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[10]Note that although Γ is one-particle irreducible, T_{fn} need not be.
[11]An exception to this occurs when T_{fn} is an elastic on-shell amplitude in a theory where a single particle can be exchanged. One consequence of this is that all legs of a three-point function cannot simultaneously be kept on-shell in our argument. Indeed, our conclusions must fail in this case, because when all legs are on-shell, the three-point function has nothing else on which to depend.
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Radiative corrections to the nonleptonic Ξ decays and the $\Delta I = \frac{1}{2}$ selection rule

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The effects of radiative corrections on the $\Delta I = 1/2$ rule predictions for the nonleptonic Ξ decays are studied. The radiative corrections are estimated using standard perturbation theory, assuming that all particles are structureless. Two different Lagrangian models are considered and it is found that the radiative corrections are sensitive to the choice of the Lagrangian. Most of the results have the usual divergence difficulties which are overcome with the aid of a cutoff. The results for the branching ratio $\Gamma(\Xi^- \to \Lambda^0\pi^-)/\Gamma(\Xi^0 \to \Lambda^0\pi^0)$ give a corrected value in disagreement with the latest experimental data by as much as 5.3 standard deviations. The corrections to the asymmetry parameters lead to good agreement between $\Delta I = \frac{1}{2}$ and experiment.

I. INTRODUCTION

One of the more intriguing regularities in the weak, nonleptonic decays of strange particles is that they experimentally obey the isospin selection rule $\Delta I = \frac{1}{2}$ to a surprising accuracy. This selection rule, proposed several years ago,[1] requires that the weak-interaction Lagrangian responsible for these decays transforms as a spinor under the isospin group SU(2). Over the past years this selection rule has been tested for both K-meson and hyperon decays with results suggesting its general validity. However, with recent experiments[2] furnishing very precise measurements of these decays, it has become increasingly important to take into account electromagnetic corrections to the $\Delta I = \frac{1}{2}$ rule if one wishes to test the limits of its validity to the order of one percent. Such corrections have been estimated[3] for K^0, Λ^0, and $\Sigma^\pm$ decays and the results tend to suggest that $\Delta I = \frac{1}{2}$ may be violated as the result of a small admixture (~4%) of $\Delta I = \frac{3}{2}$ in the decay amplitudes.

In this paper we consider the radiative corrections to the $\Delta I = \frac{1}{2}$ rule for nonleptonic Ξ decay. Our calculations are made using standard perturbation theory and treating all particles as point particles, thus neglecting all effects due to strong interactions.[4] This feature of the calculations involving structureless particles may present a rather naive picture of these hadronic decays, but it has been adopted in order to obtain at least an estimate of the effects of the radiative corrections to Ξ decay which has not been reported before. Furthermore, we hope that with an understanding of the corrections presented here more discussion can be stimulated on the possible role of structure-dependent effects in the radiative corrections.

We consider two different Lagrangian models, a vector-axial-vector (V-A) interaction associated with the customary current$\times$current description of nonleptonic decays and a scalar-pseudoscalar (S-P) interaction. The main reason for choosing two different models is to determine how sensitive the radiative corrections may be to the form of the nonleptonic interaction.

We begin in Sec. II with a review of the properties of Ξ decay. In Sec. III we present a comparison of the predictions of the $\Delta I=\frac{1}{2}$ rule and the latest experimental data on these decays. In Sec. IV we discuss the two Lagrangian models used in our calculations and present the results of the radiative corrections based on these models in Sec. V. Section VI is devoted to a discussion of the results.

II. CHARACTERISTICS OF Ξ DECAY

The principal decays of the Ξ hyperons are the nonleptonic modes

$$\Xi^- \to \Lambda^0\pi^- , \tag{1a}$$

$$\Xi^0 \to \Lambda^0\pi^0 . \tag{1b}$$

Since the Ξ hyperon probably has spin $\frac{1}{2}$,[5] each of its two-body decays may be described by the complex amplitudes S^i and P^i corresponding to S and P waves and the superscript i refers to the charge of the Ξ. With proper normalization the decay rates for processes (1) are given by

$$\Gamma^i = f^i(|S^i|^2+|P^i|^2) , \tag{2}$$

where f^i represents a phase-space factor given by

$$f^i = \frac{q^i}{4\pi}\,\frac{2E^i_\Lambda}{m_\Xi} , \tag{3}$$

and q^i is the pion momentum in the Ξ^i rest frame and E^i_Λ is the energy of the emitted Λ^0.

Since the over-all phase is unmeasurable, only two other independent real parameters are necessary to characterize the decay. It is convenient to define decay-asymmetry parameters α^i, β^i, and γ^i in terms of S^i and P^i,

$$\alpha^i = \frac{2\,\mathrm{Re}(S^{i*}P^i)}{|S^i|^2+|P^i|^2} , \tag{4a}$$

$$\beta^i = \frac{2\,\mathrm{Im}(S^{i*}P^i)}{|S^i|^2+|P^i|^2} , \tag{4b}$$

$$\gamma^i = \frac{|S^i|^2-|P^i|^2}{|S^i|^2+|P^i|^2} , \tag{4c}$$

in which these parameters satisfy the relation $(\alpha^i)^2+(\beta^i)^2+(\gamma^i)^2=1$. Parity nonconservation implies that either $\alpha^i\neq 0$ or $\beta^i\neq 0$; time-reversal invariance of the decay would require the phases of the amplitudes S^i and P^i to be given by the corresponding $\Lambda\pi$ phase shifts.

III. PREDICTIONS OF THE $\Delta I = \frac{1}{2}$ RULE AND SUMMARY OF EXPERIMENTAL RESULTS

The $\Delta I=\frac{1}{2}$ rule places an additional restriction on the amplitudes for Ξ decay by relating the matrix elements of the decay processes (1). Since the Ξ hyperon has isospin $\frac{1}{2}$ and the Λ-π final state must be in an isospin state 1, the change in isospin, ΔI, may assume the values $\frac{1}{2}$, $\frac{3}{2}$. The $\Delta I=\frac{1}{2}$ rule prohibits the latter possibility. For each angular momentum state l, one has

$$\langle\Lambda^0\pi^-|T_l|\Xi^-\rangle = T_{l,1} = A_l^- , \tag{5a}$$

$$\langle\Lambda^0\pi^0|T_l|\Xi^0\rangle = \frac{1}{\sqrt{2}}\,T_{l,1} = A_l^0 , \tag{5b}$$

where $T_{l,I}$ is the transition matrix element corresponding to the isospin state I and $A^i_0=S^i$, $A^i_1=P^i$.

From Eq. (5) we conclude that $A_l^-=\sqrt{2}A_l^0$, or that

$$|S^-|^2=2|S^0|^2 ,\qquad |P^-|^2=2|P^0|^2 . \tag{6}$$

Combining Eqs. (2) and (6) and neglecting phase-space differences give

$$\frac{\Gamma^-}{\Gamma^0} = \frac{|S^-|^2+|P^-|^2}{|S^0|^2+|P^0|^2} = 2 . \tag{7}$$

Also, from Eqs. (4) and (6) one finds

$$\frac{\alpha^-}{\alpha^0} = \frac{\beta^-}{\beta^0} = \frac{\gamma^-}{\gamma^0} = 1 . \tag{8}$$

Equations (7) and (8) represent the predictions of the $\Delta I=\frac{1}{2}$ rule for Ξ decay.

The above predictions of the $\Delta I=\frac{1}{2}$ rule for Ξ decay must be compared with the current experimental data. The latest experimental measurements on Ξ decays have yielded the present world averages[6]

$$\Gamma^-/\Gamma^0 = 1.77\pm 0.06,\qquad \alpha^-/\alpha^0 = 0.783\pm 0.124 ,\qquad \beta^-/\beta^0 = 0.14\pm 0.32 . \tag{9}$$

By comparing Eqs. (7) and (8) with Eq. (9) we observe that there is nearly a 4-standard-deviation disagreement in the decay rate branching ratio between $\Delta I=\frac{1}{2}$ and experiment and about a 2- and 3-standard-deviation disagreement for the α and β decay-parameter ratios, respectively.

In making this comparison any discrepancies are expected to be due to (a) differences in phase space between the two final states, (b) electromagnetic radiative corrections to order $\alpha=\frac{1}{137}$, the fine structure constant, (c) structure-dependent effects due to strong interactions, (d) any $\Delta I=\frac{3}{2}$ transitions, and (e) final-state interactions. With regard to final-state interactions, the Ξ decays have the unique property that there are no effects of such interactions on the predictions of the $\Delta I=\frac{1}{2}$ rule. This feature is attributed to the fact that the $\Lambda\pi$ system must be in a $I=1$ state so that final-state interaction effects described by the $\Lambda\pi$ phase shifts cancel in the ratios appearing in Eqs. (7) and (8). In the next two sections we shall focus our attention on the corrections to the $\Delta I=\frac{1}{2}$ rule predictions for Ξ decay arising from (a) and (b).

IV. CHOICE OF LAGRANGIAN MODEL

In order to calculate the radiative corrections to nonleptonic Ξ decay we must first assume some form for the basic nonleptonic weak interaction. We shall describe processes (1) by a phenomenological point interaction. We have chosen two different Lagrangian models for this interaction. We first consider a simple scalar-pseudoscalar (S-P) type of interaction described by the Lagrangian (neglecting the anomalous magnetic moment contributions)

$$\mathcal{L}_{\rm int}=\bar\psi_\Lambda(A+B\gamma_5)\psi_\Xi\phi_\pi+\text{H.c.}\ , \tag{10}$$

where ψ_Ξ (ψ_Λ) represents the spinor field of the Ξ hyperon (Λ hyperon), ϕ_π is the field of the pion, and the constants A and B are the parity-violating and parity-conserving amplitudes, respectively, and are directly related to the S- and P-wave amplitudes

$$A=\left(\frac{2E_\Lambda}{E_\Lambda+m_\Lambda}\right)^{1/2}S\ ,\qquad B=\left(\frac{2E_\Lambda(E_\Lambda+m_\Lambda)}{p'^2}\right)^{1/2}P\ , \tag{11}$$

where E_Λ and p' represent the energy and three-momentum of the Λ hyperon in the Ξ rest frame. The matrix element and decay rate corresponding to Eq. (10) in the Ξ rest frame are given by

$$\mathfrak{M}_0=\bar u_\Lambda(p')(A+B\gamma_5)u_\Xi(p)\ , \tag{12}$$

$$\Gamma=\frac{q}{8\pi m_\Xi^2}\left[(m_+^2-m_\pi^2)|A|^2+(m_-^2-m_\pi^2)|B|^2\right], \tag{13}$$

where we have used an obvious notation for the masses and $m_+=m_\Xi+m_\Lambda$, $m_-=m_\Xi-m_\Lambda$.

We next consider the more customary vector-axial-vector (V-A) type of interaction based on the current × current model and described by the Lagrangian

$$\mathcal{L}'_{\rm int}=\bar\psi_\Lambda(A'+B'\gamma_5)\gamma_\lambda\psi_\Xi\frac{\partial\phi_\pi}{\partial x_\lambda}\ , \tag{14}$$

with the matrix element and decay rate given by

$$\mathfrak{M}'_0=\bar u_\Lambda(p')[m_-A'+m_+B'\gamma_5]u_\Xi(p)\ , \tag{15}$$

$$\Gamma'=\frac{q}{8\pi m_\Xi^2}\left[m_-^2(m_+^2-m_\pi^2)|A'|^2+m_+^2(m_-^2-m_\pi^2)|B'|^2\right]\ . \tag{16}$$

By inspection of Eqs. (12) and (15) it is clear that the two Lagrangian models are essentially equivalent[7] with the V-A Lagrangian generated from the S-P interaction by redefining the constants A and B by $A=A'm_-$ and $B=B'm_+$. This equivalence holds as long as all of the particles involved are kept on the mass shell. However, when using these Lagrangians to calculate the radiative corrections, the nonleptonic interaction will necessarily involve some particles off the mass shell, and thus we expect these corrections to be different for the two interaction models and this shall be borne out in the next section.

V. CALCULATION OF RADIATIVE CORRECTIONS

Although there is no completely satisfactory technique for calculating the radiative corrections to processes (1), we shall estimate these corrections by using one of the Lagrangian models for the nonleptonic interaction discussed in the preceding section and employing ordinary perturbation theory. Since we are ignoring structure-dependent effects the electromagnetic corrections to the strong-interaction renormalization diagrams will be ignored. One feature of the calculation to be expected is the presence of an ultraviolet cutoff.

A. Scalar-pseudoscalar model

We first calculate the radiative corrections using the S-P interaction model. The zeroth-order process is shown in Fig. 1(a) and the corresponding matrix element and decay rates are given by Eqs. (12) and (13). The radiative corrections to first order in α result from the contributions of the perturbation diagrams shown in Figs. 1(b)–1(g). We first consider the virtual corrections arising from diagrams 1(b)–1(e). The corrected matrix element for Ξ^- decay from these virtual effects to order α can be written as

$$\mathcal{M} = \mathcal{M}_0 + \mathcal{M}_b + \mathcal{M}_c + \mathcal{M}_d + \mathcal{M}_e \ . \tag{17}$$

The corrections due to the Ξ-hyperon self-energy contribution [diagram 1(a)] can be written as

$$\mathcal{M}_b = \bar{u}_\Lambda(p')(A + B\gamma_5)\frac{1}{\not{p} - m_\Xi}\Sigma(p)u_\Xi(p) \ , \tag{18}$$

where

$$\Sigma(p) = \frac{-i\alpha}{4\pi^3}\int d^4k\, \frac{\gamma^\mu(\not{p} - \not{k} + m_\Xi)\gamma_\mu}{k^2[(p-k)^2 - m_\Xi{}^2]} \ . \tag{19}$$

$\Sigma(p)$ is calculated by the usual regularization procedure

$$\Sigma(p) \to \lim_{\Lambda\to\infty;\ \lambda\to 0}\left\{\frac{-i\alpha}{4\pi^3}\int d^4k\, \frac{\gamma^\mu(\not{p} - \not{k} + m_\Xi)\gamma_\mu}{(k^2 - \lambda^2)[(p-k)^2 - m_\Xi{}^2]} - \frac{-i\alpha}{4\pi^3}\int d^4k\, \frac{\gamma^\mu(\not{p} - \not{k} + m_\Xi)\gamma_\mu}{(k^2 - \Lambda^2)[(p-k)^2 - m_\Xi{}^2]}\right\} , \tag{20}$$

where Λ is the ultraviolet cutoff and λ represents the fictitious photon mass introduced as an infrared cutoff. The evaluation of Eq. (20) must be carried out for $p^2 \neq m_\Xi{}^2$. Only after $\Sigma(p)$ is inserted between the spinor and the propagator do we take $p^2 = m_\Xi{}^2$. As usual we write

$$\Sigma(p) = A_1 + A_2(\not{p} - m_\Xi) + A_3(\not{p} - m_\Xi)^2 \ , \tag{21}$$

where A_1 and A_2 are numbers independent of p; A_3 is a 4×4 matrix finite at $\Lambda\to\infty$ and $p^2 \to m_\Xi{}^2$. Thus, between a propagator $1/(\not{p} - m_\Xi)$ and a spinor $u_\Xi(p)$, the A_3 term will not contribute. A standard calculation gives

$$A_1 = \frac{3\alpha}{2\pi} m_\Xi\left(\ln\frac{\Lambda}{m_\Xi} + \frac{1}{4}\right) \ , \tag{22}$$

$$A_2 = -\frac{\alpha}{2\pi}\left(\frac{9}{4} + \ln\frac{\Lambda}{m_\Xi} + 2\ln\frac{\lambda}{m_\Xi}\right) . \tag{23}$$

The contribution from A_1 is canceled from the mass counterterm $\delta m = A_1$. The contribution from A_2 is obtained by identifying the wave-function renormalization in this order and taking $(\not{p} - m_\Xi)^{-1}A_2(\not{p} - m_\Xi)u_\Xi(p) = \frac{1}{2}A_2 u_\Xi(p)$. Thus Eq. (18) becomes

$$\mathcal{M}_b = -\frac{\alpha}{4\pi}\left(\ln\frac{\Lambda}{m_\Xi} + 2\ln\frac{\lambda}{m_\Xi} + \frac{9}{4}\right)\mathcal{M}_0 \ . \tag{24}$$

The corrections to $\mathcal{M}$ from the pion self-energy [diagrams 1(c) and 1(d)] can be evaluated in much the same way. One finds

$$\mathcal{M}_c + \mathcal{M}_d = \frac{\alpha}{4\pi}\left(2\ln\frac{\Lambda}{m_\pi} - 2\ln\frac{\lambda}{m_\pi} - \frac{3}{4}\right)\mathcal{M}_0 \ . \tag{25}$$

Thus, for both the Ξ-hyperon and pion cases the self-energy effects give the same contribution to both the S- and P-wave amplitudes.

We now turn to the contribution from graph 1(e) which represents the vertex renormalization. This correction to the matrix element has the form

$$\mathcal{M}_e = \bar{u}_\Lambda(p')(A + B\gamma_5)\Gamma u_\Xi(p) = \bar{u}_\Lambda(p')(\tilde{A} + \tilde{B}\gamma_5)u_\Xi(p) \ , \tag{26}$$

where

$$\Gamma = \frac{-ie^2}{(2\pi)^4}\int \frac{(\not{p} - \not{k} + m_\Xi)(2\not{q} + \not{k})d^4k}{(k^2 - \lambda^2)[(q+k)^2 - m_\pi{}^2][(p-k)^2 - m_\Xi{}^2]} \ . \tag{27}$$

After a lengthy evaluation of Eq. (27) we find different corrections to the S- and P-wave amplitudes with

$$\tilde{A} = A\frac{\alpha}{4\pi}\xi_+ \ , \tag{28}$$

$$\tilde{B} = B\frac{\alpha}{4\pi}\xi_- \ , \tag{29}$$

where

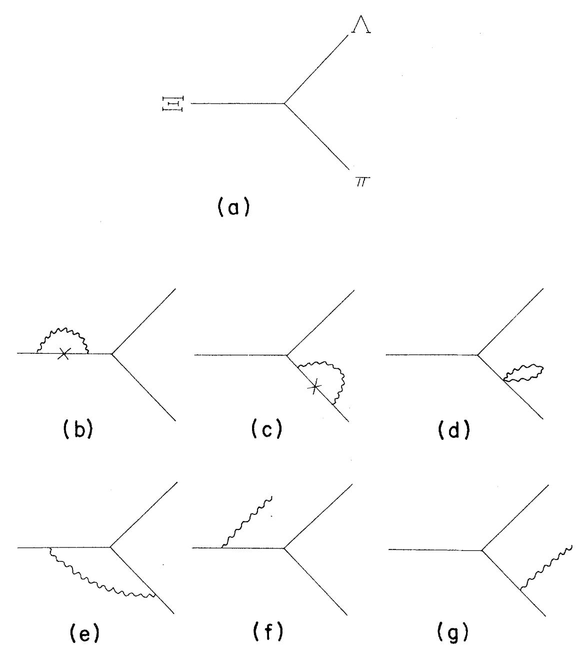

FIG. 1. Lowest-order perturbation diagrams in the scalar-pseudoscalar interaction model.

$$\xi_\pm = \frac{2E_\pi}{q}\left[\ln\frac{m_\Lambda}{\lambda}\ln\left(\frac{E_\pi - q}{E_\pi + q}\right) + Y\right] - \frac{E_\pi}{q}\ln\left(\frac{E_\pi - q}{E_\pi + q}\right) + \frac{13}{4} - 2\ln\frac{m_\Xi}{m_\pi} + 5\ln\frac{\Lambda}{m_\pi} \pm \frac{m_\Xi}{m_\Lambda}\left[\frac{x^2}{2m_\Xi q}\ln\left(\frac{E_\pi - q}{E_\pi + q}\right) + 2\ln\frac{m_\Xi}{m_\pi}\right], \tag{30}$$

where E_π represents the pion energy in the Ξ rest frame and

$$Y = \ln\left(\frac{E_\pi - q}{E_\pi + q}\right)\ln\left(\frac{2m_\Xi q}{m_\Lambda^2}\right) - \frac{1}{4}\ln^2\left(\frac{E_\Lambda - q}{E_\Lambda + q}\right) + \frac{1}{4}\ln^2\left(\frac{x^2 - 2m_\Xi q}{x^2 + 2m_\Xi q}\right) + \Phi\left(-\frac{E_\Lambda - q}{E_\Lambda + q}\right) - \Phi\left(-\frac{x^2 - 2m_\Xi q}{x^2 + 2m_\Xi q}\right), \tag{31}$$

$$x^2 = m_\Xi^2 - m_\Lambda^2 - m_\pi^2, \tag{32}$$

$$\Phi(z) = \int_1^z \frac{\ln|1+t|}{t}\,dt, \tag{33}$$

where $\Phi(z)$ is the Spence function.[8]

Finally, in order to deal with the infrared divergences we must include effects of inner bremsstrahlung (IB) photons [diagrams 1(f), 1(g)] with energies less than a maximum value ω which depends on the experimental resolution.[9] One finds that, neglecting terms of ω and higher,

$$|\mathfrak{M}_{IB}|^2 = |\mathfrak{M}_f + \mathfrak{M}_g|^2 = \frac{\alpha}{2\pi}\epsilon|\mathfrak{M}_0|^2, \tag{34}$$

where

$$\epsilon = \frac{E_\pi}{q}\ln\left(\frac{E_\pi + q}{E_\pi - q}\right)\left[1 + 2\ln\left(\frac{\omega}{\lambda}\right)\right] + 2 - 4\ln\left(\frac{2\omega}{\lambda}\right). \tag{35}$$

Then in calculating the Ξ^- decay rate $|\mathfrak{M}_0|^2$ is replaced by $|\mathfrak{M}|^2 + |\mathfrak{M}_{IB}|^2$ and as a result the infrared cutoff λ disappears.

We now state the radiative corrections to the Ξ^- decay rate. We find

$$\Gamma(\Xi^- \to \Lambda^0\pi^-, \gamma(\omega)) = \frac{q}{4\pi m_\Xi}\left[|S^-|^2\left(1 + \frac{\alpha}{2\pi}C_+\right) + |P^-|^2\left(1 + \frac{\alpha}{2\pi}C_-\right)\right], \tag{36}$$

where

$$C_+ = C_1 + C_2, \quad C_- = C_1 - C_2, \tag{37}$$

$$C_1 = \frac{2E_\pi}{q}\left[\ln\left(\frac{E_\pi - q}{E_\pi + q}\right)\ln\left(\frac{m_\Lambda}{\omega}\right) + Y\right] - \frac{2E_\pi}{q}\ln\left(\frac{E_\pi - q}{E_\pi + q}\right) + \ln\left(\frac{m_\Xi}{m_\pi}\right) + 3\ln\left(\frac{\Lambda}{m_\pi}\right) + 4\ln\left(\frac{m_\pi}{2\omega}\right), \tag{38}$$

$$C_2 = \frac{x^2}{2m_\Lambda q}\ln\left(\frac{E_\pi - q}{E_\pi + q}\right) + 2\frac{m_\Xi}{m_\Lambda}\ln\left(\frac{m_\Xi}{m_\pi}\right). \tag{39}$$

The expression $\Gamma(\Xi^- \to \Lambda^0\pi^-, \gamma(\omega))$ is the decay rate $\Gamma(\Xi^- \to \Lambda^0\pi^-)$ plus $\Gamma(\Xi^- \to \Lambda^0\pi^-\gamma)$ for photons with energy in the Ξ rest frame below ω.

We thus find for the Ξ-hyperon decay branching ratio with radiative corrections included

$$R = \frac{\Gamma(\Xi^- \to \Lambda^0\pi^-, \gamma(\omega))}{\Gamma(\Xi^0 \to \Lambda^0\pi^0)} = 2\left[1 + 0.034 + \frac{\alpha}{2\pi}(C_1 + \gamma^0 C_2)\right], \tag{40}$$

with the factor of 0.034 representing the phase-space corrections.

In order to obtain a numerical value for Eq. (40) we must choose values for ω and Λ. Setting[10] $\omega = 2$ MeV and $\Lambda = 2$ GeV and using $\gamma^0 = 0.84$, we obtain $(\alpha/2\pi)C_1 = 0.0049$ and $(\alpha/2\pi)\gamma^0 C_2 = 0.0025$. From these results it is evident that the over-all radiative corrections are slightly smaller than the phase-space correction and of the same sign. Incorporating our numerical results into Eq. (40) yields for the $\Delta I = \frac{1}{2}$ rule prediction for the Ξ decay rates with radiative corrections included

$$R = \frac{\Gamma(\Xi^- \to \Lambda^0\pi^-, \gamma(\omega))}{\Gamma(\Xi^0 \to \Lambda^0\pi^0)} = 2.083. \tag{41}$$

In Table I we present our numerical results for different values of the cutoff, Λ.

In a similar way the radiative corrections to the decay parameters can be calculated, and we find that the corrections to α^- and β^- are equal and given by

$$\frac{\alpha^-}{\alpha^0} = \frac{\beta^-}{\beta^0} = 1 + \gamma^0\frac{\Delta q}{q} - \frac{\alpha D}{2\pi}, \tag{42}$$

TABLE I. Radiative corrections to Ξ hyperon decay branching ratio $R=\Gamma(\Xi^- \to \Lambda^0\pi^-, \gamma(\omega))/\Gamma(\Xi^0\to\Lambda^0\pi^0)$ for various values of the ultraviolet cutoff, Λ, using the S-P interaction model.

Λ (GeV)	$\frac{\alpha}{2\pi}C_1$	$\frac{\alpha}{2\pi}\gamma_0 C_2$	R
1	0.0025	0.0025	2.078
2	0.0049	0.0025	2.083
4	0.0074	0.0025	2.088
8	0.0098	0.0025	2.093

where

$$D=\gamma^0 \frac{m_\Xi}{m_\Lambda}\left[\frac{x^2}{2m_\Xi q}\ln\left(\frac{E_\pi - q}{E_\pi + q}\right)+2\ln\left(\frac{m_\Xi}{m_\pi}\right)\right] , \qquad (43)$$

and Δq is the difference in momentum of the decay products between the $\Lambda^0\pi^-$ and $\Lambda^0\pi^0$ modes. Numerically we obtain $\gamma^0\Delta q/q=0.024$ and $\alpha D/2\pi=0.002$. Thus, for the decay parameters, the radiative corrections are cutoff-independent in the P-S interaction model but are also very small, being an order of magnitude smaller than the phase-space corrections. For the decay parameters we find

$$\frac{\alpha^-}{\alpha^0}=\frac{\beta^-}{\beta^0}=1.022 . \qquad (44)$$

B. Vector-axial-vector model

We now turn to the calculation of the radiative corrections to Ξ decay using the V-A interaction model. In this case the uncorrected matrix element and decay rates are given by Eq. (15) and (16). The lowest-order corrections arise from the perturbation diagrams shown in Fig. 2. In addition to the diagrams which arose in the S-P model there will be additional diagrams [2(e), 2(f), 2(h)] which have their origin in the extra derivative on the pion field appearing in $\mathcal{L}'_{\rm int}$. Thus, the matrix element with all virtual corrections added can be written as

$$\mathfrak{M}'=\mathfrak{M}'_a+\mathfrak{M}'_b+\mathfrak{M}'_c+\mathfrak{M}'_d+\mathfrak{M}'_e+\mathfrak{M}'_f+\mathfrak{M}'_g . \qquad (45)$$

In the case of diagram 2(e) its contribution to the corrected matrix element is given by

$$\mathfrak{M}'_e=\bar{u}_\Lambda(p')(A'+B'\gamma_5)\Gamma'_e u_\Xi(p)$$
$$=\bar{u}_\Lambda(p')(m_-\tilde{A}'+m_+\tilde{B}'\gamma_5)u_\Xi , \qquad (46)$$

where

$$\Gamma'_e=\frac{-ie^2}{(2\pi)^4}\int\frac{\gamma^\mu(-\not p-\not k+m_\Xi)\gamma_\mu}{(k^2-\lambda^2)[(p-k)^2-m_\Xi{}^2]}\,d^4k . \qquad (47)$$

A standard calculation similar to that for $\Sigma(p)$ yields

$$\Gamma'_e=\frac{3\alpha}{2\pi}m_\Xi\left(\ln\frac{\Lambda}{m_\Xi}+\frac{1}{4}\right) , \qquad (48)$$

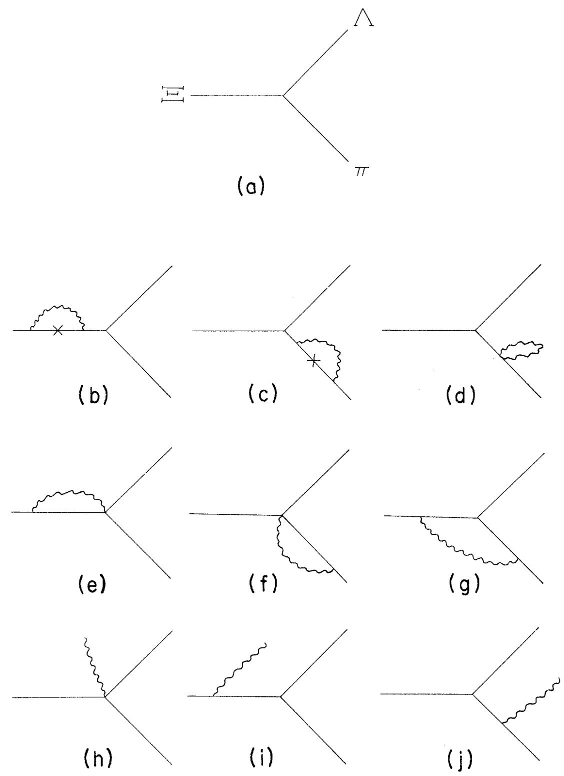

FIG. 2. Lowest-order perturbation diagrams in the vector–axial-vector interaction model.

so that

$$\tilde{A}'=\frac{\alpha}{4\pi}\kappa_+A' , \qquad (49)$$

$$\tilde{B}'=\frac{\alpha}{4\pi}\kappa_-B' , \qquad (50)$$

where

$$\kappa_\pm=\frac{-6m_\Xi}{m_\mp}\left(\ln\frac{\Lambda}{m_\Xi}+\frac{1}{4}\right) . \qquad (51)$$

Diagram 2(f) contributes to the corrected matrix element

$$\mathfrak{M}'_f=\bar{u}_\Lambda(p')(A'+B'\gamma_5)\Gamma'_f u_\Xi(p) , \qquad (52)$$

where

$$\Gamma'_f=\frac{-ie^2}{(2\pi)^4}\int\frac{(2\not q+\not k)d^4k}{(k^2-\lambda^2)[(q+k)^2-m_\pi{}^2]} . \qquad (53)$$

A straightforward calculation of Eq. (53) gives

$$\Gamma'_f=\frac{-3\alpha}{4\pi}\left(\ln\frac{\Lambda}{m_\pi}+\frac{3}{4}\right)\not q , \qquad (54)$$

so that Eq. (52) becomes

$$\mathfrak{M}_f' = \frac{-3\alpha}{4\pi}\left(\ln\frac{\Lambda}{m_\pi}+\frac{3}{4}\right)\mathfrak{M}_0' \,. \tag{55}$$

We note that in the case of diagram 2(f) the S- and P-wave amplitudes receive the same amount of contribution.

Finally, the contribution from diagram 2(h) which involves direct soft-photon emission can be shown to be very small compared to the contributions from the inner bremsstralung [diagrams 2(i) and 2(j)] and is thus neglected.

Upon adding all the virtual corrections and taking into account the inner bremsstrahlung emission, one can then calculate the corrected Ξ decay rate using $|\mathfrak{M}'|^2+|\mathfrak{M}'_{\rm IB}|$.[2] We find

$$\Gamma(\Xi^-\to\Lambda^0\pi^-,\gamma(\omega)) = \frac{q}{4\pi m_\Xi}\left[|S^-|^2\left(1+\frac{\alpha}{2\pi}A_+\right)+|P^-|^2\left(1+\frac{\alpha}{2\pi}A_-\right)\right], \tag{56}$$

where

$$A_+=C_+-G_+\,,\qquad A_-=C_--G_-\,, \tag{57}$$

and C_+, C_- are given by Eqs. (37)–(39) and

$$G_\pm=\frac{6m_\Xi}{m_\mp}\left(\ln\frac{\Lambda}{m_\Xi}+\frac{1}{4}\right)\,. \tag{58}$$

We thus obtain for the Ξ decay branching ratio with radiative corrections added

$$R=\frac{\Gamma(\Xi^-\to\Lambda^0\pi^-,\gamma(\omega))}{\Gamma(\Xi^0\to\Lambda^0\pi^0)}=2\left\{1+0.034+\frac{\alpha}{2\pi}\left[C_1-\frac{am_\Xi}{m_\Xi^2-m_\Lambda^2}+\gamma_0\left(C_2-\frac{am_\Lambda}{m_\Xi^2-m_\Lambda^2}\right)\right]\right\}\,, \tag{59}$$

with

$$a=6m_\Xi\left(\ln\frac{\Lambda}{m_\Xi}+\frac{1}{4}\right)\,. \tag{60}$$

Numerically, one finds for $\omega=2$ MeV, $\Lambda=2$ GeV that $a=3.98m_\Xi$ and that the over-all radiative correction to the branching ratio in Eq. (59) equals -0.02. Thus, in the case of the V-A interaction model the over-all radiative corrections are of the same order of magnitude as the phase-space correction but of opposite sign. Once again incorporating our numerical results into Eq. (59) yields for the $\Delta I=\frac{1}{2}$ rule prediction for the Ξ decay rates with radiative corrections included

$$R=\frac{\Gamma(\Xi^-\to\Lambda^0\pi^-,\gamma(\omega))}{\Gamma(\Xi^0\to\Lambda^0\pi^0)}=2.028\,. \tag{61}$$

For the decay-asymmetry parameters we find

$$\frac{\alpha^-}{\alpha^0}=\frac{\beta^-}{\beta^0}=1+\gamma^0\frac{\Delta q}{q}-\frac{\alpha}{2\pi}D'\,, \tag{62}$$

where

$$D'=\gamma^0\frac{m_\Xi}{m_\Lambda}\left[\frac{x^2}{2m_\Xi q}\ln\left(\frac{E_\pi-q}{E_\pi+q}\right)+2\ln\frac{m_\Xi}{m_\pi}-\frac{6m_\Lambda^2}{m_\Xi^2-m_\Lambda^2}\left(\ln\frac{\Lambda}{m_\Xi}+\frac{1}{4}\right)\right]\,. \tag{63}$$

Using the value $\Lambda=2$ GeV one obtains $(\alpha/2\pi)D'=-0.009$. Thus, in the V-A interaction model the radiative corrections to the decay parameters are cutoff-dependent and are also very small compared to the phase-space correction. For the decay-parameter ratios we find

$$\frac{\alpha^-}{\alpha^0}=\frac{\beta^-}{\beta^0}=1.033\,. \tag{64}$$

In Table II we present our numerical results for both R and the decay parameters for different values of Λ.

VI. DISCUSSION AND CONCLUSIONS

We now wish to examine how well the $\Delta I=\frac{1}{2}$ rule predictions for Ξ decays agree with the experimental data after the radiative corrections have been included.

Let us first consider the Ξ decay rate branching ratio. Using the S-P interaction we find from Table I values for R ranging from 2.078 to 2.093. Upon comparing these results with the experimental value of 1.77 ± 0.06, we observe that there still exists a discrepancy of up to 5.3 standard deviations. On the other hand, the V-A interaction yields, for the same values of the cutoff, corrections to R ranging from 2.080 to 1.923. These latter results reflect a smaller discrepancy with

TABLE II. Radiative corrections to Ξ hyperon decay branching ratio $R=\Gamma(\Xi^-\to\Lambda^0\pi^-,\gamma(\omega))/\Gamma(\Xi^0\to\Lambda^0\pi^0)$ and decay-asymmetry parameter ratios for various values of the ultraviolet cutoff, Λ, using the V-A interaction model.

Λ (GeV)	R	α^-/α^0	β^-/β^0
1	2.080	1.002	1.002
2	2.027	1.003	1.003
4	1.975	1.004	1.004
8	1.923	1.006	1.006

experiment, in fact, one as low as 2.5 standard deviations. This noticeable difference between the two models we have considered is due to the fact that in the S-P interaction the radiative corrections were of the same sign as the phase space correction, whereas in the V-A interaction the same two corrections were of opposite sign. As a result, the radiative corrections, assuming a vector–axial-vector interaction, tend to lower the predicted value for R and thus to decrease the discrepancy between theory and experiment.[11]

Let us now turn to a discussion of the decay-asymmetry parameters. Using the S-P interaction we obtain 1.022 for the cutoff-independent corrections to α^-/α^0 and β^-/β^0. Upon comparing this result with the experimental value for α^-/α^0 in Eq. (9) we observe that the agreement is good with only a 2 standard deviation discrepancy remaining. In contrast, the V-A interaction yields radiative corrections to the decay parameters which are cutoff-dependent with values for α^-/α^0 and β^-/β^0 ranging from 1.002 to 1.006 but still in good agreement with the α^-/α^0 data. A meaningful comparison in both interaction models with the β^-/β^0 data must await more precise measurements of β^0.

The remaining discrepancies revealed in the above discussion may be attributed to either structure-dependent effects or to the presence of $\Delta I=\frac{3}{2}$ transitions. The effects of possible $\Delta I\neq\frac{1}{2}$ transitions in hyperon decays have been previously studied[12] and we find that the results presented here tend to suggest that, if the S-P interaction correctly describes hyperon decays and if structure-dependent effects can be neglected, there is about a 5% contribution of $\Delta I=\frac{3}{2}$ in the S-wave amplitude of Ξ decay.[13] If in the event the current $\times$ current model is the correct picture for hyperon decays, our results suggest that only about a 2% contribution of $\Delta I=\frac{3}{2}$ in the S-wave amplitude is necessary to account fully for the remaining discrepancies.

In conclusion, we have found that the radiative corrections to the $\Delta I=\frac{1}{2}$ rule predictions for Ξ hyperon decay are small but cannot completely explain the existing discrepancy between theory and experiment. Our results suggest that there may indeed be a small amount of $\Delta I=\frac{3}{2}$ occurring in Ξ decay. However, a more definitive conclusion must await a better understanding of any structure-dependent effects which we have neglected throughout our calculations.

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[4] Final-state strong-interaction effects are taken into account by means of the $\Lambda\pi$ phase shifts if time-reversal invariance holds.

[5] A recently precise measurement of the Ξ hyperon spin has yielded the value of $\frac{1}{2}$ with higher spins excluded by about seven standard deviations. See C. Baltay *et al.*, Phys. Rev. D 9, 49 (1974).

[6] These averages were determined from the data of Ref. 5 and from the previous world averages taken from the Particle Data Group, Rev. Mod. Phys. 45, S1 (1973).

[7] In a similar way, other more complicated Lagrangians can also be shown to be equivalent to the S-P interaction. This is due to the fact that in Eq. (1) each decay is characterized by only two amplitudes so that the most general matrix element is of the form given by Eq. (12) where A and B are effective form factors. As long as all the particles involved are on the mass shell, A and B can be treated as constants and the choice of the Lagrangian made is of no physical importance.

[8] K. Mitchell, Philos. Mag. 40, 351 (1949).

[9] This is due to the fact that in the two-body decay processes (1) the Λ hyperon and the pion have definite energies E_Λ and E_π. Normally only the charged pion is observed and its energy is measured. If a real photon has also been emitted, then the pion energy will be less than E_π. However, due to experimental uncertanties in the measured pion energy, emission of a soft photon of maximum energy equal to this uncertainty becomes possible.

[10] In choosing this value we have taken the experimental uncertainty in the charged pion momentum to be about 2 MeV.

[11] It is in fact clear from our results that by choosing a large enough value for Λ it is possible to remove all of the discrepancy between theory and experiment.

[12] O. E. Overseth and S. Pakvasa, Phys. Rev. 184, 1663 (1969).

[13] It is difficult at this time to estimate the $\Delta I=\frac{3}{2}$ contribution in the P-wave amplitude owing to the uncertainties in the measured values of the Ξ^0 decay parameters.