[2]Z. Koba, H. B. Nielsen, and P. Olesen, Nucl. Phys. B40, 317 (1972); A. Białas, K. Fiałkowski, R. Wit, and K. Zalewski, Phys. Lett. 39B, 211 (1972); K. Fiałkowski, *ibid.* 41B, 379 (1972); Acta Phys. Pol. B4, 545 (1973); H. Harari and E. Rabinovici, Phys. Lett. 43B, 49 (1973).

[3]A. Jabs, Phys. Rev. D 7, 3368 (1973).

[4]A. Białas, K. Fiałkowski, and R. Wit, Nucl. Phys. B43, 413 (1972); K. Fiałkowski, and H. I. Miettinen, Phys. Lett. 43B, 61 (1973); L. Van Hove, *ibid.* 43B, 65 (1973); P. Pirilä and S. Pokorski, *ibid.* 43B, 502 (1973); Z. Koba, Acta Phys. Pol. B4, 95 (1973); D. Robertson and N. Sakai, Nuovo Cimento Lett. 7, 227 (1973).

[5]A. Białas, K. Fiałkowski, and K. Zalewski, Nucl. Phys. B48, 237 (1972).

[5a]*Note added in proof* P. K. Malhotra, D. S. Narayan, B. S. Chaudhary, and V. Gupta recently also considered both mechanisms occurring simultaneously; they use the term "mixed two-component model." See Tata Institute Reports No. TIFR-BC-74-8, TIFR-TH-74-18, TIFR-BC-73-8, TIFR-TH-73-40 (unpublished).

[6]M. G. Albrow, A. Bagchus, D. P. Barber, A. Bogaerts, B. Bošnjaković, J. R. Brooks, A. B. Clegg, F. C. Erné, C. N. P. Gee, D. H. Locke, F. K. Loebinger, P. G. Murphy, A. Rudge, and J. C. Sens, Phys. Lett. 44B, 207 (1973); 44B, 518 (1973).

[7]The two formulas which follow formula (4) in Ref. 3 contain a numerical error and must be corrected to read $2\langle n\rangle = \langle n\rangle_P + \langle n\rangle_F = \langle n\rangle_P + 1$, $f_2 = -\frac{11}{4}\langle n\rangle + \langle n\rangle^2 + \frac{7}{8}$.

[8]J. Karczmarczuk, Nuovo Cimento Lett. 6, 524 (1973).

[9]A. Jabs, University of Kaiserslautern report, 1973 (unpublished); Nucl. Phys. B52, 554 (1973).

[10]The values in Ref. 3 are derived for the asymptotic cases of zero particle mass and large total energy of the statistical ensemble. As we do not know where the asymptotic region begins, we assume that the results are valid also in the cases considered in this note.

PHYSICAL REVIEW D VOLUME 10, NUMBER 11 1 DECEMBER 1974

$f^0 \longrightarrow 4\pi$ decay in a relativistic quark model*

Dayanand Parashar†

Department of Physics and Astronomy, University of Hawaii, Honolulu, Hawaii 96822

(Received 20 June 1974)

Based on a relativistic version of the phenomenological couplings among hadrons in the broken SU(6) $\times$ O(3) quark model developed largely by Mitra, we calculate the decay width and the corresponding branching ratio for the four-body decay process $f^0 \longrightarrow 4\pi$. In addition, we *assume* that the dominant contribution to the process $f^0 \longrightarrow 4\pi$ comes from the mechanism $f^0 \rightarrow A_1\pi \rightarrow (\rho\pi)\pi \rightarrow 4\pi$. The model makes a very interesting prediction for the branching ratio $\Gamma(f^0 \rightarrow \pi^+\pi^+\pi^-\pi^-)/\Gamma(f^0 \rightarrow \pi\pi)$ which turns out to be 3.1%, in excellent agreement with the recent experimental value of $(3.7 \pm 0.7)\%$ determined by Anderson *et al*.

Ever since its discovery,[1] the f^0 meson has been a subject of extensive investigation. Although its existence has been confirmed and its quantum numbers $I^G(J^P) = 0^+(2^+)$ have been fairly well established in many experiments,[2] the information about its decay modes has been rather sparse. A particularly interesting process is the four-body decay mode $f^0 \rightarrow 4\pi$ which seems to have received some attention during the past few years. Some of the earlier estimates of this decay have been made by Ascoli *et al.*,[3] who obtain the branching ratio $\Gamma(f^0 \rightarrow \pi^+\pi^+\pi^-\pi^-)/\Gamma(f^0 \rightarrow \pi\pi) \sim 7\%$. The most likely mechanism suggested by these authors for this process is via $f^0 \rightarrow \rho\rho$, where the ρ mesons are emitted mainly in a relative s wave, and then each ρ meson decays into two pions thereby yielding a 4π final state. It is pointed out that if the d-wave amplitude is neglected, the $f^0 \rightarrow 4\pi$ amplitude can be completely determined[4] in the $\rho \rightarrow \pi\pi$ decay width. The other experimental estimate for this decay, reported by Oh and collaborators,[5] corresponds to the branching ratio $\sim(4.7 \pm 1.1)\%$. These investigators, however, make no mention about the mechanism for the decay in question. The branching ratio was further revised most recently by Anderson and collaborators[6] from a study of the reaction $\pi^+ d \rightarrow \pi^+\pi^+\pi^-\pi^- p p_s$, where p_s is the spectator proton. The value of the branching ratio established in this experiment is $(3.7 \pm 0.7)\%$.

Unfortunately, there have been comparatively few attempts to compute this decay process in a theoretical framework. A model for the $f^0 \rightarrow 4\pi$ decay was proposed by Banyai and Rittenberg[7] using the effective Lagrangian techniques, chiral dynamics, and vector-meson dominance. Contrary to the $\rho\rho$ mechanism advocated by Ascoli *et al.*,[3] the decay $f^0 \rightarrow 4\pi$ is assumed to proceed mainly via A_1 and ρ mesons in the intermediate state. The resulting branching ratio in this model is predicted to be 2.9%, in reasonable agreement with the experiment.

The purpose of the present investigation is to compute this decay and the corresponding branching ratio of the f^0 meson in a simple relativistic quark model developed largely by Mitra.[8] The relativistic couplings appropriate for the evaluation of these vertices are constructed in a phenomenological sense, thereby incorporating some degree of parametrization,[8,9] by making extensive use of the broken SU(6) × O(3) group structure. Here the rotation group O(3) is designed to incorporate the internal orbital excitation (L) of the quarks in the hadrons. The main advantage of this extended group over SU(6) lies in its remarkable capacity to accommodate experimentally known resonances up to $L=2$ in the most economical representation. The chief ingredient in this scheme is provided by a single-quark transition with the emission of a meson, regarded as a quantum of radiation (mass μ, energy ω_k) corresponding to the basic $\bar{q}qP$ coupling. The resulting "direct" and "recoil" terms are combined suitably in the form factor which not only gives a better form of parametrization but also introduces the desired symmetry breaking through the masses of the hadrons participating in the interaction. The empirical structure of the multiplying form factor for a supermultiplet transition from an initial $\bar{q}q$ state (mass M) to a final $\bar{q}q$ state (mass m) is assumed to have the general form

$$f_L(k^2)=g_L\mu^{-L-1}(\mu/m_\pi)^{1/2}(\mu/\omega_k)^{L\pm 1}(2M)\,, \tag{1}$$

the plausibility of whose structure is based on physical grounds.[8] Here g_L is the only adjustable dimensionless parameter. The exponent $(L+1)$ is used for the higher-wave and $(L-1)$ for the lower-wave supermultiplet transitions.

Now we apply the general prescriptions of the model to the study of the 4π decay mode of the f^0 meson. The four-body decay process $f^0 \to 4\pi$ is assumed to proceed mainly via A_1 and ρ mesons in the intermediate state according to the suggestion of Banyai and Rittenberg[7] and is represented by

$$f^0 \to A_1\pi \to (\rho\pi)\pi \to 4\pi.$$

The coupling structures governing this process are $f^0A_1\pi$, $A_1\rho\pi$, and $\rho\pi\pi$ for which explicit constructions have already been worked out[8] for the purpose of application. Since f^0 and A_1 mesons are supposed to belong to the $L=1$ supermultiplet, the vertex $f^0A_1\pi$ is governed by the self-coupling of the form

$$f^0A_1\pi:\ \frac{1}{\sqrt{3}}f^{\mu\nu}A_{1\nu}(\partial_\mu\pi)\,, \tag{2}$$

with the corresponding multiplying form factor given by

$$f_{f^0A_1\pi}=g_0(2m_f/\mu),\quad \mu=\text{pion mass}\,. \tag{3}$$

The vertex $A_1\rho\pi$ corresponds to a supermultiplet transition from $L=1$ to $L=0$, and can, therefore, be written as

$$A_1\rho\pi:\ \frac{1}{\sqrt{2}}(k_\mu\rho^\mu k_\nu+\mu^2\rho_\nu)A^\nu \tag{4}$$

to be multiplied by the form factor

$$f_{A_1\rho\pi}=g_1(2m_{A_1}/\mu^2)\,. \tag{5}$$

Similarly, the structure for the $\rho\pi\pi$ coupling can be written in the form

$$\rho\pi\pi:\ i\epsilon_{\alpha\beta\gamma}\rho^\alpha_\mu\pi^\beta\partial^\mu\pi^\gamma\,, \tag{6}$$

with the relevant form factor being

$$f_{\rho\pi\pi}=g_0(2m_\rho/\mu)\,. \tag{7}$$

It is now a matter of straightforward application to write down the invariant T-matrix element squared, dominated by the A_1- and ρ-meson poles, for the process $f^0\to 4\pi$. Using the four-body covariant phase-space calculations,[11,12] the expression for the decay width is given explicitly by

$$\Gamma(f^0\to\pi^+\pi^+\pi^-\pi^-)=\left(\frac{C_1C_2C_3}{2\sqrt{6}}\right)^2\left(\frac{g_0^{\,2}}{4\pi}\right)^2\left(\frac{g_1^{\,2}}{4\pi}\right)\left(\frac{4\pi^2}{\mu^2 m_f}\right)^3 I_p\,. \tag{8}$$

Here C_1, C_2, and C_3 are the SU(3) factors for the $fA_1\pi$, $A_1\rho\pi$, and $\rho\pi\pi$ vertices. The coupling constants g_0 and g_1 are given by

$$g_0^{\,2}/4\pi=0.03 \text{ and } g_1^{\,2}/4\pi=0.08\,. \tag{9}$$

These values are fixed by independent experimental comparisons. The phase-space integral I_p, appearing in Eq. (8), has the form

$$I_p=\int_{9\mu^2}^{(m_f-\mu)^2}\left(\frac{ds_1}{s_1}\right)\frac{\lambda^{3/2}(s_1,m_f^{\,2},\mu^2)}{[(m_{A_1}^{\,2}-s_1)^2+m_{A_1}^{\,2}\Gamma_{A_1}^{\,2}]}$$

$$\times\int_{4\mu^2}^{(\sqrt{s_1}-\mu)^2}\left(\frac{ds_2}{s_2}\right)\frac{\lambda^{1/2}(s_2,\mu^2,\mu^2)\lambda^{1/2}(s_1,s_2,\mu^2)}{[(m_\rho^{\,2}-s_2)^2+m_\rho^{\,2}\Gamma_\rho^{\,2}]}\,[-4\mu^2m_{A_1}^{\,2}+(s_2-s_1-\mu^2)^2]\,[-4\mu^2m_\rho^{\,2}+(s_2-s_1+\mu^2)^2]\,, \tag{10}$$

where s_1 and s_2 are Mandelstam-like variables defined in Ref. 10. The function $\lambda(a, b, c)$ is defined as

$$\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ca. \quad (11)$$

The phase-space integration, Eq. (10), is computed numerically to obtain an estimate for the decay width $f^0 \to 4\pi$. As a result of our calculations, we find the decay width[13]

$$\Gamma(f^0 \to \pi^+\pi^+\pi^-\pi^-) = 2.32 \text{ MeV}. \quad (12)$$

This result is based on the further assumption that f^0 and $f^{0\prime}$, both belonging to the same tensor nonet in the $L=1$ supermultiplet, are mixed in accordance with the ideal mixing angle such that f^0 contains only nonstrange quarks. The branching ratio corresponding to the process $f^0 \to 4\pi$ is predicted to be

$$\Gamma(f^0 \to \pi^+\pi^+\pi^-\pi^-)/\Gamma(f^0 \to \pi\pi) = 3.1\%, \quad (13)$$

in excellent agreement with the experimental value[6] of $(3.7 \pm 0.7)\%$. We also note agreement of our result with that of Ref. 7 because the SU(3) coupling scheme is the same for both approaches. The advantage of the present scheme is that it fits into a very inclusive framework which works well for a large class of decay processes.

Finally, we report briefly on the three-body decay mode $f^0 \to K\bar{K}\pi$ which may be regarded as proceeding via a two-body $K^*(892)\bar{K}$ channel in the intermediate state such that

$$f^0 \to K^*(892)\bar{K} \to (K\pi)\bar{K}.$$

The couplings appropriate for the evaluation of these vertices can be easily written down in the context of our model.[8] Since f^0 and $K^*(892)$ belong respectively to the $L=1$ and $L=0$ supermultiplets, the vertex $f^0K^*(892)\bar{K}$ is governed by the transition from $L=1$ to $L=0$ and the structural form of the coupling can be written as[8]

$$f^0K^*\bar{K}:\ i\epsilon_{\alpha\beta\gamma} f^{\alpha\mu}(\partial_\mu \bar{K})(\partial_\beta K^*_\gamma), \quad (14)$$

and the corresponding form factor is

$$f_{f^0K^*\bar{K}} = g_1(m_{\bar{K}}/\mu)^{1/2}(2m_f/m_{\bar{K}}^2). \quad (15)$$

Similarly, the $K^*(892)K\pi$ vertex is governed by the self-coupling ($L=0$ to $L=0$):

$$K^*K\pi:\ \bar{K}\tau_a K^{*\mu}(\partial_\mu \pi^a) + \text{H.c.}, \quad (16)$$

with the form factor

$$f_{K^*K\pi} = g_0(m_K/\mu)^{1/2}(2m_f/m_K). \quad (17)$$

Proceeding as before, the appropriate expressions for the decay width for the process $f^0 \to K\bar{K}\pi$ can be easily written down. After doing the necessary computation, we obtain the result

$$\Gamma(f^0 \to K\bar{K}\pi) = 1.61 \text{ MeV}, \quad (18)$$

and the corresponding branching ratio turns out to be

$$\Gamma(f^0 \to K\bar{K}\pi)/\Gamma(f^0 \to \pi\pi) = 2.1\%, \quad (19)$$

which is within the experimental upper bound[5] of about 7%.

Finally, we take a look at the $\eta\eta$ mode of the f^0 meson. The decay width for this mode turns out to be about 5.7 MeV, so that the branching ratio is predicted to be

$$\Gamma(f^0 \to \eta\eta)/\Gamma(f^0 \to \pi\pi) = 7.6\%. \quad (20)$$

This ratio can be compared whenever the experimental measurements become available.

I am indebted to Professor Peter N. Dobson, Jr. for very helpful and stimulating discussions. I also thank Professor S. F. Tuan and Professor Sandip Pakvasa for useful comments on the manuscript. Interest shown by Rose Nakamura and Sumi Makey of the East-West Center is highly appreciated.

---

*Work supported in part by the U. S. Atomic Energy Commission under Contract No. AT(04-3)-511.

†On leave of absence from Atma Ram Sanatan Dharma College, University of Delhi, Delhi, India.

[1]W. Selove *et al.*, Phys. Rev. Lett. 9, 272 (1962); J. J. Viellet *et al.*, *ibid.* 10, 29 (1963).

[2]M. Wahlig, E. Shibata, D. Gordon, D. Frisch, and I. Mannelli, Phys. Rev. 147, 941 (1966).

[3]G. Ascoli, H. B. Crawley, D. W. Mortara, and A. Shapiro, Phys. Rev. Lett. 21, 1712 (1968).

[4]There may, however, be uncertainties in the $\rho \to 2\pi$ decay width and also perhaps in the form factor for the $f^0 \to \rho\rho$ decay amplitude.

[5]B. Y. Oh *et al.*, Phys. Rev. D 1, 2494 (1970); see also, for instance, W. M. Bugg *et al.*, *ibid.* 7, 3264 (1973).

[6]J. C. Anderson *et al.*, Phys. Rev. Lett. 31, 562 (1973).

[7]L. Banyai and V. Rittenberg, Nucl. Phys. B15, 199 (1970).

[8]A. N. Mitra, in *Particles and Fields*, edited by H. H. Aly (Gordon and Breach, New York, 1970), pp. 125–211. Also see D. Parashar, Ph.D. thesis, University of Hawaii, 1974 (unpublished).

[9]D. Parashar and P. N. Dobson, Jr., Phys. Rev. D 9, 185 (1974); D. Parashar and D. L. Katyal, *ibid.* 9, 1420 (1974).

[10]K. T. Mahanthappa and E. C. G. Sudarshan, Phys. Rev. Lett. 14, 163 (1965).

[11]R. Kumar, Phys. Rev. 185, 1865 (1969).

[12]D. Parashar and P. N. Dobson, Jr., Phys. Rev. D (to be published).

[13]Strictly speaking, contributions due to other possible mechanisms such as $f^0 \to \rho\rho \to (\pi\pi)(\pi\pi)$, $f^0 \to \epsilon\epsilon \to (\pi\pi)(\pi\pi)$, ϵ being the $I=0$ scalar meson, etc., should be taken into account in the calculation of the decay rate for the process $f^0 \to 4\pi$. Unfortunately at this stage the coupling structures for these channels are unknown, at least in the context of the quark-model formulation adopted in the present investigation. Moreover, Banyai and Rittenberg (Ref. 7) have shown that the $\rho\rho$ model of Ascoli *et al.* (Ref. 3) is in some sense complementary to their own model. Our result [Eq.(12)], therefore, admits of appropriate corrections due to contributions from these mechanisms.

PHYSICAL REVIEW D VOLUME 10, NUMBER 11 1 DECEMBER 1974

Relation among structure functions for massive currents*

Myron Bander and Mythili Rangachari
Department of Physics, University of California, Irvine, Irvine, California 92664
(Received 18 March 1974)

A relation among structure functions for processes involving massive photons (deep-inelastic electron scattering, $e^+ + e^-$ annihilation into hadrons and massive lepton pair production) is presented. This relation is valid in a variety of models. A rough comparison with experimental data shows this relation to be grossly violated. Some speculations on this violation are presented.

Reactions involving currents with large momentum squared are believed to probe fundamental constituents of hadronic physics. Among these we find not only deep-inelastic electron scattering, but likewise $e^+ + e^-$ annihilation into hadrons and the production of massive lepton pairs. This pointlike constituent structure supposedly manifests itself in scaling behavior of the rates for these processes.

Though the parton picture,[1] or to a lesser extent the operator production expansion on the light cone,[2] may be most directly tied to the concept of fundamental point constituents, the hypothesized scaling behavior is consistent with more prosaic views of hadronic reactions: multiperipheral, Regge-pole exchange, etc. In this note we obtain a relation between the above-mentioned current processes that, under specific assumptions, may be common to all models.

At this juncture we must point out that the entire scaling structure may be suspect in light of recent experiments on $e^+ + e^-$ annihilation,[3] which (at least at present energies) do not indicate the desired scaling. We shall have more to say on this later; for the moment let us proceed on the assumption that all reactions will scale.

Before stating the relation central to this article we review and establish the necessary notation. $e^+ + e^-$ annihilation into hadrons is described in terms of $R(q^2)$, the ratio of the cross section for this process to the cross section for $e^+ + e^- \to \mu^+ + \mu^-$. In the scaling limit $R(q^2)$ is taken to be independent of q^2. Deep-inelastic electron scattering is presented in terms of the usual structure functions $mW_1(\nu, q^2)$, $\nu W_2(\nu, q^2)$ which are to approach the scaling limits $F_1(x)$, $F_2(x)$ with $x = q^2/2m\nu$. The third process we study is inclusive lepton pair production, specifically $p + p \to \mu^+ + \mu^- + \cdots$. Let q^μ be the four-momentum of the lepton pair and p_1, p_2 the momenta of the incident protons. This process is governed by the structure function

$$W = -(2\pi)^6 \frac{E_1 E_2}{m^2} \sum_n \langle p_1, p_2 | J_\mu | n \rangle \langle n | J^\mu | p_1, p_2 \rangle \times \delta^4(p_1 + p_2 - p_n - q) . \quad (1)$$

Letting $x_i = q^2/2p_i \cdot q$, $s = 4p_1 \cdot p_2$, we obtain

$$\frac{d\sigma}{dx_1 dx_2} = \frac{4\pi\alpha^2}{3s} \frac{V(s; x_1, x_2)}{(x_1 x_2)^2} , \quad (2)$$

with

$$\frac{V(s; x_1, x_2)}{x_1 x_2} = 4m^2 \int \frac{d^4 q}{q^2} \delta(x_1 - q^2/2p_1 \cdot q) \times \delta(x_2 - q^2/2p_2 \cdot q) W . \quad (3)$$

In the scaling limit $V(s; x_1, x_2)$ ceases to depend on s.

The following relation between the above structure functions is proposed:

$$\lim_{x_1, x_2 \to 0} \frac{[2x_1 F_1(x_1) + F_2(x_1)][2x_2 F_1(x_2) + F_2(x_2)]}{8V(x_1 x_2)} = R . \quad (4)$$

We shall briefly indicate, in two classes of models, the conditions necessary for the validity of this relation: