in the same way as the integrals in Appendix B. The results are

$$I_1 = -\frac{\pi}{2}(1+\beta)^2[M_N{}^2\mu^2 + \mu^2(p_1\cdot p_2) - 2(p_1\cdot k)(p_2\cdot k)] \times R^2\left(\frac{R^2-4M_K{}^2}{R^2}\right)^{3/2}, \tag{C7}$$

$$I_2 = -\frac{2\pi}{3}\beta(M_N{}^2 + p_1\cdot p_2)[(R\cdot k)^2 - \mu^2 R^2] \times\left(\frac{R^2-4M_K{}^2}{R^2}\right)^{3/2}, \tag{C8}$$

$$I_3 = -\frac{\mu^2\pi}{3}(1+\beta)^2[(R\cdot p_1)(R\cdot p_2) - R^2(p_1\cdot p_2)] \times\left(\frac{R^2-4M_K{}^2}{R^2}\right)^{3/2}, \tag{C9}$$

$$I_4 = \frac{2\pi}{3}(1+\beta)(p_1\cdot k)[(R\cdot k)(R\cdot p_2) - R^2(p_2\cdot k)] \times\left(\frac{R^2-4M_K{}^2}{R^2}\right)^{3/2}, \tag{C10}$$

$$I_5 = \frac{2\pi}{3}(1+\beta)(p_2\cdot k)[(R\cdot k)(R\cdot p_1) - R^2(p_1\cdot k)] \times\left(\frac{R^2-4M_K{}^2}{R^2}\right)^{3/2}. \tag{C11}$$

Upon evaluating these expressions in the $\bar{p}p$ rest frame, substituting the results into Eq. (C1), and using Eq. (2.22), we arrive at our result described by Eqs. (2.23)–(2.25).

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$$S_{fi} = \delta_{fi} + i(2\pi)^4\delta(P_i - P_f)M_{fi}/N,$$

where $N^2 = \prod_i N_i{}^2$; $N_i{}^2 = 2E$ for bosons, $N_i{}^2 = E/m$ for fermions. The integrals in Eq. (2.5) are related to S_{fi} through the LSZ (Lehmann-Symanzik-Zimmermann) reduction formulas. In this way Eq. (2.5) can be expressed in terms of the Lorentz-invariant amplitudes, which will be utilized from now on.

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Theoretical models for proton-proton elastic polarization*

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We apply various theoretical models to proton-proton elastic polarization for $-t \lesssim 6$ (GeV/c)^2 at 12.33 GeV/c, where pronounced structure has recently been observed.

I. INTRODUCTION

Due to the discovery of several unexpected phenomena[1] at CERN Intersecting Storage Ring energies [such as rising σ_{tot}, the change of slope in $d\sigma/dt$ at $-t \approx 0.15$ (GeV/c)^2, and a dip at $-t \approx 1.5$ (GeV/c)^2], proton-proton scattering has recently become one of the most exciting of scattering processes. This is in sharp contrast to the general feeling previously held about pp scattering at much lower energies, where σ_{tot} is flat and $d\sigma/dt$ comparatively smooth. The new wide-angle pp polarization measurements,[2] however, made recently at the Argonne ZGS, now seem to indicate that, even at comparatively low energies, many exciting things are happening. In particular, the new 12.33-GeV/c data, together with some earlier data[3,4] at small t, show (see our Fig. 1) that the polarization has some very pronounced structures: The polarization seems to have "double zeros" located at $-t \approx 0.8$ and 2.4 (GeV/c)^2. For larger values of $-t$, the polarization again swings positive, and there are indications of a broad third dip in the region $-t \approx 4$–5 (GeV/c)^2.

It is clearly important to try to understand these new structures in the polarization. Since polarization is a sensitive probe of amplitudes, these structures clearly provide us with an important clue about the amplitudes in this region. In this paper we briefly examine what various theoretical models have to say about this new pp polarization structure. We consider in turn several Regge models,[5-7] the diffraction model of Chou and Yang[8] as developed by Durand and Lipes,[9] and an optical type of model developed by Chu and Hendry.[10] Since all of these models have been well described in the literature, we shall not expand on them here, but refer the readers to the original articles for details.

II. SOME REGGE MODELS

We start first with a consideration of Regge models. Any significant structure at all in pp elastic polarization, which is given essentially[11] by

$$P\,\frac{d\sigma}{dt}\sim \mathrm{Im}(NF^*)$$

where N, F are the helicity-nonflip $(\Phi_1+\Phi_3)$ and helicity-single-flip (Φ_5) amplitudes respectively, is of course somewhat surprising from the point of view of the simplest kinds of traditional Regge pole models—the nonflip amplitude N is dominated by the smooth diffractive Pomeron, while the usual ideas of s-channel exoticity and exchange degeneracy (EXD) make the flip amplitude F also smooth in t. To fit the polarization data, one must relax the condition of EXD, and we discuss below several specific models of this kind which have been proposed.

Regge A (Austin et al., Ref. 5). Here the t-channel helicity amplitudes are parametrized in terms of 3, 4, and 5 $(P, P', P'', \omega, \omega')$ poles, with various kinds of ghost-eliminating mechanisms considered. We show in Fig. 1(a) their 4-pole prediction[12] (labeled as R1) for the polarization. (As with other fits in the present paper, $d\sigma/dt$ and σ_{tot} are also fitted simultaneously; these essentially serve to fit the nonflip amplitude N, and we do not show them here.) The oscillations of the prediction occur in the wrong places, and clearly give an unsatisfactory fit.

Keeping the same formalism, we tried to generate a better fit to the new data by further variation of the parameters involved. The best fit we were able to achieve is shown as R2 in Fig. 1(a). However, we note that this fit is somewhat pathological, in that the oscillations come about from fortuitous cancellations of various pieces, rather than through signature zeros as one would like.

Regge B (Parry et al., Ref. 6). A formula for

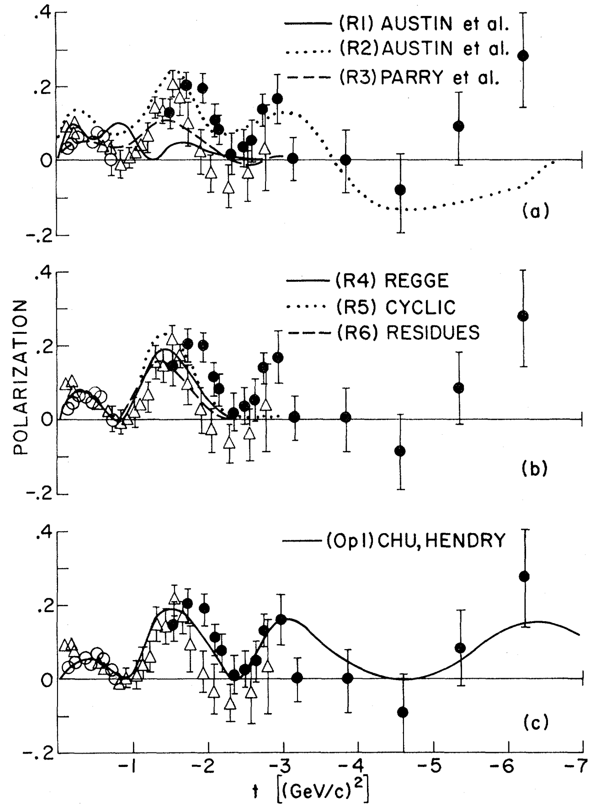

FIG. 1. Model fits to the new 12.33 GeV/c elastic polarization data. (R1) Austin *et al.*, quoted in Ref. 5. (R2) Austin *et al.* with different parameter values. (R3) Parry *et al.*, Ref. 6, with linear EXD breaking. (R4), (R5), (R6) Regge fits with cylic residues. (Op1) Chu-Hendry optical model, Ref. 10. Data are from Ref. 2 (●) at 12.33 GeV/c, Ref. 3 (○) at 12.0 GeV/c and Ref. 4 (△) at 10.0 GeV/c.

pp polarization has recently been suggested by Parry *et al.*,[6] namely,

$$P = Cp_{\mathrm{lab}}^{-1}(-t^*/4M^2)^{1/2}\cos\theta_s[1+Bt^*\cos\pi\alpha(t)]\times\left(\frac{s}{s_0}\right)^{\alpha(t)}\exp(-5t^*)\,,$$

where C, B are constants, $t^*=-tu/s$, and $\alpha(t)=\alpha_0+\alpha' t$. This formula ensures that the polarization vanishes at $\theta_s=0°$ and $90°$, and incorporates the Krisch[13] parametrization $d\sigma/dt\sim\exp(10t^*)$ for the differential cross section. Structure in the polarization then comes about from the linear EXD breaking factor $[1+Bt^*\cos\pi\alpha(t)]$, where $B=0$ for exact EXD.

A typical fit, with $B=1.0$ and $\alpha(t)=0.5+t$, is shown as R3 in Fig. 1(a). B and $\alpha(t)$ are chosen to give a reasonable fit for $-t\lesssim 2.5$ (GeV/c)^2. However, because of the $(s/s_0)^{\alpha(t)}=\exp[\alpha(t)\times\ln(s/s_0)]$ factor, the magnitude of the polarization rapidly dies out and is extremely small beyond $-t \approx 2.5$ (GeV/c)^2. The parametrization is unable to give the pronounced two-sided valley around $-t \approx 2.4$ (GeV/c)^2.

Regge C. Another possible way of achieving oscillations in the polarization is to use the method of cyclic residues, as suggested by Barger and Phillips.[7] We have considered three such models,[14] with the flip amplitude taken as

$$F = \left(\frac{-t}{4M^2}\right)^{1/2} g \exp(ht)\, C(t) \left(\frac{s}{s_0}\right)^{\alpha(t)},$$

with g, h constants and $C(t)$ as

$$C_1(t) = \sin^2[\pi\alpha(t)/2],$$

$$C_2(t) = \sin^2[\pi\alpha(t)/2]\cos^2[\pi\alpha(t)/2],$$

$$C_3(t) = \cos^2[\pi\alpha(t)/2]$$

for the three models.

The structure in the polarization comes about through the coefficients $C(t)$, the positions of the double zeros at $-t \approx 0.8$, 2.4 (GeV/c)^2 determining the intercept and slope of the trajectory function $\alpha(t)$ in each case. The trajectory functions for the three cases $C_1(t)$, $C_2(t)$, $C_3(t)$ considered then turn out to be $\alpha(t) = 0.77 + 1.1t$, $0.4 + 0.58t$, and $-0.23 + 1.1t$, respectively, and the corresponding fits to the polarization with these trajectories are shown as R4, R5, and R6 of Fig. 1(b). (These trajectories are not meant to have physical particles on them for positive t, but are presumably effective trajectories which are necessary to get the correct structure in the polarization.)

We see from Fig. 1(b) that good fits can be obtained for $-t \lesssim 2$ (GeV/c)^2, but again because of the damping $(s/s_0)^{\alpha(t)}$ factor, the calculated polarization is negligible beyond $-t \approx 2.5$ (GeV/c)^2.

None of the Regge fits discussed in this section is very satisfactory. As we have seen, trajectory and residue functions can certainly be chosen to get the polarization structures at the correct places, but typically the magnitude dies away rather rapidly for larger $-t$. Moreover, as the authors[5,6] of the various models themselves observe, the models do not give the correct energy dependence for the polarization at fixed t; that is, taking the values of the parameters as determined above by the 12.33-GeV/c data, the calculated polarizations for other energies disagree with the data. It does not seem clear how to modify these models, by means of cuts or otherwise, in order to get better agreement with the data. We have not pursued this particular aspect.

III. TWO OPTICAL MODELS

We turn next to a basically different kind of model, namely, the optical or geometric model. The major difference between Regge and optical models is that, in the latter, the (s-channel) diffraction amplitude has a well-defined set of single zeros just as in classical optics. (In contrast, the Pomeron is traditionally taken to have no zeros.) Double zeros in the polarization then come about from the coincidence, or near coincidence, of the zeros in the diffraction amplitude and a corresponding set in the single-flip amplitude must be substantially peripheral.

We shall briefly describe two optical models, and it is convenient to describe them in terms of the partial-wave expansions

$$k^2 N(s,t) = \sum (l + \tfrac{1}{2}) n_l\, d^l_{00}(\theta_s),$$

$$k^2 F(s,t) = \sum (l + \tfrac{1}{2}) f_l\, d^l_{10}(\theta_s)$$

for the nonflip and flip amplitudes, respectively. Here n_l, f_l, or equivalently $n(b), f(b)$ where $b = (l+\frac{1}{2})/k$ is the impact parameter, are the corresponding partial-wave amplitudes.

Optical A (Chou and Yang, Ref. 8; Durand and Lipes, Ref. 9). In this model, perhaps the best known in the optical category, the collision is envisaged as coming about through the overlap of the two proton mass distributions, each mass distribution being taken proportional to the proton electromagnetic form factor $G_p(t)$. Taking the dipole form $G_p(t) = (1 - t/\mu^2)^{-2}$ and writing $n_l = i(1 - S_l)$, $f_l = S_l \rho_l'$, one finds

$$S_l = \exp\left[A\left(\frac{\mu}{2k}\right)^3 Q_l^{(3)}\left(\cosh\frac{\mu}{k}\right)\right],$$

$$\rho_l' = Cl\left(\frac{\mu}{2k}\right)^4 Q_l^{(3)}\left(\cosh\frac{\mu}{k}\right).$$

The ReA is adjusted to fit $\sigma_{\text{tot}}(pp)$, and in the electromagnetic form factor $\mu^2 = 0.71$ (GeV/c)^2.

This parametrization, as it stands, is unable to fit the new polarization data, even if μ^2 is varied. It is easy to see from the original Durand-Lipes paper how this comes about. From their Figs. 1 and 2 one can see that the imaginary part of their nonflip amplitude has only two zeros, at $-t \approx 1.4$ and 5.8 (GeV/c)^2, for $0 < \theta_s < 90°$; the flip amplitude has only one zero, at $-t \approx 0.9$ (GeV/c)^2. Thus while it is possible to arrange for a dip in the polarization around $-t \approx 1$(GeV/c)^2 with this model, one does not generate the subsequent dips. Beyond $-t \approx 1$ (GeV/c)^2 the polarization is positive until it changes sign at $-t \approx 5.8$ (GeV/c)^2, remaining negative beyond this point.

The lack of recurring zeros in $N(s,t)$, $F(s,t)$ can be easily understood in terms of the impact-parameter profiles $n(b), f(b)$. The b-distributions that are generated from the dipole form of the electromagnetic form factor have too much low-b

component; to fit the data therefore $n(b), f(b)$ need to be more peripheral.

Optical B (Chu and Hendry, Ref. 10). This last model which we consider is based upon the direct parametrization of the impact-parameter distributions $n(b), f(b)$. Schematically

$$n(b)=D(b)+P_n(b),\ f(b)=P_f(b),$$

where $D(b)$ is a Fermi distribution in b (which gives rise to the diffraction), and $P_n(b), P_f(b)$ are Gaussian distributions in b centered around the periphery $b=R$. One can then get a fit such as that shown in Fig. 1(c), with $R=0.9$ F. The oscillations of ImN and ReF approximately coincide, thus yielding a sequence of double zeros in the polarization.

The Chu-Hendry model is actually just a sophisticated version of the gray-disk approximation, in which the disk is given a rounded edge (Fermi distribution in b). Since this model seems rather successful in duplicating [Fig. 1(c)] the pp polarization, it is worthwhile to see how this might come about in the cruder gray-disk version. Here we have

$$D(b)=\begin{cases} iD & \text{for } b\leqslant R \\ 0 & \text{for } b>R \end{cases}$$

and

$$P(b)=P\delta(b-R),$$

and in the classical limit we get

$$N(s,t)\sim iJ_1(R\sqrt{-t})/R\sqrt{-t}\ ,$$

$$F(s,t)\sim J_1(R\sqrt{-t}\,).$$

This yields a polarization proportional to $[J_1(R\sqrt{-t}\,)]^2/R\sqrt{-t}$, with double zeros located at the zeros of the J_1 Bessel function. With $R \approx 0.9$ F, these occur at $-t\approx 0.7$, 2.5, 5.2... $(\mathrm{GeV}/c)^2$, rather close in fact to the locations of the structure observed experimentally. As can be seen from our fit in Fig. 1(c), these features essentially remain, even when a smooth Fermi distribution is used for the central b region and Gaussians are used for the peripheral piece.

IV. CONCLUDING REMARKS

We have applied various models which have appeared in the recent literature to the case of proton-proton elastic polarization. Regge models seem to have great difficulty in fitting the newly observed polarization structures. On the other hand, at least one relatively simple optical model can fit the data with ease. Moreover, it has the appealing feature of relating the locations of the successive structures rather nicely in terms of a single interaction radius.

It seems to be generally agreed[1] that the dip which is observed in the pp differential cross section at ISR energies is a clear indication that optical diffraction is certainly taking place at these very high energies. It seems quite conceivable that the new measurements of the elastic pp polarization, which is a rather more sensitive probe of amplitudes than the differential cross section, indicate that optical behavior may very well be occurring at considerably lower energies.

It will also be very interesting to compare the various models with the forthcoming detailed data from Argonne using polarized proton beams on polarized proton targets (some early results are reported in Ref. 15). Regge predictions have previously been discussed by Rarita *et al.*[16] For the optical model, one expects (to the extent that the double-flip amplitudes ϕ_2, ϕ_4 may be ignored) that $C_{NN}\approx K_{NN}$ with a sequence of double zeros at the same positions as the double zeros of the polarization; the correlation $(1-D_{NN})$ which measures the difference between the nonflip amplitudes ϕ_1 and ϕ_3 is expected to be very small at small $-t$, but will steadily deviate from zero as $-t$ increases [we estimate about + 10% at $-t\approx 0.6$ $(\mathrm{GeV}/c)^2$ for a beam momentum of 5 GeV/c in the lab].

Also at Argonne the near-forward proton-neutron polarization will be measured[17] in the near future. At lower energies ($p_{\mathrm{lab}}\lesssim 1.5$ GeV/c), this polarization is known[18] to be large and positive, similar to proton-proton polarization. From an optical standpoint, one would expect the two polarizations to remain similar (that is, still quite large and positive) at least for $|t|\lesssim 0.6$ $(\mathrm{GeV}/c)^2$ as the beam momentum is increased. A naive Regge pole model, on the other hand, would expect proton-proton and proton-neutron forward polarizations to be mirror symmetric (just like $\pi^\pm$-proton polarizations). This is already in conflict with the low-energy data, and it would be surprising (though not inconceivable) if the forward np polarization changes sign as the energy is increased. However, not much weight should be given to this oversimplified Regge prediction since it is well known to have failed previously in the case of antiproton-proton forward polarization, which was also initially predicted[16] to be the mirror symmetry of pp polarization (experimentally, forward $\bar{p}p$ polarization is positive). Presumably the appropriate Regge analysis is much more complicated.

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$$N = i[\beta_1(t)(s/s_0)^{\alpha_1(t)} + \beta_2(t)(s/s_0)^{\alpha_2(t)}],$$

where each $\beta(t) = B \exp(Ct)$ and each $\alpha(t) = \alpha_0 + \alpha' t$. The first part of this expression is the usual Pomeron contribution giving the forward diffraction peak, while the second part is necessary to allow for the flattening of $d\sigma/dt$ beyond $-t \approx 1.5$ (GeV/c)^2.

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Effects of final-state interaction in lepton-pair production in proton-proton collisions, and in the massive quark model for electroproduction*

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We point out that analysis of a particular structure function for massive muon-pair production in proton-proton collisions may be capable of signaling the diffractive corrections which are expected, in the parton model, to be present in addition to the "bare" Drell-Yan term. A similar analysis of single-particle semi-inclusive electroproduction in the massive quark model suggests that the azimuthal angle dependence should be suppressed at most by logarithms of q^2, at variance with the behavior expected in the parton model. This investigation reveals the importance of a particular assumption which is necessary in the derivation of the parton-model result.

I. INTRODUCTION

Perhaps the most controversial extension of the parton model[1,2] has been to the process

$$p + p \to \mu^+ \mu^- X, \tag{1.1}$$

where the muon pair is emitted with large invariant mass. Drell and Yan[3] proposed a mechanism for this process in which a parton and an antiparton, one from each of the incoming particles, annihilated into a virtual photon. The cross section is then calculated as the appropriate discontinuity of Fig. 1. Landshoff and Polkinghorne[4] pointed out that one would also expect contributions from more complicated diagrams involving more general interaction between the two hadronic parts. The dominant contribution, which might be expected to be of the same order of magnitude as the Drell-Yan term, would be given by a diffractive interaction as represented by Pomeron exchange in Fig. 2.

Terazawa[5] has pointed out that extra information, in particular confirming the one-photon origin of the muon pair, might be obtained by separating out structure functions for the process (1.1). In Sec. II we give an alternative set of structure functions, and derive the asymptotic limits of these structure functions in the parton model. One of these structure functions is suppressed by a factor of s^{-1} in the squared amplitude corresponding to Fig. 1, but is expected to be suppressed at most by logarithms as a result