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Deep-inelastic electroproduction and Koba-Nielsen-Olesen scaling*

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The multiplicity distribution measured in hadron-hadron collisions is related to the parton distribution measured in deep-inelastic leptoproduction. This relation results from a parton model in which bunches of particles are created through parton-parton collisions. The data on $pp \to n$ charged particles are compared with the model, and some further predictions are discussed.

New experimental results at high energy seem to indicate a universal behavior for the multiplicity distribution in hadron-hadron collisions.[1] Such a universal behavior was anticipated by Koba, Nielsen, and Olesen[2] (KNO), who predicted (relying on Feynman scaling arguments augmented by further technical assumptions) the following asymptotic law for the multiplicity distribution:

$$\langle n\rangle \frac{\sigma_n}{\sigma_{\rm in}} = \psi\left(\frac{n}{\langle n\rangle}\right) . \qquad (1)$$

The KNO scaling function ψ obeys the normalization conditions[3]

$$\int \psi(z)dz = 1 , \quad \int z\psi(z)dz = 1 , \qquad (2)$$

where $z \equiv n/\langle n\rangle$. Although relation (1) is expected to hold only asymptotically, recent analyses of the experimental data indicate that it is valid (up to 10%) in the range $p_{\rm lab} = 30-400$ GeV/c,[1,4,5] with slightly different ψ for different processes.

Several functional forms for ψ and fits to the experimental data performed with them exist in the literature.[4-6] Furthermore, several models from which KNO scaling emerges and in which ψ is calculable have been suggested; for comprehensive lists and descriptions of models the reader is referred to recent review articles.[1]

In this paper we present a model for the particle production mechanism which reflects the dynamical structure of the hadrons. In contrast with the geometrical models for which the multiplicity distribution is related to the distribution of inelastic collisions at a given impact parameter,[1] we suggest here that the multiplicity distribution reflects the momentum distribution of the constituents (partons) of the hadron. As is well known, this parton distribution is measured in deep-inelastic leptoproduction. We then prove that KNO scaling holds in such a model and derive an explicit relation [Eq. (12)] between the scaling function ψ and $F_2 = \nu W_2$.

Let us consider the process $A + B \to n$ particles in the framework of a parton model. Following Nielsen and Olesen,[7] we assume that as the hadrons pass through each other, a bunch of particles is produced whenever one parton from A hits a parton from B. All particles detected come from these bunches.[8] More specifically, we assume the production of particles in a parton-parton collision to proceed in two stages. The first stage is a "small fireball" production, weakly dependent on energy, followed by a decay stage.

Let us denote by $f(x)dx$ the number of partons having longitudinal fraction of the hadron momentum between x and $x+dx$. Let us consider only partons with x larger than some x_0 such that

$$x = \left(\frac{s}{s_c}\right)^{1/2} , \qquad (3)$$

where $\frac{1}{2}\sqrt{s}$ is the parton energy and $\frac{1}{2}\sqrt{s_c}$ is the hadron energy in the c.m. system of the hadrons. The cross section for inelastic collision per dx_1 interval of partons from hadron A, per dx_2 interval of partons from hadron B, is then

$$\frac{d^2\sigma_{\rm in}}{dx_1 dx_2} = Cf(x_1)f(x_2) \qquad (4)$$

(assuming for simplicity $A = B$), where C is approximately constant. Thus

$$\sigma_{\rm in} = C \int_{x_0}^{1} f(x_1)f(x_2)dx_1 dx_2 . \qquad (5)$$

The next step is to extract σ_n from $\sigma_{\rm in}$ through[3]

$$\sigma_n = \frac{d\sigma_{\rm in}}{dn} = \int_{x_0}^{1} \frac{d}{dx_2}\left(\frac{d\sigma_{\rm in}}{dx_1}\right)\frac{dx_2}{dn}dx_1 . \qquad (6)$$

In order to obtain dx_2/dn we further specify the mechanism for particle production by adding the following assumptions[9]:

(1) The bunch produced when two partons collide is emitted after an intermediate hydrodynamical expansion stage.[10] Thus it follows that particles are emitted from the "small fireball" which is produced when a parton-parton collision occurs with a multiplicity proportional to $s^{1/4}$.[11]

(2) The multiplicity distribution of the particles emitted in each parton-parton collision is narrow

compared with the total multiplicity distribution, i.e., there are only small fluctuations around the number of particles created in any bunch.[12]

From the last two assumptions it follows that there exists an (*a priori* unknown) constant k' such that

$$s=x_1x_2s_c=k'n^4\,. \tag{7}$$

Let us furthermore adopt the so-called universal multiplicity formula,[13] which agrees well with the existing data. Then

$$s_c=k\langle n\rangle^4\,, \tag{8}$$

where k is a constant well determined from experiment. Therefore, we conclude from Eqs. (7) and (8) that

$$x_1x_2=\frac{1}{K^4}\frac{n^4}{\langle n\rangle^4}=\frac{1}{K^4}z^4\,, \tag{9}$$

where $K^4\equiv k/k'$.[14] Thus, using Eqs. (4) and (6)

$$\langle n\rangle\sigma_n=\frac{4z^3C}{K^4}\int\frac{f(x_1)}{x_1}f(x_2)dx_1\,. \tag{10}$$

From Eqs. (5), (9), and (10) it is clear that KNO scaling is a direct consequence of our model and that a specific form is predicted for the KNO scaling function.

We now turn to the experimentally interesting case where one measures n charged particles in the final state. Then the two normalization conditions read

$$\int\psi(z)dz=2\,,\quad\int z\psi(z)dz=2\,, \tag{11}$$

and the KNO scaling function is predicted to be

$$\psi(z)=\frac{az^3}{K^2}\int\frac{f(x_1)}{x_1}f(x_2)dx_1\,, \tag{12}$$

where the connection between x_2 and z is given in Eq. (9). Equation (12), which is the main result of the model presented here, is a no-parameter relation between $\psi(z)$ and $f(x)$ since a and K are determined from the normalization conditions [Eq. (11)] as follows:

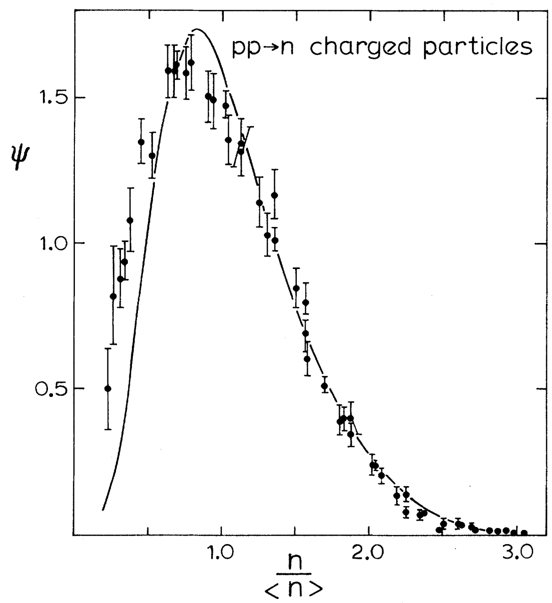

FIG. 1. $\psi(z)=\langle n\rangle\sigma_n/\sigma_{\rm in}$ for $pp\to n$ charged particles, $p_{\rm lab}\gtrsim 50$ GeV/c (see Ref. 4) as a function of $z=n/\langle n\rangle$. The curve is the prediction from Eq. (12) with $F_2(x)$ from Ref. 17, and—for illustration only—$F_2(x)\sim x$ below the lowest experimental point.

$$a=\frac{8[\int_{x_0}^1 x_1^{1/4}x_2^{1/4}f(x_1)f(x_2)dx_1dx_2]^2}{[\int_{x_0}^1 f(x_1)f(x_2)dx_1dx_2]^3}\,, \tag{13}$$

$$K=\frac{\int_{x_0}^1 f(x_1)f(x_2)dx_1dx_2}{\int_{x_0}^1 x_1^{1/4}x_2^{1/4}f(x_1)f(x_2)dx_1dx_2}\,. \tag{14}$$

It is clear that $K>1$, i.e., $k>k'$.

A direct consequence of KNO scaling is energy independence of the moments $\langle n^q\rangle/\langle n\rangle^q$ ($q=2,3,\cdots$), which are given by[2]

$$\frac{\langle n^q\rangle}{\langle n\rangle^q}=\tfrac{1}{2}\int z^q\psi(z)dz\,. \tag{15}$$

Thus in our model the above moments are given in terms of the parton distribution function by the sum rule [see Eqs. (9) and (12)]

$$\frac{\langle n^q\rangle}{\langle n\rangle^q}=\frac{[\int_{x_0}^1 f(x_1)f(x_2)dx_1dx_2]^{q-1}\int_{x_0}^1 x_1^{q/4}x_2^{q/4}f(x_1)f(x_2)dx_1dx_2}{[\int_{x_0}^1 x_1^{1/4}x_2^{1/4}f(x_1)f(x_2)dx_1dx_2]^q}\,. \tag{16}$$

In the parton model for inelastic leptoproduction[15] Bjorken scaling[16] is obeyed, and $x=-q^2/2M$ (in the usual notation) is identified with the longitudinal fraction of the hadron momentum. To compare Eqs. (12) and (16) with experiment, $f_i(x)$, where i denotes the parton type, must be known. These functions appear in Eq. (4), which should be interpreted as

$$\frac{d^2\sigma_{\rm in}}{dx_1dx_2}\sim\sum_{i,j}f_i(x_1)f_j(x_2)\,. \tag{17}$$

Unfortunately, the knowledge of $f_i(x)$ requires

TABLE I. Values of the moments $\langle n^q \rangle / \langle n \rangle^q$ for $pp \to n$ charged particles (Ref. 4) compared with the prediction from Eq. (16) with $F_2(x)$ from Ref. 17, and — for illustration only — $F_2(x) \sim x$ below the lowest experimental point.

q	Data	Sum rule
2	1.2438 ± 0.0056	1.3
3	1.813 ± 0.020	1.9
4	2.973 ± 0.057	3.2
5	5.36 ± 0.15	5.9
6	10.43 ± 0.36	11.7
7	21.6 ± 1.1	24.7
8	47.0 ± 2.8	54.7
9	107.4 ± 7.8	126.8
10	252 ± 22	305.0

complete knowledge of inelastic scattering of electron and neutrino from protons and neutrons. Let us assume that an average parton coupling strength can be pulled out from Eq. (17), and since it cancels in Eqs. (12) and (16) we can identify

$$f(x) = F_2(x)/x = \nu W_2/x.$$

To keep the symmetry between x_1 and x_2 the limits of integration in Eq. (12) are $z^4/K^4 \leq x_1 \leq 1$ [see Eq. (9)]. Since it is clear that our model is not valid for the small- (wee-) x region, this could tell us that KNO scaling may be violated in the small-z region. Unfortunately, there are no experimental data for $f(x)$ if $x < 0.03$ (see Ref. 17) and no experimental data for $\psi(z)$ if $z < 0.22$ (see Ref. 4) for $pp \to n$ charged particles.

Let us now compare Eq. (12) with the available data. With data for $F_2(x)$ over the whole x region Eq. (12) is a no-parameter sum rule; because of the lack of data for $F_2(x)$ at small x we can only compare using some *ad hoc* continuation.[18] It is clear that the assumption $F_2(x) \to \text{const}$ $(x \to 0)$ contradicts our model, and thus $F_2(x)$ has to decrease for $x \to 0$ (this may indeed be the trend of the data[17]). Taking, just for the sake of illustration, $F_2(x) \sim x$ below the lowest experimental point, we can determine a, K, and thus $\psi(z)$.

The resulting KNO scaling function is compared with the data for $pp \to n$ charged particles for $p_{\text{lab}} \geq 50$ GeV/c.[4] The important result is the agreement for the large-z region, since this region is not as sensitive to data on $F_2(x)$ at small x as the small-z region.

Turning now to Eq. (16), it is clear that in using the presently available data for $F_2(x)$ to compare with the experimental results for $\langle n^q \rangle / \langle n \rangle^q$, the small-$x$ problem mentioned above should get worse as q gets large.[19] This is indeed clear from the comparison of the experimental data[4] with Eq. (16), again using $F_2(x) \sim x$ below the lowest experimental point. One should further note that in our model $\langle n^q \rangle / \langle n \rangle^q > 1$, indicating the presence of long-range correlations.[2]

A further prediction is the value of K deduced from Eq. (14). From Eq. (9) it follows that

$$z_{\max} = K \,, \tag{18}$$

and now using the data on $F_2(x)$,[17] again using $F_2(x) \sim x$ below the lowest experimental point, we predict

$$z_{\max} = 3.68 \Rightarrow n_{\max} = 3.68 \langle n \rangle \,, \tag{19}$$

in agreement with experiment. This result depends only weakly on the continuation of $F_2(x)$ below the lowest experimental point.

Another prediction follows from the Drell-Yan relation[21]; from this together with Eq. (12) it follows that

$$\psi(z) \to z \left(1 - \frac{z^2}{K^2}\right)^6 \tag{20}$$

for $z \to z_{\max}$. Because of the small cross sections at large n, it is impossible to check this prediction with the present experimental data. In terms of σ_n we predict that for $n \to n_{\max}$

$$\sigma_n \to \frac{\sigma_{\text{in}}}{\langle n \rangle^2} n \left(1 - \frac{n^2}{K^2 \langle n \rangle^2}\right)^6 . \tag{21}$$

Since there is no deep-inelastic electroproduction experiment on pions, one cannot predict the KNO scaling function ψ for πN processes. However, since it appears that ψ for $\pi^- p \to n$ charged particles is only slightly different from ψ for $pp \to n$ charged particles,[5] one expects a similar parton distribution in a pion as in a proton. It will be interesting to see if $\psi(\pi^- p \to n$ charged particles) will differ for large z from the behavior predicted in Eq. (20) for $\psi(pp \to n$ charged particles), because of the different Drell-Yan relations for the pion and proton inelastic structure functions.

Another prediction regarding neutron processes such as $pn \to n$ charged particles follows from our model. In the neutron case, the experimental situation is the reverse of the pion case, since there are some data on the parton distribution[20] but no data on the KNO scaling function. At this stage we can only predict that ψ^n/ψ^p (ψ^p, ψ^n are the KNO scaling functions for $pp \to n$ charged particles, $pn \to n$ charged particles, respectively) should be a decreasing function of z as $z \to z_{\max}$. It is clear that a detailed comparison with experiment of the prediction for ψ^n/ψ^p that follows from Eq. (12) would be an important test of our model.

Let us end with the following remarks:

(1) It seems that since the lower limit x_0 is

s-dependent, the constants a and K [defined in Eqs. (13) and (14)] depend on s. However, it can easily be checked numerically that a and K are almost independent of s as long as $F_2(x) \not\to$ const and is a decreasing function of x for $x \to 0$. If a and K are calculated with $F_2(x) \to$ const and then used to calculate ψ, then KNO scaling is badly violated, and the experimental ψ cannot be reproduced. The exact form of $F_2(x)$ at small x is not that important for determining a, K, and then ψ, if $F_2(x)$ decreases as $x \to 0$.

(2) It seems that we are faced with a similar problem in the integrals used to calculate ψ [see Eq. (12)]. However, starting with $x_0 \leq x_1 \leq 1$ and then using Eq. (9), $z^4/K^4 \leq x_2 \leq z^4/x_0 K^4$, thus introducing an asymmetry between x_1 and x_2. The only way to avoid such an asymmetry is to integrate in the limits $z^4/K^4 \leq x_1 \leq 1$. Therefore there is no s dependence in the limits of the integrals used to calculate ψ.

(3) An upper limit for x_0 and for E_0 (the lowest possible energy for partons to be in the game) is obtained since one measures only charged particles; thus $z \geq z_0 = 2/\langle n \rangle$. Since $z^4/K^4 \geq x_0^2$, and for $p_{\text{lab}} = 50$ GeV/c $z_0 \simeq 0.376$, then $x_0 \lesssim 0.01$ ($p_{\text{lab}} \geq 50$ GeV/c) and $E_0 \lesssim 0.05$ GeV.

(4) It is clear that we miss some energy-carrying partons in the model described here. We are unable to account for them quantitatively, and we assume that they contribute to the leading particle effect.

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[1]P. Olesen, in *Proceedings of the Fourth International Symposium on Multiparticle Hadrodynamics, Pavia, 1973*, edited by F. Duimio *et al.* (Pavia, INFN, 1974), p. 463; A. Białas, *ibid.*, p. 93, and references therein.

[2]Z. Koba, H. B. Nielsen, and P. Olesen, Nucl. Phys. B40, 317 (1972).

[3]We use the asymptotic approximation $\sum_n \to \int dn$; see, for instance, A. J. Buras and Z. Koba, Nuovo Cimento Lett. 6, 629 (1973).

[4]P. Slattery, Phys. Rev. Lett. 29, 1624 (1972); Phys. Rev. D 7, 2073 (1973).

[5]F. T. Dao, J. Lach, and J. Whitmore, Phys. Lett. 45B, 513 (1973).

[6]G. Bozoki, E. Gombosi, M. Posch, and L. Vanicsek, Nuovo Cimento 64A, 881 (1969); H. Weisberg, Phys. Rev. D 8, 331 (1973).

[7]H. B. Nielsen and P. Olesen, Phys. Lett. 43B, 37 (1973).

[8]This assumption is identical to the set of pictures given in Ref. 7.

[9]It is clear that x_1 and x_2 are interchangeable.

[10]L. D. Landau, Izv. Akad. Nauk. SSSR 17, 51 (1953).

[11]P. Carruthers and Minh Duong-Van, Phys. Lett. 44, 507 (1973).

[12]For discussion of this assumption see Ref. 7.

[13]This expression (see Ref. 11) is used here since it provides a successful description of the data; we do not adopt here the complete hydrodynamical model for the hadron-hadron collision.

[14]Since $k = 0.107$ GeV^2 for $pp \to n$ charged particles (see Ref. 11), knowledge of K amounts to knowledge of k'.

[15]See, for instance, R. P. Feynman, *Photon-Hadron Interactions* (Benjamin, New York, 1972).

[16]J. D. Bjorken, Phys. Rev. 179, 1574 (1969).

[17]J. S. Poucher *et al.*, Phys. Rev. Lett. 32, 118 (1974).

[18]A similar sensitivity to small x exists in all sum rules which involve $F_2(x)/x$.

[19]Other problems connected with high q are discussed in A. Chodos, M. Rubin, and R. L. Sugar, Phys. Rev. D 8, 1620 (1973).

[20]A. Bodek *et al.*, Phys. Rev. Lett. 30, 1087 (1973).

[21]S. D. Drell and T.-M. Yan, Phys. Rev. Lett. 24, 181 (1970).