Tadpole term and (8, 8) chiral-symmetry breaking

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A Coleman-Glashow-type sum rule is obtained for the electromagnetic mass differences of the pseudoscalar mesons using the SU(2)-breaking term in the (8, 8) chiral-symmetry-breaking scheme. However, the agreement with the experimental data is not as good as in the case of the $(3,\bar{3})\oplus(\bar{3},3)$ model.

It is fairly well established now that the chiral symmetry, first proposed by Gell-Mann[1] about a decade ago, is a good approximation for the study of strong interactions. Since the chiral symmetry holds only approximately, it is important to know the SU(3)⊗SU(3) structure of the symmetry-breaking part of the Hamiltonian density. For many years, the most widely used form of the symmetry-breaking Hamiltonian was a single representation of SU(3)⊗SU(3), viz., the $(3,\bar{3})\oplus(\bar{3},3)$ representation,[2,3] and it was fairly successful. However, some recent experimental data have indicated the above-mentioned form of symmetry breaking to be unsatisfactory and attempts have been made to find alternative symmetry-breaking schemes[4-8] to explain certain experimental difficulties. In this connection, the (8,8) symmetry-breaking model, proposed by Genz and Katz[4,5] and several other workers, is an attractive possibility. One of the nice features of the (8,8) model is that it is possible to get a negative value for the parameter λ_0 of the K_{l3} decays.[4]

Recently Cicogna, Strocchi, and Vergara Caffarelli have derived a Coleman-Glashow-type sum rule[9] for the electromagnetic mass differences of the pseudoscalar mesons in a model-independent way.[10] For the derivation of this sum rule, these authors have included the $(\epsilon_3 u_3)$ term in the well-known $(3,\bar{3})\oplus(\bar{3},3)$ symmetry-breaking scheme[2] of SU(3)⊗SU(3) and have assumed the vacuum to be SU(3)-non-invariant. The agreement of their sum rule with the experimental data is excellent, since its accuracy is of the order of 1%.

In this paper we adopt the approach of Cicogna, Strocchi, and Vergara Caffarelli for the (8,8) form of chiral-symmetry breaking with an SU(2)-breaking term. Consequently, the symmetry breaking is assumed to be due to the Hamiltonian density $\mathcal{H}'$ belonging to the (8,8) representation of SU(3)⊗SU(3). $\mathcal{H}'$ is assumed to be of the form

$$\mathcal{H}' = d_0 z_0 + d_3 z_3 + d_8 z_8 \,, \tag{1}$$

where z_0 and z_α are defined[6] in the suggestive form of the products of SU(3)⊗SU(3) currents as follows:

$$\begin{aligned} z_0 &= \frac{1}{2\sqrt{2}}\,(V-A)^{\mu}_{\beta}(V+A)_{\mu\beta}\,, \\ z_\alpha &= (3/5)^{1/2} d_{\alpha\beta\gamma}(V-A)^{\mu}_{\beta}(V+A)_{\mu\gamma}\,, \end{aligned} \tag{2}$$

with

$$\alpha,\beta,\gamma = 1,2,3,\ldots,8\,; \quad \mu = 0,1,2,3\,.$$

Here d_0, d_3, and d_8 are constants and z_0 and z_α belong to the singlet and an octet, respectively, of the SU(3) obtained in the SU(3) decomposition of $\underline{8}\times\underline{8}$. The pseudoscalar operators of this model transform as the $\underline{8}+\underline{10}+\overline{\underline{10}}$ representation under SU(3). It is worth mentioning at this point that the (8,8) model does not contain an SU(3) singlet.

Now Eq. (32) of Ref. 10 is

$$m_{(i)}{}^2[\underline{G}^\alpha,\lambda]_i = [\underline{G}^\alpha, d]_i\,. \tag{3}$$

Here

$$\lambda = \lambda_i \hat{z}_i\,,$$

$$d = d_i \hat{z}_i\,,$$

where $\hat{z}_i$ is the unit vector in the direction i and λ_i are the vacuum expectation values of $\hat{z}_i$:

$$\lambda_i = \langle 0|\hat{z}_i|0\rangle\,.$$

With the help of Eq. (3), we get the following expressions for the masses of the pseudoscalar mesons:

$$\begin{aligned}
\pi^+ &= \frac{(3\sqrt{3}/\sqrt{10}\,)d_0 + (3\sqrt{3}/5)d_8}{(3\sqrt{3}/\sqrt{10}\,)\lambda_0 + (3\sqrt{3}/5)\lambda_8} = \pi^-\,, \\
K^+ &= \frac{(3\sqrt{3}/\sqrt{10}\,)d_0 + (9/10)d_3 - (3\sqrt{3}/10)d_8}{(3\sqrt{3}/\sqrt{10}\,)\lambda_0 + (9/10)\lambda_3 - (3\sqrt{3}/10)\lambda_8} = K^-\,, \\
K^0 &= \frac{(3\sqrt{3}/\sqrt{10}\,)d_0 - (9/10)d_3 - (3\sqrt{3}/10)d_8}{(3\sqrt{3}/\sqrt{10}\,)\lambda_0 - (9/10)\lambda_3 - (3\sqrt{3}/10)\lambda_8} = \bar{K}^0\,, \\
\pi^0 &= \frac{[(3\sqrt{3}/\sqrt{10}\,)d_0 + (3\sqrt{3}/5)d_8]a_{33} + [(3\sqrt{3}/5)d_3]a_{83}}{[(3\sqrt{3}/\sqrt{10}\,)\lambda_0 + (3\sqrt{3}/5)\lambda_8]a_{33} + [(3\sqrt{3}/5)\lambda_3]a_{83}}\,, \\
\eta &= \frac{[(3\sqrt{3}/\sqrt{10}\,)d_0 - (3\sqrt{3}/5)d_8]a_{88} + [(3\sqrt{3}/5)d_3]a_{38}}{[(3\sqrt{3}/\sqrt{10}\,)\lambda_0 - (3\sqrt{3}/5)\lambda_8]a_{88} + [(3\sqrt{3}/5)\lambda_3]a_{38}}\,.
\end{aligned} \tag{4}$$

In the above expressions, the mixing between π^0 and η is defined by

$$\hat{z}_3 = a_{38}\hat{\eta} + a_{33}\hat{\pi}^0 ,$$
$$\hat{z}_8 = a_{83}\hat{\pi}^0 + a_{88}\hat{\eta} . \qquad (5)$$

The above expressions for the pseudoscalar meson masses, displayed in Eq. (4), are obtained under the assumption that the pseudoscalar octet does not mix with the decuplet states. We note that the PCAC (partially conserved axial-vector current) hypothesis provides an independent way for the evaluation of the parameters λ_0 and λ_8 [with no SU(2)-breaking term in the Hamiltonian]. The PCAC hypothesis gives

$$F_\pi = (3\sqrt{3}/\sqrt{10})\lambda_0 + (3\sqrt{3}/5)\lambda_8 ,$$
$$F_K = (3\sqrt{3}/\sqrt{10})\lambda_0 - (3\sqrt{3}/10)\lambda_8 . \qquad (6)$$

Thus, by using the experimental estimates for the pion and kaon decay constants [$F_\pi = 0.95 m_\pi$ and $(F_K/F_\pi) = 1.28$], we obtain from (6)

$$\lambda_0 = 0.69 m_\pi , \quad \lambda_8 = -0.17 m_\pi ;$$
$$d_0 = 6.36 m_\pi{}^3 , \quad d_8 = -9.15 m_\pi{}^3 . \qquad (7)$$

It is obvious now that the symmetry breaking due to the SU(3) noninvariance of the vacuum is not negligible compared with the explicit SU(3) breaking in the Hamiltonian, since $(\lambda_8/\lambda_0)/(\epsilon_8/\epsilon_0)$ is of the order of 17%. This shows that the SU(3) results based on the naive application of the Wigner-Eckart theorem might get reasonable corrections. A similar effect is, therefore, expected in the case of SU(2) chiral-symmetry breaking.

The Ward-identity technique of Glashow and Weinberg now yields the following three sum rules:

$$\frac{F_K}{F_\pi}(K^+ - K^0)_{d_3} + \frac{F_\pi}{F_K}(\pi^0 - \pi^+)_{d_3} + \frac{9}{10}\frac{\lambda_3 K_{av}}{F_\pi} = \sqrt{3}M_{38}\left[\frac{1}{3}\left(4\frac{F_K}{F_\pi} - 1\right) - \frac{9}{5}\frac{\lambda_3}{F_K}\right] + \frac{9}{5}M_{33}\frac{\lambda_3}{F_\pi} + (\pi^0 - M_{33})\frac{F_\pi}{F_K} , \qquad (8)$$

$$M_{83} = \frac{3\sqrt{3}}{5F_\pi}(d_3 - M_{88}\lambda_3) , \qquad (9)$$

$$\frac{9}{5}\frac{\lambda_3}{F_\pi} \simeq \frac{1}{K_{av}}\left[\frac{9}{5}\frac{d_3}{F_\pi} + \frac{F_K}{F_\pi}(K^0 - K^+) - \frac{F_K}{F_\pi}(\pi^0 - \pi^+)\right] . \qquad (10)$$

In the above relations (8), (9), and (10),[11] M_{ij} is the mass matrix of the pseudoscalar mesons. $(K^+ - K^0)_{d_3}$ and $(\pi^0 - \pi^+)_{d_3}$ denote the contribution to the mass difference due to the term $d_3 z_3$. K_{av} is defined by

$$K_{av} = K^0 - \tfrac{1}{2}(K^+ - K^0)_{d_3} .$$

It is not difficult to recognize that the first two equations are analogous, respectively, to the following two relations obtained earlier:

$$(K^+ - K^0) + (\pi^0 - \pi^+) = \sqrt{3}\, M_{\eta\pi} , \qquad (8')$$

$$\sqrt{3}\, M_{\eta\pi} = \frac{\sqrt{2}\,\epsilon_3}{F_\pi} . \qquad (9')$$

It is interesting to notice that Eq. (8) reduces to Eq. (8′) in the limit $F_K = F_\pi$ [SU(3) invariance of the vacuum] and $\lambda_3 = 0$ [SU(2) invariance of the vacuum]. Similarly, Eq. (9) reduces to an equation similar to (9′) under the same limits.

The corrections to these simple relations (8′) and (9′) arise from the SU(3) noninvariance of the vacuum ($F_K \neq F_\pi$) and SU(2) noninvariance of the vacuum ($\lambda_3 \neq 0$).

By using a generalization of the Dashen theorem, which takes into account the noninvariance of the vacuum, one may obtain an equation equivalent to Eq. (8), but involving only the observed masses:

$$\frac{F_K}{F_\pi}(K^+ - K^0) + \frac{F_\pi}{F_K}(\pi^0 - \pi^+) + \frac{9}{10}\frac{\lambda_3 K_{av}}{F_\pi} = \sqrt{3}M_{83}\left[\frac{1}{3}\left(4\frac{F_K}{F_\pi} - 1\right) - \frac{9}{5}\frac{\lambda_3}{F_K}\right] + \tfrac{9}{5}M_{33}\frac{\lambda_3}{F_\pi} + (\pi^0 - M_{33})\frac{F_\pi}{F_K} . \qquad (11)$$

In order to check the accuracy of Eq. (11), we make the assumption[12] that

$$\frac{d_3}{d_8} = \frac{\lambda_3}{\lambda_8} . \qquad (12)$$

Then using the observed values of masses and decay constants in Eq. (10), we obtain

$$\lambda_3 = -3.28\times 10^{-4}\ \mathrm{GeV} ,$$
$$d_3 = -3.48\times 10^{-4}\ \mathrm{GeV}^3 . \qquad (13)$$

Once again, in deriving (11), we have neglected the possible mixing of the octet states with the decuplet states, obtained in the SU(3) decomposition of $\underline{8}\times\underline{8}$.

Substituting the observed masses and decay constants in Eq. (11), we have

$$\text{right-hand side of Eq. (11)} = \begin{cases} -5.78\times 10^{-3} \text{ for } (F_K/F_\pi) = 1.28 , \\ -5.88\times 10^{-3} \text{ for } (F_K/F_\pi) = 1.22 ; \end{cases}$$

$$\text{left-hand side of Eq. (11)} = \begin{cases} -6.81\times 10^{-3} \text{ for } (F_K/F_\pi) = 1.28 , \\ -6.49\times 10^{-3} \text{ for } (F_K/F_\pi) = 1.22 . \end{cases}$$

Thus the agreement is of the order of 16% for $(F_K/F_\pi)=1.28$ and 10% for $(F_K/F_\pi)=1.22$. The agreement is not as spectacular as in the case of the $(3,\bar{3})\oplus(\bar{3},3)$ model. Most probably, it is due to the fact that the mixing between the octet and the decuplet states has not been taken into account or that Eq. (12) is not exact.

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[11]Equations (8), (9), and (10) are the equations corresponding to Eqs. (5), (6), and (8) of Ref. 9. Here M_{ij} is the mass matrix defined by $M_{ij}[\underline{G}^\alpha,\lambda]_j=[\underline{G}^\alpha,d]_i$ and the summation over j is limited to the octet states only.

[12]It has been shown in Ref. 9 that Eq. (12) is indeed very nearly satisfied by explicit calculation of the parameters with the help of the scalar mesons. Moreover, it is seen that M_{83} becomes equal to M_{38} with the help of Eq. (12) after an approximate calculation, neglecting the octet-decuplet mixing.