Multiplicity behavior of $\pi^- p \to pX$: A multiperipheral-model description*

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For the inclusive reaction $\pi^- p \to pX$, the average charged multiplicity $\langle n\rangle$ of the system X as a function of missing mass M^2 and momentum transfer t is studied in terms of a multiperipheral model. For $M^2 \gtrsim 20$ GeV^2 (above the low-mass pion diffraction region) and for $|t| \lesssim 1$ GeV^2, the model is found to be in good agreement with data at 205 GeV/c, which show $\langle n(M^2,t)\rangle \sim \ln M^2$ for fixed t, and $\langle n(M^2,t)\rangle$ only weakly dependent on t for fixed M^2. Further experimental and theoretical investigation is suggested.

I. INTRODUCTION

The multiperipheral model[1] has had considerable success in describing many features of multiparticle production at high energy. Here we apply a simple version of the model to the inclusive inelastic reaction

$$\pi^- p \to pX . \tag{1}$$

We study the behavior of the average charged multiplicity $\langle n\rangle$ of the produced system X as a function of M^2 and t, where M^2 is the mass-squared of X and t is the four-momentum transfer squared to the recoil proton. The model is shown to provide a good description of data[2] at 205 GeV/c, which, for $M^2 \gtrsim 20$ GeV^2 and $|t| \lesssim 1$ GeV^2, show $\langle n(M^2, t)\rangle \sim \ln M^2$ at fixed t, and $\langle n(M^2, t)\rangle$ approximately independent of t for fixed M^2.

We first discuss some theoretical details and then compare model predictions with average multiplicity data for reaction (1). Although we specifically treat reaction (1), the model is applicable to the general class of reactions of the form $a + p \to pX$, as well as to reactions involving charge exchange at the nucleon vertex, such as $a + p \to nX$ and $a + p \to \Delta^{++}X$.

II. MULTIPERIPHERAL MODEL

To obtain $d\sigma/dM^2dt$ for reaction (1) we make two basic assumptions: (i) The M^2 dependence of $d\sigma/dM^2dt$ can be described by a *single Pomeron-pole term* for sufficiently large values of M^2. (ii) The system X is produced mainly by a *non-Pomeron* exchange mechanism for $M^2 \gtrsim 20$ GeV^2, as suggested by the observed inclusive M^2 distribution[2] for reaction (1) at 205 GeV/c.

A detailed multiperipheral model embodying these assumptions has been described in Ref. 3. For the present, we employ a simplified version of this model which assumes that the system X is produced by pion exchange (ignoring G-parity restrictions). Although other exchanges are clearly possible, this simplified model predicts essentially the same M^2 and t behavior for the average charged multiplicity of X as the more realistic model, and has the advantage of being calculationally considerably easier to use.

The inclusive differential cross section for reaction (1) for 20 GeV$^2 \lesssim M^2 \lesssim s$ is then

$$\frac{d\sigma}{dM^2dt} = \frac{g^2}{16\pi^3 s^2}\frac{1}{(t-m_\pi{}^2)^2}\,\overline{\beta}_\alpha\left(\frac{s_0}{s_0-t}\right)^{\alpha+1} \times\left(\frac{M^2}{s_0}\right)^{\alpha} . \tag{2}$$

Here g^2 is the effective coupling at the nucleon vertex, s is the center-of-mass energy squared, and α is the intercept of the Pomeron pole. The last three factors represent the high-energy off-shell $\pi\pi$ scattering amplitude; this form for the $\pi\pi$ amplitude is suggested by an approximate analytic solution to the multiperipheral integral equation.[4] The factor $\overline{\beta}_\alpha$ depends only on α, and s_0 is related to the squared masses of the prominent $\pi\pi$ resonances (thus $s_0 \approx 1$ GeV^2).

We now calculate $\langle n\rangle$, the average charged multiplicity of X, as a function of M^2 and t. Assuming that $\langle n\rangle$ is a constant fraction of the overall charged plus neutral multiplicity of X, $\langle n\rangle$ can be obtained from the inclusive differential cross section using[5,3]

$$\langle n(M^2, t)\rangle = C\,\frac{\partial}{\partial\alpha} D(M^2, t)\Big/ D(M^2, t) ,$$

where C is a constant related to the $\pi\pi$ coupling strength and $D(M^2, t) = d\sigma/dM^2dt$. Inserting (2) we find

$$\langle n(M^2, t)\rangle = C\left[\ln M^2 + \ln\left(\frac{s_0}{s_0-t}\right)\right] + d , \tag{3}$$

with $d = C[\partial(\ln\overline{\beta}_\alpha)/\partial\alpha - \ln s_0]$. Thus, for positive C, *$\langle n(M^2, t)\rangle$ is predicted to increase logarithmically with M^2 at fixed t, and to decrease very slowly with t for fixed M^2.* Furthermore, $\langle n(M^2, t)\rangle$ is predicted to be independent of s. Although both constants C and d in (3) are calculable from the multiperipheral model, they depend strongly on the details of the model and we therefore choose to obtain them by fitting the data, as described below.

We now proceed, by integrating (3) with respect to t or M^2, to calculate two other quantities of interest. The average charged multiplicity at mass M^2 is

$$\langle n(M^2)\rangle = \int \langle n(M^2, t)\rangle D(M^2, t)dt \Big/ \int D(M^2, t)dt$$
$$= C[\ln M^2 + \varphi(u_m)] + d, \qquad (4)$$

where u_m is the minimum allowed $|t|$ value for given M^2 and s, and

$$\varphi(u_m) \approx \ln\left[\left(\frac{s_0}{s_0+u_m}\right)\left(\frac{u_m}{s_0+u_m}\right)^{u_m/s_0}\right].$$

The function $u_m(M^2)$ represents the Chew-Low boundary: For values of M^2 not too close to s, $u_m \approx (M^2/s)^2 m_p{}^2(1-M^2/s)^{-1}$, where m_p is the proton mass. When $M^2/s \ll 1$, u_m is close to zero and the term $\varphi(u_m)$ in (4) has negligible contribution. However, as M^2, and therefore u_m, increases, $\varphi(u_m)$ gives a small negative contribution, so that for $s/2 \lesssim M^2 \lesssim s$, $\langle n(M^2)\rangle$ is predicted to deviate downward slightly from a simple $\ln M^2$ dependence.

The average charged multiplicity of X as a function of t, for $M^2 > 20$ GeV^2, is similarly obtained by integrating over M^2:

$$\langle n(t)\rangle = \frac{\int_{M^2_{\min}}^{M^2_{\max}} \langle n(M^2, t)\rangle D(M^2, t)dM^2}{\int_{M^2_{\min}}^{M^2_{\max}} D(M^2, t)dM^2}.$$

Here $M^2_{\min} = 20$ GeV^2 and

$$M^2_{\max} \approx (s/2m_p{}^2)(-t)^{1/2}[(4m_p{}^2-t)^{1/2}-(-t)^{1/2}]$$
$$\approx (s/m_p)(-t)^{1/2}$$

for $-t \ll 4m_p{}^2$. Using $\langle n(M^2, t)\rangle$ as given by Eq. (3) we find

$$\langle n(t)\rangle = C\left[\frac{(M^2_{\max})^{\alpha+1}\ln M^2_{\max} - (M^2_{\min})^{\alpha+1}\ln M^2_{\min}}{(M^2_{\max})^{\alpha+1} - (M^2_{\min})^{\alpha+1}} + \ln\left(\frac{s_0}{s_0-t}\right) - \frac{1}{\alpha+1}\right] + d. \qquad (5)$$

For $-t \gtrsim 0.1$ GeV^2, the $M^2_{\min}$ terms in (5) can be neglected and we obtain, using $M^2_{\max} \approx (s/m_p)(-t)^{1/2}$,

$$\langle n(t)\rangle \approx C\left[\ln\frac{(-t)^{1/2}s_0}{m_p(s_0-t)} + \ln s - \frac{1}{\alpha+1}\right] + d. \qquad (6)$$

Thus, for fixed s, $\langle n(t)\rangle$ is predicted to rise with $-t$ from $t \approx 0$, reaching a maximum at $-t = s_0 \approx 1$ GeV^2.

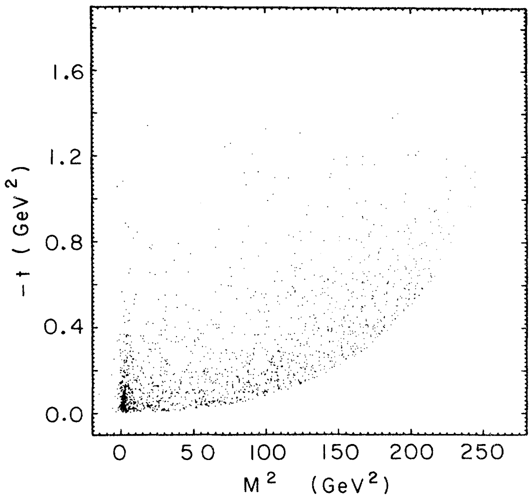

FIG. 1. Two-dimensional plot of missing mass (M^2) vs momentum transfer (t) for 1566 inelastic events of the reaction $\pi^- p \to pX$ at 205 GeV/c. A cutoff of 1.4 GeV/c in the recoil proton momentum limits $-t$ to ≤ 1.4 GeV^2.

III. COMPARISON WITH DATA

The data to be considered consist of 1566 inelastic events of reaction (1) at 205 GeV/c ($s = 385$ GeV^2), observed in the Fermi National Accelerator Laboratory 30-in. hydrogen bubble chamber. The outgoing proton, identified by ionization, has momentum ≤ 1.4 GeV/c. The quantities M^2 and t were calculated from measurements of the beam and recoil proton. Further experimental details are given in Ref. 2.

In order to apply the present version of the multiperipheral model, we assume that the produced system X consists entirely of charged and neutral pions. We assume, furthermore, that the charged/neutral pion ratio is independent of M^2 and t, so that the average charged multiplicity of X is a constant fraction of the average overall multiplicity of X.

Figure 1 shows $d\sigma/dM^2dt$ as a two-dimensional scatter plot of M^2 vs t. [As discussed in Ref. 2, the observed exponential falloff with t of $d\sigma/dt\,dM^2$ is such that biases introduced by the 1.4-GeV/c cutoff in proton momentum ($-t \leq 1.4$ GeV^2) are negligible up to $M^2 \approx 180$ GeV^2.] For fixed t, events extend in M^2 up to the Chew-Low boundary, $M^2_{\max} \approx (s/m_p)(-t)^{1/2}$. The low-$t$ cluster of events which peaks at $M^2 \approx 2$ GeV^2 is produced by diffraction dissociation of the incoming pion.[2] In the following, pion diffraction dissociation [i.e., production of the system X in reaction (1) via Pomeron exchange] is assumed to be unimportant for M^2 above 20 GeV^2.

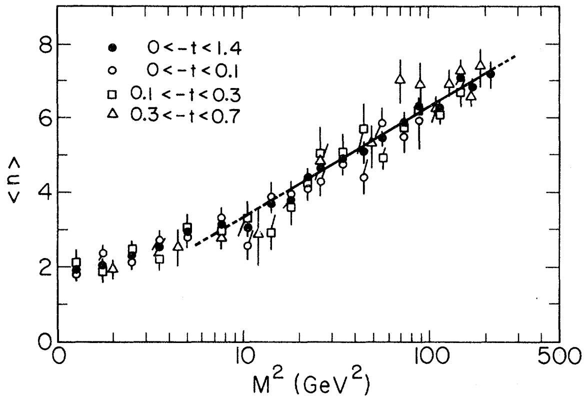

FIG. 2. Average charged multiplicity $\langle n\rangle$ as a function of M^2 for $-t<1.4$ GeV^2 (solid circles). The straight line is the functional form $\langle n\rangle = C\ln M^2 + d$ predicted by the multiperipheral model, with fitted parameters $C=1.3 \pm 0.1$ and $d=0.3\pm 0.3$ for $20 \le M^2 \le 180$ GeV^2. Also shown is $\langle n\rangle$ vs M^2 for three smaller intervals of momentum transfer: $0<-t<0.1$ GeV^2 (open circles), $0.1<-t<0.3$ GeV^2 (squares), and $0.3<-t<0.7$ GeV^2 (triangles).

A. M^2 dependence

Figure 2 shows the average charged multiplicity, $\langle n(M^2)\rangle$, of the system X for all t values combined (solid circles). In agreement with the model, $\langle n(M^2)\rangle \sim \ln M^2$ for $M^2 \gtrsim 10$ GeV^2. Fitting $\langle n(M^2)\rangle$ over the region $20<M^2<180$ GeV^2 using Eq. (4) without the small $\varphi(u_m)$ term, we find $\langle n(M^2)\rangle = C\ln M^2 + d$ with $C=1.3\pm 0.1$ and $d=0.3\pm 0.3$, as shown by the straight line in Fig. 2.

Figure 2 also gives $\langle n(M^2, t)\rangle$ vs M^2 for $-t = 0.0-0.1$, $0.1-0.3$, and $0.3-0.7$ GeV^2. The behavior of $\langle n(M^2,t)\rangle$ for each of these t intervals is similar, indicating at most a weak t dependence.

B. t dependence

Figure 3 shows $\langle n\rangle$ as a function of t for $M^2 < 20$ GeV^2 (pion diffractive region) and for $M^2 > 20$ GeV^2. We observe that $\langle n(t)\rangle$ for the $M^2 < 20$ GeV^2 region is nearly constant for $-t \lesssim 0.4$ GeV^2 and may be rising at higher t. On the other hand, for $M^2 > 20$ GeV^2, $\langle n(t)\rangle$ rises rapidly for $-t \lesssim 0.1$ GeV^2, and increases slowly for $0.1 \lesssim -t \lesssim 1$ GeV^2; the depression of $\langle n(t)\rangle$ at small t is a Chew-Low boundary effect.

The solid curve in Fig. 3 is the prediction for $M^2 > 20$ GeV^2 using Eq. (6) with $s_0 = 1$ GeV^2 and $\alpha = 1$, and with $C=1.3$, $d=0.3$ [as determined from the above fit to $\langle n(M^2)\rangle$]. Good agreement with the data is observed. The prediction is not sensitive to the precise value of α. Using an effective Pomeron[6] with an intercept of 0.85, for example, would shift the entire curve downward by only 0.05 units.

To investigate the t dependence of $\langle n(M^2, t)\rangle$ at

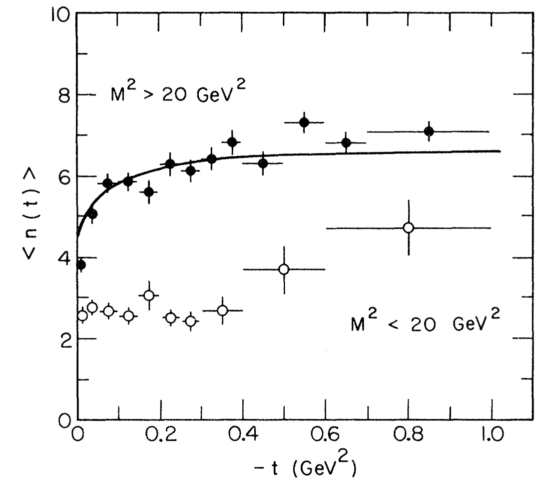

FIG. 3. Average charged multiplicity $\langle n\rangle$ vs t for $M^2<20$ GeV^2 (open circles) and for $M^2>20$ GeV^2 (solid circles). The curve is the prediction of the multiperipheral model for $M^2>20$ GeV^2.

fixed M^2, we show in Fig. 4 $\langle n\rangle$ vs t for several representative small intervals in M^2 above the pion diffractive region. In general, we find that *$\langle n\rangle$ at fixed M^2 is nearly independent of t for $-t \lesssim 1$ GeV^2*. The solid curves in Fig. 4 are the predictions of Eq. (3) with $s_0 = 1$ GeV^2, again using the previously determined values of C and d. Reasonable agreement with the data is observed.

For the most part, therefore, the observed weak dependence of $\langle n\rangle$ on t can be understood by

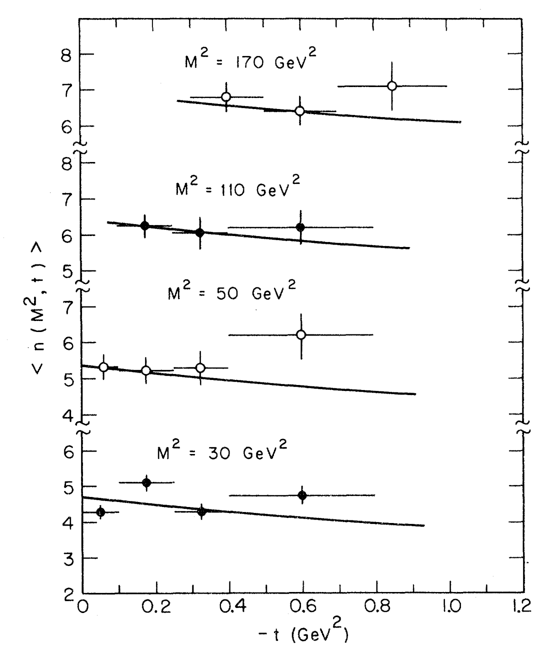

FIG. 4. Average charged multiplicity $\langle n\rangle$ vs t for $M^2 = 20-40$, $40-60$, $100-120$, and $160-180$ GeV^2. The curves are the predictions of the multiperipheral model.

the $[s_0/(s_0-t)]^{\alpha+1}$ factor in Eq. (2). However, while the statistical significance is not overwhelming, Figs. 3 and 4 do show that the data for $M^2 \gtrsim 20$ GeV^2 at high $|t|$ (i.e., $|t| \gtrsim 0.5$ GeV^2) are consistently above the model predictions. This suggests that other than pion exchange may contribute appreciably in this not-very-peripheral region. Further high-statistics data at the higher t values would be desirable.

IV. CONCLUSIONS

We have shown that the average charged multiplicity data for the reaction $\pi^- p \to pX$ at 205 GeV/c can be reasonably understood with a simple version of the multiperipheral model for $20 \lesssim M^2 \lesssim 200$ GeV^2 and $|t| \lesssim 1$ GeV^2. It would clearly be of interest to extend the present multiperipheral analysis to higher values of s, M^2, and $|t|$; to the higher multiplicity moments of X, such as f_2 and f_3; and to other reactions of the form $a+p \to pX$.

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Structure of the hadronic neutral current

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On the basis of general phenomenological arguments, it is suggested that the neutral weak hadronic current should transform as a U-spin scalar. Possible tests for this hypothesis are proposed, and the implication of the available data is discussed. The question whether such a structure can be generated from unified gauge theories is also examined.

Several recent experiments[1-5] have established the existence of neutral currents in neutrino-induced reactions. Qualitatively at least, such neutral currents have been proposed in some gauge-theory models of weak and electromagnetic interactions. Not surprisingly, a great deal of effort has been directed recently at testing the original Weinberg-Salam model,[6] and although nothing conclusive can be said at the present time, more definitive and elaborate comparisons will undoubtedly be forthcoming as the experimental data grow.

At this stage, however, it is worthwhile to keep an open mind on various theoretical or phenomenological possibilities. Even within the context of unified gauge theories such an attitude is useful, due to the fact that several models exist or can be constructed with different structures for neutral currents, although probably none is as simple in appeal (at least for the leptonic sector) as the original Weinberg-Salam model.

In this note we propose a general structure for the weak hadronic neutral current from a phenomenological point of view. This structure is a generalization of a recent proposal due to Sakurai.[7] We suggest possible tests for this current structure, and also discuss the implications of the available data. The question whether such struc-