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Construction of the Reggeon calculus in $4-\epsilon$ dimensions

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We study the Reggeon field theory in $4-\epsilon$ dimensions. When the Pomeranchuk singularity has intercept 1, the theory cannot be renormalized order by order in the perturbation series. Nevertheless we are able to develop systematic techniques for constructing the Pomeranchuk Green's functions. An integral representation is obtained for the Pomeranchuk propagator which allows us to explicitly display its infrared ($l \approx 1$, $t \approx 0$) behavior and to show that the perturbation series is an asymptotic expansion for small values of the coupling constant and for large values of the angular momentum or momentum transfer. We also obtain an integral representation for the intercept renormalization counterterm. We find that for the renormalized Pomeranchuk singularity to have intercept 1, the bare Pomeranchuk pole must have intercept greater than 1.

I. INTRODUCTION

Gribov's Reggeon field theory[1] provides an elegant framework for the study of high-energy reactions. In this theory Reggeons are treated as quasi-particles and associated with fields in two space and one "time" dimension. The space variables are conjugate to the transverse momentum, $\vec{k}$, and the "time" to the "energy" variable, $E=1-l$, where l is the angular momentum.

The field-theory approach is most useful in discussing problems involving the Pomeranchuk singularity where interactions among l-plane poles and cuts are important. In these problems it seems necessary to take into account the full Reggeon unitarity relations.[2] The field theory has the advantage of guaranteeing that these relations are satisfied identically.

The purpose of this paper is to develop systematic techniques for constructing solutions to the Reggeon field theory when the Pomeranchuk singularity has intercept one. In this case the theory has a nontrivial infrared ($E,\vec{k}^2\approx 0$) behavior, and the standard renormalization program cannot be carried through order by order in perturbation theory. The problem is analogous to the one encountered in relativistic field theories with massless scalar particles.

Recently considerable progress has been made in studying the infrared behavior of the Reggeon field theory by making use of the renormalization group and the ϵ expansion.[3,4] Here one studies the field theory in $D=4-\epsilon$ space dimensions. We shall do likewise. Our construction of the Pomeron propagator leads directly to the renormalization-group scaling law when an infrared-stable Gell-Mann–Low eigenvalue exists. More importantly, we are able to justify the neglect of intercept renormalization in the ϵ-expansion calculations.

In this paper we consider only a bare triple-Pomeranchuk coupling and a linear trajectory function. Higher-order couplings and higher powers in $\vec{k}^2$ in the trajectory function are generated by the interaction. Wilson has shown that in the analogous problem of Euclidean ϕ^4 theory the inclusion of such terms directly into the bare Lagrangian will in general not disturb the infrared behavior of the theory.[5] The techniques we develop in this paper are also applicable to massless ϕ^4 theory in $4-\epsilon$ dimensions. A discussion of ϕ^4 can be found in a companion paper.[6]

In Sec. II we discuss the structure of a general perturbation-theory diagram which contributes to the Pomeron self-energy part. In general the interactions among the Pomerons will lead to a shift in their intercept. This can be compensated by adding an intercept renormalization counterterm to the Lagrangian just as one adds a mass counterterm in conventional field theories. When the Pomeron intercept is below one the counterterm can be adjusted at each order in perturbation theory so that the intercept takes on its physical value. However, when the intercept is at one the counterterm develops a branch point at zero value of the coupling constant, and it is no longer possible to carry out the intercept renormalization order by order in perturbation theory. Another difficulty with the perturbation theory is that as the Pomeron intercept approaches one, the second-order approximation to the progagator develops a tachyon (an l-plane pole to the right of one). This spurious pole disappears only after an infinite class of diagrams has been summed. Despite these difficulties it is possible to systematically construct the renormalized Pomeron propagator by making use of the skeleton expansion for the self-energy part. For $\epsilon>0$ the field theory is superrenormalizable, and once the renormalized propagator has been constructed, all Green's functions can in principle be calculated.

In Sec. III we develop an alternative method for constructing the renormalized propagator which appears to have considerable calculational advantages. We derive integral representations for the intercept renormalization counterterm and for the full propagator in terms of the other (finite) renormalization constants in the theory. The latter can be calculated systematically using renormalization-group techniques. This approach has several advantages. It enables us to explicitly display the infrared behavior of the propagator; it allows us to see how the tachyon, which is present in lowest-order perturbation theory, is removed from the full solution to the field theory; and it enables us to show that the perturbation series is an asymptotic expansion when the coupling constant becomes small for fixed values of E and $\vec{k}^2$, and when E or $\vec{k}^2$ become large for fixed values of the coupling constant.

From the integral representation for the intercept counterterm we are able to show that the intercept of the bare Pomeranchuk pole lies above one. We should emphasize that this bare Pomeranchuk pole is not necessarily experimentally observable at low energies, since there are many effects responsible for the complete intercept renormalization which are not accurately represented by the Reggeon calculus.

In Sec. IV we review our results and briefly discuss their application to the ϵ expansion and to direct calculations in two space dimensions.

II. PERTURBATION THEORY AND THE SKELETON EXPANSION

In the Reggeon field theory a noninteracting Reggeon is treated as a quasiparticle with energy-momentum dispersion relation

$$E = 1 - \alpha_0(-\vec{k}^2), \tag{1}$$

where $E = 1 - l$ and $\alpha_0(\vec{k}^2)$ is the bare Regge trajectory. We take the bare Pomeron trajectory to be linear and write

$$\alpha_0(-\vec{k}^2) = \alpha(0) - \alpha_0'\vec{k}^2. \tag{2}$$

The free Lagrangian density is then given by

$$\begin{aligned}\mathcal{L}_0(\vec{x}, t) = {} & \tfrac{1}{2} i \psi_0^\dagger(\vec{x}, t) \overleftrightarrow{\frac{\partial}{\partial t}} \psi_0(\vec{x}, t) \\ & - \alpha_0' \vec{\nabla}\psi_0^\dagger(\vec{x}, t) \cdot \vec{\nabla}\psi_0(\vec{x}, t) \\ & - \Delta \psi_0^\dagger(\vec{x}, t)\psi_0(\vec{x}, t).\end{aligned} \tag{3}$$

Here $\psi_0(\vec{x}, t)$ is the bare Pomeron field and $\Delta = 1 - \alpha(0)$ is the intercept gap. We shall work in $D = 4 - \epsilon$ space dimensions keeping in mind that the point of physical interest is $D = \epsilon = 2$.

The interaction Lagrangian density will be taken to be

$$\begin{aligned}\mathcal{L}_I(\vec{x}, t) = {} & -\tfrac{1}{2} i r_0[\psi_0^\dagger(\vec{x}, t)\psi_0(\vec{x}, t)^2 + \psi_0^\dagger(\vec{x}, t)^2\psi_0(\vec{x}, t)] \\ & + \delta\Delta\, \psi_0^\dagger(\vec{x}, t)\psi_0(\vec{x}, t).\end{aligned} \tag{4}$$

It follows from Gribov's signature analysis,[1] or equivalently the negative sign of the two-Pomeron cut,[2,7] that the bare triple-Pomeron coupling constant, r_0, is real. $\delta\Delta$ is the intercept renormalization counterterm which is to be adjusted so that the intercept gap, Δ, retains its physical value.

Our task is to construct the Green's functions, $G^{n,m}(E_i, \vec{k}_i)$, for n incoming and m outgoing Pomerons. The rules for evaluating the contribution of an individual Feynman diagram to $G^{n,m}$ are as follows:

1. For each Pomeron of momentum $\vec{k}$ and energy E use the bare propagator

$$G_0(E, \vec{k}^2) = i(E - \alpha_0'\vec{k}^2 - \Delta + i\epsilon)^{-1}.$$

2. For each triple-Pomeron vertex put a factor of $r_0/(2\pi)^{(D+1)/2}$.

3. For each intercept renormalization counterterm put a factor of $i\delta\Delta$.

4. For each two-Pomeron loop with both momenta in the same direction multiply by $\frac{1}{2}$.

5. Conserve energy and momentum at all vertices.

6. Evaluate $\int d^D\vec{k}\, dE$ around each closed loop.

Let us start by studying the structure of the Pomeron propagator in perturbation theory. It is convenient to write

$$\begin{aligned} i\Gamma^{1,1}(E, \vec{k}^2) &= iG^{1,1}(E, \vec{k}^2)^{-1} \\ &= E - \alpha_0'\vec{k}^2 - \Delta - \Sigma(E, \vec{k}^2) + \delta\Delta. \end{aligned} \tag{5}$$

$\Sigma(E, \vec{k}^2)$ is the proper self-energy part. The lowest-order diagram that contributes to $\Sigma(E, \vec{k}^2)$ is the bubble graph shown in Fig. 1. It gives[8]

$$\begin{aligned}\Sigma_2(E, \vec{k}^2) &= \frac{-i}{2}\frac{r_0^2}{(2\pi)^{D+1}} \int d^D\vec{k}'\, dE' (E' - \alpha_0'\vec{k}'^2 - \Delta + i\epsilon)^{-1} [E - E' - \alpha_0'(\vec{k} - \vec{k}')^2 - \Delta + i\epsilon]^{-1} \\ &= c r_0^2 (\tfrac{1}{2}\alpha_0'\vec{k}^2 + 2\Delta - E)^{1-\epsilon/2},\end{aligned} \tag{6}$$

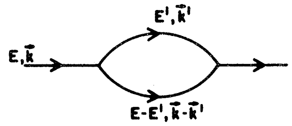

FIG. 1. Lowest-order contribution to the self-energy part.

with

$$c=\frac{\Gamma(-1+\epsilon/2)}{2(8\pi\alpha_0')^{D/2}}\,. \tag{7}$$

Clearly if the Pomeron intercept is to remain at $\alpha(0)$ one must have

$$\Gamma^{1,1}(\Delta,0)=0\,, \tag{8}$$

which requires that

$$\delta\Delta_2=cr_0^{\,2}\Delta^{1-\epsilon/2}\,. \tag{9}$$

Many of the difficulties with the perturbation theory already appear in this second-order calculation. First notice that the limits $\Delta\to 0$ and $\epsilon\to 2$ do not commute. In fact for $\Delta=0$ and $\epsilon\geq 2$ there is no choice for $\delta\Delta_2$ such that Eq. (8) holds. Furthermore there is a second solution to the equation

$$\Gamma^{1,1}(E,0)=0\,. \tag{10}$$

This spurious solution occurs for $E<\Delta$ when $\Delta^{\epsilon/2}<(-1+\epsilon/2)cr_0^{\,2}$ and for $E<0$ when $\Delta^{\epsilon/2}<-cr_0^{\,2}(2^{1-\epsilon/2}-1)$. In the second case the propagator has a tachyon, an l-plane pole to the right of $l=1$. As we shall see, this tachyon is not present in the full solution to the field theory. However, its presence in second-order perturbation theory indicates that the Δ goes to zero limit is a subtle one. In higher orders of perturbation theory one will find tachyon-tachyon and Pomeron-tachyon cuts. The tachyon pole and cuts disappear only after an infinite class of diagrams are summed.

To analyze higher-order terms in the expansion of $\Sigma(E,\vec{k}^2)$ it is convenient to use Rayleigh-Schrödinger perturbation theory (which is equivalent to performing all the E integrations by picking up propagator poles). Then momentum, but not energy, is conserved at each vertex, and one integrates only over the loop momenta. For each triple-Pomeron vertex one now puts a factor of $r_0/(2\pi)^{D/2}$ and for each n-Pomeron intermediate state the free n-Pomeron Green's function

$$G_n(E,\vec{k}_1,\ldots,\vec{k}_n)=i\left(E-n\Delta-\alpha_0'\sum_{i=1}^{n}\vec{k}_i^{\,2}+i\epsilon\right)^{-1}\,. \tag{11}$$

Let us consider a diagram of order $2n$ in the triple-Pomeron coupling but with no intercept counterterms. Making use of the Feynman identity to combine the Green's-function denominators one can write the contribution of such a diagram in the form[9]

$$\Sigma_n(E,\vec{k}^2)=(-)^{n+1}[r_0^{\,2}/(2\pi)^D]^n \times\tfrac{1}{2}\int\prod_{i=1}^{n}d^D\vec{k}_i\int_0^1\prod_{i=1}^{2n-1}dz_i\,\delta\!\left(1-\sum_{i=1}^{2n-1}z_i\right)\left[\sum_{i,j=1}^{n}A_{ij}(z)\alpha_0'\vec{k}_i\cdot\vec{k}_j+a(z)\vec{k}^2+b(z)\Delta-E-i\epsilon\right]^{-(2n-1)}\,. \tag{12}$$

Here $A_{ij}(z)$ is a positive semidefinite, symmetric matrix, $a(z)>0$ and $b(z)\geq 2$ for all allowed values of the z_i. Notice that $\Sigma_n(E,\vec{k}^2)$ has no singularities as a function of ϵ arising from the infrared $(\vec{k}_i^{\,2}\approx 0)$ region of integration even when $\Delta=0$. Such infrared singularities are present in relativistic field theories with massless scalar particles which makes the analysis of these theories more difficult.[6]

The only singularities in $\Sigma_n(E,\vec{k}^2)$ as a function of ϵ are poles arising from ultraviolet divergences. Performing the momentum integrations in Eq. (12) gives

$$\Sigma_n(E,\vec{k}^2)=(-)^{n+1}[r_0^{\,2}/(4\pi\alpha_0')^{D/2}]^n\,\frac{\Gamma(-1+n\epsilon/2)}{\Gamma(2n-1)} \times\int_0^1\prod_{i=1}^{2n-1}dz_i\,\delta\!\left(1-\sum_{1}^{2n-1}z_i\right)[\det A(z)]^{-D/2}[a(z)\vec{k}^2+b(z)\Delta-E-i\epsilon]^{1-n\epsilon/2}\,. \tag{13}$$

The pole at $\epsilon=2/n$, which arises from the overall ultraviolet divergence of the diagram, appears explicitly in the Γ function in Eq. (13). If the topology of the diagram is such that one of the internal lines contains a self-energy insertion of order $2m$, then there will be an additional pole at $\epsilon=2/m$. It will arise from a vanishing of $\det A(z)$ when one or more of the Feynman parameters is zero. Some specific examples illustrated in Figs. 2 and 3 are worked out in the Appendix.

From simple power counting one sees that for $\epsilon>0$ the only ultraviolet divergences in the theory are those that arise in the self-energy part. Once they have been removed by performing the intercept renormalization all Green's functions are finite. The renormalization can be carried out order by order in perturbation theory for $\Delta>0$, but not for $\Delta=0$ which is the theory of interest. To see this let us set $\Delta=0$ in Eq. (13). We can then write

$$\Sigma_n(E,0)=(-)^n(r_0^2/\alpha_0'^{\,D/2})^n(-E)^{1-n\epsilon/2}\times C_n(1-n\epsilon/2)^{-1}. \tag{14}$$

Clearly for $n\geq 2/\epsilon$ there is no choice for $\delta\Delta_n$ such that $\Sigma_n(0,0)-\delta\Delta_n=0$. This is the generalization of the difficulty encountered in the second-order diagram for $\epsilon\geq 2$. The problem is that when $\Delta=0$, $\delta\Delta$ has a branch point at $r_0=0$. This follows from the fact that when $\Delta=0$ the only quantity with the dimensions of energy is $(r_0^2/\alpha_0'^{\,D/2})^{2/\epsilon}$. As a result, $\delta\Delta$ must have the form

$$\delta\Delta=(r_0^2/\alpha_0'^{\,D/2})^{2/\epsilon}f(\epsilon), \tag{15}$$

where $f(\epsilon)$ is a dimensionless quantity independent of r_0 and α_0'.

It is clear from Eq. (15) that one must develop nonperturbative techniques for calculating $\delta\Delta$. (This would be the case even if we introduced a large E or $\vec{k}^2$ cutoff, since the nonanalyticity of $\delta\Delta$ in r_0 arises from the infrared behavior of the theory.) One approach is to make use of the skeleton expansion for the self-energy part. One starts by summing all graphs in which the internal lines have no self-energy insertions and no intercept counterterm insertions. Examples are the ladder graphs shown in Fig. 2. For this class of graphs there is no difficulty in carrying out the intercept renormalization after the summation has been performed, since the only singularity of each diagram is the overall ultraviolet divergence. One then obtains a first-order approximation to the propagator. The next step is to recalculate the skeleton graphs using the improved propagator in place of the bare one, and to again renormalize after summing the graphs. By carrying out this process an infinite number of times one generates all the diagrams in the theory. At each step of the calculation the inverse propagator is finite and vanishes at $E=\vec{k}^2=0$.

Let us consider the first-round calculation in more detail. We can write

$$\Sigma^{(1)}(E,0)=\sum_{n=1}^{\infty}(-)^n(r_0^2/\alpha_0'^{\,D/2})^n(-E)^{1-n\epsilon/2}D^{(1)}(n)\times(1-n\epsilon/2)^{-1}, \tag{16}$$

where the $D^{(1)}(n)$ are positive and have no singularities in n for $n\geq 2/\epsilon$. If the $D^{(1)}(n)$ are sufficiently well behaved for large values of n as is indicated by the examples of the Appendix and by renormalization group calculations, then the series can be summed using the Sommerfeld-Watson transformation:

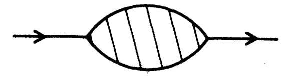

FIG. 2. A typical "ladder" graph.

$$\Sigma^{(1)}(E,0)=\tfrac{1}{2}i\int_c\frac{dn}{\sin\pi n}(r_0^2/\alpha_0'^{\,D/2})^n(-E)^{1-n\epsilon/2}D^{(1)}(n)\times(1-n\epsilon/2)^{-1}. \tag{17}$$

The contour c circles all of the positive integers in the clockwise direction. Let us now pull back the integration contour. The right-most singularity of the integral that one encounters is the pole at $n=2/\epsilon$, so one can write

$$\Sigma^{(1)}(E,0)=(r_0^2/\alpha_0'^{\,D/2})^{2/\epsilon}\frac{2}{\epsilon}\frac{\pi}{\sin(\pi 2/\epsilon)}D^{(1)}\left(\frac{2}{\epsilon}\right)+\bar{\Sigma}^{(1)}(E,0), \tag{18}$$

where $\bar{\Sigma}^{(1)}(E,0)$ vanishes as E goes to zero. As a result,

$$\delta\Delta^{(1)}=(r_0^2/\alpha_0'^{\,D/2})^{2/\epsilon}\frac{2}{\epsilon}\frac{\pi}{\sin\pi(2/\epsilon)}D^{(1)}\left(\frac{2}{\epsilon}\right). \tag{19}$$

The task of calculating the $D^{(1)}(n)$ is a formidable one, to say nothing of the problem of carrying out the higher-order iterations. In the next section we shall present a more practical calculational scheme. The point we wish to emphasize here is the interplay between the ultraviolet and infrared singularities. It is the fact that the overall ultraviolet divergence of the graphs occurs at the same value of ϵ that the infrared difficulties set in that makes our construction work. This will continue to happen in the higher-order iterations.

Although the skeleton expansion does not seem to be of practical use, the Sommerfeld-Watson representation of the self-energy part does. For example, in the next section we shall show that for small values of ϵ the full self-energy part can be written in the form of Eq. (17), with $D^{(1)}(n)$ replaced by

$$C(n)=\left(\frac{6}{(8\pi)^2\epsilon}\right)^n\frac{\Gamma(n+\frac{1}{6})}{n!\,\Gamma(\frac{1}{6})}. \tag{20}$$

In this approximation

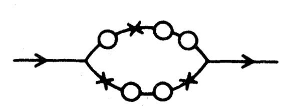

FIG. 3. A typical "bubble" graph. $\times$ denotes an intercept counterterm.

$$\delta\Delta = [\,6r_0^{\,2}/(8\pi)^2\alpha_0{}'^{D/2}\epsilon]^{2/\epsilon}\,\frac{2}{\epsilon}\,\frac{\pi}{\sin\pi(2/\epsilon)}\times\frac{\Gamma(2/\epsilon+\frac{1}{6})}{\Gamma(1+2/\epsilon)\Gamma(\frac{1}{6})}\,. \tag{21}$$

The pole at $n=0$ arising from the vanishing of $\sin\pi n$ cancels the linear term in E in the bare propagator, so the small-E behavior of $\Gamma^{1,1}(E,0)$ is controlled by the right-most pole in $\Gamma(n+\frac{1}{6})$. One finds

$$i\Gamma^{1,1}(E,0)\underset{E\to 0}{\sim} -[\,6r_0^{\,2}/(8\pi)^2\alpha_0{}'^{D/2}\epsilon]^{1/6}\times(-E)^{1+\epsilon/12}, \tag{22}$$

in agreement with the results of Refs. 3 and 4.

III. INTEGRAL REPRESENTATIONS FOR THE POMERON PROPAGATOR

Up to now we have concentrated on the intercept renormalization. However, in order to derive integral representations for the Pomeron propagator it will be useful to carry out the wave-function, coupling-constant, and slope parameter renormalizations. We start by introducing the renormalized Pomeron field operator $\psi(\vec{x},t)$,

$$\psi(\vec{x},t)=Z_3^{-1/2}\psi_0(\vec{x},t). \tag{23}$$

Then the Lagrangian density of Eqs. (3) and (4) can be rewritten in the form

$$\mathcal{L}=\tfrac{1}{2}iZ_3\psi^\dagger(\vec{x},t)\,\frac{\overleftrightarrow{\partial}}{\partial t}\psi(\vec{x},t)-Z_2\alpha'\vec{\nabla}\psi^\dagger(\vec{x},t)\cdot\vec{\nabla}\psi(\vec{x},t)-\tfrac{1}{2}irZ_1[\,\psi^\dagger(\vec{x},t)\psi(\vec{x},t)^2+\psi^\dagger(\vec{x},t)^2\psi(\vec{x},t)]+Z_3\delta\Delta\psi^\dagger(\vec{x},t)\psi(\vec{x},t). \tag{24}$$

For the remainder of this paper we shall take $\Delta=0$. Z_1, Z_2, and Z_3 will be referred to respectively as the coupling-constant, slope, and wave-function renormalization constants:

$$\alpha'=Z_3Z_2^{-1}\alpha_0{}', \tag{25}$$

$$r=Z_3^{3/2}Z_1^{-1}r_0. \tag{26}$$

The Z_i are determined by the following normalization conditions on the renormalized propagator and three-point function:

$$\frac{\partial}{\partial E}i\Gamma_R^{1,1}(E,\vec{k}^2)\bigg|_{E=-E_N;\,\vec{k}^2=\vec{k}_N^{\,2}}=1, \tag{27}$$

$$\frac{\partial}{\partial\vec{k}^2}i\Gamma_R^{1,1}(E,\vec{k}^2)\bigg|_{E=-E_N;\,\vec{k}^2=\vec{k}_N^{\,2}}=-\alpha', \tag{28}$$

$$\Gamma_R^{1,2}(E_1,\vec{k}_1,E_2,\vec{k}_2,E_3,\vec{k}_3)\,|_{E_1=2E_2=2E_3=-E_N;\,\vec{k}_1=2\vec{k}_2=2\vec{k}_3=\vec{k}_N}=ir/(2\pi)^{(D+1)/2}. \tag{29}$$

Here $(-E_N,\vec{k}_N)$ is a general point away from the singularities of the Green's functions. Our normalization conditions differ from those of Abarbanel and Bronzan in that we have not set $\vec{k}_N^{\,2}=0$. This generalization is necessary in order for us to study the $\vec{k}^2$ dependence of the propagator. With the above definitions, the renormalized proper vertex function for n incoming and m outgoing Pomerons, $\Gamma_R^{n,m}$, is related to the unrenormalized one by

$$\Gamma_R^{n,m}(E_i,\vec{k}_i,r,\alpha',E_N,\vec{k}_N^{\,2})=Z_3^{(n+m)/2}\Gamma^{n,m}(E_i,\vec{k}_i,r_0,\alpha_0{}'). \tag{30}$$

Let us now turn to the problem of finding an integral representation for the Pomeron propagator. We start with the unrenormalized self-energy part with $\vec{k}^2=0$. From dimensional analysis it can be written in the form

$$\Sigma(E,0)=\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}(r_0^{\,2}/\alpha_0{}'^{D/2})^{n+2m/\epsilon}(-E)^{1-n\epsilon/2-m}\times\frac{C_{n,m}}{n\epsilon/2+m-1}. \tag{31}$$

The (n,m) term in the sum arises from diagrams with n triple-Pomeron vertices and m intercept renormalization counterterms. We have used Eq. (15) for $\delta\Delta$ and absorbed $f(\epsilon)$ into $C_{n,m}$. The pole in ϵ arising from the overall ultraviolet divergence of the graphs has been displayed explicitly in Eq. (31). The $C_{n,m}$, which are functions of ϵ only, also contain poles in ϵ, but they cancel among themselves when the sum is taken. It is convenient to introduce the auxiliary function

$$F(x)=\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}x^{n\epsilon/2+m}C_{n,m}. \tag{32}$$

Then Eq. (3.1) can be rewritten in the form

$$\Sigma(E,0)=(r_0^2/\alpha_0'^{D/2})^{2/\epsilon}\int_0^{x_1}\frac{dx}{x^2}F(x), \tag{33}$$

where

$$x_1=(r_0^2/\alpha_0'^{D/2})^{2/\epsilon}(-E)^{-1}. \tag{34}$$

The integral in Eq. (33) is well defined for $\epsilon>2$. It can be continued to smaller values of ϵ by making use of Eq. (32) to explicitly remove the poles at $\epsilon=(2/n)(1-m)$. The residues of these poles can be calculated in perturbation theory.

The function $F(x)$ can be related to the wave-function renormalization constant, Z_3, provided we carry out the renormalization at $\vec{k}_N^2=0$. In that case

$$Z_3^{-1}(x_{1N})=\frac{\partial}{\partial E}i\Gamma^{1,1}(E,\vec{k}^2)\bigg|_{E=-E_N;\,\vec{k}^2=0} =1-F(x_{1N}), \tag{35}$$

where

$$x_{1N}=(r_0^2/\alpha_0'^{D/2})^{2/\epsilon}E_N^{-1}. \tag{36}$$

That Z_3 is a function of x_{1N} only follows from the fact that this is the only dimensionless variable one can make from r_0, α_0', and E_N. We now have

$$\delta\Delta=\Sigma(0,0) =(r_0^2/\alpha_0'^{D/2})^{2/\epsilon}\int_0^{\infty}\frac{dx}{x^2}[1-Z_3^{-1}(x)], \tag{37}$$

and from Eq. (5)

$$i\Gamma^{1,1}(E,0)=-(r_0^2/\alpha_0'^{D/2})^{2/\epsilon}\int_{x_1}^{\infty}\frac{dx}{x^2}Z_3^{-1}(x). \tag{38}$$

Notice that the bare propagator has been canceled by the term in $\Sigma(E,0)$ which is linear in E.

The next step is to evaluate Z_3. This can be done using renormalization-group techniques. Following Abarbanel and Bronzan[3] we introduce a dimensionless renormalized coupling constant

$$g=r\alpha'^{-D/4}E_N^{-\epsilon/4}, \tag{39}$$

and the renormalization-group functions

$$\beta=E_N\frac{\partial}{\partial E_N}g\bigg|_{r_0,\alpha_0'\text{ fixed}}, \tag{40}$$

$$\gamma=E_N\frac{\partial}{\partial E_N}\ln Z_3\bigg|_{r_0,\alpha_0'\text{ fixed}}, \tag{41}$$

$$\zeta=E_N\frac{\partial}{\partial E_N}\alpha'\bigg|_{r_0,\alpha_0'\text{ fixed}}. \tag{42}$$

Since g is the only dimensionless renormalized quantity in the problem when $\vec{k}_N^2=0$, β, γ, ζ/α', and the Z_i must be functions of g only. This tells us that Eqs. (41) and (42) can be written in the form

$$\gamma=\beta\frac{d}{dg}\ln Z_3 \tag{43}$$

and

$$\zeta=\alpha'\beta\frac{d}{dg}\ln(Z_2^{-1}Z_3). \tag{44}$$

Furthermore, from Eqs. (26) and (39) we find that

$$g^{-1}+\tfrac{1}{4}\epsilon\beta^{-1}=\frac{d}{dg}\ln(Z_3^{3/2-D/4}Z_1^{-1}Z_2^{D/4}). \tag{45}$$

Since the Z_i are all one at $g=0$ our final results are

$$Z_3(g)=\exp\left[\int_0^g dg'\gamma(g')\beta^{-1}(g')\right], \tag{46}$$

$$Z_2^{-1}(g)Z_3(g)=\exp\left[\int_0^g dg'(\zeta/\alpha')\beta^{-1}(g')\right], \tag{47}$$

and

$$r_0\alpha_0'^{-D/4}E_N^{-\epsilon/4}=g\exp\left[-\int_0^g dg'\,[g'^{-1}+\tfrac{1}{4}\epsilon\beta^{-1}(g')]\right]. \tag{48}$$

Several important results can be read off without detailed calculations. First, since γ and β are real analytic functions, Z_3 is positive-definite for real values of g. As a result, we see from Eq. (38) that $\Gamma^{1,1}(E,0)$ cannot vanish in the range $-\infty<E<0$. Thus the tachyon which we found in second-order perturbation theory is not present in the full propagator. Its cancellation is clearly a nonperturbative effect.

Next let us suppose that $\beta(g)$ has an infrared-stable zero, i.e., a zero of the form

$$\beta(g)\underset{g\to g_1}{\sim}b(g-g_1), \tag{49}$$

with $b>0$. This is the case at least for small values of ϵ.[3,4] Then for $g\lesssim g_1$ Eqs. (46) and (48) yield

$$x_{1N}\simeq c_1(g_1-g)^{-1/b}, \tag{50}$$

$$Z_3\simeq c_3'(g_1-g)^{\gamma/b}=c_3x_{1N}^{-\gamma}, \tag{51}$$

where $\gamma\equiv\gamma(g_1)$ and the c's are constants. We now see that the integrals in Eqs. (37) and (38) will converge provided

$$\gamma<1. \tag{52}$$

Notice that the limit $g\to g_1$ requires either $r_0^2\to\infty$ or $E_N\to 0$. The small-E behavior of $\Gamma^{1,1}(E,0)$ is just

$$i\Gamma^{1,1}(E,0)\underset{E\to 0}{\sim}-c_3(r_0^2/\alpha_0'^{D/2})^{(2/\epsilon)\gamma}(-E)^{1-\gamma}, \tag{53}$$

in agreement with the renormalization-group result.[3,4]

The small-r_0 and large-E behaviors of $\Gamma^{1,1}(E,0)$

can also be read off from Eq. (38). Let us denote by $\bar{x}_1$ a value of x small enough so that the series expansion for $Z_3^{-1}(x)$ given by Eqs. (32) and (35) is uniformly convergent for $x_1 \le x \le \bar{x}_1$. Then in this range one can integrate the series term by term. The dominant contribution will come from those terms in Eq. (32) with $m=0$ and $n<2/\epsilon$. They will just give rise to the first n terms in the perturbation expansion of the propagator plus a connection of order $(r_0^2/\alpha_0'^{D/2})^{2/\epsilon}$. The integral from $\bar{x}_1$ to infinity will also be of order $(r_0^2/\alpha_0'^{D/2})^{2/\epsilon}$. Thus the perturbation series provides an asymptotic expansion of the propagator for large E or small r_0. However, it is only legitimate to work to order $n<2/\epsilon$. No diagrams with intercept counterterms need be included. This result is crucial for the ϵ-expansion calculations of the critical exponents.[3,4] They are generally assumed to have a power-series expansion about $\epsilon=0$. If one wishes to calculate the first n terms in this expansion one need merely imagine working with $\epsilon<2/n$. One can then calculate the renormalization-group functions in perturbation theory, and no diagrams with intercept renormalization counterterms will enter. At the end of the calculation one can of course attempt to continue the answer to $\epsilon>2/n$.

In order to proceed further it is necessary to compute the renormalization-group functions. In the one-loop approximation one need only calculate the bubble diagram of Fig. 1 and the vertex graphs shown in Fig. 4. We find that

$$\beta(g) = -\tfrac{1}{4}\epsilon g(1-g^2/g_1^2)\,, \tag{54}$$

$$\gamma(g) = \gamma g^2/g_1^2\,, \tag{55}$$

$$\zeta/\alpha' = \delta g^2/g_1^2\,. \tag{56}$$

Here g_1, γ, and δ are functions of ϵ only and γ and δ are negative. The one-loop approximation gives the correct behavior of the renormalization-group functions to leading order in ϵ. Working to this order one finds[3,4]

$$\frac{g_1^2}{(8\pi)^2} = \tfrac{1}{6}\epsilon\,, \tag{57}$$

$$\gamma = -\tfrac{1}{12}\epsilon\,, \tag{58}$$

$$\delta = -\tfrac{1}{24}\epsilon\,. \tag{59}$$

From Eqs. (46) and (48) we see that in the one-loop approximation

$$x_{1N}^{\epsilon/2} = g^2(1-g^2/g_1^2)^{-1} \tag{60}$$

and[10]

$$\begin{aligned} Z_3 &= (1-g^2/g_1^2)^{(2/\epsilon)\gamma} \\ &= (1+x_{1N}^{\epsilon/2}/g_1^2)^{-(2/\epsilon)\gamma}\,. \end{aligned} \tag{61}$$

Notice that the region $-g_1 \le g \le g_1$ corresponds to $-\infty < r_0 < \infty$. This will always be the case if an infrared-stable zero exists since $\beta(g)$ has the form

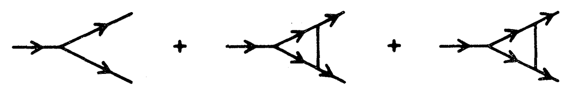

FIG. 4. Lowest-order contributions to the vertex function needed to calculate $\beta(g)$ in the one-loop approximation.

$-\frac{1}{4}\epsilon g f(g^2)$ with $f(0)=1$. Substituting Eq. (61) into Eq. (37) gives

$$\delta\Delta = \gamma(r_0^2/\alpha_0'^{D/2}g_1^2)^{2/\epsilon}\,\frac{2}{\epsilon}\,\frac{\Gamma(1-2/\epsilon)\Gamma((2/\epsilon)(1-\gamma))}{\Gamma(1-(2/\epsilon)\gamma)}\,, \tag{62}$$

which reduces to Eq. (21) in the small-ϵ limit. For small values of ϵ we also find that

$$\begin{aligned} i\Gamma^{1,1}(E,0) &= -(r_0^2/\alpha_0'^{D/2})^{2/\epsilon} \\ &\quad\times \int_{x_1}^{\infty} dx\, x^{-2}(1+x^{\epsilon/2}/g_1^2)^{-(2/\epsilon)\gamma} \\ &\simeq E[1+6r_0^2(-E)^{-\epsilon/2}/(8\pi)^2\epsilon]^{-1/6}\,. \end{aligned} \tag{63}$$

This result was first obtained by Bronzan using the renormalization group directly.[11] He has shown that it corresponds to summing all terms in the theory of order $(r_0^2/\epsilon)^n$, but neglecting those of order $\epsilon^m(r_0^2/\epsilon)^n$. One sees directly from Eq. (63) that there is a tachyon in second-order perturbation theory but not in the full theory.

From Eq. (62) we see that $\delta\Delta$ develops poles at $\epsilon=2/n$, $n=1,2,\ldots$ and so the bare intercept is undefined at these values of ϵ, and in particular at the physical value $\epsilon=2$. This is because we are calculating within a pure renormalizable field theory. In practice, of course, there will be a natural cutoff in the angular momenta and momentum transfers we consider provided by, say, secondary trajectories and the two-pion threshold. This cutoff does not affect the infrared behavior of the theory, but it does give a meaning to the bare intercept. The introduction of the cutoff can be carried out using the renormalization-group method of Wilson.[5] This is essentially the method used by Migdal, Polyakov, and Ter-Martirosyan.[4] In this approach higher-order couplings can also be simply introduced from the start since the renormalizability of the theory is not a requirement when the cutoff is present. As we discussed in the Introduction the critical exponents should be the same, whether or not higher-order couplings are introduced.

If we suppose there is a cutoff Λ in the angular momentum then this cutoff will apply also to the representation of $\delta\Delta$ given in Eq. (37). That is,

$$\begin{aligned} \delta\Delta(\Lambda) &= (r_0^2/\alpha_0'^{D/2})^{2/\epsilon} \\ &\quad\times \int_{(r_0^2/\alpha_0'^{D/2})^{2/\epsilon}\Lambda^{-1}}^{\infty} dx\, x^{-2}[1-Z_3^{-1}(x)]\,. \end{aligned} \tag{64}$$

At least in the small-ϵ limit $Z_3 \geq 1$, and so we see that $\delta\Delta > 1$ and hence $\alpha_0(0) - 1 > 0$, where $\alpha_0(0)$ is the bare Pomeron intercept. At $\epsilon = 2$

$$\delta\Delta = \frac{r_0^2}{\alpha_0'} \int_{r_0^2/\alpha_0'\Lambda}^{\bar{x}_1} \frac{dx}{x^2}\left[\frac{x}{16\pi} + O(x^2)\right] + \frac{r_0^2}{\alpha_0'}\int_{\bar{x}_1}^{\infty}\frac{dx}{x^2}[1 - Z_3^{-1}(x)] \tag{65}$$

$$= \frac{-1}{16\pi}\frac{r_0^2}{\alpha_0'}\ln\left(\frac{r_0^2}{\alpha_0'\Lambda}\right) + O\left(\frac{r_0^2}{\alpha_0'}\right) . \tag{66}$$

Similarly one can show that $\delta\Delta > 0$, for general ϵ, when $(r_0^2/\alpha_0'^{D/2})^{2/\epsilon}(1/\Lambda)$ is sufficiently small. Equation (66) agrees with that obtained by Migdal, Polyakov, and Ter-Martirosyan.[4] Note, however, that corrections to Eq. (66) are $O(r_0^2/\alpha_0')$ and not $O(r_0^4/\alpha_0'^2)$ as suggested in Ref. 4. It should be clear from our discussion that we do not agree with the suggestion in Ref. 4, that $\delta\Delta$ be determined perturbatively by a self-consistency equation. The second-order self-consistency equation given by Migdal *et al.* is

$$-\delta\Delta + \frac{1}{16\pi}\frac{r_0^2}{\alpha_0'}\ln\left[\frac{r_0^2}{\alpha_0'(-\delta\Delta)}\right] = 0 , \tag{67}$$

which does not have a real solution for $\delta\Delta$.

We can now envision a systematic calculation of the propagator. We were able to calculate the renormalization-group functions in the one-loop approximation because that did not involve the intercept renormalization counterterm. We were then led to the first-order approximation to $\delta\Delta$ given in Eq. (62). This in turn can be used to construct an improved propagator which is exact to order g^2 and has the Pomeranchuk singularity intercept at 1. The improved propagator can in turn be used to calculate a new approximation to the renormalization-group functions in which all one- and two-loop diagrams are treated correctly.[12] This procedure can in principle be repeated indefinitely.[13] In the n-loop approximation the critical exponents will be given correctly to nth order in ϵ.

In order to carry out this program it is necessary to calculate $\Gamma^{1,1}(E,\vec{k}^2)$ for $\vec{k}^2 \neq 0$. It can be expressed in terms of the variable x_1 defined in Eq. (34) and

$$x_2 = \alpha_0'\vec{k}^2/(-E) . \tag{68}$$

Introducing the vector notation $\vec{x} = (x_1, x_2)$ and $\vec{\nabla} = (\partial/\partial x_1, \partial/\partial x_2)$ and noting that the origin of the (x_1, x_2) plane corresponds to $r_0 = 0$, we can write the self-energy part as a line integral

$$\Sigma(E,\vec{k}^2) = \int_{\vec{0}}^{\vec{x}} d\vec{x}' \cdot \vec{\nabla}\Sigma . \tag{69}$$

In order to evaluate $\vec{\nabla}\Sigma$ we must carry out the renormalization at a general point $E_N, \vec{k}_N^2 > 0$. Then from Eqs. (27) and (28) we have

$$Z_3^{-1} = 1 - \frac{\partial}{\partial E}\Sigma(E,\vec{k}^2)\bigg|_{E=-E_N;\vec{k}^2=\vec{k}_N^2} , \tag{70}$$

$$Z_2^{-1} = 1 + \frac{1}{\alpha_0'}\frac{\partial}{\partial \vec{k}^2}\Sigma(E,\vec{k}^2)\bigg|_{E=-E_N;\vec{k}^2=\vec{k}_N^2} , \tag{71}$$

so

$$\frac{\partial}{\partial x_1}\Sigma(x_1,x_2)\bigg|_{x_1=x_{1N};x_2=x_{2N}} = (r_0^2/\alpha_0'^{D/2})^{2/\epsilon} x_{1N}^{-2} \times [(1 - Z_3^{-1}) + x_{2N}(1 - Z_2^{-1})] \tag{72}$$

and

$$\frac{\partial}{\partial x_2}\Sigma(x_1,x_2)\bigg|_{x_1=x_{1N};x_2=x_{2N}} = -(r_0^2/\alpha_0'^{D/2})^{2/\epsilon} \times x_{1N}^{-1}(1 - Z_2^{-1}) , \tag{73}$$

where x_{1N} is defined in Eq. (36) and

$$x_{2N} = \alpha_0'\vec{k}_N^2/E_N . \tag{74}$$

That the Z_i can be expressed as functions of x_{1N} and x_{2N} only follows from dimensional analysis. They can also be written in terms of two dimensionless renormalized parameters which we shall choose to be g and[14]

$$y = \alpha'\vec{k}_N^2/E_N . \tag{75}$$

The intercept renormalization counterterm is now given by

$$\delta\Delta = \int_0^{\vec{\infty}} d\vec{x}' \cdot \vec{\nabla}\Sigma . \tag{76}$$

The integral in Eq. (76) can run over any path from the origin to infinity lying in the first quadrant of the (x_1, x_2) plane. Equation (37) corresponds to the special choice of having the contour of integration run along the x_1 axis. The full propagator takes the form

$$i\Gamma^{1,1}(E,\vec{k}^2) = (r_0^2/\alpha_0'^{D/2})^{2/\epsilon}\int_{\vec{x}}^{\vec{\infty}} d\vec{x}' \cdot \vec{A}(x_1', x_2')/x_1'^2 , \tag{77}$$

with

$$A_1(x_1,x_2) = -[Z_3^{-1}(x_1,x_2) + x_2 Z_2^{-1}(x_1,x_2)] \tag{78}$$

and

$$A_2(x_1,x_2) = x_1 Z_2^{-1}(x_1,x_2) . \tag{79}$$

Just as in Eq. (38) the bare propagator has been canceled by the terms in $\Sigma(E,\vec{k}^2)$ which do not involve the Z_i. Equation (77) reduces to Eq. (38) when $\vec{k}^2 = 0$, provided one again takes the path of integration along the x_1 axis. It will often be convenient to express Eq. (77) as a one-dimensional integral by simultaneously scaling E and $\vec{k}^2$.[15] Writing

$$x_1 = x_{1N}\xi^{-1} , \tag{80}$$

$$x_2 = x_{2N}\xi^{a} \tag{81}$$

gives

$$i\Gamma^{1,1}(E,\vec{k}^2) = -E_N \int_0^{\xi} d\xi' [Z_3^{-1}(\xi') + (a+1)x_{2N}\xi'^a Z_2^{-1}(\xi')]$$

$$= -\int_0^{-E} dE_N' Z_3^{-1}(E_N',\vec{k}_N'^2) - \int_0^{\alpha_0'\vec{k}^2} d\alpha_0' \vec{k}_N'^2 Z_2^{-1}(E_N',\vec{k}_N'^2) . \tag{82}$$

In the last line of Eq. (82) the integrals are to be taken along the path $E_N'/E_N = (\vec{k}_N'^2/\vec{k}_N^2)^{1/(1+a)}$.

The Z_i can again be calculated using renormalization-group techniques. There are now two β functions:

$$\beta_E(g,y) = E_N \frac{\partial}{\partial E_N} g \bigg|_{r_0,\alpha_0',W_N \text{ fixed}} , \tag{83}$$

$$\beta_W(g,y) = W_N \frac{\partial}{\partial W_N} g \bigg|_{r_0,\alpha_0',E_N \text{ fixed}} , \tag{84}$$

where $W_N = \vec{k}_N^2/E_N$. Similarly, we define γ_E, γ_W, ζ_E, and ζ_W in analogy with Eqs. (43) and (44). In keeping with Eqs. (80) and (81) we let $E_N \to E_N\xi$ and $\vec{k}_N^2 \to \vec{k}_N^2\xi^{a+1}$. The ξ dependence of g and y can be determined from the differential equations ($t \equiv \ln\xi$)

$$\frac{dg(t)}{dt} = \beta_E(g(t),y(t)) + a\beta_W(g(t),y(t)) \tag{85}$$

and

$$\frac{dy(t)}{dt} = y(t)\{a + \alpha'(t)^{-1}[\zeta_E(g(t),y(t)) + a\zeta_W(g(t),y(t))]\} , \tag{86}$$

with the boundary conditions

$$g(0) = g , \tag{87}$$

$$y(0) = y . \tag{88}$$

The renormalization constants of interest can be obtained from the differential equations

$$\frac{\partial}{\partial t}\ln Z_3(t) = \gamma_E(t) + a\gamma_W(t) \tag{89}$$

and

$$\frac{\partial}{\partial t}\ln[Z_2(t)^{-1}Z_3(t)] = \alpha'(t)^{-1}[\zeta_E(t) + a\zeta_W(t)] , \tag{90}$$

with

$$Z_i(0) = Z_i . \tag{91}$$

Since $\xi \to \infty$ corresponds to $r_0 \to 0$ we also know that

$$Z_i(\infty) = 1 . \tag{92}$$

In order to see how the calculation goes let us consider the one-loop approximation. In order to simplify the algebra we shall work in the small-ϵ limit. Then the renormalization-group functions can be written in the form

$$\gamma_E = \gamma G^2/g_1^2 , \tag{93}$$

$$\gamma_W = \gamma \tfrac{1}{2}y(1+\tfrac{1}{2}y)^{-1}G^2/g_1^2 , \tag{94}$$

$$\zeta_E/\alpha' = \delta G^2/g_1^2 , \tag{95}$$

$$\zeta_W/\alpha' = \delta \tfrac{1}{2}y(1+\tfrac{1}{2}y)^{-1}G^2/g_1^2 , \tag{96}$$

$$\beta_E = -\tfrac{1}{4}\epsilon g(1 - G^2/g_1^2) , \tag{97}$$

$$\beta_W = \tfrac{1}{4}\epsilon g \tfrac{1}{2}y(1+\tfrac{1}{2}y)^{-1}G^2/g_1^2 , \tag{98}$$

with

$$G^2 = g^2(1+\tfrac{1}{2}y)^{-\epsilon/2} . \tag{99}$$

g_1, γ, and δ are again given by Eqs. (57)–(59). Equations (85) and (86) can be rewritten in the form

$$\frac{\partial}{\partial t}G(t)^2 = -\tfrac{1}{2}\epsilon G(t)^2\{1 + a\tfrac{1}{2}y(t)[1+\tfrac{1}{2}y(t)]^{-1}\} \times [1 - G(t)^2/g_1^2] \tag{100}$$

and

$$\frac{\partial}{\partial t}y(t) = y(t)(a + \delta[G(t)^2/g_1^2]\{1 + a\tfrac{1}{2}y(t)[1+\tfrac{1}{2}y(t)]^{-1}\}) \tag{101}$$

As long as the quantity

$$\{1 + a\tfrac{1}{2}y(t)[1+\tfrac{1}{2}y(t)]^{-1}\}$$

remains positive, $G(t)^2$ will approach the fixed point limit g_1^2 as $t \to -\infty (\xi \to 0)$. Combining Eq. (100) with Eqs. (89), (90), and (91) gives

$$\frac{\partial}{\partial t}\ln Z_3(t) = -\frac{2}{\epsilon}\gamma \frac{\partial}{\partial t}G(t)^2 \Big/ g_1^2 , \tag{102}$$

so

$$Z_3(t) = [1 - G(t)^2/g_1^2]^{(2/\epsilon)\gamma} . \tag{103}$$

Similarly we find that

$$Z_2(t) = [1 - G(t)^2/g_1^2]^{-(2/\epsilon)(\delta-\gamma)} . \tag{104}$$

There are three cases of interest. First, if $a < -\delta = \frac{1}{24}\epsilon$, then $y(t) \to_{t\to-\infty} 0$. From Eq. (100)

$$1 - G(t)^2/g_1^2 \underset{t\to-\infty}{\sim} C_0 e^{(\epsilon/2)t} , \tag{105}$$

where C_0 depends on g and y, but not on t. Equation (82) then yields

$$i\Gamma^{1,1}(E,\vec{k}^2) \xrightarrow[E,\vec{k}^2\to 0]{} -E_N C_0^{-(2/\epsilon)\gamma}\xi^{1-\gamma} \times (1 + x_{2N}C_0^{(2/\epsilon)\delta}\xi^{a+\delta}) . \tag{106}$$

Only terms of leading order in ϵ have been retained in Eq. (106). The renormalized propagator takes the form

$$i\Gamma_R^{1,1}(E,\vec{k}^2) \underset{E,\vec{k}^2\to 0}{\sim} -E_N C_0'^{-2/\epsilon}(-E/E_N)^{1-\gamma} \times (1+C_0'^{(2/\epsilon)\delta}\rho), \tag{107}$$

where

$$\rho = \frac{\alpha'\vec{k}^2}{E_N(-E/E_N)^{1-\delta}}, \tag{108}$$

and

$$C_0' = C_0[1-G(0)^2/g_1^2]^{-1}. \tag{109}$$

Equation (107) is in agreement with the result of Abarbanel and Bronzan.[3] Again, it holds for $a<-\delta$, i.e., $\rho \ll 1$.

For $a>-\delta$, $y(t) \to_{t\to-\infty} \infty$. Then

$$1-G(t)^2/g_1^2 \underset{t\to-\infty}{\sim} C_\infty e^{(\epsilon/2)(1+a)t}, \tag{110}$$

and

$$i\Gamma_R^{1,1}(E,\vec{k}^2) \underset{t\to-\infty}{\sim} -\alpha'\vec{k}_N^2 C_\infty'^{-(2/\epsilon)(\gamma-\delta)}(\vec{k}^2/\vec{k}_N^2)^{1-\gamma+\delta} \times[1+C_\infty'^{-(2/\epsilon)\delta}\rho^{-1-\delta}], \tag{111}$$

for $\rho \gg 1$. Again C_∞ and $C_\infty' = C_\infty[1-G(0)^2/g_1^2]^{-1}$ depend only on g and y. Finally for $a=-\delta+O(\epsilon^2)$, $y(t)$ approaches a constant as $t\to-\infty$, as does the scaling variable ρ. Working to leading order in ϵ, the y dependence in Eq. (100) can be dropped and we find

$$1-G(t)^2/g_1^2 \underset{t\to-\infty}{\sim} [G(0)^2/g_1^2]^{-1}[1-G(0)^2/g_1^2]e^{(\epsilon/2)t} \tag{112}$$

and

$$\begin{aligned} i\Gamma_R^{1,1}(E,\vec{k}^2) &\underset{E,\vec{k}^2\to 0}{\sim} -E_N[G(0)^2/g_1^2]^{(2/\epsilon)\gamma}(-E/E_N)^{1-\gamma}\{1+[G(0)^2/g_1^2]^{(2/\epsilon)\delta}\rho\} \\ &= -\alpha'\vec{k}_N^2[G(0)^2/g_1^2]^{(2/\epsilon)(\gamma-\delta)}(\vec{k}^2/\vec{k}_N^2)^{1-\gamma+\delta}\{1+[G(0)^2/g_1^2]^{(2/\epsilon)\delta}\rho^{-1-\delta}\}, \end{aligned} \tag{113}$$

for all values of ρ.

Equations (107), (111), and (113) provide an example of a general relation among the scaling indices. If we write

$$\begin{aligned} i\Gamma_R^{1,1}(E,\vec{k}^2) &= (-E/E_N)^{1-\gamma}F(\rho) \\ &= (\vec{k}^2/\vec{k}_N^2)^\nu G(\rho), \end{aligned} \tag{114}$$

then

$$\nu = \frac{1-\gamma}{1-\delta}, \tag{115}$$

in agreement with our results to leading order in ϵ.

If we go beyond the one-loop approximation we will still obtain scaling results analogous to Eqs. (107), (111), and (113) provided the effective β function $\beta_E + a\beta_W$ has an infrared-stable zero. The scaling indices γ and δ will again be given by the values of $\gamma_E + a\gamma_W$ and $(\zeta_E + a\zeta_W)/\alpha'$ at the Gell-Mann–Low zero. The integral representation of Eq. (82) will converge provided $\gamma<1$ and $[\gamma-(a+\delta)]<1$. Under these circumstances we can again show that the perturbation series is an asymptotic expansion both for small values of r_0 and for large values of $-E$ and (or) $\vec{k}^2$. Again the perturbation series is valid only to order $n<2/\epsilon$. The argument is essentially the same as for the $\vec{k}^2=0$ case.

IV. DISCUSSION

In this paper we have studied the problem of constructing solutions to the Reggeon field theory when the Pomeranchuk singularity has intercept 1. We have seen that this theory cannot be renormalized order by order in the perturbation series. In fact our integral representation for the intercept renormalization counterterm

$$\delta\Delta = (r_0^2/\alpha_0'^{D/2})^{2/\epsilon}\int_0^\infty \frac{dx}{x^2}[1-Z_3(x)^{-1}] \tag{116}$$

shows explicitly that $\delta\Delta$ has a branch point at $r_0=0$. This is the case even for $\epsilon=2$. The integral in Eq. (116) has a pole in ϵ at this point so $\delta\Delta$ is well defined only if one introduces a cutoff into the theory. One then finds that

$$\delta\Delta = -\frac{(r_0^2/\alpha_0')}{16\pi}\ln(r_0^2/\alpha_0'\Lambda) + O(r_0^2/\alpha_0'). \tag{117}$$

At least for small values of ϵ, or $(r_0^2/\alpha_0'^{D/2})(1/\Lambda)$, $\delta\Delta$ is positive, which means that the intercept of the bare Pomeranchuk pole must be greater than 1.

We have given a nonperturbative prescription for constructing the Pomeranchuk propagator by use of the integral representation

$$i\Gamma^{1,1}(E,\vec{k}^2) = -\int_0^\xi d\xi'[E_N Z_3^{-1}(\xi') + \alpha_0'\vec{k}_N^2(a+1)\xi'^a Z_2^{-1}(\xi')], \tag{118}$$

where

$$\xi = (-E/E_N) = (\vec{k}^2/\vec{k}_N^2)^{1/(1+a)}.$$

The renormalization constants Z_2 and Z_3 which enter into Eqs. (116) and (118) can be computed systematically using renormalization-group techniques as is described in Sec. III.

Equation (118) is useful for a variety of purposes. In Sec. III we saw how it could be used to derive the infrared behavior of the propagator. It also allowed us to see how the tachyon which is present in second order in the perturbation series is removed from the exact solution. The tachyon arose from a cancellation between the bare inverse propagator and the second-order approximation to the self-energy part. However, our integral representation for $\Sigma(E,\vec{k}^2)$ contained linear terms in E and $\vec{k}^2$ which exactly canceled the bare inverse propagator and left us with Eq. (118) which is explicitly free of tachyons. The cancellation of the bare inverse propagator is also crucial for the infrared behavior, since it goes to zero for small E and $\vec{k}^2$ more slowly than the full inverse propagator. One might be concerned that this cancellation would be destroyed if one altered the large E and $\vec{k}^2$ behavior of the theory by, for example, introducing a cutoff or introducing E and $\vec{k}^2$ behavior into r_0.[3] Such effects must be taken into account in a more general formulation of the Reggeon field theory. In fact under these circumstances we can obtain integral representations analogous to the ones given in Sec. III, and again demonstrate the cancellation of the bare inverse propagator. The infrared behavior of the theory appears to be unchanged provided r_0 approaches a constant for small values of E and $\vec{k}^2$.

We have also used our integral representation for the propagator to show that the perturbation series is an asymptotic expansion for small values of r_0 as well as for large values of E and/or $\vec{k}^2$. The perturbation series is accurate only to order $n < 2/\epsilon$, and to this order one can neglect all diagrams involving intercept renormalization counterterms. The small-r_0 result is important for the ϵ-expansion calculations of the critical exponents.[3,4] If one wishes to calculate them to nth order in ϵ, then one can imagine working with $\epsilon < 2n$ and use perturbation theory to calculate the renormalization-group functions. Of course at the end one may attempt to continue the answers to larger values of ϵ.[16]

The large $(E,\vec{k}^2)$ result is also of considerable importance. The full propagator will approach the bare one for large negative values of E and $\vec{k}^2$ provided one stays away from the cuts, i.e., provided $\alpha'\vec{k}^2/E < 2$. This means that for large positive values of t (negative values of $\vec{k}^2$) the leading l-plane singularity will be a pole. It is only in this case that one is guaranteed that the solution to the field theory satisfies full multiparticle t-channel unitarity in the l plane.[2]

Since the order ϵ^2 calculations of the critical exponents indicate that the ϵ expansion is not converging rapidly, it seems worthwhile to ask whether the approach presented here can be used to perform practical calculations directly in two dimensions. It is crucial for our procedure that an infrared-stable Gell-Mann–Low eigenvalue exist at each step of the calculation. As we have seen, this is the case in the one-loop approximation. If one wishes to go beyond the one-loop approximation in two dimensions, it is not possible to use perturbation theory to calculate the renormalization-group functions. Instead one must use the latest approximation to the propagator at each step of the calculation. This may actually be an advantage. Since the improved propagator has the type of infrared behavior that one expects in the exact solution, our iteration scheme may converge more rapidly than the perturbation series for small values of ϵ. Even if this is the case, the higher-order calculations will certainly be difficult.

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APPENDIX

We have seen in the text that for many purposes it is sufficient to study the self-energy part at $\vec{k}^2=0$. For simplicity we shall do so here. Let us start with the bubble diagrams of Fig. 3. We denote by $\Sigma_{n_1k_1;n_2k_2}$ the contribution to Σ of the diagrams in which the upper and lower lines have n_1 and n_2 inserted bubbles and k_1 and k_2 intercept counterterms, respectively. From Eq. (6) we see that

$$\Sigma_{n_1k_1;n_2k_2}(E,0)=\tfrac{1}{2}i(Cr_0{}^2)^{n_1+n_2}\frac{r_0{}^2}{(2\pi)^{D+1}}\frac{(n_1+k_1)!}{n_1!k_1!}\frac{(n_2+k_2)!}{n_2!k_2!}(-\delta\Delta)^{k_1+k_2}$$
$$\times\int dE'd^D\vec{q}(\alpha_0'\vec{q}^2+\Delta-E'-i\epsilon)^{-(n_1+k_1+1)}(\alpha_0'\vec{q}^{\,2}+\Delta-E+E'-i\epsilon)^{-(n_2+k_2+1)}$$
$$\times(\tfrac{1}{2}\alpha_0'\vec{q}^{\,2}+2\Delta-E'-i\epsilon)^{n_1(1-\epsilon/2)}(\tfrac{1}{2}\alpha_0'\vec{q}^{\,2}+2\Delta-E+E'-i\epsilon)^{n_2(1-\epsilon/2)}\,. \tag{A1}$$

The factorials count the number of ways of ordering the bubbles and counterterms on each line. Making use of the identity

$$x^{\alpha}=-\frac{\sin\pi\alpha}{\pi}\int_0^{\infty}dx'\frac{x'^{\alpha}}{x'+x} \tag{A2}$$

and introducing Feynman parameters to combine denominators with the same E dependence we find

$$\begin{aligned}\Sigma_{n_1k_1;n_2k_2}(E,0)=&\tfrac{1}{2}i(Cr_0^{\,2})^{n_1+n_2}\frac{r_0^{\,2}}{(2\pi)^{D+1}}\frac{(n_1+k_1)!}{n_1!k_1!}\frac{(n_2+k_2)!}{n_2!k_2!}(-\delta\Delta)^{k_1+k_2}\frac{\sin\pi n_1(1-\frac{1}{2}\epsilon)}{\pi}\frac{\sin\pi n_2(1-\frac{1}{2}\epsilon)}{\pi}\\&\times\int_0^1dz_1z_1^{\,n_1+k_1}\int_0^1dz_2z_2^{\,n_2+k_2}\int_0^{\infty}dx_1x_1^{\,n_1(1-\epsilon/2)}\int_0^{\infty}dx_2x_2^{\,n_2(1-\epsilon/2)}\int dE'd^D\vec{q}\\&\times[z_1(\alpha_0'\vec{q}^{\,2}+\Delta-E')+(1-z_1)(x_1+\tfrac{1}{2}\alpha_0'\vec{q}^{\,2}+2\Delta-E')-i\epsilon]^{-(n_1+k_1+2)}\\&\times[z_2(\alpha_0'\vec{q}^{\,2}+\Delta-E+E')+(1-z_2)(x_2+\tfrac{1}{2}\alpha_0'\vec{q}^{\,2}+2\Delta-E+E')-i\epsilon]^{-(n_2+k_2+2)}\,.\end{aligned} \tag{A3}$$

The E' integration can now be performed by closing the contour in either the upper or lower half plane and picking up the pole arising from the appropriate denominator. We next use the fact that

$$\int d^D\vec{q}(A\vec{q}^{\,2}+B)^{-N}=\pi^{D/2}A^{-D/2}B^{D/2-N}\frac{\Gamma(N-D/2)}{\Gamma(N)} \tag{A4}$$

to perform the $\vec{q}$ integration. The x_1 and x_2 integrations then yield β functions, and we finally find

$$\begin{aligned}\Sigma_{n_1k_1;n_2k_2}(E,0)=&(Cr_0^{\,2})^{n_1+n_2+1}\frac{4^{D/2}}{\Gamma(-1+\epsilon/2)}\frac{\Gamma(k_1+k_2-1+\frac{1}{2}\epsilon(n_1+n_2+1))}{\Gamma((-1+\frac{1}{2}\epsilon)n_1)\Gamma((-1+\frac{1}{2}\epsilon)n_2)}\frac{(-\delta\Delta)^{k_1+k_2}}{n_1!k_1!n_2!k_2!}\\&\times\int_0^1dz_1\int_0^1dz_2z_1^{\,n_1+k_1}z_2^{\,n_2+k_2}(1-z_1)^{-(1-\epsilon/2)n_1-1}(1-z_2)^{-(1-\epsilon/2)n_2-1}\\&\times(2+z_1+z_2)^{-D/2}[(4-z_1-z_2)\Delta-E]^{-[k_1+k_2-1+(\epsilon/2)(n_1+n_2+1)]}\,.\end{aligned} \tag{A5}$$

We see explicitly from Eq. (A5) that there is no difficulty in taking the limit $\Delta\to0$ provided E is off the positive real axis. Then using the mean-value theorem to remove the harmless factor $(2+z_1+z_2)^{-D/2}$ from the integrand, we find

$$\begin{aligned}\Sigma_{n_1k_1;n_2k_2}(E,0)=&A^{-D/2}(Cr_0^{\,2})^{n_1+n_2+1}\frac{4^{D/2}}{\Gamma(-1+\epsilon/2)}\\&\times\frac{\Gamma(k_1+k_2-1+\frac{1}{2}\epsilon(n_1+n_2+1))}{\Gamma(k_1+1+n_1\frac{1}{2}\epsilon)\Gamma(k_2+1+n_2\frac{1}{2}\epsilon)}\frac{(n_1+k_1)!}{n_1!k_1!}\frac{(n_2+k_2)!}{n_2!k_2!}(-\delta\Delta)^{k_1+k_2}(-E)^{1-k_1-k_2-(\epsilon/2)(n_1+n_2+1)}\,,\end{aligned} \tag{A6}$$

where $2\le A\le4$. Recalling that the graph is of order $n=n_1+n_2+1$ in $r_0^{\,2}$ and has $m=k_1+k_2$ factors of $\delta\Delta$, we see that the E dependence and the position of the pole arising from the overall ultraviolet divergence of the graph are in agreement with the general result given in the text. Clearly the position of the poles in ϵ would be unaffected by taking Δ and k^2 different from zero.

In addition to the simple pole at $\epsilon=2(1-k_1-k_2)/(1+n_1+n_2)$ arising from the overall ultraviolet divergence of the diagram, there is a pole of order $n_1+n_2+k_1+k_2$ at $\epsilon=2$. n_1+n_2 powers of $(\epsilon-2)^{-1}$ come from the inserted bubbles and k_1+k_2 powers from the intercept counterterms. Notice that in the neighborhood of $\epsilon=2$ the simple bubble graph is proportional to $r_0^{\,2}/(\epsilon-2)$. For $\epsilon=2$ this pole leads to a $\ln(-E)$ contribution to $\Sigma(E,0)$ as $E\to0$. On the other hand, the graphs with $N=n_1+n_2+k_1+k_2>0$ are proportional to $r_0^{\,2}[r_0^{\,2}/(\epsilon-2)]^N$ for $\Delta>0$. At each order in the coupling constant, they are the most divergent graphs in the limit $\epsilon\to2$, so one might hope that their sum would soften the small-E behavior of the simple bubble graph and lead to a reasonable approximation for Σ. In ϕ^4 theory the corresponding set of graphs does soften the infrared behavior and leads to the screening approximation.[17] Unfortunately, this is not the case in the Reggeon calculus because of the presence of the tachyon in the second-order approximation to the propagator. The leading l-plane singularity of the sum of the $\Sigma_{n_1k_1;n_2k_2}$ is a tachyon-tachyon cut, which has all of the bad features of the Pomeron-Pomeron cut of the simple bubble graph plus the added difficulty of having its intercept to the right of $l=1$.

We now turn to the ladder graphs of Fig. 2. They are a subset of the skeleton graphs discussed in the text. Denoting by Σ_n the contribution to Σ of those graphs with $n-1$ rungs on the ladder we have

$$\Sigma_n(E,0)=\frac{1}{2}\frac{(-ir_0{}^2)^n}{(2\pi)^{(D+1)n}}\times\int\prod_{i=1}^{n}d^D\vec{q}_i dE_i\prod_{i=1}^{n}(\tfrac{1}{2}E+E_i-\alpha_0'\vec{q}_i{}^2+i\epsilon)^{-1}(\tfrac{1}{2}E-E_i-\alpha_0'\vec{q}_i{}^2+i\epsilon)^{-1}\times\prod_{i=1}^{n}\{[E_{i+1}-E_i-\alpha_0'(\vec{q}_i-\vec{q}_{i+1})^2+i\epsilon]^{-1}+[E_i-E_{i+1}-\alpha_0'(\vec{q}_i-\vec{q}_{i+1})^2+i\epsilon]^{-1}\}\ . \tag{A7}$$

The first product of Green's functions in Eq. (A7) arises from the sides of the ladder and the second product from the rungs. A sum of two Green's functions is associated with each rung because of the two possible directions of propagation of these Pomerons.

For simplicity we shall consider the case $\epsilon \ll 1$. Then the integrals in Eq. (A7) can be evaluated in the following way. We first perform the E_1 integration. For the term containing the rung $[E_1-E_2-\alpha_0'(\vec{q}_1-\vec{q}_2)^2+i\epsilon]^{-1}$ we close the E_1 contour in the upper half plane, and for the rung $[E_2-E_1-\alpha_0'(\vec{q}_1-\vec{q}_2)^2+i\epsilon]^{-1}$ we close in the lower half plane. In either case we pick up only one pole from a Green's function associated with a side of the ladder, and we are left with a $\vec{q}_1$ integral of the form

$$\frac{1}{(2\pi)^D}\int d^D\vec{q}_1(2\alpha_0'\vec{q}_1{}^2-E-i\epsilon)^{-1}[\alpha_0'\vec{q}_1{}^2+\alpha_0'(\vec{q}_1-\vec{q}_2)^2-E_2-\tfrac{1}{2}E-i\epsilon]^{-1}=\frac{\Gamma(\frac{1}{2}\epsilon)}{(8\pi\alpha_0')^{D/2}}\int_0^1 dz[\alpha_0'q_2{}^2z(1-\tfrac{1}{2}z)-zE_2-E(1-\tfrac{1}{2}z)-i\epsilon]^{-\epsilon/2}\ . \tag{A8}$$

Now when we perform the E_2 integration we can ignore the contribution of the cut in E_2 arising from the term in Eq. (A8), because the discontinuity across this cut is of order ϵ. As a result, we can close the E_2 contour so as to pick up a single pole arising from a side of the ladder. The $\vec{q}_2$ integration can then be performed using Eqs. (A2) and (A4). Proceeding in this way we find that to leading order in ϵ

$$\Sigma_n(E,0)=\tfrac{1}{4}[-4r_0{}^2/(8\pi\alpha_0')^{D/2}\epsilon]^n\times\frac{(-E)^{1-n\epsilon/2}}{n!(1-n\epsilon/2)}\ . \tag{A9}$$

As a result, from Eqs. (35) and (38)

$$Z_3{}^{-1}(x)=\tfrac{3}{4}+\tfrac{1}{4}\exp[-4x^{\epsilon/2}/(8\pi)^{D/2}\epsilon] \tag{A10}$$

and

$$i\Gamma^{1,1}(E,0)=-(r_0{}^2/\alpha_0'^{D/2})^{2/\epsilon}\int_{x_1}^{\infty}\frac{dx}{x^2}Z_3{}^{-1}(x)\simeq E\left\{\frac{3}{4}+\frac{1}{4}\exp\left[-\frac{r_0{}^2}{(8\pi\alpha_0')^{D/2}\epsilon}(-E)^{-\epsilon/2}\right]\right\}\ . \tag{A11}$$

Again the tachyon, which is present in second order, disappears when the infinite set of graphs is summed. The essential singularity in $\Gamma^{1,1}(E,0)$ at $E=0$ was also found by Bronzan.[18] As we have seen in the text, it is not present in the full solution of the theory. The ladder graphs simply do not give a good indication of the small-E behavior of the propagator.

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[8]Throughout this paper we shall make use of dimensional regularization. All integrals we shall encounter are well defined for $\epsilon > 2$. After the integration has been carried out we can continue the answers to the value of ϵ of interest.

[9]Depending on the topology of the diagram, there may be additional factors of $\frac{1}{2}$ arising from closed loops.

[10]Equation (20) follows directly from Eq. (61) coupled with Eqs. (57)–(59).

[11]J. B. Bronzan (unpublished).

[12]Of course, we can treat the n-loop graphs by ordinary perturbation theory for $n < 2/\epsilon$.

[13]The use of this iteration procedure to construct both massless ϕ^4 theory and the Reggeon calculus is described in more detail in Ref. 6.

[14]The second dimensionless variable that we use here differs from that used in Ref. 6. The variables used in Ref. 6 are more suitable for a general construction of the theory, emphasizing the symmetry in E and $\vec{k}^2$. The variables used here seem to be more suitable for explicit calculation and in particular for a discussion of the scaling laws.

[15]This was pointed out to us by J. B. Bronzan (private communication).

[16]At first glance one might be concerned that the branch point in the propagator at $g=0$ would lead to nonanalyticity in the critical indices at $\epsilon=0$; however, this does not appear to be the case. The point is discussed in Ref. 6 for $\lambda\phi^4$, but the argument given there cannot be carried over directly due to the added complication that, in the present problem, the propagator depends on two variables. Probably the most convincing way to argue that the critical indices have a power-series expansion in ϵ is to reformulate the problem using the Wilson approach to the renormalization group. [J. B. Bronzan (unpublished).]

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Are two-dimensional Yang-Mills gauge theories "color" dependent?*

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If the symmetry of the theory under global transformations generated by the charges is normal, the physical states of the system must be "color" singlets. (This is analogous to the physical states of two-dimensional quantum electrodynamics being neutral.) Consequently, the local color currents vanish in physical states. The (two-dimensional) inhomogeneous Lorentz invariance of the theory is also discussed.

I. INTRODUCTION

In the past year, the discovery of asymptotic freedom in non-Abelian gauge theories[1] has been accompanied by enormous enthusiasm over the tantalizing possibility that this class of theory might also provide a mechanism for confining quarks. The hopes that exist in this direction arise from the observation that such theories are very infrared singular.[2] Calculations employing renormalization-group techniques indicate the effective coupling constant grows at large distances, which suggests it may be energetically favorable for the quanta of the theory to condense locally in regions of space. We have here a sort of Orwellian liberty, where one is free only as long as one does not wander off too far.

So far there are no firm calculations that actually support these hopes, or more ambitious speculations based upon them, in four-dimensional space-time. Of course entrapment might also occur in theories which are not of non-Abelian–gauge type, as indicated by several recent investigations.[3] Nevertheless, the basic aesthetic reasoning underlying Yang-Mills theories[4] is so appealing that it is urgent to explore further whether the behavior