

of  $\rho(b)$ , and are independent of the energy at which it takes on that form. If, however, we accept the energy dependence of the two-pion-exchange model hypothesized above, we predict  $\sigma_{pp}$ ,  $\sigma_{A=14}^{\text{incl}}$ , and  $\sigma_{A=64}^{\text{incl}}$  to increase by 36%, 27%, and 20% between  $E=10^3$  GeV and  $E=10^8$  GeV. The corresponding percentages based on the model of Cheng, Wu,

and Walker for  $pp$  scattering are 200%, 44%, and 22%. If either of these models is approximately correct, it will be difficult to detect the energy dependence of  $\sigma_{pp}^{\text{incl}}$ , because of the inherent difficulties of cosmic-ray experiments. The main hope of such experiments would be to detect (or rule out) a more rapid rise of the cross section.

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## Chiral perturbation theory and the magnetic moments of the baryon octet\*

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We have calculated, exactly to leading order in the chiral perturbation symmetry-breaking parameter  $\epsilon$ , the corrections to the Coleman-Glashow SU(3) relations among the magnetic moments of the baryon octet. The corrections are of order  $\epsilon^{1/2}$ . In general the corrections turn out to be at least as large as the zeroth-order term and hence the perturbation expansion breaks down. We discuss the reasons for this breakdown. We also derive sum rules which are independent of the leading-order corrections and hence valid to next order in chiral perturbation symmetry breaking, i.e.,  $\epsilon \ln \epsilon$ .

### I. INTRODUCTION

In the limit that SU(3) is an exact symmetry and assuming octet transformation properties for the photon, that is,  $Q=Q_3+(1/\sqrt{3})Q_8$ , it is well known that one obtains the Coleman-Glashow formulas<sup>1</sup> for the magnetic moments of the baryon octet.

Of course, SU(3) is not an exact symmetry. For those magnetic moments which have been experimentally measured,<sup>2</sup> that for the  $\Xi^-$  shows an especially large deviation from the SU(3) prediction, the difference being 1.7 standard deviations.<sup>3</sup>

Also, the recent measurement of the  $\Sigma^-$  magnetic moment differs by 1.6 standard deviations from the SU(3) value.<sup>4</sup> If this kind of discrepancy continues upon further experimental determinations, then the corrections to the Coleman-Glashow formulas must, in some cases at least, be large.

There have been a number of attempts to estimate the SU(3)-breaking corrections. For example, Bég and Pais<sup>5</sup> conjectured that the dominant first-order effect is given by mass corrections, that is, that the Coleman-Glashow formulas should be interpreted as conditions on magnetic moments

measured in natural magnetons rather than nuclear magnetons. This conjecture was supported by the calculation of Cheng and Pagels<sup>6</sup> which involved saturating the Drell-Hearn-Gerasimov sum rule for anomalous moments.<sup>7</sup> An alternate approach, provided by the naive quark model, is based on the additivity of quark magnetic moments. Symmetry breaking is effected by giving the strange quark a magneton different from that of the non-strange quarks.<sup>8</sup>

The main point of this paper is to explicitly calculate the exact leading-order corrections to the Coleman-Glashow formulas in the framework of chiral perturbation theory. This theory is based on the assumption that the strong-interaction Hamiltonian can be written

$$H = H_0 + \epsilon H', \quad (1)$$

where  $H_0$  is  $SU(3) \times SU(3)$ -invariant but the vacuum is  $SU(3)$ -symmetric so that the octet  $\pi$ ,  $K$ , and  $\eta$  mesons play the role of ground-state Nambu-Goldstone bosons.<sup>9</sup>  $H'$  removes the  $SU(3)$  degeneracy of the states and also gives the ground-state Nambu-Goldstone bosons a mass,  $\mu$ . Hence  $\epsilon$  is proportional to  $\mu^2$ . This approach has been successful in establishing symmetry breaking in the baryon octet mass spectrum as discussed by Li and Pagels,<sup>10</sup> and Langacker and Pagels.<sup>11</sup>

If one expands an anomalous moment in  $\epsilon$  one obtains

$$\kappa(\epsilon) = \kappa(0) + C_1 \epsilon^{1/2} + C_2 \epsilon \ln \epsilon + C_3 \epsilon + \dots, \quad (2)$$

with  $\kappa(0)$  the value in the  $SU(3)$  symmetry limit. The terms of  $O(\epsilon^{1/2})$  can be calculated exactly since they are nonanalytic quantities. The results of our calculation are

$$\begin{aligned} \kappa_\Lambda &= \frac{1}{2} \kappa_n + \frac{g^2}{8\pi} \left( \frac{\mu_K - \mu_\pi}{M} \right) \left( \frac{1}{2} \right), \\ \kappa_{\Sigma^+} &= \kappa_p + \frac{g^2}{8\pi} \left( \frac{\mu_K - \mu_\pi}{M} \right) \left( \frac{8}{3} \alpha^2 - 4\alpha + 1 \right), \\ \kappa_{\Sigma^0} &= -\frac{1}{2} \kappa_n + \frac{g^2}{8\pi} \left( \frac{\mu_K - \mu_\pi}{M} \right) \left( -\frac{1}{2} \right), \\ \kappa_{\Sigma^-} &= -(\kappa_p + \kappa_n) + \frac{g^2}{8\pi} \left( \frac{\mu_K - \mu_\pi}{M} \right) \left( -\frac{8}{3} \alpha^2 + 4\alpha - 2 \right), \\ \kappa_{\Xi^0} &= \kappa_n + \frac{g^2}{8\pi} \left( \frac{\mu_K - \mu_\pi}{M} \right) (4\alpha^2 - 4\alpha + 2), \\ \kappa_{\Xi^-} &= -(\kappa_p + \kappa_n) + \frac{g^2}{8\pi} \left( \frac{\mu_K - \mu_\pi}{M} \right) (-4\alpha^2 + 4\alpha - 1), \\ \kappa_{\Lambda\Sigma^0} &= -\frac{\sqrt{3}}{2} \kappa_n + \frac{g^2}{8\pi} \left( \frac{\mu_K - \mu_\pi}{M} \right) \left( -\frac{4}{\sqrt{3}} (\alpha^2 - \alpha) - \frac{\sqrt{3}}{2} \right), \end{aligned} \quad (3)$$

where  $g$  is the pion-nucleon coupling constant,  $\alpha$  is related to the  $f$  and  $d$  coupling of the pseudo-

scalar mesons to the baryons by  $(f/d) = (1 - \alpha)/\alpha$ , and  $M$  is a common baryon mass which we take to be the nucleon mass when evaluating our results. We note that kinematical effects due to different baryon masses<sup>10</sup> are of  $O(\epsilon)$ . The first terms of Eqs. (3) are just the Coleman-Glashow formulas to which our equations reduce in the chiral symmetry limit. The corrections to Eqs. (3) are of  $O(\epsilon \ln \epsilon)$ . No assumption about the representation content of  $H'$  was made in obtaining our results.

From Eqs. (3) one can also derive the following sum rules:

$$\kappa_{\Sigma^+} + \kappa_{\Sigma^-} = 2\kappa_{\Sigma^0} = -2\kappa_\Lambda, \quad (4a)$$

$$\kappa_{\Xi^-} + \kappa_{\Xi^0} = 2\kappa_\Lambda - \kappa_n - \kappa_p, \quad (4b)$$

$$\kappa_{\Lambda\Sigma^0} = \frac{1}{\sqrt{3}} (\kappa_\Lambda - \kappa_{\Xi^0} - \kappa_n). \quad (4c)$$

Since these sum rules are independent of a term proportional to  $\epsilon^{1/2}$ , the corrections to them are of  $O(\epsilon \ln \epsilon)$ .

The remainder of this article will be devoted to the derivation and a discussion of the implications of our results.

## II. CALCULATIONS AND DISCUSSION

We calculate relations among the baryon octet anomalous magnetic moments by employing threshold dominance via pairs of pseudoscalar Goldstone bosons in the intermediate states of a dispersion integral.

The matrix element under consideration is that of the electromagnetic current between baryon states,

$$\begin{aligned} \langle B^a(p_2) | J_\mu^b(0) | B^c(p_1) \rangle \\ = \bar{U}(p_2) e \left[ \gamma_\mu F_1^{abc}(t) + \frac{i\sigma_{\mu\nu}}{2M} (p_2 - p_1)^\nu F_2^{abc}(t) \right] U(p_1), \end{aligned} \quad t = q^2 \quad (5)$$

where  $J_\mu^b(x) = V_\mu^3(x)$  or  $(1/\sqrt{3})V_\mu^8(x)$ .  $F_2^{aba}(0) = \kappa_a$ , the anomalous magnetic moment of baryon  $a$ . We assume it obeys an unsubtracted dispersion relation

$$F_2^{abc}(0) = \frac{1}{\pi} \int_{4\mu^2}^{\infty} \frac{\text{Im} F_2^{abc}(t)}{t} dt; \quad (6)$$

$4\mu^2$  corresponds to the two-pseudoscalar production threshold, with  $\mu = \mu_\pi$  or  $\mu_K$ , depending on which intermediate state is being considered. At such a production threshold for two mesons of momentum  $q_1$  and  $q_2$ , unitarity implies

$$\text{Im} F_2^{abc}(t) = \frac{1}{4} i \phi(t) P_2^\mu f_\mu^{bef}(t) M_{ac}^{ef}, \quad (7)$$

where the two-body phase space is

$$\phi(t) = \frac{(t - 4\mu^2)^{1/2}}{8\pi\sqrt{t}},$$

and

$$if_{\mu}^{bef}(t) = ef^{bef}(q_1 - q_2)_{\mu} F_{ef}^*(t)$$

is the matrix element of the electromagnetic current between meson states, with  $F_{ef}(t)$ , the meson form factor, normalized to  $F_{ef}(0) = 1$ .  $P_2^{\mu}$  is the projection operator to pick out the  $F_2$  term;

$$M_{ac}^{ef}(t) = \langle B^a(p_2) \bar{B}^c(-p_1) | M^e(q_1) M^f(q_2) \rangle$$

is the  $p$ -wave projection of the amplitude for  $M_e(q_1) + M_f(q_2) \rightarrow B_a(p_2) + \bar{B}_c(-p_1)$ . The contribution from baryon exchange is the most singular term

as  $\mu \rightarrow 0$  and one obtains

$$M_{ac}^{ef}(t) \rightarrow M(t) (\Gamma_{eam} \Gamma_{fmc} - \Gamma_{fam} \Gamma_{emc}),$$

$$\Gamma_{aba} = \alpha d_{abc} + i(1 - a) f_{abc} \quad (8)$$

with  $f = (1 - \alpha)$ ,  $f + d = 1$  the  $f$  and  $d$  coupling of the pseudoscalar mesons to the baryons.

After expanding out the trace over the SU(3) coefficients one finds that only pairs of pions and pairs of kaons contribute to the two-meson intermediate states for the isovector or 3 term, and only pairs of kaons contribute to the isoscalar or 8 term. Putting these results into the dispersion integral, calculating up to a cutoff  $4L^2$ , and keeping only the leading term as the meson masses go to zero, we obtain

$$\begin{aligned} \kappa_p &= C(L) \left( \frac{8}{3} \alpha^2 - 4\alpha + 3 \right) - \frac{g^2}{8\pi} \left[ \frac{\mu_{\pi}}{M} (1) + \frac{\mu_K}{M} \left( \frac{8}{3} \alpha^2 - 4\alpha + 2 \right) \right], \\ \kappa_n &= C(L) (4\alpha^2 - 4\alpha) - \frac{g^2}{8\pi} \left[ \frac{\mu_{\pi}}{M} (-1) + \frac{\mu_K}{M} (4\alpha^2 - 4\alpha + 1) \right], \\ \kappa_{\Lambda} &= C(L) (2\alpha^2 - 2\alpha) - \frac{g^2}{8\pi} \left[ \frac{\mu_K}{M} (2\alpha^2 - 2\alpha) \right], \\ \kappa_{\Sigma^+} &= C(L) \left( \frac{8}{3} \alpha^2 - 4\alpha + 3 \right) - \frac{g^2}{8\pi} \left[ \frac{\mu_{\pi}}{M} \left( \frac{8}{3} \alpha^2 - 4\alpha + 2 \right) + \frac{\mu_K}{M} (1) \right], \\ \kappa_{\Sigma^0} &= C(L) (-2\alpha^2 + 2\alpha) - \frac{g^2}{8\pi} \left[ \frac{\mu_K}{M} (-2\alpha^2 + 2\alpha) \right], \\ \kappa_{\Sigma^-} &= C(L) \left( -\frac{20}{3} \alpha^2 + 8\alpha - 3 \right) - \frac{g^2}{8\pi} \left[ \frac{\mu_{\pi}}{M} \left( -\frac{8}{3} \alpha^2 + 4\alpha - 2 \right) + \frac{\mu_K}{M} (-4\alpha^2 + 4\alpha - 1) \right], \\ \kappa_{\Xi^0} &= C(L) (4\alpha^2 - 4\alpha) - \frac{g^2}{8\pi} \left[ \frac{\mu_{\pi}}{M} (4\alpha^2 - 4\alpha + 1) + \frac{\mu_K}{M} (-1) \right], \\ \kappa_{\Xi^-} &= C(L) \left( -\frac{20}{3} \alpha^2 + 8\alpha - 3 \right) - \frac{g^2}{8\pi} \left[ \frac{\mu_{\pi}}{M} (-4\alpha^2 + 4\alpha - 1) + \frac{\mu_K}{M} \left( -\frac{8}{3} \alpha^2 + 4\alpha - 2 \right) \right], \\ \kappa_{\Lambda\Sigma^0} &= C(L) \left( -2\sqrt{3} (\alpha^2 - \alpha) - \frac{g^2}{8\pi} \left[ \frac{\mu_{\pi}}{M} \left( -\frac{4}{\sqrt{3}} (\alpha^2 - \alpha) \right) + \frac{\mu_K}{M} \left( -\frac{2}{\sqrt{3}} (\alpha^2 - \alpha) \right) \right] \right). \end{aligned} \quad (9)$$

We eliminate the cutoff dependent term by forming relations among the anomalous moments, which turn out to be the Coleman-Glashow formulas and the leading-order corrections thereto, given in Eqs. (3). It should be emphasized that these equations (3) are exact to leading order in chiral symmetry breaking. The next terms are of  $O(\mu^2 \ln \mu^2)$ . However, all the contributions to this next-order term are not determinable in a model-independent way.

It may also be mentioned that Eqs. (3) are not very sensitive functions of  $\alpha$ , unlike the corrections to the Gell-Mann-Okubo formula.<sup>11</sup>

We also obtain an extra relation beyond the

Coleman-Glashow formulas,

$$\begin{aligned} \kappa_n &= \kappa_p \left( \frac{4\alpha^2 - 4\alpha}{\frac{8}{3}\alpha^2 - 4\alpha + 3} \right) \\ &+ \frac{g^2}{8\pi} \left( \frac{\mu_K - \mu_{\pi}}{M} \right) \left( \frac{-\frac{20}{3}\alpha^2 + 8\alpha - 3}{\frac{8}{3}\alpha^2 - 4\alpha + 3} \right). \end{aligned} \quad (10)$$

Keeping only the zeroth-order term, we may re-express it in terms of the anomalous magnetic moments parameterized by a  $d$  and  $f$  coupling with  $(f/d)_{\kappa} = -(\kappa_n + 2\kappa_p)/3\kappa_n$ . Then from the zeroth-order term of Eq. (10) we obtain

$$\left( \frac{f}{d} \right)_{\kappa} = \frac{-(14\alpha^2 - 18\alpha + 9)}{18(\alpha^2 - \alpha)}. \quad (11)$$

With  $\alpha = \frac{2}{3}$  we obtain  $(f/d)_\kappa = 0.805$  as compared to the experimental result<sup>12</sup> of  $(f/d)_\kappa = 0.292$ . The gross disagreement with the experimentally determined ratio is to be contrasted with the case of the extra relation obtained by Li and Pagels<sup>10</sup> on the baryon mass splitting  $f/d$  ratio,  $(f/d)_B$ . Their relation

$$\frac{3}{10}(f/d)_B = (f/d)_A/[3(f/d)_A^2 - 1], \quad (12)$$

with  $(f/d)_A = (1 - \alpha)/\alpha$ , is in excellent agreement with the experimentally determined ratio.

The baryon mass difference calculation of Li and Pagels is similar to this calculation of the anomalous moments but with the essential difference that their calculation was for a  $d$  coupling and ours is for an  $f$  coupling. Consequently in the baryon mass difference calculation the intermediate pair state always contained one strange ground-state meson, and so a difference of  $\mu_K^2 - \mu_\pi^2$  or  $\mu_K^2 - \mu_\eta^2$  appears in their formulas. These differences of squared meson masses are small quantities on the scale of SU(3) breaking. For the anomalous moment calculation here the relevant states are the two-pion and two-kaon states. While in the symmetry limit the thresholds for pair production are equal for pions and kaons, in the real world of the broken symmetry the thresholds are very different. Hence we expect large symmetry-breaking effects, from the viewpoint of chiral perturbation theory.

One can conclude from the failure of the zeroth-order relation [Eq. (11)] that the corrections must be large. Indeed, if one returns to Eq. (10) and evaluates  $\kappa_n$  using  $g^2/4\pi = 14.6$ ,  $\alpha = \frac{2}{3}$ , and  $\kappa_p = 1.79$ , the result is  $\kappa_n = -2.10$ , which is within 10% of the experimentally determined moment,<sup>2</sup>  $\kappa_n = -1.91$ . While this result is almost as accurate as the SU(6) prediction<sup>13</sup> of  $\mu_p/\mu_n = -\frac{3}{2}$  which is valid to within 2%, it probably cannot be trusted for reasons we shall now discuss.

Returning to our main result [Eqs. (3)], let us focus on the first equation

$$\kappa_\Lambda = \frac{1}{2}\kappa_n + \frac{g^2}{16\pi} \left( \frac{\mu_K - \mu_\pi}{M} \right).$$

It turns out that the correction term is actually larger than the experimental value for the zeroth-order term,  $\frac{1}{2}\kappa_n$ . Thus the perturbation expansion breaks down. The numerical answer we obtain,  $\kappa_\Lambda = +0.44$ , is also far from the experimentally determined result<sup>2</sup>  $\kappa_\Lambda = -0.67 \pm 0.06$ . Breakdown of the perturbation expansion also holds for most of the other formulas in Eqs. (3) and for Eq. (10).

The reason for the failure of chiral perturbation theory in this application might be attributed to the large mass of the kaon compared to the chiral-

symmetry limit value of zero and the fact that the leading term is of  $O(\epsilon^{1/2})$ . Hence the leading symmetry-breaking effects are large compared to the zeroth-order term.

We can eliminate these large leading-order correction terms in Eqs. (3) by forming the sum rules given in Eqs. (4). Although the leading corrections to these sum rules are of  $O(\mu^2 \ln \mu^2)$ , we are not able to calculate them in a representation-independent way. Furthermore, not all the moments in these sum rules are yet determined by experiment. However, using the recent  $\kappa_\Sigma^-$  determination<sup>4</sup> and the average of the experimental measurements<sup>3</sup> for  $\kappa_{\Sigma^+}$  and  $\kappa_\Lambda$  we find that for Eq. (4a),

$$\kappa_{\Sigma^+} + \kappa_{\Sigma^-} = -2\kappa_\Lambda,$$

the left-hand side is

$$1.59 \pm 0.46 + (-0.48 \pm 0.37) = 1.11 \pm 0.59,$$

while the right-hand side is  $1.34 \pm 0.12$ . So within the admittedly large experimental errors the sum rule is verified. In addition, Eq. (4c),

$$\kappa_{\Lambda\Sigma^0} = \frac{1}{\sqrt{3}}(\kappa_\Lambda - \kappa_{\Sigma^0} - \kappa_n),$$

is consistent with, but stronger than, the Okubo formula,<sup>14</sup>

$$\mu_{\Lambda\Sigma^0} = \frac{1}{2\sqrt{3}}[\mu_{\Sigma^0} + 3\mu_\Lambda - 2\mu_{\Sigma^0} - 2\mu_n], \quad (13)$$

derived from SU(3) symmetry, broken to first order. The agreement follows from Eq. (4a),  $\kappa_{\Sigma^0} = -\kappa_\Lambda$ .

### III. CONCLUSIONS

We have done an exact calculation of the leading-order corrections in chiral perturbation theory to the Coleman-Glashow formulas for the baryon octet magnetic moments. We find that these corrections are, in general, large, and therefore the perturbation expansion breaks down.

According to our approach, the baryon anomalous magnetic moment arises from the electric current of the ground-state pseudoscalar mesons surrounding the baryon. The large  $K-\pi$  mass difference [which is large on the standard of SU(3) breaking] induces large symmetry-breaking effects, at least to the leading order of  $\epsilon^{1/2}$ . Evidently other terms in the perturbation series are not negligible, but we do not yet know how to control them. However, we can understand why the approach adopted here fails for magnetic moments but works for the baryon mass differences.

The picture we present of baryon magnetic moments and symmetry breaking is not easily reconciled with the naive quark model and simple ad-

ditivity. According to the quark model, symmetry-breaking effects in the *total* magnetic moment are accommodated by retaining a magneton for the strange quark different from that for the non-strange quarks. From our viewpoint the *anomalous* magnetic moment is predominantly due to the interactions of Nambu-Goldstone boson pair states. Since the range of these bosons is very different in the broken  $SU(3) \times SU(3)$  world, there is substantial symmetry breaking in the anomalous moments to leading order in perturbation theory. We emphasize that our results [Eqs. (3)] are necessarily exact in any theory that embraces the

assumption that the symmetry limit is realized with Nambu-Goldstone bosons and an  $SU(3)$ -degenerate vacuum. It is our opinion that progress in understanding symmetry breaking will be made once the idea of a chiral symmetry realized by a ground-state octet of Nambu-Goldstone bosons is reconciled with the quark model and its group structure.

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