the elastic slope ($R^{*2} \simeq 5$–$6\ \mathrm{GeV}^{-2}$).[12]

The fact that R^* is distinct from R means that the inclusive process $A+B \rightarrow B+X$ gives no direct information on the hypothetical increase of R with increasing mass. Such information may be forthcoming when one adopts a specific model for the decay of the large radius excitations. It is plausible that the momentum distribution of the decay products of these unusual objects may be rather different from that observed from objects of lower mass. In particular one can argue that the effective temperature of these states may grow with their radii, leading perhaps to an energy-dependent component in the production of particles at large transverse momenta. Further discussion of these ideas—and their implementation in a specific model—is reserved to a separate publication.

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Note on two-particle correlations in two-component models

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The two-particle correlation function $f_2 = (\Delta n)^2 - \langle n \rangle$ is calculated for the case where two independent incoherent mechanisms simultaneously contribute to high-energy particle production. It is pointed out that a possible discrepancy between f_2 values observed in pp collisions and in pure $\bar{p}p$ annihilation events could be explained in terms of the phase-space description of the pionization component.

Many correlation phenomena in high-energy hadronic particle production are easily understood in terms of a two-component picture, i.e., by the idea that particle production proceeds via two distinct dynamical mechanisms or processes. Consider, for example, the fully integrated inclusive two-particle correlation function for undiscriminated particles $f_2 = \langle n(n-1)\rangle - \langle n\rangle^2 = (\Delta n)^2 - \langle n\rangle$. Experimentally, as a function of $\langle n\rangle$, f_2 first decreases and then increases.[1,2] This increase in f_2 can be obtained by the superposition of two particle-number distributions either of which can have arbitrary (within their possible range) values f_{21}, f_{22}, respectively. The only requirement is that the distributions yield different average numbers $\langle n\rangle_1 \neq \langle n\rangle_2$.[2–5]

However, this only works if one excludes the possibility that both mechanisms act only simultaneously. Moreover, the derivations of the two-component formula for f_2 so far excluded not only this case but in fact considered only the case where either the first or the second mechanism acts, but never both simultaneously, in a collision.[5a] Besides the theoretical interest in the general case there are some experimental data[6] which can be interpreted as the simultaneous occurrence of the two mechanisms of pionization and fragmentation as specified, for example, in Ref. 9. Therefore in this note we present the formulas for the general case.

We have

$$\sigma = \sum_{n=1}^{(\infty)} \sigma_n ,$$

where σ is the cross section for the type of events considered (inelastic collisions, inelastic collisions with fixed number of neutrals, etc., elastic collisions excluded), and σ_n is the cross section for producing n new hadrons in such an event. We then write

$$\sigma_n = \int \left| \sum_{k=0}^{n} A_{k,\,n-k}^{(n)}(\xi_1, \xi_2, \ldots, \xi_n) \right|^2 \prod_{i=1}^{n} d\xi_i , \tag{1}$$

where $A_{k,\,n-k}^{(n)}(\xi_1, \xi_2, \ldots, \xi_n)$ is the amplitude for production of k particles by mechanism 1 and (simultaneously) $n-k$ particles by mechanism 2 in a state characterized by $\xi_1, \xi_2, \ldots, \xi_n$. The quantum numbers of the initial state are omitted in our notation.

We assume that the amplitudes $A_{k,\,n-k}^{(n)}$ that belong to different values of k are incoherent, i.e., interference terms between them vanish. That is, k does not form a quantum number of an intermediate state but represents an additional quantum number of the final state. Hence

$$\sigma_n = \sum_{k=0}^{n} \int |A_{k,\,n-k}^{(n)}(\xi_1, \xi_2, \ldots, \xi_n)|^2 \prod_{i=1}^{n} d\xi_i$$

$$\equiv \sum_{k=0}^{n} \alpha_{k,\,n-k}^{(n)} \tag{2}$$

$$= \alpha_{n,\,0}^{(n)} + \alpha_{0,\,n}^{(n)} + \sum_{k=1}^{n-1} \alpha_{k,\,n-k}^{(n)} . \tag{3}$$

Physically, this means that in each collision we can, in principle, decide how many of the n produced particles are produced via mechanism 1 and how many via mechanism 2. One may imagine that this is achieved by process 1 populating different phase-space regions from process 2, or by observing other characteristics (charge, isospin, baryon number, etc.) of the two mechanisms. Such an incoherence assumption has been made in all two-component calculations so far. Whether this is possible for mechanisms actually conceived in high-energy multiparticle production is another question not dealt with here. We write further

$$\begin{aligned} \alpha_{n,\,0}^{(n)} &= \alpha_1 w_1(n) , \\ \alpha_{0,\,n}^{(n)} &= \alpha_2 w_2(n) , \\ \alpha_{k,\,n-k}^{(n)} &= \alpha_{12} w_1(k) w_2(n-k) , \end{aligned} \tag{4}$$

where α_1 now is proportional to the probability that mechanism 1 acts but 2 does not ("exclusive" cross section), and $w_1(n)$ is the normalized probability that a total number of n particles is produced if only mechanism 1 acts. α_2 means the same for mechanism 2, and α_{12} is proportional to the probability that both mechanisms 1 and 2 act simultaneously in a collision. It is

$$\alpha_1 + \alpha_2 + \alpha_{12} = \sigma . \tag{5}$$

In writing $\alpha_{k,\,n-k}^{(n)} = \alpha_{12} w_1(k) w_2(n-k)$ we make the additional assumption that the two mechanisms are independent of one another. Correlations induced by energy-momentum conservation can be neglected in first order since normally the kinematical boundaries are far away.

In order to be able to recover the previous results in their old form, we want to express our result in terms of "inclusive" cross sections. We define $\sigma_1 \equiv \alpha_1 + \alpha_{12}$ = ("inclusive") cross section that in a collision particle production proceeds via mechanism 1, irrespective of mechanism 2 acting or not in the same collision. $\sigma_2 \equiv \alpha_2 + \alpha_{12}$ = the same for mechanism 2. Using (5) we obtain $\alpha_1 = \sigma - \sigma_2$, $\alpha_2 = \sigma - \sigma_1$, $\alpha_{12} = \sigma_1 + \sigma_2 - \sigma$. We have $\sigma_1 + \sigma_2 \geq \sigma$ because by our definition σ_1 includes partly the same final states as σ_2. One may say that the inclusive cross sections σ_1 and σ_2 partially overlap.

Division by σ gives us probabilities: $\alpha_1/\sigma = 1 - \sigma_2/\sigma$ = probability that only mechanism 1 acts, and so on. The normalized probability that a total number of n particles are produced per collision can then be written in the form

$$w(n) = (1 - \sigma_2/\sigma) w_1(n) + (1 - \sigma_1/\sigma) w_2(n) + (\sigma_1/\sigma + \sigma_2/\sigma - 1) \sum_{\nu=1}^{n-1} w_1(\nu) w_2(n-\nu) \tag{6}$$

and

$$\langle n \rangle \equiv \sum_{n=1}^{\infty} n w(n) = (1 - \sigma_2/\sigma) \sum_{n=1}^{\infty} n w_1(n) + (1 - \sigma_1/\sigma) \sum_{n=1}^{\infty} n w_2(n) + (\sigma_1/\sigma + \sigma_2/\sigma - 1) \sum_{n=2}^{\infty} \sum_{\nu=1}^{n-1} n w_1(\nu) w_2(n-\nu) .$$

Realizing that the last double sum can be written as

$$\sum_{n=2}^{\infty}\sum_{l=1}^{n}\sum_{\substack{m=1\\ l+m=n}}^{n}(l+m)w_1(l)w_2(m)=\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}[\,lw_1(l)w_2(m)+mw_1(l)w_2(m)]$$

$$=\langle l\rangle_1+\langle m\rangle_2$$

we have

$$\langle n\rangle=(\sigma_1/\sigma)\langle n\rangle_1+(\sigma_2/\sigma)\langle n\rangle_2\,. \tag{7}$$

An analogous procedure for $(\Delta n)^2\equiv\sum_{n=1}^{\infty}(n-\langle n\rangle)^2w(n)$ gives us finally our general result

$$f_2=(\sigma_1/\sigma)f_{21}+(\sigma_2/\sigma)f_{22}+\{[(\sigma_1/\sigma)(1-\sigma_1/\sigma)]^{1/2}\langle n\rangle_1-[(\sigma_2/\sigma)(1-\sigma_2/\sigma)]^{1/2}\langle n\rangle_2\}^2$$
$$+2\{[(\sigma_1\sigma_2/\sigma^2)(1-\sigma_1/\sigma)(1-\sigma_2/\sigma)]^{1/2}-(1-\sigma_1/\sigma)(1-\sigma_2/\sigma)\}\langle n\rangle_1\langle n\rangle_2\,. \tag{8}$$

The previous calculations mentioned above meant the specific cases $A_{k,\,n-k}^{(n)}=0$ for $k\neq 0,\,n$ in (1), $\sigma_n=\alpha_{n,\,0}^{(n)}+\alpha_{0,\,n}^{(n)}$ in (3) (omitting the sum), $\alpha_{12}=0$ in (5), and $\sigma_1/\sigma+\sigma_2/\sigma=1$ in (6). This can be interpreted as those cases where particle production proceeds only via mechanism 1 or mechanism 2 but never via both simultaneously. It results in the well-known formula

$$f_2=(\sigma_1/\sigma)f_{21}+(\sigma_2/\sigma)f_{22}+(\sigma_1\sigma_2/\sigma^2)(\langle n\rangle_1-\langle n\rangle_2)^2\,, \tag{9}$$

which is identical with formula (4) of Ref. 3 with $\sigma_1/\sigma=\sigma_2/\sigma=\frac{1}{2}$.[7]

Consider now the opposite extreme, i.e., that in each collision both processes occur simultaneously. In this case, $\sigma_1/\sigma=\sigma_2/\sigma=1$ and hence

$$f_2=f_{21}+f_{22}\,. \tag{10}$$

Here f_2 increases with rising $\langle n\rangle$ only if at least either of f_{21} or f_{22} increases.

In all other cases the above statement that f_2 increases while f_{21} and f_{22} are arbitrary remains valid because the last curly-bracket term in (8) never is negative. The quantitative behavior of the increase in f_2, however, is modified by that term and also by the different coefficients in the first curly-bracket term.

It would be interesting to know the f_2 values of either process separately. Generally it is not easy to disentangle the contributions from either process in a model-independent way. In Ref. 8 an attempt was made to isolate the pionization contribution (mechanism 1) from the fragmentation contribution (mechanism 2) by considering only pp events with at least six or at least eight prongs. The data thus selected showed an increasing f_2 in the range $\langle n^-\rangle\approx 3-5$. Assuming that fragmentation only contributes to 2- and 4-prong events this result would mean that the pionization value f_{21} itself increases. On the other hand, in Ref. 5 data on $\bar{p}p$ collisions with and without annihilation have been used to calculate f_2 values. The result is that f_2 seems to rise with increasing $\langle n\rangle$ for $\bar{p}p$ events without annihilation but decreases in pure annihilation events. Obviously, in pure annihilation events there is only pionization but no fragmentation, according to the conception of these processes.[9] So, here pionization shows decreasing instead of increasing f_2 values. This contradiction could easily be resolved in terms of our statistical-model calculations as presented in Ref. 3. There, pionization was described by phase-space computations, and it was shown that a statistical ensemble yields $f_{21}\sim-\langle n\rangle$ if the total energy of the ensemble is kept fixed, but that it yields $f_{21}\sim+\langle n\rangle^2$ if some fluctuation in the total energy is admitted.[10] In pp collisions such a fluctuation can easily arise from fluctuations in inelasticity, i.e., from the fact that the escaping protons (excited or not) retain a variable fraction of their initial energy. In $\bar{p}p$ annihilation, on the contrary, pionization takes all of the available energy and no fluctuations are possible. In fact, the annihilation data on f_{21} presented in Ref. 5 within their error bars are consistent with $f_{21}=-\frac{2}{3}\langle n\rangle$ and $f_{21}=-\frac{3}{4}\langle n\rangle$ as calculated in Ref. 3 for different versions of phase space.

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$f^0 \longrightarrow 4\pi$ decay in a relativistic quark model*

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Based on a relativistic version of the phenomenological couplings among hadrons in the broken SU(6) × O(3) quark model developed largely by Mitra, we calculate the decay width and the corresponding branching ratio for the four-body decay process $f^0 \longrightarrow 4\pi$. In addition, we *assume* that the dominant contribution to the process $f^0 \longrightarrow 4\pi$ comes from the mechanism $f^0 \to A_1\pi \to (\rho\pi)\pi \to 4\pi$. The model makes a very interesting prediction for the branching ratio $\Gamma(f^0 \to \pi^+\pi^+\pi^-\pi^-)/\Gamma(f^0 \to \pi\pi)$ which turns out to be 3.1%, in excellent agreement with the recent experimental value of (3.7 ± 0.7)% determined by Anderson *et al*.

Ever since its discovery,[1] the f^0 meson has been a subject of extensive investigation. Although its existence has been confirmed and its quantum numbers $I^G(J^P) = 0^+(2^+)$ have been fairly well established in many experiments,[2] the information about its decay modes has been rather sparse. A particularly interesting process is the four-body decay mode $f^0 \to 4\pi$ which seems to have received some attention during the past few years. Some of the earlier estimates of this decay have been made by Ascoli *et al.*,[3] who obtain the branching ratio $\Gamma(f^0 \to \pi^+\pi^+\pi^-\pi^-)/\Gamma(f^0 \to \pi\pi) \sim 7\%$. The most likely mechanism suggested by these authors for this process is via $f^0 \to \rho\rho$, where the ρ mesons are emitted mainly in a relative s wave, and then each ρ meson decays into two pions thereby yielding a 4π final state. It is pointed out that if the d-wave amplitude is neglected, the $f^0 \to 4\pi$ amplitude can be completely determined[4] in the $\rho \to \pi\pi$ decay width. The other experimental estimate for this decay, reported by Oh and collaborators,[5] corresponds to the branching ratio ~(4.7 ± 1.1)%. These investigators, however, make no mention about the mechanism for the decay in question. The branching ratio was further revised most recently by Anderson and collaborators[6] from a study of the reaction $\pi^+d \to \pi^+\pi^+\pi^-\pi^-pp_s$, where p_s is the spectator proton. The value of the branching ratio established in this experiment is (3.7 ± 0.7)%.

Unfortunately, there have been comparatively few attempts to compute this decay process in a theoretical framework. A model for the $f^0 \to 4\pi$ decay was proposed by Banyai and Rittenberg[7] using the effective Lagrangian techniques, chiral dynamics, and vector-meson dominance. Contrary to the $\rho\rho$ mechanism advocated by Ascoli *et al.*,[3] the decay $f^0 \to 4\pi$ is assumed to proceed mainly via A_1 and ρ mesons in the intermediate state. The resulting branching ratio in this model is predicted to be 2.9%, in reasonable agreement with the experiment.

The purpose of the present investigation is to