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Proton-nucleus cross sections and the peripheral component of elastic scattering*

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A model of proton-proton scattering is presented in which σ_{pp} rises with energy due to large-impact-parameter effects which are associated with the two-pion-exchange cut. The model is applied to the question of determining σ_{pp} from proton-nucleus interactions of cosmic rays.

The total proton-proton cross section σ_{pp} is observed to increase with energy, at equivalent laboratory energies up to 3×10^3 GeV.[1] This observation and the variety of theories which encompass it arouse curiosity about σ_{pp} at still higher energies. One is led to ask whether cosmic-ray protons, with energies E up to $\sim10^{10}$ GeV, can be used to satisfy this curiosity.

Because of the small flux involved, cosmic-ray experiments at extreme energies necessarily employ interactions with nuclei, either in dense targets, or in the atmosphere itself. One therefore faces the problem of extracting σ_{pp} from measurements of σ_{pA}, where A refers to a nucleus with mass number A. This problem is the subject of the present paper. Other difficulties inherent in cosmic-ray experiments have been discussed elsewhere.[2] These difficulties are considerable, but should not be overwhelming, because total inelastic cross sections are the simplest of all to measure. Furthermore, the possible effects being sought are large: E.g., a particular $(\ln s)^2$ extrapolation of existing data predicts σ_{pp} to rise from 42 mb at $E=10^3$ GeV to 138 mb at $E=10^8$ GeV.[3]

Multiple-scattering theory,[4] or an optical model, suggests that $\sigma_{pA}\simeq A\sigma_{pp}$ for small A, while $\sigma_{pA}\simeq 2\pi R^2$ for large A, where R is the radius of an effective "black disk." The black-disk limit is a useful guide to the results of the complete optical model, which we use for actual numerical work. The effective radius R represents an impact parameter b at which the interaction probability is $\simeq\frac{1}{2}$. It is determined by the radius associated with the nuclear matter distribution, combined with the range and strength of the pp interaction. (At the energies considered here, pn and pp interactions can be considered equal.) A component of the pp interaction which has a range comparable to the nuclear radius will increase R, and hence σ_{pA}, significantly. This will be true even if the long-range component is relatively weak, because the density of the nucleus is so large.

Maor and Nussinov[5] calculate significant effects on σ_{pA} due to such a long-range component. They parametrize the pp amplitude as a function of impact parameter with the form[3]

$$\rho(b)=\sum_{i=1}^{2} C_i \exp(-b^2/2R_i^{\,2}) . \qquad (1)$$

$\rho(b)$ refers to the imaginary part of the amplitude, normalized so that $\rho=1$ corresponds to complete absorption. Thus $\rho(b)=1-\exp[i\chi(b)]$ in the eikonal model, and the total cross section is given by $\sigma_{pp}=4\pi\int_0^\infty b\,db\,\rho(b)$. The parameters $C_1=0.73$, $C_2=0.021$, and $R_1=0.65$ F are fixed, while R_2 is determined by $\sigma_{pp}=4\pi(C_1R_1^{\,2}+C_2R_2^{\,2})$. Thus $R_2=0.62(\sigma_{pp}-38.4)^{1/2}$, where σ_{pp} is in millibarns and R_2 in fermis.

The shape of $\rho(b)$ determines the connection between σ_{pp} and σ_{pA}. The dependence on energy is entirely implicit. Maor and Nussinov assume $\sigma_{pp}=38.4+0.49[\ln(s/122)]^2$, where $s=2m_p(E+m_p)$ is in GeV^2.[3] Thus at $E=10^4$, 10^5, 10^6, 10^7 GeV, the cross section is 51, 65, 84, 108 mb. The forward slopes at these four energies, defined by $d\sigma/dt\propto\exp(Bt)$ as $t\to0$, are $B=43$, 119, 257, 459 GeV^{-2}. These slopes reflect the extremely large radius R_2.

According to the uncertainty principle, the amplitude at large b is determined by the lowest possible exchanged mass in the t channel. Its energy dependence is governed by the location of the Pomeron trajectory at that mass. We now construct a model according to these precepts. We find that a relatively rapid rise of the slope B with energy, such as assumed by Maor and Nussinov, is unlikely.

The singularity of the imaginary part of the elastic amplitude which lies nearest to $t=0$ is the two-pion-exchange cut at $t=4m_\pi^{\,2}$. This singularity implies $\rho(b)\propto\exp(-2m_\pi b)/b^n$ as $b\to\infty$. The power n is determined by the threshold behavior of the cut, and is close to 3.[6] One of the proposed explanations for the rise of σ_{pp} is that the strength of this two-pion-exchange tail increases with energy, while the central part remains approximately constant.[6,7] We define a phenomenological model of this type by

$$\rho(b)=c_1\exp(-b^2/2c_2)+c_3\exp(-u)(u^{-1}+3u^{-2}+3u^{-3})/[(2m_\pi b)^2+c_4{}^2],\quad u=[(2m_\pi b)^2+c_5{}^2]^{1/2}\ . \tag{2}$$

The nonleading terms $3u^{-2}+3u^{-3}$ make the Hankel transform of (2) simple in the limit $c_4=c_5$.[8] They aid in fitting the data at small b, but are not essential. We set $c_1=0.4$, $c_2=13\ \text{GeV}^{-2}$, and $c_4=1.0$. We determine c_5 by requiring $\rho(0)=0.77$. The remaining parameter c_3 determines the total cross section, and the strength of the tail: $\rho(b)\to c_3\exp(-2m_\pi b)/(2m_\pi b)^3$ as $b\to\infty$. This parameterization agrees with determinations[9] of $\rho(b)$ from CERN ISR data to an accuracy of $\simeq 5\%$. [The model does not reproduce the diffraction dip observed at $t=-1.4\ \text{GeV}^2$,[10] for readers interested in that question. The dip is very sensitive to $\rho(b)$ at small b, which is irrelevant to our present purpose.] The value of c_3 is 5.1 at $s=900\ \text{GeV}^2$ and 16.6 at $s=2800\ \text{GeV}^2$, corresponding to cross sections of 40.6 and 43.2 mb. The cross section given by this model can be approximated by $\sigma_{pp}\simeq[25+11.6\ln(2+\ln c_3)]$ mb, to an accuracy of better than 1% when $c_3>10$. The slope is given to 10% accuracy by $B=150\ln(c_3+75)/\sigma_{pp}$, with σ_{pp} in mb and B in GeV^{-2}.

To speculate on an energy dependence, we fit c_3 by a power law in the energy, using the ISR data. We obtain $c_3\simeq 0.012s^{0.9}$, which corresponds to a Pomeron trajectory with $\alpha=1.9$ at $t=4m_\pi{}^2$. This involves a considerable increase from $\alpha\simeq 1.0$ at $t=0$, and hence corresponds to a fairly rapid rise of the two-pion cut contribution as a function of energy. From it, we obtain $\sigma_{pp}=47$, 50, 52, 54 mb at $E=10^4$, 10^5, 10^6, 10^7 GeV. The forward slopes are $B=17.5$, 19, 26, and 30 GeV^{-2}. The asymptotic cross section rises very slowly—like $\ln(\ln c_3)$, and hence like $\ln(\ln s)$—in this model, because the rise does not begin until $b\gtrsim(\ln c_3)^{1/2}$, and the asymptotic form $\rho(b)\simeq c_3\exp(-2m_\pi b)/(2m_\pi b)^3$ holds only for $2m_\pi b\gtrsim(\ln c_3)^2$.

The very slow rise is not a necessary characteristic of models in which σ_{pp} rises due to the two-pion cut. Rather, we have constructed this particular model in such a way that the rise at large s does not begin until large b. This was done in order to examine the effect on σ_{pA} of a rise in σ_{pp} which is extremely peripheral, but which nevertheless corresponds to reasonable behavior in the t channel.

For comparison, we consider a model in which the assumed increase in $\rho(b)$ is less peripheral. We use the eikonal fit of Cheng, Wu, and Walker[11]:

$$\begin{aligned}\rho(b)&=1-\exp[i\chi(b)]\ ,\\ i\chi(b)&=-6.58[E\exp(-i\pi/2)]^{0.083}\\ &\quad\times\exp[-0.60(b^2+15.0)^{1/2}]\ ,\end{aligned}\tag{3}$$

where b and E are expressed in GeV = 1 units. This model has $\rho(b)\propto\exp(-4.3m_\pi b)$ at large b, which is not very different from the two-pion cut form $\exp(-2m_\pi b)/b^3$, in the important region of $1\lesssim b\lesssim 5$ F. The coefficient of this term rises very slowly with E, and hence with σ_{pp}, however. The rise of σ_{pp} in this model is thus basically *central* in impact parameter. Indeed as $s\to\infty$ the model corresponds to a black disk with radius $\propto\ln s$.

To calculate the nuclear cross section, we use the large A, or optical model, limit of Glauber's multiple-scattering theory.[4] The relevant nuclear cross section is the inelastic one, $\sigma_{pA}^{\text{inel}}=\sigma_{pA}^{\text{total}}-\sigma_{pA}^{\text{elastic}}$, since elastic scattering involves small momentum transfer to the cosmic-ray proton, and is experimentally indistinguishable from no interaction at all. The formulas are

$$\begin{aligned}T(b)&=A\int_{-\infty}^{\infty}dz\,D[(b^2+z^2)^{1/2}]\ ,\\ \Omega(b)&=\int d^2b'\rho(|\vec{b}'|)T(|\vec{b}-\vec{b}'|)\ ,\\ \sigma_{pA}^{\text{inel}}&=\int d^2b\{1-\exp[-2\,\text{Re}\,\Omega(b)]\}\ .\end{aligned}\tag{4}$$

$D(r)$ represents the nuclear density, normalized to $\int d^3r D(|\vec{r}|)=1$. We use $D(r)=N/\{1+\exp[(r-r_0A^{1/3})/d]\}$, with $r_0=1.05$ F and $d=0.55$ F. $T(b)$ is the nuclear density as a function of impact parameter. $\rho(b)$ is the pp amplitude discussed above. $\Omega(b)$ is the p-nucleus eikonal function. It involves a convolution of ρ and T, and therefore has an effective radius which combines their radii.

The multiple-scattering theory could be invalid at very high energies, for example, due to the increasing importance of inelastic shadow effects. However, the essential content of Eqs. (4), namely, that the p-nucleus eikonal function is proportional to the nuclear density, integrated over the longitudinal variable z and smeared out by the impact-parameter distribution of elastic scattering, appears extremely reasonable, and may survive a breakdown of the detailed theory. It has been shown that Eqs. (4) *may* hold in the presence of fairly strong inelastic shadow effects.[12]

A comparison of the models for pp scattering is shown in Fig. 1. The model of Cheng, Wu, and Walker is, as expected, considerably more central in impact parameter than ours, although the black-disk limit is still far away when $E=10^7$ GeV and $\sigma_{pp}=73$ mb. The model of Maor and Nussinov is more peripheral than ours. However, at large σ_{pp}, this model develops an abrupt break in $\rho(b)$

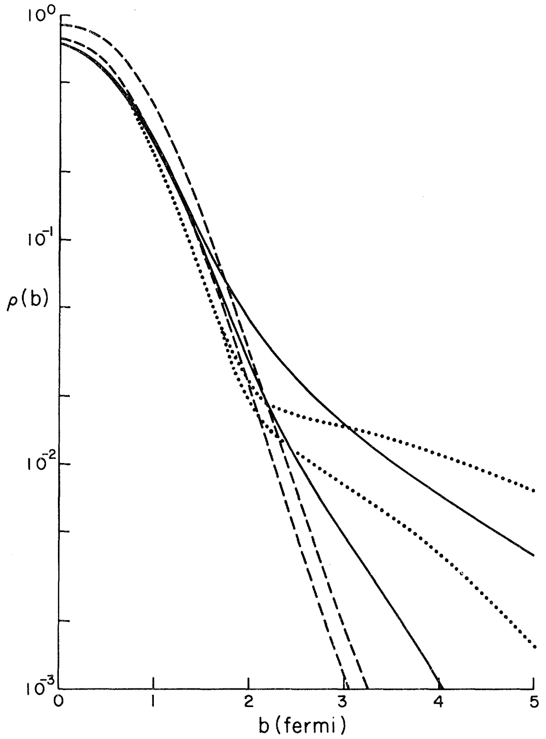

FIG. 1. Models of the imaginary part of the proton-proton elastic amplitude as a function of impact parameter b. The *solid curve* is the two-pion-cut model [Eq. (2)]. The *dashed curve* is the model of Cheng, Wu, and Walker (Ref. 11). The *dotted curve* is the model of Leader and Maor (Ref. 3). In each case, the *lower curve* corresponds to $\sigma_{pp} = 50$ mb, and the *upper curve* to $\sigma_{pp} = 70$ mb.

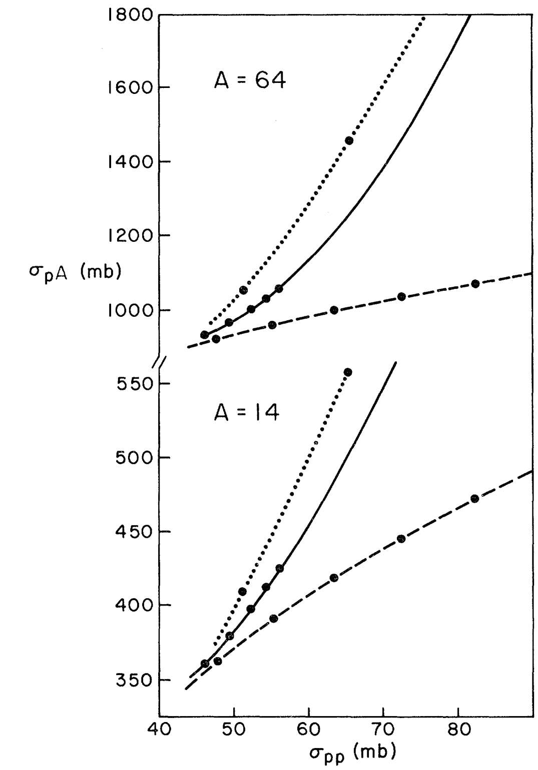

FIG. 2. Inelastic proton cross sections on nitrogen ($A = 14$) and copper ($A = 64$), as a function of the proton-proton cross section. *Solid, dashed,* and *dotted* curves correspond to the three models for *pp* scattering shown in Fig. 1. Note that σ_{pA} rises rapidly with σ_{pp}, if the increase in σ_{pp} comes from large impact parameter. The circles correspond to laboratory energies $E = 10^4$, $10^5, \ldots, 10^8$ GeV, according to particular models of the energy dependence discussed in the text.

at large b, which is out of keeping with the expected $\exp(-2m_\pi b)/b^n$ asymptotic behavior. The energy dependence of the tail is much stronger than in our model, which we have already argued is rather strong in the sense of corresponding to $\alpha_P(4m_\pi^2) = 1.9$. The model used by Maor and Nussinov therefore appears to be an unlikely speculation for the behavior of $\rho(b)$. This somewhat narrows the range of σ_{pA} which can be expected for a given σ_{pp}.

Results of the optical-model calculation are shown in Fig. 2. One sees that the change in $\sigma_{pA}^{\text{inel}}$ produced by an increase in σ_{pp} can be quite large, if the increase comes from large impact parameter. Therefore, *if a rise in σ_{pA} is observed at cosmic-ray energies, it may correspond to a relatively small rise of σ_{pp}, if that rise is peripheral, as it would be if associated with the two-pion-exchange cut.* Unlike other calculations of this effect,[5,13] we use models of the *pp* amplitude which agree with experiment at ISR energies, and which respect the asymptotic behavior in b which is required by analyticity.

We note that *the fractional increase in $\sigma_{pA}^{\text{inel}}$ is less than the fractional increase in σ_{pp} which produces it,* even for light nuclei where the effect is largest. Thus the early indications of a possible rise in σ_{pp} from cosmic-ray satellite measurements,[14] using a carbon target at $E \lesssim 10^3$ GeV, were apparently accidental, since these experiments were not accurate to the level of <10% required in that energy range.

The variation of $\sigma_{pA}^{\text{inel}}$ with A, for fixed σ_{pp}, differs somewhat between the models. Thus in principle one could use the dependence on A to determine the degree of peripherality of the *pp* amplitude, or to determine the cross section from data whose absolute normalization is unknown. This effect is too small, however, to be of practical value, as concluded also by Camillo *et al.*[13]

The curves in Fig. 2 are determined by the form of $\rho(b)$, and are independent of the energy at which it takes on that form. If, however, we accept the energy dependence of the two-pion-exchange model hypothesized above, we predict σ_{pp}, $\sigma_{A=14}^{\rm incl}$, and $\sigma_{A=64}^{\rm incl}$ to increase by 36%, 27%, and 20% between $E=10^3$ GeV and $E=10^8$ GeV. The corresponding percentages based on the model of Cheng, Wu, and Walker for pp scattering are 200%, 44%, and 22%. If either of these models is approximately correct, it will be difficult to detect the energy dependence of $\sigma_{pA}^{\rm incl}$, because of the inherent difficulties of cosmic-ray experiments. The main hope of such experiments would be to detect (or rule out) a more rapid rise of the cross section.

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Chiral perturbation theory and the magnetic moments of the baryon octet*

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We have calculated, exactly to leading order in the chiral perturbation symmetry-breaking parameter ϵ, the corrections to the Coleman-Glashow SU(3) relations among the magnetic moments of the baryon octet. The corrections are of order $\epsilon^{1/2}$. In general the corrections turn out to be at least as large as the zeroth-order term and hence the perturbation expansion breaks down. We discuss the reasons for this breakdown. We also derive sum rules which are independent of the leading-order corrections and hence valid to next order in chiral perturbation symmetry breaking, i.e., $\epsilon \ln\epsilon$.

I. INTRODUCTION

In the limit that SU(3) is an exact symmetry and assuming octet transformation properties for the photon, that is, $Q=Q_3+(1/\sqrt{3})Q_8$, it is well known that one obtains the Coleman-Glashow formulas[1] for the magnetic moments of the baryon octet.

Of course, SU(3) is not an exact symmetry. For those magnetic moments which have been experimentally measured,[2] that for the Ξ^- shows an especially large deviation from the SU(3) prediction, the difference being 1.7 standard deviations.[3] Also, the recent measurement of the Σ^- magnetic moment differs by 1.6 standard deviations from the SU(3) value.[4] If this kind of discrepancy continues upon further experimental determinations, then the corrections to the Coleman-Glashow formulas must, in some cases at least, be large.

There have been a number of attempts to estimate the SU(3)-breaking corrections. For example, Bég and Pais[5] conjectured that the dominant first-order effect is given by mass corrections, that is, that the Coleman-Glashow formulas should be interpreted as conditions on magnetic moments