi - i\varphi}{\sqrt{2}} \qquad (22)$$

we can bring our Lagrangian of Eq. (11) to the form

$$L = -\partial_\mu \phi_+ \partial^\mu \phi_- - \mu^2 \phi_+ \phi_- - \frac{\lambda^2}{3!}(\phi_+\phi_-)^2 . \qquad (23)$$

If we now proceed according to Ref. 5 we obtain the following free equations of motion for the new fields:

$$-\partial_\mu \partial^\mu \phi_- - \mu^2 \phi_- = \mu^2 \phi_+$$

and (24)

$$-\partial_\mu \partial^\mu \phi_+ - \mu^2 \phi_+ = \mu^2 \phi_- .$$

These equations can be decoupled and we recover our previous result

$$-\partial_\mu \partial^\mu (\phi_+ + \phi_-) - 2\mu^2 (\phi_+ + \phi_-) = 0$$

and (25)

$$-\partial_\mu \partial^\mu (\phi_+ - \phi_-) = 0 .$$

In the case of Ref. 5, the uncoupling can be achieved only if the degree of the equations is increased and the structure of the trajectories is thus changed in the process. In particular,

$$J = 1 \pm [\alpha' t (\alpha' t - 2\delta)]^{1/2} \qquad (26)$$

is obtained, which, contrary to conventional considerations,[8] is real for $t < 0$.

Let us now return to our starting point and reconsider our choice of the Lagrangian L_0. The standard Klein-Gordon Lagrangian is given by

$$L = \tfrac{1}{2}(\dot{\varphi}^2 - \vec{\nabla}\varphi \cdot \vec{\nabla}\varphi) - \tfrac{1}{2}\mu^2 ; \qquad (27)$$

previously, in order to obtain a Euclidean metric, we changed the sign of the energy term $\dot{\varphi}^2$. A sign change in the $\vec{\nabla}\varphi \cdot \vec{\nabla}\varphi$ term would lead to a form

$$L = \tfrac{1}{2}(\partial_\mu \varphi \partial^\mu \varphi + \partial_\mu \psi \partial^\mu \psi) - \tfrac{1}{2}\mu^2(\varphi^2 + \psi^2) - \frac{1}{4!}\lambda^2(\varphi^2 + \psi^2)^2 , \qquad (28)$$

with the metric given again by Eq. (7). This would imply that the initial trajectories are given by

$$\alpha_\pm = 1 \pm (\alpha' t + \mu^2)^{1/2} , \qquad (29)$$

where for $\mu^2 > 0$ the trajectory with positive slope would violate the Froissart bound, and, by Eqs. (15), there is no "force" to bring this trajectory below one. If, on the other hand, μ^2 is less than zero the trajectory does not violate the Froissart bound, but spontaneous symmetry breakdown sets in. We obtain a trajectory with intercept one, and another pair with

$$\alpha_\pm = 1 \pm (\alpha' t - 2\mu^2)^{1/2} . \qquad (30)$$

The intercept of the trajectory with positive slope lies again *above* one. This approach is obviously not very fruitful, since the Lagrangian of Eq. (28) violates s-channel unitarity for either choice of the sign of μ^2.

However, the question remains, whether either of the Lagrangians considered leads to a reasonable field theory. Recall that the standard Klein-Gordon field is given by

$$\varphi(x) = \int d^4k\, \delta(k^2 - \mu^2)\theta(E)[A(E, \vec{k})e^{-ik\cdot x} + A^\dagger(E, \vec{k})e^{ik\cdot x}] . \qquad (31)$$

Our first choice, Eq. (9), corresponds to replacing E^2 by $-E^2$, that is, $E \to \pm iE$. Therefore the field becomes

$$\varphi(x)=\pm\int d^3k\, d(iE)\delta((iE-\omega_k)(iE+\omega_k))\theta(\pm iE)[A(\pm iE,\vec{k})e^{\pm Et+i\vec{k}\cdot\vec{x}}+A^\dagger(\pm iE,\vec{k})e^{\mp Et-i\vec{k}\cdot\vec{x}}], \tag{32}$$

where $\omega_k=(\vec{k}^2+\mu^2)^{1/2}$ (we take $\alpha'=1$); thus

$$1-J=E=\pm i\omega_k. \tag{33}$$

We define the square-root cut such that $E=-i\omega_k$ leads, for $t>\mu^2$, to a trajectory with positive slope, which we identify with the "physical state." Note that here too, as in the conventional Klein-Gordon theory, a second-sheet "unphysical" pole at $E=i\omega_k$ is required by real analyticity. The choice of $E=-i\omega_k$ amounts to selecting the $E\to iE$ rotation, and therefore

$$\varphi(x)=\int\frac{d^3k}{2\omega_k}\theta(\omega_k)[A(\omega_k,\vec{k})e^{-i\omega_k t+i\vec{k}\cdot\vec{x}}+A^\dagger(\omega_k,\vec{k})e^{i\omega_k t-i\vec{k}\cdot\vec{x}}]. \tag{34}$$

It appears that there are no obvious problems with this field theory.

Finally, let us consider the Lagrangian of Eq. (28), which corresponds to $\vec{k}\to\pm i\vec{k}$. The appropriate field becomes

$$\varphi(x)=\mp i\int d^3k\,dE\,\theta(E)\delta(E^2+k^2-\mu^2)[A(E,\pm i\vec{k})e^{-iEt\mp\vec{k}\cdot\vec{x}}+A^\dagger(E,\pm i\vec{k})e^{iEt\pm\vec{k}\cdot\vec{x}}], \tag{35}$$

leading to a requirement $k^2<\mu^2$ and therefore an unacceptable Reggeon field.

Let us note that the two conjugate states $E=\pm i\omega_k$ might be treated in a fermion theory. This will be considered in more detail elsewhere.

The Reggeon field that we have discussed here clearly has a number of tantalizing features, which warrant additional study (for example, through the renormalization group). Furthermore, many of the ideas seem sufficiently general to be included in other Reggeon field theories, in particular the simultaneous presence of both the Pomeronlike and mesonlike trajectories. The appearance of a new conserved charge should also be pursued.

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