To obtain a quantitative value of the multiplicity, we used the q^2, ω' parametrization averaging over all q^2 since essentially no q^2 dependence was observed. The highest ω' bin was eliminated since it had poor acceptance at forward angles. Figure 3 shows that a simple exponential function of the recoil angle appears adequate to fit the data and continue satisfactorily to the forward direction.[4] The estimated charged-particle multiplicity was 1.12 ± 0.11 at the low-ω' point and 1.70 ± 0.17 at the high-ω' point. An estimated systematic correction of $\sim(14 \pm 10)\%$ was added for absorption of slow protons in the target and this gave final values of 1.24 ± 0.15 and 1.96 ± 0.20 (Table I). These values are in quite good agreement with the Cornell[5] and DESY[6] measurements using incident electrons.

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[2] See F. Brasse, rapporteur's talk, in *Proceedings of the Sixth International Symposium on Electron and Photon Interactions at High Energy, Bonn, Germany, 1973*, edited by H. Rollnik and W. Pfeil (North-Holland, Amsterdam, 1974). See also Refs. 5–7.

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PHYSICAL REVIEW D VOLUME 10, NUMBER 11 1 DECEMBER 1974

Pion production in high-energy collisions*

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It is noted that the pion production by 200- and 300-GeV/c protons is in agreement with the prediction of an empirical formula, which is generally applicable to high-energy proton–light-nucleus collisions as well as to proton-proton collisions.

Recently, Baker *et al.*[1] have measured the pion production by 200- and 300-GeV/c protons from a Be target and noted that the model of Hagedorn and Ranft[2] disagrees with the data by a factor of 2 to 3, and an old empirical formula[3] disagrees somewhat more with the data. In this addendum, I wish to note that (1) the above data are in good agreement with the prediction of an empirical formula[4] published in 1973 which was revised to include data from 20 to 1500 GeV/c, and (2) the formula can be applied to proton–light-nucleus collisions as well as to proton-proton collisions.

In general, the number of pions produced in an inclusive reaction is related to the double differential cross section by[2]

$$\frac{d^2N}{dPd\Omega}=\frac{1}{\sigma_a}\frac{d^2\sigma}{dPd\Omega}\left[\frac{\text{pions}}{\text{sr(GeV}/c)\text{ interacting proton}}\right], \qquad (1)$$

where σ_a is the inelastic cross section of the reaction. Data at lower energies show that[5,6]

$$\frac{d^2N}{dPd\Omega}(p+Z\to\pi^\pm+\text{anything}) \approx \frac{d^2N}{dPd\Omega}(p+p\to\pi^\pm+\text{anything}), \qquad (2)$$

where Z represents a light nucleus such as Be or Al. It is expected that Eq. (2) holds at higher energies. We may thus obtain the pion yields in p-Be collisions from those in p-p collisions.

For p-p collisions, $\sigma_a = 32.7$ mb at 200 and 300 GeV, and we have, up to 1500 GeV,[4]

$$\frac{d^2\sigma}{dPd\Omega} = AP_mX(1-X)\exp(-BX^C - DP_t)\left[\frac{\text{mb}}{\text{sr}(\text{GeV}/c)}\right], \tag{3}$$

where X is the longitudinal momentum P_l divided by P_m, the maximum kinematically allowed value of P_l, P_t is the transverse momentum of the pion ($= P\sin\theta \approx P\theta$, θ being the production angle in radians and P the pion momentum in GeV/c), and the four parameters are given in Table I. Thus,

$$\frac{d^2N}{dPd\Omega} = A'P_mX(1-X)\exp(-BX^C - DP_t)\left[\frac{\text{pions}}{\text{sr}(\text{GeV}/c)\text{ interacting proton}}\right], \tag{4}$$

where $A'(\pi^+) = 2.385$ and $A'(\pi^-) = 1.572$ is expected to hold for proton-light nucleus collisions as well as for p-p collisions.

To compare formula (4), which is for a thin target and in terms of interacting protons, with the one-collision-length target data of Baker *et al.* in terms of incident protons, one multiplies (4) by 0.37, the often quoted maximum target efficiency for an external target.[7] Figure 1 shows the comparison of the prediction of the empirical formula with the recently obtained experimental data.[1] In view of the good agreement, one concludes that formula (4) represents a good estimate of pion production in high-energy p-light-nucleus collisions as well as p-p collisions. In light of the current interest in high-energy neutrino physics, formula (4) should be particularly useful in estimating neutrino spectra and fluxes at high-energy accelerators.

It is important to note that for given P_t and X, $d^2N/dPd\Omega$ is linearly proportional to P_m, which is consistent with scaling[8] and limiting fragmentation.[9] (For details see Ref. 4.) Therefore the pion production data in p-light-nucleus collisions between 20 and 300 GeV (see Refs. 1 and 6) do support the scaling and limiting fragmentation, as do the data in p-p collisions between 20 and 1500 GeV.[6,10] The early data on pion production in p-nucleus collisions between 10 and 70 GeV,[5,11] however, are inconsistent[12] with scaling and limit-

TABLE I. Values of parameters.

Pion	Parameters A	B	C	D
Negative	51.403	5.732	1.333	4.247
Positive	77.793	3.558	1.333	4.727

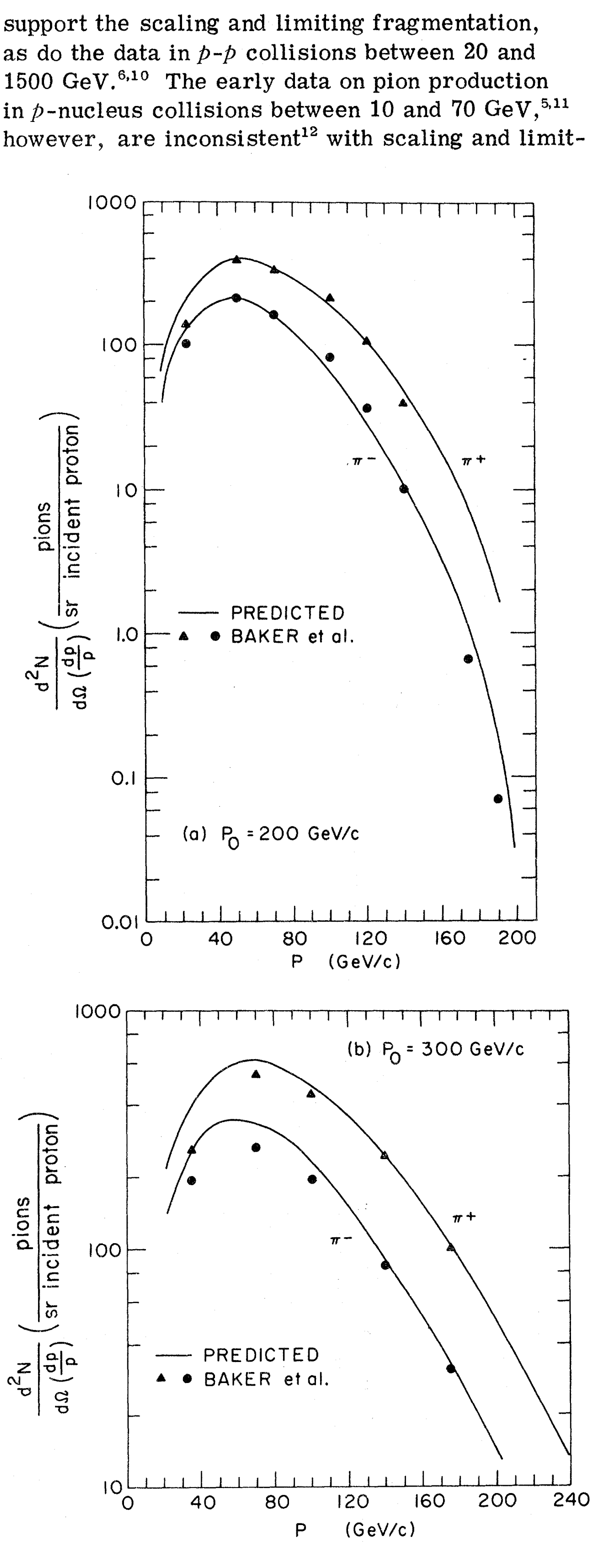

FIG. 1. Predicted and measured pion yields at 3.6 mrad at (a) 200 and (b) 300 GeV/c incident momenta. The curves are applicable to p-light-nucleus as well as p-p collisions.

ing fragmentation. It is most likely that the normalizations of these experiments[5,11] are not consistent with one another, and therefore an extrapolation[3] based on these data disagrees with the data at 200 and 300 GeV.[1]

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Semiclassical model for rising cross sections*

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The optical model is extended to incorporate diffractive excitations. The physical significance of cross sections rising indefinitely with energy becomes manifest in this model, and predictions result for various inclusive cross sections. These predictions accord well with existing data and open new avenues of approach to problems such as the production of particles with large transverse momenta.

Recent experiments[1] indicate that a general feature of high-energy scattering cross sections may be a slow but steady increase in magnitude with increasing energy. Such behavior is of course consistent with general asymptotic bounds[2] and has been demonstrated to be the plausible consequence of a number of field-theoretic and dispersion-theoretic models.[3] What has not been clear is whether rising cross sections can be consistent with a simple optical picture[4] of high-energy scattering. This is a disturbing point for two reasons:

(i) The existence of a large number of open channels would seem to make high-energy scattering ideally suited to a semiclassical treatment.

(ii) For fixed large energies, such a model seems to provide quite a good description[5,6] of the angular dependence of the differential cross sections.

In this paper we will show how rising cross sections can be accommodated in the optical model of Chou and Yang.[4] We obtain as a result several predictions for inclusive cross sections which admit a simple intuitive explanation and accord rather well with available data. Some further speculations are also made linking these effects to the production of particles with very large transverse momenta.

We begin by reviewing briefly the structure of the Chou-Yang model.[4] A particle A is assumed to be characterized in impact parameter space by a matter distribution $D_A(\vec{b})$. For the scattering of particles A and B one calculates an effective den-