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Light-cone algebra and the structure of weak neutral current*

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Light-cone algebra of Fritzsch and Gell-Mann is used to suggest tests for the structure of recently discovered weak neutral current.

Experimenters at CERN[1] and NAL[2] have recently reported numerous muonless events in neutrino-nucleon collisions at high energies. These events have been interpreted as the neutral-current processes $\nu_\mu + N \to \nu_\mu + X$ and $\bar{\nu}_\mu + N \to \bar{\nu}_\mu + X$. The CERN group[3] has also reported one purely leptonic event which looks like a neutral-current effect and is interpretable as $\bar{\nu}_\mu + e \to \bar{\nu}_\mu + e$. Although such events arise naturally in the currently fashionable Salam[4]-Weinberg[5] (SW) gauge theory model of weak and electromagnetic interactions, it is clear that the neutral current could exist even if the model turned out to be incorrect. Therefore, it is important to experimentally ascertain various characteristics of the neutral current, such as isospin, SU(3) transformation properties, chiral structure, etc. The problem has attracted urgent attention of many people, and in recent papers Sakurai,[6] Pais and Treiman,[7] and Freedman[8] have suggested various ways in which one should determine properties of the neutral current. In the present note, we employ the techniques of light-cone algebra of Fritzsch and Gell-Mann[9] to derive a number of relations involving the structure functions of $\nu_\mu(\bar{\nu}_\mu) + N \to \nu_\mu(\bar{\nu}_\mu) + X$. If the existence of these processes is confirmed and the data on their structure functions become available, our relations should be useful in studying characteristics of the neutral current.

In the SW model, the neutral current is a linear combination of the electromagnetic current J_μ^{em} and the $I=1$ neutral sum $J_\mu^3 + J_\mu^{5,3}$ of the weak vector and axial-vector currents.[10] It is, however, conceivable that the neutral current has also a piece like $J_\mu^3 - J_\mu^{5,3}$ in addition to J_μ^{em} and $J_\mu^3 + J_\mu^{5,3}$. Next, we have the simple and elegant hypothesis put forward by Sakurai[6] that the weak hadronic neutral current is the baryonic current J_μ^0 [i.e., a singlet in SU(3)], possibly accompanied by its chiral partner $J_\mu^{5,0}$. In fact, Sakurai has suggested that one should not rule out the possibility that the whole neutral current is given by the baryon current J_μ^0 alone. We may also mention the model of Bég and Zee[11] (BZ) in which the neutral current is simply proportional to the electromagnetic current. Accordingly, we write for the neutral current J_μ^Z

$$J_\mu^Z = a\left(J_\mu^3 + \frac{1}{\sqrt{3}} J_\mu^8\right) + b\,(J_\mu^3 + J_\mu^{5,3}) + c\,(J_\mu^3 - J_\mu^{5,3}) + dJ_\mu^0 + d' J_\mu^{5,0} . \tag{1}$$

It is true that *a priori* there is no reason why the neutral current could not have totally new isovector and isoscalar pieces, i.e., pieces that are new operators and are not proportional to J_μ^i or $J_\mu^{5,i}$. At the same time, it is an attractive idea that neutral currents should be built up of vector and axial-vector currents which are already familiar to us in one way or another. At any rate, one should first examine the consequences of such an idea before adding hitherto-unknown pieces. The light-cone commutator of the neutral current (1) reads

$$[J_\mu^Z(x), J_\nu^Z(0)] \hat{=} \tfrac{1}{2}\partial_\rho D(x)\{s_{\mu\nu\rho\sigma}[(\tfrac{2}{3})^{1/2}\alpha_1 J_\sigma^0(A;x,0) + (\alpha_2/3^{1/2})J_\sigma^3(A;x,0) + (\alpha_3/3^{1/2})J_\sigma^8(A;x,0)]$$
$$-\epsilon_{\mu\nu\rho\sigma}[(\tfrac{2}{3})^{1/2}\beta_1 J_\sigma^0(S;x,0) + (\beta_2/3^{1/2})J_\sigma^3(S;x,0) + (\beta_3/3^{1/2})J_\sigma^8(S;x,0)]\} + \cdots . \tag{2}$$

The symbol $\hat{=}$ means that the commutator is evaluated near $x^2=0$. The dots represent the terms which are axial-vector in character. Since we are going to sandwich the commutator (2) between nucleon states of the same momentum and sum over spins, such terms do not contribute. Also,

$$D(x) = -\frac{1}{2\pi}\epsilon(x_0)\delta(x^2),$$

$$J(S;x,0) \equiv J(x,0) + J(0,x),$$

$$J(A;x,0) \equiv J(x,0) - J(0,x),$$

$$s_{\mu\nu\rho\sigma} = -\delta_{\mu\nu}\delta_{\rho\sigma} + \delta_{\mu\rho}\delta_{\nu\sigma} + \delta_{\mu\sigma}\delta_{\nu\rho} .$$

The coefficients α_i and β_i are given by the following expressions:

$$\begin{aligned}
\alpha_1 &= \tfrac{1}{3}a^2 + (a+b+c)^2 + (b-c)^2 + d^2 + d'^2 ,\\
\alpha_2 &= (2/3^{1/2})\,[(a+6^{1/2}d)(a+b+c) + 6^{1/2}d'(b-c)] ,\\
\alpha_3 &= -\tfrac{1}{3}a^2 + (a+b+c)^2 + (b-c)^2 + 2(2^{1/2}/3^{1/2})ad,\\
\beta_1 &= -2\,[(b-c)(a+b+c) + dd'],\\
\beta_2 &= (-2/3^{1/2})\,[(a+6^{1/2}d)(b-c) + 6^{1/2}d'(a+b+c)],\\
\beta_3 &= -2\,[(b-c)(a+b+c) + (2^{1/2}/3^{1/2})ad'] .
\end{aligned} \tag{3}$$

It may be noted that in the SW model $c=d=d'=0$, $b=1$, and a is fixed by the process $\nu_\mu + e \to \nu_\mu + e$. In the BZ model, $b=c=d=d'=0$ and a is a free parameter and is not fixed by $\nu_\mu + e \to \nu_\mu + e$. In Sakurai's model, if the chiral partner of the baryon current is also participating with full strength (S_1 model), we have $d=d'=1$, $a=b=c=0$, and if the chiral partner is altogether absent (S_2 model), we have $d=1$, $a=b=c=d'=0$.

Let us denote by G_i $(i=1,2,3)$ the scale functions of the processes $\nu_\mu(\bar\nu_\mu) + N \to \nu_\mu(\bar\nu_\mu) + X$. On using (2) and defining

$$\begin{aligned}
\langle p|J^i_\sigma(S;x,0)|p\rangle &= p_\sigma \int_{-1}^{+1} d\xi\, e^{-i\xi p\cdot x} S^i(\xi)\\
&\quad + \text{trace terms},\\
\langle p|J^i_\sigma(A;x,0)|p\rangle &= p_\sigma \int d\xi\, e^{-i\xi p\cdot x} A^i(\xi)\\
&\quad + \text{trace terms},
\end{aligned} \tag{4}$$

where nucleon spin summation is understood, we obtain the following relations:

$$G_2^{\nu p}(\xi) = \tfrac{1}{4}\xi\,[(\tfrac{2}{3})^{1/2}\alpha_1 A^0(\xi) + (\alpha_2/3^{1/2})A^3(\xi) + (\alpha_3/3^{1/2})A^8(\xi)] , \tag{5a}$$

$$G_2^{\nu n}(\xi) = \tfrac{1}{4}\xi\,[(\tfrac{2}{3})^{1/2}\alpha_1 A^0(\xi) - (\alpha_2/3^{1/2})A^3(\xi) + (\alpha_3/3^{1/2})A^8(\xi)], \tag{5b}$$

$$G_3^{\nu p}(\xi) = \tfrac{1}{4}\,[(\tfrac{2}{3})^{1/2}\beta_1 S^0(\xi) + (\beta_2/3^{1/2})S^3(\xi) + (\beta_3/3^{1/2})S^8(\xi)] , \tag{5c}$$

$$G_3^{\nu n}(\xi) = \tfrac{1}{4}\,[(\tfrac{2}{3})^{1/2}\beta_1 S^0(\xi) - (\beta_2/3^{1/2})S^3(\xi) + (\beta_3/3^{1/2})S^8(\xi)] . \tag{5d}$$

Note that if the neutral current is in fact proportional to the electromagnetic current, we must have $G_3^{\nu p} = 0 = G_3^{\nu n}$ (BZ model). If the neutral current is given by $J^0_\mu + J^{5,0}_\mu$, we have $G^{\nu p}_{2,3} = G^{\nu n}_{2,3}$ (S_1 model) and if the axial-vector part is absent, we get $G_3^{\nu p} = 0 = G_3^{\nu n}$, $G_2^{\nu p} = G_2^{\nu n}$ (S_2 model).

For further discussion, we also need the scale functions of $e + N \to e + X$ and $\nu + N \to l + X$. With our definitions, we get

$$F_2^{ep}(\xi) = \tfrac{1}{6}\xi\,[2(\tfrac{2}{3})^{1/2}A^0(\xi) + A^3(\xi) + (1/3^{1/2})A^8(\xi)], \tag{6a}$$

$$F_2^{en}(\xi) = \tfrac{1}{6}\xi\,[2(\tfrac{2}{3})^{1/2}A^0(\xi) - A^3(\xi) + (1/3^{1/2})A^8(\xi)] , \tag{6b}$$

$$F_2^{\nu p}(\xi) = \xi\,[(\tfrac{2}{3})^{1/2}A^0(\xi) - S^3(\xi) + (1/3^{1/2})A^8(\xi)] , \tag{6c}$$

$$F_2^{\nu n}(\xi) = \xi\,[(\tfrac{2}{3})^{1/2}A^0(\xi) + S^3(\xi) + (1/3^{1/2})A^8(\xi)] , \tag{6d}$$

$$F_3^{\nu p}(\xi) = -(\tfrac{2}{3})^{1/2}S^0(\xi) + A^3(\xi) - (1/3^{1/2})S^8(\xi), \tag{6e}$$

$$F_3^{\nu n}(\xi) = -(\tfrac{2}{3})^{1/2}S^0(\xi) - A^3(\xi) - (1/3^{1/2})S^8(\xi) . \tag{6f}$$

Using Eqs. (5a), (5b), (6a), (6b), (6e), and (6f), we get

$$\frac{G_2^{\nu p} - G_2^{\nu n}}{F_2^{ep} - F_2^{en}} = \frac{6(G_2^{\nu p} - G_2^{\nu n})}{\xi(F_3^{\nu p} - F_3^{\nu n})} = \frac{3^{1/2}}{2}\,\alpha_2 = \begin{cases} a(1+a) & (\text{SW}),\\ a^2 & (\text{BZ}),\\ 0 & (\text{S}_1 \text{ and } \text{S}_2).\end{cases}$$

One could regard (7) either as determining the parameter a of the SW or BZ models or as a way of distinguishing between these two models. The point is that if we suppose that the SW model is correctly describing the events seen at CERN and NAL, we would have[1] $a = -0.6$ to -0.8, so that $a(1+a) < 0$. The right-hand side of (7) in the BZ case is positive, however.

From Eqs. (5c), (5d), (6c), and (6d), we have

$$\frac{\xi(G_3^{\nu p} - G_3^{\nu n})}{F_2^{\nu p} - F_2^{\nu n}} = -\frac{\beta_2}{4\sqrt{3}} = \begin{cases} \tfrac{1}{6}a < 0 & (\text{SW}),\\ 0 & (\text{BZ, S}_1, \text{ and S}_2).\end{cases} \tag{8}$$

Equations (5c), (5d), (6e), and (6f) give

$$\frac{G_3^{\nu p} + G_3^{\nu n}}{F_3^{\nu p} + F_3^{\nu n}} = \begin{cases} \tfrac{1}{2}(1+a) > 0 & (\text{SW}),\\ 0 & (\text{BZ}).\end{cases} \tag{9}$$

We have no such relation in the Sakurai model. Combining (7), (8), and (9), we obtain the interesting relation

$$2\xi^2\left(\frac{G_3^{\nu p} + G_3^{\nu n}}{F_3^{\nu p} + F_3^{\nu n}}\right) = \frac{(F_2^{\nu p} - F_2^{\nu n})(G_2^{\nu p} - G_2^{\nu n})}{(F_3^{\nu p} - F_3^{\nu n})(G_3^{\nu p} - G_3^{\nu n})} \quad (\text{SW}). \tag{10}$$

Next, define the ratios

$$R_1(\xi) = \frac{F_2^{\nu p} + F_2^{\nu n}}{F_2^{ep} + F_2^{en}} , \quad R_2(\xi) = \frac{G_2^{\nu p} + G_2^{\nu n}}{F_2^{ep} + F_2^{en}} .$$

Using Eqs. (5a), (5b), and (6a)–(6d), we get

$$4R_2(\xi) = 6(\alpha_1 - \alpha_3) + (2\alpha_3 - \alpha_1)R_1(\xi) . \tag{11}$$

In a recent publication, Lipkin and Paschos[12] have studied the ratio $R_1(\xi)$. They find that this ratio can be narrowed down to lie in the range $3 \leq R_1(\xi) \leq \frac{18}{5}$ to within 10%. Using this result in (11), we find that

$$\tfrac{3}{4}\alpha_1 \leq R_2(\xi) \leq \tfrac{3}{10}(2\alpha_1+\alpha_3)\,. \tag{12}$$

As special cases, we have

$$a^2+\tfrac{3}{2}(1+a) \leq R_2(\xi) \leq a^2+\tfrac{9}{5}(1+a)\,. \quad \text{(SW)} \tag{13}$$

The limits on $R_2(\xi)$ are more explicit in Sakurai's model, where we get

$$1.2 \leq R_2(\xi) \leq 1.5\,, \quad (\mathrm{S}_1) \tag{14a}$$

$$0.6 \leq R_2(\xi) \leq 0.75\,. \quad (\mathrm{S}_2) \tag{14b}$$

In the BZ model, we have the equality

$$R_2(\xi)=a^2\,. \quad \text{(BZ)} \tag{15}$$

Thus if the ratio $R_2(\xi)$ exhibits ξ dependence, the BZ model will be ruled out. If it is independent of ξ, Eqs. (13) and (15) are not useful in distinguishing between the SW and BZ models. They can, however, be used as consistency checks for the respective models when studied in conjunction with Eq. (7).

Before proceeding further, we consider an integrated version of (9). From the recent experiments by Hasert *et al.*[13] we find that

$$\frac{\int_0^1 d\xi\, \xi^2(F_3^{\nu p}+F_3^{\nu n})}{\int_0^1 d\xi\, \xi(F_2^{\nu p}+F_2^{\nu n})} = -0.87 \pm 0.08\,.$$

Using this result, we obtain from Eq. (9)

$$\frac{\int_0^1 d\xi\, \xi^2(G_3^{\nu p}+G_3^{\nu n})}{\int_0^1 d\xi\, \xi(F_2^{\nu p}+F_2^{\nu n})} = (-0.87 \pm 0.08)\tfrac{1}{2}(1+a)<0 \quad \text{(SW)}\,. \tag{16}$$

Similarly, we can also obtain an integrated version of (11). That is, we get

$$\begin{aligned}4\int_0^1 d\xi(G_2^{\nu p}+G_2^{\nu n}) &= 6(\alpha_1-\alpha_3)\int_0^1 d\xi(F_2^{ep}+F_2^{en}) \\ &\quad +(2\alpha_3-\alpha_1)\int_0^1 d\xi(F_2^{\nu p}+F_2^{\nu n}) \\ &= 6(\alpha_1-\alpha_3)(0.28\pm0.04) \\ &\quad +(2\alpha_3-\alpha_1)(1.00\pm0.04)\,,\end{aligned} \tag{17}$$

where the data used are from the recent CERN bubble chamber experiments[13] and from SLAC.[14] Hence, we get

$$\int_0^1 d\xi(G_2^{\nu p}+G_2^{\nu n}) = \begin{cases} 0.28a^2+\tfrac{1}{2}(1+a) & \text{(SW)}, \\ 0.28a^2 & \text{(BZ)}, \\ 0.34 & (\mathrm{S}_1), \\ 0.17 & (\mathrm{S}_2). \end{cases} \tag{18}$$

It is also possible to write the integrated version of (7). A recent estimate gives[15]

$$\int_0^1 \frac{d\xi}{\xi}(F_2^{ep}-F_2^{en})=0.27\,.$$

Therefore, we get

$$\int_0^1 \frac{d\xi}{\xi}(G_2^{\nu p}-G_2^{\nu n}) = \tfrac{1}{2}\sqrt{3}\,\alpha_2 \times 0.27 = \begin{cases} 0.27a(1+a) & \text{(SW)}, \\ 0.27a^2 & \text{(BZ)}, \\ 0 & (\mathrm{S}_1 \text{ and } \mathrm{S}_2). \end{cases} \tag{19}$$

Lastly, we note that S^0, S^3, and S^8 are, respectively, matrix elements of the baryon number, isotopic spin, and hypercharge currents. It is, therefore, possible to normalize them. Indeed, we have

$$\begin{aligned} &\int_0^1 d\xi\, S^0(\xi)=\sqrt{6}\,, \\ &\int_0^1 d\xi\, S^3(\xi)=1\,, \\ &\int_0^1 d\xi\, S^8(\xi)=\sqrt{3}\,. \end{aligned} \tag{20}$$

Thus

$$\int_0^1 d\xi(G_3^{\nu p}+G_3^{\nu n}) = \tfrac{1}{2}(2\beta_1+\beta_3) = \begin{cases} -3(1+a) & \text{(SW)}, \\ 0 & \text{(BZ)}, \\ -2 & (\mathrm{S}_1), \\ 0 & (\mathrm{S}_2). \end{cases} \tag{21}$$

In a similar fashion, we get

$$\int_0^1 d\xi(G_3^{\nu p}-G_3^{\nu n}) = \frac{\beta_2}{2\sqrt{3}} = \begin{cases} -\tfrac{1}{3}a & \text{(SW)}, \\ 0 & (\mathrm{BZ},\ \mathrm{S}_1, \text{ and } \mathrm{S}_2). \end{cases} \tag{22}$$

We have considered consequences of various models which are special cases of (1). However, it also seems possible to examine the structure of (1) in its generality from our relations (7), (8), (12), (17), (19), (21), and (22). First of all, we note that the inequalities (12) determine α_1 and α_3 in terms of the minimum and maximum values of the ratio $R_2(\xi)$, provided, of course, that the ratio $R_1(\xi)$ does in fact achieve its minimum and maximum values of 3 and $\frac{18}{5}$, respectively. Equation (17) is then simply a consistency check for the values of α_1 and α_3 so determined. If, on the other hand, the ξ dependence of the ratio $R_1(\xi)$ can really be ignored, (11) and (17) represent one and the same equation for α_1 and α_3. Obviously, then we need another relation between α_1 and α_3 to determine them separately. Using SU(3), we get

$$\int_0^1 \frac{d\xi}{\xi}[4(G_2^{\nu p}+G_2^{\nu n})-3\alpha_1(F_2^{ep}+F_2^{en})] = 0.8(2\alpha_3-\alpha_1), \quad (23)$$

where we have made use of the numerical value for the quantity

$$\int_0^1 \frac{d\xi}{\xi}(F_2^{ep}-F_2^{en})$$

quoted earlier. The quantities α_2 and β_2 are determined separately, though we have just one relation [Eq. (21] for β_1 and β_3. Again using SU(3) and the experimental estimate[13]

$$\int_0^1 d\xi\, \xi(F_3^{\nu p}+F_3^{\nu n}) = -0.88 \pm 0.05$$

we obtain another relation connecting β_1 and β_3, namely,

$$\int_0^1 d\xi[4\xi(G_3^{\nu p}+G_3^{\nu n})+(\beta_3-\beta_1)(F_2^{\nu p}-F_2^{\nu n})] = 0.88\beta_1 . \quad (24)$$

This enables one to determine β_1 and β_3 separately.

When the quantities α_i, β_i are determined, we have six equations for five unknowns a, b, c, d, d' [cf. Eq. (3)], i.e., we have one extra equation which should serve as a consistency test. We believe that this procedure should determine the five unknowns unambiguously. However, such an exercise is feasible only when sufficient data on the inelastic nucleon form factors become available.

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[1] F. J. Hasert *et al.*, Phys. Lett. 46B, 138 (1973).
[2] A. Benvenuti (unpublished).
[3] F. J. Hasert *et al.*, Phys. Lett. 46B, 121 (1973).
[4] A. Salam and J. C. Ward, Phys. Lett. 13, 168 (1964); A. Salam, in *Elementary Particle Thoery: Relativistic Groups and Analyticity (Nobel Symposium No. 8)*, edited by N. Svartholm (Wiley, New York, 1969), p. 367.
[5] S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); 27, 1688 (1971).
[6] J. J. Sakurai, Phys. Rev. D 9, 250 (1974).
[7] A. Pais and S. B. Treiman, Phys. Rev. D 9, 1459 (1974).
[8] D. Z. Freedman, Phys. Rev. D 9, 1389 (1974).
[9] H. Fritzsch and M. Gell-Mann, in *Broken Scale Invariance and the Light Cone*, 1971 Coral Gables Conference, edited by M. Dal Cin, G. J. Iverson, and A. Perlmutter (Gordon and Breach, New York, 1971), Vol. 2, p. 1; and in *Proceedings of the International Conference on Duality and Symmetry in Hadron Physics*, edited by E. Gotsman (Weizmann Science Press, Jerusalem, 1971).
[10] It must be understood, however, that the SW model is a model of leptons only. Identification of the "weak" isospin with the "strong" isospin is an additional assumption. However, this assumption has been made by all authors who suggest tests for neutral currents. See, for example, E. A. Paschos and L. Wolfenstein, Phys. Rev. D 7, 2220 (1973), and references quoted therein. See, also, Ref. 7.
[11] M. A. B. Bég and A. Zee, Phys. Rev. Lett. 30, 675 (1973).
[12] H. J. Lipkin and E. Paschos, Phys. Rev. D 8, 2325 (1973).
[13] F. J. Hasert *et al.*, Phys. Lett. 46B, 274 (1973).
[14] G. Miller *et al.*, Phys. Rev. D 5, 528 (1972).
[15] E. D. Bloom, rapporteur's talk at the International Symposium on Electron and Photon Interactions at High Energies, Bonn, 1973 [SLAC Report No. SLAC-PUB-1319, 1973 (unpublished)].