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Inelasticity parameters of $\pi\pi$ scattering

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We predict the inelasticity η of $\pi\pi$ P and F waves using the Veneziano model for $\pi\pi \to \pi\omega$. The scale of the Veneziano amplitude is fixed from the ω width in $\omega \to 3\pi$ decay.

INTRODUCTION

The Veneziano model[1] for elastic $\pi\pi$ scattering gives good results for the low-energy amplitudes, although it is not quite correct, as shown by the large couplings of daughter trajectories. This makes us hopeful that the Veneziano model[2] for $\pi^+\pi^- \to \omega^0\pi^0$ may also work reasonably well at low energies, thus enabling us to calculate inelasticity parameters η for $\pi\pi$ partial waves in the isovector state, i.e., P and F waves. The assumption that the dominant inelastic channel is $\pi\pi \to \pi\omega$ is crucial to the credibility of our results. Simple $\pi\pi \to \pi\pi\pi\pi$ is clearly negligible, as the P wave shows no sudden inelasticity at the 4π threshold. Besides $\pi\omega$, the remaining quasi-two-body states are $\rho\rho$ and πA_2. Our calculations should thus be good to the πA_2 threshold at 1450 MeV, and perhaps beyond.

We shall show that the $g(1680)$ and $\rho(2275)$ couplings to $\pi\omega$ in this model are reasonable and that our results for inelasticities are consistent with the data available,[3-5] within their large errors. These two facts indicate that our predictions may be approximately correct, and hence can be useful to $\pi\pi$ phase-shift investigators, who want to know, e.g., whether they can safely assume elasticity of the F wave at a given energy. This is important as it is impossible to extract many parameters from the inaccurate data used in phase-shift analysis. Conversely, more detailed data on P and F waves could be used as a check on the Veneziano model.

Below we describe the determination of the scale λ of the model, and present the P- and F-wave inelasticities. Finally we calculate $g(1680)$ and $\rho(2275)$ couplings and compare results with what data are available.

THE SCALE FACTOR λ

Writing the amplitude for $\pi\pi \to \pi\omega$, $T(s,t,u)$, as $\epsilon_{\alpha\beta\gamma\delta}P_1^\alpha P_2^\beta P_3^\gamma \epsilon^\delta(M)A(s,t,u)$ with notation as in Fig. 1, the amplitude A is given by

$$\lambda[\mathrm{B}(1-\alpha(s), 1-\alpha(t)) + \mathrm{B}(1-\alpha(t), 1-\alpha(u)) + \mathrm{B}(1-\alpha(u), 1-\alpha(s))]\,.$$

B is the beta function, s, t, and u are Mandelstam variables, and $\alpha(s)+\alpha(t)+\alpha(u)=2$. On continuing the amplitude to the region where s, t, and u are all positive, it describes the decay of ω^0 into three pions, and s, t, and u become the effective masses[2] of the three pion pairs.

As pointed out by Goldberg *et al.*,[6] the Veneziano model reduces essentially to the Gell-Mann–Sharp–Wagner[7] pole model for ω decay. Taking the partial width as 8.97 MeV, we find $\lambda^2 = 8586$ GeV^{-6}. Here $\lambda/\alpha' = G_{\rho\pi\pi}G_{\omega\rho\pi}$, so that λ essentially fixes the ρ coupling to $\omega\pi$.[3]

THE P- AND F-WAVE INELASTICITIES

The unitarity relation $T - T^\dagger = iTT^\dagger$, when taken between elastic $\pi\pi$ states and saturated with only $\pi\pi$ and $\pi\omega$ intermediate states, gives

$$1-\eta_l^2 = \frac{q_1^3 q_3^3}{64\pi^3}\int(\cos\theta - \cos\theta'\cos\theta'')A(s,t,u) \times A^*(s',t',u')P_l(\cos\theta) \times d\cos\theta\, d\cos\theta''\, d\phi''\,, \tag{1}$$

where η_l is the inelasticity of the lth partial wave, θ is the $\pi\pi \to \pi\pi$ scattering angle, θ'' and ϕ'' are the scattering angles from the initial $\pi\pi$ state to the intermediate $\pi\omega$ state, and

$$\cos\theta' = \cos\theta\cos\theta'' + \sin\theta\sin\theta''\cos\phi''\,. \tag{2}$$

q_1 and q_3 are P_1 and P_3 in the center-of-mass system. We took the Veneziano[2] form for A, writing the $\alpha(s)$-dependent terms as

$$\frac{\lambda}{\Gamma(\alpha(s))}\left[\frac{\Gamma(1-\alpha(t))}{\Gamma(\alpha(u))} + \frac{\Gamma(1-\alpha(u))}{\Gamma(\alpha(t))}\right] \times \frac{\pi}{\sin\pi\alpha(s)\cosh\pi\alpha_I + i\cos\pi\alpha(s)\sinh\pi\alpha_I}\,, \tag{3}$$

where $\alpha(s)$ and α_I are real and imaginary parts of the trajectory, and α_I depends linearly on s, being $\alpha' m_\rho\Gamma_\rho$ and $\alpha' m_g\Gamma_g$ at ρ and g masses, respectively. The results are only weakly dependent on this assumption, which is the simplest giving the correct widths to the ρ and g mesons.

Figure 2 shows the results for P and F waves

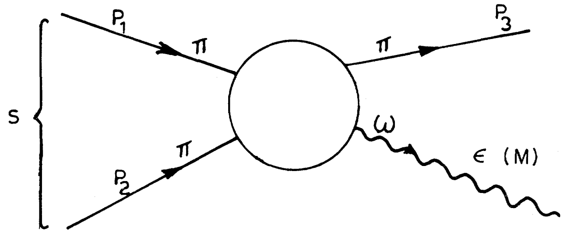

FIG. 1. Kinematic notation.

for $1-\eta^2$ proportional to the inelastic cross section.

Below 1.13 GeV the P wave is very nearly elastic, η decreasing from threshold to a value of 0.997 at 99 GeV, then increasing again to a maximum at 1.09 GeV. This is due to cancellation between the polar term and the tu beta function, and results in an elastic P wave up to 1.13 GeV, a characteristic feature of the model. Above that the inelasticity rapidly increases to a maximum at 1.3 GeV.

The authors in Ref. 5 find $1-\eta^2$ to be about $1\frac{1}{2}$ times our value at 1.13–1.15 GeV, which agrees within the errors, although this does not mean much as the data are consistent with an elastic P wave also. They also find a sudden onset of inelasticity at 1.13 GeV, but the data are too few to make a meaningful test of our predictions. Oh *et al.*[3] find a P wave more inelastic than ours, but their inelasticities may be too large; for example, their S_2 wave is much more inelastic than the more recent data of Cohen *et al.*[8] Carroll *et al.*[4] again find an inelastic P wave (in disagreement with Ref. 5). It has a maximum inelasticity at 1.3 GeV, as does our model, but is more inelastic. The experimental data are clearly inconsistent, but our model seems qualitatively correct. It will be interesting to see whether future analyses agree with our values.

Our F wave does not become inelastic until 1.35 GeV. The authors in Ref. 5 find a highly inelastic wave, which they attribute either to a large $\pi\omega$ cross section or to absorption effects distorting. On the other hand, those in Ref. 4 find an elastic and negligible F wave, agreeing with our prediction.

CALCULATION OF $g(1680)$ AND $\rho(2275)$ COUPLINGS

To check whether we can trust the predictions of the model at low energies, we use it to compute the couplings of the $g(1680)$ and its probable recurrence, $\rho(2275)$ (see Ref. 9) to $\pi\omega$. We calculate the cross section for $\pi\pi\to\pi\omega$ at the resonant mass, and equate this to the inelastic cross section of $\pi\pi$ scattering into this channel due to the

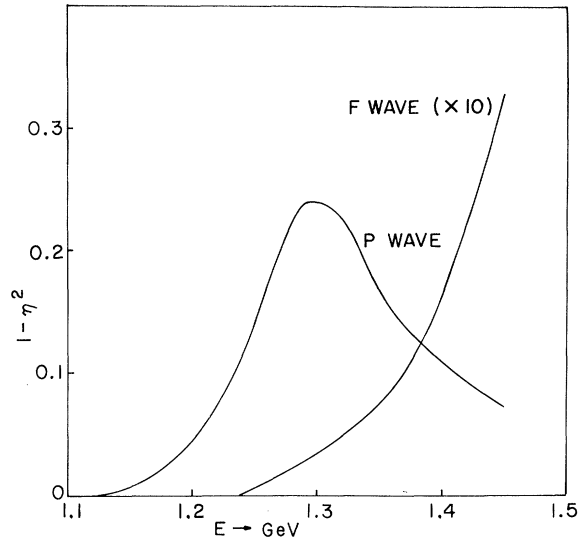

FIG. 2. Inelasticities of P and F waves.

resonance. At the g mass we have

$$\sigma_{\pi\pi\to\pi\omega}=\frac{\lambda^2 q_3^{\,2}}{32\pi q_1}\int_{-1}^{1}\frac{(1-\cos^2\theta)\,\alpha^2(u)\,\alpha^2(t)\,d\cos\theta}{[\alpha(s)-3]^2} \tag{4}$$

and

$$\sigma_{\pi\pi}(\text{inelastic})=\frac{4\pi m_g^{\,2}\,\Gamma_g^{\,2}}{q_1^{\,2}}\;\frac{7x_g\,x_g^{\pi\omega}+3x_{\rho''}\,x_{\rho''}^{\pi\omega}}{(s-m_g^{\,2})^2+m_g^{\,2}\,\Gamma_g^{\,2}}\,, \tag{5}$$

where we include the ρ'' daughter of the g meson. Assuming negligible daughter coupling, with $\Gamma_g=160\pm30$ MeV and the branching ratio to $\pi\pi$, x_g, taken as 0.4, we predict $x_g^{\pi\omega}$, the branching ratio to $\pi\omega$, to be 0.41. Varying Γ_g to its limits of 190 and 130 MeV, $x_g^{\pi\omega}$ becomes 0.29 to 0.62, respectively.

It has been claimed[8] that the $\rho(1710)$ has a $\pi\omega$ branching ratio of only 0.12 of the 4π cross section. However, the same authors cannot find any 2π decay of $\rho(1710)$ and so conclude that this resonance cannot be the $g(1680)$. Our result is consistent with present data[11] which give 0.5 for the 4π decay of the g meson. Although $\pi\omega$ is only one possible decay mode of $g(1680)$, along with $\rho\rho$, $\pi\pi\rho$, and $A_2\pi$, it is not surprising that the coupling to $\pi\omega$ nearly saturates the 4π coupling, as some of the other decays should be dual to $\pi\omega$, and we would commit double counting if we simply added them all together.

For the $\rho(2275)$ we predict $x^{2\pi}x^{\pi\omega}=0.062$ assuming a total width of 160 MeV.[9] This seems reasonable in view of the large number of channels open to the ρ.

Daughters should be included along with the $g(1680)$ and $\rho(2275)$ mesons. However, even if $\rho(1710)$ and $\rho(2100)$ are daughters, there is no evidence of appreciable coupling. The authors in Ref. 9 find $\rho(2100)$ decaying to 2π, while those in Ref. 10 find $\pi(1710)$ decaying to $\pi\omega$ but not $\pi\pi$. We therefore neglect daughters.

CONCLUSION

We have derived inelasticity parameters η_l for P and F waves in agreement with the scanty data available, using the Veneziano model for $\pi\pi \to \pi\omega$. The results should be fairly accurate as the model gives good results[6] for ω decay into 3π and $\pi^0\gamma$, and the predicted g coupling agrees with experiment. The results can be useful as a guide to phase-shift investigators, and as an indirect test of the model. The sudden onset of inelasticity in the P wave only at 1.13 GeV is a result of the presence of the nonpolar real part (t- and u-channel exchanges) and its verification would be a good test.

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