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Comments on *CP* violation through phase angles in weak currents, the $\Delta I=\frac{1}{2}$ rule, and the relation $\eta_{+-}=\eta_{00}$

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It is shown explicitly in a relativistic quark model that the approximate $\Delta I=\frac{1}{2}$ rule requires that the phase angles ϕ and ξ of the axial-vector weak currents, which introduce *CP* violation, be opposite.

The equality between the *CP*-violation parameters $\eta_{+-}=\eta_{00}$ has been shown recently[1] to follow from the isospin transformation properties of the weak currents, if their phase angles are opposite, $\phi=-\xi$. Since the soft-pion limit is not used, the authors stress that the result does not involve the $\Delta I=\frac{1}{2}$ rule. In fact, from such simple symmetry arguments, one cannot draw definite conclusions about the $\Delta I=\frac{1}{2}$ rule, as pointed out by Lee.[2]

We shall show, by means of explicit dynamical computations in a relativistic quark model,[3] that the relation $\phi=-\xi$ not only implies $\eta_{+-}=\eta_{00}$ (as proven in Ref. 1), but it is required by the approximate $\Delta I=\frac{1}{2}$ rule. We show, moreover, that $\eta_{+-}=\eta_{00}$ holds even if the rule is broken to allow, for instance, for $K^+\to\pi^+\pi^0$ decay.

In the relativistic quark model that we use, the mesons are bound states of spin-$\frac{1}{2}$ quark-antiquark pairs ($q\bar{q}$), described by the Bethe-Salpeter (BS) equation with an harmonic oscillator kernel. When symmetry breaking is introduced through quark mass differences, one can explain all the K-meson decays, including the $\Delta I=\frac{1}{2}$ rule and its deviations.[4] The BS amplitude for the pseudoscalar mesons reads

$$\chi(P,r)=\frac{4\pi}{(3\beta)^{1/2}}\left(1+\frac{\not{P}}{M}\right)\gamma_5\exp\left(-\frac{r_E{}^2}{2\sqrt{\beta}}\right)|q\bar{q}\rangle. \quad (1)$$

$2\sqrt{\beta}\simeq 1\ \text{GeV}^2$ is the inverse of the Regge slope, P is the meson momentum, and M is the quark mass, which is related to the K decay constant by

$$F_K=\frac{4\sqrt{\beta}}{\pi\sqrt{3}\,M}. \quad (2)$$

$|q\bar{q}\rangle$ is the SU(3) wave function and r_E is the Wick-rotated relative momentum of the $q\bar{q}$ inside the meson.

The relevant diagrams for the $K\to 2\pi$ decays are shown in Figs. 1, where the circles are BS amplitudes[5] and the triangles stand for the effective weak current

$$J_q^\alpha=\bar{\mathcal{P}}\gamma^\alpha(1+e^{i\phi}\gamma_5)\mathfrak{N}\cos\theta_C+\bar{\mathcal{P}}(\gamma^\alpha(1+e^{i\xi}\gamma_5)+\delta k^\alpha)\lambda\sin\theta_C, \quad (3)$$

$$\delta=-\frac{3}{4M}\,\frac{m_K{}^2-m_\pi{}^2}{m_{K^*}{}^2}.$$

The diagrams Figs. 1(a) and 1(b) vanish identically for $K^+\to\pi^+\pi^0$. The diagram Fig. 1(c) does not contribute to $K^0\to\pi^0\pi^0$, whereas the one in Fig. 1(d) does not contribute to $K^0\to\pi^+\pi^-$. The

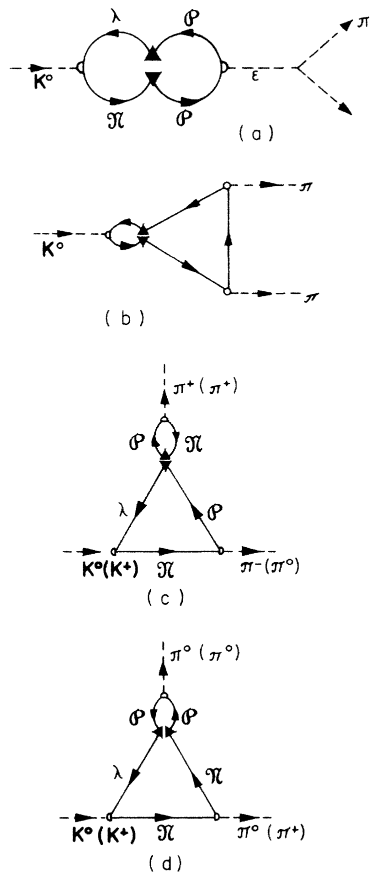

FIG. 1. Diagrams for the $K^0 \to \pi\pi$ decays. The circles stand for BS amplitudes and the triangles for the effective weak currents.

details of the computations can be found in Ref. 4 and the results of the diagrams are summarized in Table I.

The $\Delta I = \frac{1}{2}$ rule, which suppresses the $\Delta I = \frac{3}{2}$ decay $K^+ \to \pi^+\pi^0$, is realized in the CP-conserving case ($\phi = \xi = 0$) through the cancellation of the first terms in diagrams 1(c) and 1(d).[6] For these terms to cancel it is necessary now that $\phi = -\xi$. In this CP-violating case two additional terms appear in 1(b) and 1(d) with coefficients $e^{i\phi} - e^{i\xi}$ which depend explicitly on the quark mass M. These terms, which vanish for $\phi = -\xi$, would be too big by orders of magnitude and would break the $\Delta I = \frac{1}{2}$ prediction for the ratio $(K_S \to \pi^+\pi^-)/(K_S \to \pi^0\pi^0)$. The approximate $\Delta I = \frac{1}{2}$ rule, as realized in the relativistic quark model, requires therefore $\phi = -\xi$. One can see at once in Table I that if one sets $\phi = -\xi$, there is a common phase in all amplitudes $e^{i\phi}$. In this case one has

TABLE I. Traces of the diagrams for $K^0 \to 2\pi$ decays.

(a)	$\frac{16\beta^{5/4}}{3\sqrt{3}\pi^2}\left(1-\frac{m_K^2}{4m_\epsilon^2}\right)\frac{g_{\epsilon\pi\pi}}{(m_K^2-m_\epsilon^2+\frac{1}{4}\Gamma_\epsilon^2)+im_\epsilon\Gamma_\epsilon} \times\left[\delta e^{i\phi}+\frac{4}{M}(e^{i\phi}-e^{-i\xi})\right]$
(b)	$(e^{i\phi}-e^{-i\xi})\frac{16M\sqrt{\beta}}{3\sqrt{3}\pi}+\delta e^{i\phi}[\frac{3}{4}(m_K^2-4m_\pi^2)+8\sqrt{\beta}]\frac{1}{3\pi}\left(\frac{\beta}{3}\right)^{1/4}$
(c)	$e^{i\phi}[-F_K(m_K^2-m_\pi^2)]-\frac{16\delta\sqrt{\beta}}{3\pi\sqrt{3}}m_\pi^2 e^{i\phi}$
(d)	$\frac{e^{i\phi}+e^{-i\xi}}{2}F_K(m_K^2-m_\pi^2)+\frac{4\sqrt{\beta}\,\delta}{3\pi\sqrt{3}}(2\sqrt{\beta}+\frac{5}{8}m_K^2)e^{i\phi} +(e^{i\phi}-e^{-i\xi})\frac{16M\sqrt{\beta}}{3\sqrt{3}\pi}$

$$\eta_{00} = \eta_{+-} = \frac{i\tan\phi + \epsilon}{1 + i\epsilon\tan\phi}\,, \tag{4}$$

where ϵ is the usual CP mixing parameter

$$\epsilon = \langle K_2^0 | K_S\rangle = \langle K_1^0 | K_L\rangle.$$

We have therefore $\eta_{+-} = \eta_{00}$, as discussed in Ref. 1, even in spite of the experimental deviations from the $\Delta I = \frac{1}{2}$ rule in the $K^+ \to \pi^+\pi^0$ decays and in the ratio $(K_S \to \pi^+\pi^-)/(K_S \to \pi^0\pi^0)$, which are explained in the present relativistic quark model.

Finally, we remark that our approach permits a numerical evaluation[7] of the parameters of the models of CP violation based on modifications of the weak currents (by means of phase angles[8] or through neutral currents[9]).

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[5]The graph Fig. 1(b) takes into account the strong interaction of the final-state π mesons. The S-wave resonance ϵ is given in the model by

$$\chi = \frac{4\pi\sqrt{2}}{3\beta^{3/4}}\left[\left(1+\frac{\not{P}}{M}\right)\not{r} - \frac{(P\cdot r)\not{r}}{m_\epsilon^2} - \frac{ir^2}{M} - \frac{r\cdot P}{M} + \frac{i}{M}\frac{(r\cdot P)^2}{m_\epsilon^2}\right]$$

$$\times \exp(-r_\epsilon^2/2\sqrt{\beta})\,|q\bar{q}\rangle.$$

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