Charge-symmetry violations in the Achiman model*

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We investigate the implications of the Achiman model in the deep-inelastic region using the light-cone algebra. The model predicts $\sin^2\theta_W \sim 0.7$. Charge-symmetry violations are calculated, and found to be rather large.

With the advent of recently proposed[1-4] unified theories of weak and electromagnetic interactions considerable effort has gone into extending these theories to include hadrons. In particular, the theory of Salam[1] and Weinberg,[2] which seems to be the most promising candidate, has been extended to the hadronic sector by Glashow, Iliopoulos, and Maiani (GIM)[5] in a scheme which makes use of a charmed quark in addition to the three conventional quarks. Despite the elegance of this scheme, there seems no simple way of explaining the observed hadron spectrum in the GIM model.

The advantages of using Han-Nambu quarks[6] in other domains of particle physics are well known[7]; recently, Lipkin[8] suggested the use of Han-Nambu quarks to extend gauge theories of leptons to include the hadrons. A number of authors have developed models based on various gauge groups[9-13] using three triplets of the Han-Nambu variety or the "colored" variety.

Of special interest is the model due to Achiman[9] which is based on a $SU(2)\otimes U(1)$ gauge group and gives a rather simple and economical extension of the Weinberg-Salam model to the hadronic sector. It should be pointed out that, in this model, it is necessary to introduce a heavy neutral lepton. Universality is preserved by construction, rather than occurring naturally. We study the Achiman model in the deep-inelastic region using the technique of light-cone algebra proposed by Fritzsch and Gell-Mann.[14] The data on deep-inelastic electron-nucleon scattering and the charged- and neutral-current neutrino processes $\nu_\mu(\bar\nu_\mu)+N\to\mu^\mp+X$ and $\nu_\mu(\bar\nu_\mu)+N\to\nu_\mu(\bar\nu_\mu)+X$ allow calculation of the value of the Weinberg angle. The detailed expressions for the structure functions allow explicit comparison of putative charge-symmetric pairs (such as $F_2^{\nu p}$ versus $F_2^{\bar\nu n}$ for example). In the Achiman model we find substantial violation of charge symmetry, especially above the threshold for producing $SU(3)''$ nonsinglet states.

In the Achiman model[9] the electromagnetic and weak charged and neutral currents are given by

$$J_\mu^{\rm em}=i\bar{\mathcal{P}}_2\gamma_\mu\mathcal{P}_2+i\bar{\mathcal{P}}_3\gamma_\mu\mathcal{P}_3-i\bar{\mathfrak{N}}_1\gamma_\mu\mathfrak{N}_1-i\bar\lambda_1\gamma_\mu\lambda_1\,, \tag{1}$$

$$J_\mu^{+}=[i\bar{\mathcal{P}}_2\gamma_\mu(1+\gamma_5)\lambda_3+i\bar{\mathcal{P}}_3\gamma_\mu(1+\gamma_5)\mathfrak{N}_3]\cos\theta_C+[-i\bar{\mathcal{P}}_2\gamma_\mu(1+\gamma_5)\mathfrak{N}_3+i\bar{\mathcal{P}}_3\gamma_\mu(1+\gamma_5)\lambda_3]\sin\theta_C \tag{2}$$

$$J_\mu^{Z}=\tfrac{1}{2}[i\bar{\mathcal{P}}_2\gamma_\mu(1+\gamma_5)\mathcal{P}_2+i\bar{\mathcal{P}}_3\gamma_\mu(1+\gamma_5)\mathcal{P}_3-i\bar{\mathfrak{N}}_3\gamma_\mu(1+\gamma_5)\mathfrak{N}_3-i\bar\lambda_3\gamma_\mu(1+\gamma_5)\lambda_3-4\sin^2\theta_W J_\mu^{\rm em}]\,, \tag{3}$$

where θ_C and θ_W are the Cabibbo and Weinberg angles, respectively. Each of the three triplets $(\mathcal{P}_1^0,\mathfrak{N}_1^-,\lambda_1^-)$, $(\mathcal{P}_2^+,\mathfrak{N}_2^0,\lambda_2^0)$, and $(\mathcal{P}_3^+,\mathfrak{N}_3^0,\lambda_3^0)$ transforms as the representation $\underset{\sim}{3}$ under $SU(3)'$. The "proton" quarks $(\mathcal{P}_1^0,\mathcal{P}_2^+,\mathcal{P}_3^+)$ transform as $\underset{\sim}{3}^*$ under the new group $SU(3)''$ as do the $\mathfrak{N}$ and λ quarks. We write the currents (1) through (3) in the form in which their $SU(3)'\otimes SU(3)''$ transformation properties are more transparent. Let us introduce the "generalized" vector and axial-vector currents:

$$J_\mu^{(i,j)}(x)=i\bar q(x)\gamma_\mu\tfrac{1}{2}\lambda_i\otimes\tfrac{1}{2}\rho_j\,q(x)\,, \tag{4a}$$

$$J_\mu^{5(i,j)}(x)=i\bar q(x)\gamma_\mu\gamma_5\tfrac{1}{2}\lambda_i\otimes\tfrac{1}{2}\rho_j\,q(x)\,, \tag{4b}$$

where q is the nine-quark column vector

$$q=\begin{bmatrix}\mathcal{P}_1\\ \mathcal{P}_2\\ \mathcal{P}_3\\ \mathfrak{N}_1\\ \mathfrak{N}_2\\ \mathfrak{N}_3\\ \lambda_1\\ \lambda_2\\ \lambda_3\end{bmatrix}. \tag{4c}$$

Since q transforms as $(3,3^*)$ under $SU(3)'\otimes SU(3)''$, the matrices ρ_j differ from λ_j by suitable phases. We follow the phase conventions of de Swart.[15] Defining

$$\rho_0=-\lambda_0=-(\tfrac{2}{3})^{1/2}\,1\,, \tag{5a}$$

we find that

$$\rho_i = \eta_i \lambda_i \text{ (no summation)}, \tag{5b}$$

where

$$\eta_i = \begin{cases} +1, & i = 1, 4, 7 \\ -1, & i = 0, 2, 3, 5, 6, 8 . \end{cases} \tag{5c}$$

The ρ_i satisfy the same commutation relations as the λ_i. However, the anticommutators differ by a minus sign:

$$\{\rho_i, \rho_j\} = -2 d_{ijk} \rho_k , \quad i, j, k = 0, 1, \ldots, 8 . \tag{6}$$

In terms of the currents (4a) and (4b), we now find that the currents (1) through (3) are given by

$$J_\mu^{\text{em}} = \sqrt{6}\, [J_\mu^{(0,3)} + (\tfrac{1}{3})^{1/2} J_\mu^{(0,8)} - J_\mu^{(3,0)} - (\tfrac{1}{3})^{1/2} J_\mu^{(8,0)}] , \tag{7}$$

$$\begin{aligned} J_\mu^{+} = {} & -[J_\mu^{(4+i5,6-i7)} + J_\mu^{5(4+i5,6-i7)} + (\tfrac{2}{3})^{1/2} J_\mu^{(1+i2,0)} + (\tfrac{2}{3})^{1/2} J_\mu^{5(1+i2,0)} - 2(\tfrac{1}{3})^{1/2} J_\mu^{(1+i2,8)} - 2(\tfrac{1}{3})^{1/2} J_\mu^{5(1+i2,8)}] \cos\theta_C \\ & + [J_\mu^{(1+i2,6-i7)} + J_\mu^{5(1+i2,6-i7)} - (\tfrac{2}{3})^{1/2} J_\mu^{(4+i5,0)} - (\tfrac{2}{3})^{1/2} J_\mu^{5(4+i5,0)} + 2(\tfrac{1}{3})^{1/2} J_\mu^{(4+i5,8)} + 2(\tfrac{1}{3})^{1/2} J_\mu^{5(4+i5,8)}] \sin\theta_C , \end{aligned} \tag{8}$$

$$\begin{aligned} J_\mu^{Z} = {} & (\tfrac{1}{6})^{1/2} (J_\mu^{(0,3)} + J_\mu^{5(0,3)}) - (\tfrac{3}{2})^{1/2} (J_\mu^{(3,0)} + J_\mu^{5(3,0)}) - (\tfrac{1}{2})^{1/2} (J_\mu^{(0,8)} + J_\mu^{5(0,8)}) - (\tfrac{1}{2})^{1/2} (J_\mu^{(8,0)} + J_\mu^{5(8,0)}) \\ & + \tfrac{1}{2}\sqrt{3}\, (J_\mu^{(3,8)} + J_\mu^{5(3,8)}) + \tfrac{1}{2}(\tfrac{1}{3})^{1/2} (J_\mu^{(8,3)} + J_\mu^{5(8,3)}) + \tfrac{1}{2} (J_\mu^{(3,3)} + J_\mu^{5(3,3)}) + \tfrac{1}{2} (J_\mu^{(8,8)} + J_\mu^{5(8,8)}) - 2 \sin^2\theta_W J_\mu^{\text{em}} . \end{aligned} \tag{9}$$

The currents (7)–(9) contain pieces transforming as an octet under SU(3)″. Below the threshold for producing SU(3)″ nonsinglet states (for brevity referred to as the SU(3)″ threshold), these pieces lie dormant and can be dropped. Above SU(3)″ threshold, all three currents are capable of producing SU(3)″ nonsinglet states singly when incident on SU(3)″ singlet targets, so that the octet pieces can no longer be ignored.

The light-cone commutation relations satisfied by the weak and electromagnetic currents are presented in an appendix. We obtain different expressions for the commutators depending on whether we are above or below the SU(3)″ threshold. We form the nucleon matrix elements of the commutators (A1) to (A6), and define these matrix elements as Fourier transforms

$$\begin{aligned} \langle p | J_\sigma^{(i,j)}(S; x, 0) | p \rangle &= p_\sigma \int_{-1}^{1} d\xi\, e^{-i\xi p\cdot x} S^{ij}(\xi) + \text{trace terms}, \\ \langle p | J_\sigma^{(i,j)}(A; x, 0) | p \rangle &= p_\sigma \int_{-1}^{1} d\xi\, e^{-i\xi p\cdot x} A^{ij}(\xi) + \text{trace terms}, \end{aligned} \tag{10}$$

$$\xi = q^2/2M\nu ,$$

where the proton-spin summation is understood. We can now write the fourteen scale functions F_2^{ep}, F_2^{en}, $F_{2,3}^{\nu p}$, $F_{2,3}^{\nu n}$, $F_{2,3}^{\bar\nu p}$, $F_{2,3}^{\bar\nu n}$, $G_{2,3}^{\nu p}$ and $G_{2,3}^{\nu n}$ in terms of the functions S^{i0} and A^{i0}, where $i = 0, 3, 8$. [The nucleon scale functions in the neutral-current processes $\nu(\bar\nu) + N \to \nu(\bar\nu) + X$ are denoted by G_2 and G_3.] These can easily be worked out from the relevant commutators given in the Appendix. Here we shall be mainly concerned with the "averaged" scale functions F_2^{eN}, $F_{2,3}^{\nu N}$, $F_{2,3}^{\bar\nu N}$, and $G_{2,3}^{\nu N}$, where N stands for the average of proton and neutron.

Structure functions below the SU(3)″ threshold.

$$F_2^{eN}(\xi) = -\xi[\tfrac{2}{3} A^{00} + \tfrac{1}{3}(\tfrac{1}{2})^{1/2} A^{80}] , \tag{11}$$

$$F_2^{\nu N}(\xi) = -\xi[2A^{00} + \sqrt{2}\,(1 - \tfrac{3}{2}\sin^2\theta_C) A^{80} - 3(\tfrac{1}{2})^{1/2} \sin^2\theta_C S^{80}] , \tag{12}$$

$$F_3^{\nu N}(\xi) = [2S^{00} + \sqrt{2}\,(1 - \tfrac{3}{2}\sin^2\theta_C) S^{80} - 3(\tfrac{1}{2})^{1/2} \sin^2\theta_C A^{80}] , \tag{13}$$

$$F_2^{\bar\nu N}(\xi) = -\xi[2A^{00} + \sqrt{2}\,(1 - \tfrac{3}{2}\sin^2\theta_C) A^{80} + 3(\tfrac{1}{2})^{1/2} \sin^2\theta_C S^{80}] , \tag{14}$$

$$F_3^{\bar\nu N}(\xi) = [2S^{00} + \sqrt{2}\,(1 - \tfrac{3}{2}\sin^2\theta_C) S^{80} + 3(\tfrac{1}{2})^{1/2} \sin^2\theta_C A^{80}] , \tag{15}$$

$$G_2^{\nu N}(\xi) = -\tfrac{1}{3}\xi(1 - 4z + 8z^2) \times [A^{00} + \tfrac{1}{2}(\tfrac{1}{2})^{1/2} A^{80}] , \tag{16}$$

$$G_3^{\nu N}(\xi) = \tfrac{1}{3}(1 - 4z)[S^{00} + \tfrac{1}{2}(\tfrac{1}{2})^{1/2} S^{80}] . \tag{17}$$

Here and in the following $z = \sin^2\theta_W$.

Structure functions above the SU(3)″ threshold.

$$F_2^{eN}(\xi) = -\tfrac{1}{3}\xi[4A^{00} + (\tfrac{1}{2})^{1/2} A^{80}] , \tag{18}$$

$$F_2^{\nu N}(\xi) = -3\xi[4A^{00} + (\tfrac{1}{2})^{1/2} A^{80} - 3(\tfrac{1}{2})^{1/2} S^{80}] , \tag{19}$$

$$F_3^{\nu N}(\xi) = 3[4S^{00} + (\tfrac{1}{2})^{1/2} S^{80} - 3(\tfrac{1}{2})^{1/2} A^{80}] , \tag{20}$$

$$F_2^{\bar\nu N}(\xi) = -3\xi[4A^{00} + (\tfrac{1}{2})^{1/2} A^{80} + 3(\tfrac{1}{2})^{1/2} S^{80}] , \tag{21}$$

$$F_3^{\bar\nu N}(\xi) = 3[4S^{00} + (\tfrac{1}{2})^{1/2} S^{80} + 3(\tfrac{1}{2})^{1/2} A^{80}] , \tag{22}$$

$$G_2^{\nu N}(\xi) = -\tfrac{1}{3}\xi[2(1 - 2z + 8z^2) A^{00} + \tfrac{1}{2}(\tfrac{1}{2})^{1/2} (1 - 8z + 8z^2) A^{80}] , \tag{23}$$

$$G_3^{\nu N}(\xi) = \tfrac{2}{3}(1 - 2z) S^{00} + \tfrac{1}{6}(\tfrac{1}{2})^{1/2} (1 - 8z) S^{80} . \tag{24}$$

Note that both above and below the SU(3)″ threshold we have to deal with five unknowns, namely S^{00}, S^{80}, A^{00}, A^{80}, and z. Fortunately, there is enough data to determine all five quantities. Indeed, from the SLAC data,[16]

$$2\int_0^1 d\xi\, F_2^{eN}(\xi)=0.29\pm 0.02\,,$$

while the CERN data of Eichten *et al.*[17] and Hasert *et al.*[18] give

$$\sigma_{cc}^{\nu N}=(0.48\pm 0.02)\,\frac{G^2ME}{\pi}\,,$$

$$\sigma_{cc}^{\bar\nu N}=(0.17\pm 0.01)\,\frac{G^2ME}{\pi}\,,$$

$$\sigma_{nc}^{\nu N}/\sigma_{cc}^{\nu N}=0.21\pm 0.03\,,$$

$$\sigma_{nc}^{\bar\nu N}/\sigma_{cc}^{\bar\nu N}=0.45\pm 0.09\,.$$

Here $\sigma_{cc}^{\nu,\bar\nu}$ and $\sigma_{nc}^{\nu,\bar\nu}$ stand for the cross sections for the charged-current processes $\nu_\mu(\bar\nu_\mu)+N\to\mu^\mp+X$ and the neutral-current processes $\nu_\mu(\bar\nu_\mu)+N\to\nu_\mu(\bar\nu_\mu)+X$, respectively. In terms of the structure functions F_i and G_i, the four cross sections are given by

$$\sigma_{cc}^{(\nu,\bar\nu)N}=\frac{G^2ME}{\pi}\int_0^1 d\xi\,[\tfrac{2}{3}F_2^{(\nu,\bar\nu)N}(\xi)\mp\tfrac{1}{3}\xi F_3^{(\nu,\bar\nu)N}(\xi)]\,, \tag{25}$$

$$\sigma_{nc}^{(\nu,\bar\nu)N}=\frac{G^2ME}{\pi}\int_0^1 d\xi\,[\tfrac{2}{3}G_2^{\nu N}(\xi)\mp\tfrac{1}{3}\xi G_3^{\nu N}(\xi)]\,. \tag{26}$$

The upper and lower signs are for ν and $\bar\nu$, respectively. In writing (25) and (26), and hereafter, we assume the Callan-Gross relation[19]

$$F_2(\xi)=2\xi F_1(\xi)\,.$$

Solving below the SU(3)″ threshold for $\theta_C=0.242$, we find that

$$z=0.68\pm 0.07\,, \tag{27}$$

$$S_2\equiv\int_0^1 d\xi(F_2^{\nu N}+F_2^{\bar\nu N})=0.99\pm 0.10\,, \tag{28}$$

$$D_2\equiv\int_0^1 d\xi(F_2^{\nu N}-F_2^{\bar\nu N})=-0.41\pm 0.09\,, \tag{29}$$

$$S_3\equiv\int_0^1 d\xi\,\xi(F_3^{\nu N}+F_3^{\bar\nu N})=-1.74\pm 0.50\,, \tag{30}$$

$$D_3\equiv\int_0^1 d\xi\,\xi(F_3^{\nu N}-F_3^{\bar\nu N})=0.03\pm 0.02\,. \tag{31}$$

Solving above the SU(3)″ threshold (and assuming that the aforementioned experimental results continue to hold above this threshold),

$$z=0.73\pm 0.04\,, \tag{32}$$

$$S_2=2.61\pm 0.18\,, \tag{33}$$

$$D_2=0.71\pm 0.18\,, \tag{34}$$

$$S_3=0.50\pm 0.33\,, \tag{35}$$

$$D_3=3.26\pm 0.37\,. \tag{36}$$

The Weinberg angle in the Achiman model is somewhat larger than is generally believed to be the case.[21] For example, in the possible observation of the purely leptonic event $\bar\nu_\mu+e^-\to\bar\nu_\mu+e^-$ at CERN,[20] the limits on z are given to be $0.1<z<0.6$, with a confidence level of 90%. Other estimates[21] of the parameter z also give lower values and generally put it around 0.3 to 0.4.

If charge symmetry is assumed, the strangeness-conserving part of the charged current J_μ^+ satisfies the relation

$$e^{i\pi I_2}J_\mu^+e^{-i\pi I_2}=-(J_\mu^+)^\dagger\,, \tag{37}$$

from which follow the relations $F_i^{\bar\nu N}=F_i^{\nu N}$, $i=2, 3$, if $\theta_C=0$. Below the SU(3)″ threshold, one expects only small departures (of the order of 5%) from the charge-symmetry hypothesis due to the strangeness-changing part of the weak charged current.[22] From Eqs. (12) through (15), we see that the departure from charge symmetry is indeed proportional to $\sin^2\theta_C$ below the SU(3)″ threshold. To determine the magnitude of this violation, we compute the ratios $r_i=D_i/S_i$, $i=2, 3$. We find

$$r_2=-0.42\pm 0.08\,,\quad r_3=-0.02\pm 0.01\,. \tag{38}$$

The charge-symmetry violation in F_2 is much bigger than expected. However, no significant charge-symmetry violation is apparent in F_3. Above the SU(3)″ threshold, there is charge-symmetry violation even if $\theta_C=0$, since it is independent of the value of the Cabibbo angle. We find that

$$r_2=0.27\pm 0.07\,,\quad r_3=6.52\pm 4.30 \tag{39}$$

above the SU(3)″ threshold. Here the charge-symmetry violation in F_3 is much stronger than in F_2. From (38) we see that the Achiman model leads to a rather large violation of charge symmetry even below the SU(3)″ threshold.

Experimentally, the situation is not quite clear even though the recent data of Eichten *et al.*[17] show no indication of a violation of charge symmetry. In this connection it is also useful to study the y distributions:

$$\frac{d\sigma^{(\nu,\bar{\nu})N}}{dy} = \frac{G^2ME}{\pi}\left[(1-y+\tfrac{1}{2}y^2)\int_0^1 d\xi\, F_2^{(\nu,\bar{\nu})N}(\xi) \mp y(1-\tfrac{1}{2}y)\int_0^1 d\xi\,\xi F_3^{(\nu,\bar{\nu})N}(\xi)\right]. \tag{40}$$

Using the numbers in Eqs. (28)–(31) and (33)–(36), we find that

$$\left(\frac{G^2ME}{\pi}\right)^{-1}\left(\frac{d\sigma^{\nu N}}{dy}+\frac{d\sigma^{\bar{\nu}N}}{dy}\right) = \begin{cases} 0.99\pm0.10-(1.02\pm0.10)y+(0.51\pm0.05)y^2, \\ 2.61\pm0.18-(5.87\pm0.41)y+(2.93\pm0.20)y^2; \end{cases} \tag{41}$$

$$\left(\frac{G^2ME}{\pi}\right)^{-1}\left(\frac{d\sigma^{\nu N}}{dy}-\frac{d\sigma^{\bar{\nu}N}}{dy}\right) = \begin{cases} -(0.41\pm0.09)+(2.15\pm0.51)y-(1.07\pm0.25)y^2, \\ 0.71\pm0.18-(1.21\pm0.38)y+(0.60\pm0.19)y^2. \end{cases} \tag{42}$$

Here the upper (lower) expression holds below (above) the SU(3)″ threshold. On the other hand, if charge symmetry is assumed, these distributions can be extracted from the experimental data[17] and we find that

$$\left(\frac{G^2ME}{\pi}\right)^{-1}\left(\frac{d\sigma^{\nu N}}{dy}\pm\frac{d\sigma^{\bar{\nu}N}}{dy}\right) = \begin{cases} (0.98\pm0.03)(1-y+\tfrac{1}{2}y^2), \\ (0.91\pm0.06)y(1-\tfrac{1}{2}y). \end{cases} \tag{43}$$

Further tests of the model can be formulated in terms of sum rules of the Adler and Gross–Llewellyn Smith[23] variety. The functions $S^{00}(\xi)$, $S^{30}(\xi)$, and $S^{80}(\xi)$ are related to the proton matrix elements of the baryon-number current, the third component of the isotopic-spin current, and the hypercharge currents, respectively. As a result, these functions are normalized as follows:

$$\int_0^1 d\xi\, S^{i0}(\xi) = -1,\ -(\tfrac{1}{6})^{1/2},\ -(\tfrac{1}{2})^{1/2} \tag{44}$$

for $i=0$, 3, and 8, respectively.

Then we obtain the Adler sum rule:

$$\int_0^1 \frac{d\xi}{\xi}\,(F_2^{\bar{\nu}p}-F_2^{\nu p}) = \begin{cases} 2(1+\sin^2\theta_C) \text{ below threshold}, \\ 18 \text{ above threshold}; \end{cases} \tag{45}$$

and the Gross–Llewellyn Smith sum rule:

$$\int_0^1 d\xi\,(F_3^{\bar{\nu}p}+F_3^{\nu p}) = \begin{cases} -(4+2\cos^2\theta_C) \text{ below threshold}, \\ -30 \text{ above threshold}. \end{cases} \tag{46}$$

As already mentioned, there are a total of 14 structure functions at hand, given in terms of the seven parameters S^{i0}, A^{i0} ($i=0,3,8$), and z. It is clear that we may obtain numerous relations between the various structure functions by eliminating the seven parameters. Just to give one example in each case, note that

$$(F_2^{\nu N}+F_2^{\bar{\nu}N}-6F_2^{eN})\sin^2\theta_C = \xi(\cos^2\theta_C-\tfrac{2}{3})\times(F_3^{\nu N}-F_3^{\bar{\nu}N}) \tag{47}$$

below the SU(3)″ threshold, and

$$F_2^{\nu N}+F_2^{\bar{\nu}N}=18F_2^{eN} \tag{48}$$

above the SU(3)″ threshold. These local relations are naturally more difficult to test than the integrated versions.

The masses of the vector bosons in the Achiman model are given by

$$m_W = (12.6 \text{ GeV})/\sin\theta_W \tag{49}$$

and

$$m_Z = m_W/\cos\theta_W. \tag{50}$$

The numerical values for m_W and m_Z follow from (27) or (32). Using the value of z below the SU(3)″ threshold,

$$m_W = 15.3\pm0.8,$$
$$m_Z = 27.0\pm4.1;$$

for z above SU(3)″ threshold

$$m_W = 14.7\pm0.4,$$
$$m_Z = 28.4\pm2.9.$$

These masses are smaller than the usual Salam-Weinberg-theory lower limits $m_W \gtrsim 37$ GeV, $m_Z \gtrsim 75$ GeV. This of course results from the rather large values for $\sin^2\theta_W$.

We would emphasize the consistency in the results (27) and (32). It is encouraging that the $\sin\theta_W$ values are comparable below and above SU(3)″ threshold despite the fact that the relations leading to them are quite different. We consider this an important feature of the model, the larger value of θ_W notwithstanding.

Finally, let us mention the recent parton-model calculation of Scharbach[24] in the Achiman model, in which he is unable to find a value of $\sin^2\theta_W$ consistent with both the charged- and the neutral-current data. Is this an indication of a possible difference between the leading light-cone singularity and the parton model?[25]

APPENDIX: LIGHT-CONE COMMUTATION RELATIONS OF THE WEAK AND ELECTROMAGNETIC CURRENTS

As explained in the text, we write the light-cone commutation relations separately above and below the threshold for producing SU(3)″ nonsinglet states.

Commutation relations below the SU(3)″ *threshold.* Dropping the SU(3)″ nonsinglet pieces in the electromagnetic and weak currents we find that

$$[J_\mu^{\mathrm{em}}(x),\, J_\nu^{\mathrm{em}}(0)] \hat{=} -\partial_\lambda D(x)\, \mathcal{S}_{\mu\nu\lambda\sigma}[\tfrac{4}{3}J_\sigma^{(0,0)}(A;x,0) + (\tfrac{2}{3})^{1/2} J_\sigma^{(3,0)}(A;x,0) + \tfrac{1}{3}\sqrt{2}\, J_\sigma^{(8,0)}(A;x,0)] + \cdots, \tag{A1}$$

$$\begin{aligned}[J_\mu^{+}(x),\, J_\nu^{-}(0)] \hat{=} -\tfrac{2}{3}\partial_\lambda D(x)\{\mathcal{S}_{\mu\nu\lambda\sigma}[&\tfrac{2}{3}J_\sigma^{(0,0)}(A;x,0) + (\tfrac{1}{6})^{1/2}\sin^2\theta_C J_\sigma^{(3,0)}(A;x,0) + \tfrac{1}{3}\sqrt{2}\,(1-\tfrac{3}{2}\sin^2\theta_C) J_\sigma^{(8,0)}(A;x,0) \\ &+ (\tfrac{2}{3})^{1/2}(1-\tfrac{1}{2}\sin^2\theta_C) J_\sigma^{(3,0)}(S;x,0) + (\tfrac{1}{2})^{1/2}\sin^2\theta_C J_\sigma^{(8,0)}(S;x,0)] \\ &+\epsilon_{\mu\nu\lambda\sigma}\,[\text{same as the above square bracket with the substitution } A \leftrightarrow S]\} + \cdots,\end{aligned} \tag{A2}$$

$$\begin{aligned}[J_\mu^{Z}(x),\, J_\nu^{Z}(0)] \hat{=} -\partial_\lambda D(x)\{&\mathcal{S}_{\mu\nu\lambda\sigma}(1-4z+8z^2)[\tfrac{2}{3}J_\sigma^{(0,0)}(A;x,0) + (\tfrac{1}{6})^{1/2} J_\sigma^{(3,0)}(A;x,0) + \tfrac{1}{3}(\tfrac{1}{2})^{1/2} J_\sigma^{(8,0)}(A;x,0)] \\ &+\epsilon_{\mu\nu\lambda\sigma}(1-4z)[\text{same as the above square bracket with the substitution } A \leftrightarrow S]\} \\ &+\cdots.\end{aligned} \tag{A3}$$

The symbol $\hat{=}$ means that the commutators are evaluated near $x^2=0$. Also,

$$z = \sin^2\theta_W,$$

$$D(x) = -\frac{1}{2\pi}\,\epsilon(x_0)\delta(x^2),$$

$$J_\sigma(S;x,0) = J_\sigma(x,0) + J_\sigma(0,x),$$

$$J_\sigma(A;x,0) = J_\sigma(x,0) - J_\sigma(0,x),$$

$$\mathcal{S}_{\mu\nu\lambda\sigma} = -\delta_{\mu\nu}\delta_{\lambda\sigma} + \delta_{\mu\lambda}\delta_{\nu\sigma} + \delta_{\mu\sigma}\delta_{\nu\lambda}.$$

The dots in Eqs. (A1) through (A3) denote terms which are axial-vector in character. When the commutators are sandwiched between nucleon states of the same momentum and a spin summation carried out, these terms give zero contribution.

Commutation relations above the SU(3)″ *threshold.* The currents (7)–(9) must now be used in full. We then have

$$[J_\mu^{\mathrm{em}}(x),\, J_\nu^{\mathrm{em}}(0)] \hat{=} -\partial_\lambda D(x)\, \mathcal{S}_{\mu\nu\lambda\sigma}[\tfrac{8}{3}J_\sigma^{(0,0)}(A;x,0) + (\tfrac{2}{3})^{1/2} J_\sigma^{(3,0)}(A;x,0) + \tfrac{1}{3}\sqrt{2}\, J_\sigma^{(8,0)}(A;x,0)] + \cdots, \tag{A4}$$

$$\begin{aligned}[J_\mu^{+}(x),\, J_\nu^{-}(0)] \hat{=} -\partial_\lambda D(x)\{\mathcal{S}_{\mu\nu\lambda\sigma}[&\tfrac{8}{3}J_\sigma^{(0,0)}(A;x,0) + (\tfrac{2}{3})^{1/2} J_\sigma^{(3,0)}(A;x,0) + \tfrac{1}{3}\sqrt{2}\, J_\sigma^{(8,0)}(A;x,0) + \sqrt{6}\, J_\sigma^{(3,0)}(S;x,0) \\ &+ \sqrt{2}\, J_\sigma^{(8,0)}(S;x,0)] \\ &+\epsilon_{\mu\nu\lambda\sigma}\,[\text{same as the above square bracket with the substitution } A \leftrightarrow S]\} + \cdots,\end{aligned} \tag{A5}$$

$$\begin{aligned}[J_\mu^{Z}(x),\, J_\nu^{Z}(0)] \hat{=} -\partial_\lambda D(x)(&\mathcal{S}_{\mu\nu\lambda\sigma}\{\tfrac{4}{3}(1-2z+8z^2) J_\sigma^{(0,0)}(A;x,0) + (\tfrac{1}{6})^{1/2}(1-8z+8z^2)[J_\sigma^{(3,0)}(A;x,0) \\ &\qquad + (\tfrac{1}{3})^{1/2} J_\sigma^{(8,0)}(A;x,0)]\} \\ &+\epsilon_{\mu\nu\lambda\sigma}\{\tfrac{4}{3}(1-2z) J_\sigma^{(0,0)}(S;x,0) + (\tfrac{1}{6})^{1/2}(1-8z)[J_\sigma^{(3,0)}(S;x,0) + (\tfrac{1}{3})^{1/2} J_\sigma^{(8,0)}(S;x,0)]\}) \\ &+\cdots.\end{aligned} \tag{A6}$$

The terms omitted on the right-hand sides of Eqs. (A4) through (A6) are either terms which are axial-vector in character or terms which are nonsinglets under SU(3)″. The axial-vector terms are omitted for the reason given after Eq. (A3); nonsinglet terms are omitted because the commutators are sandwiched between nucleon states which are SU(3)″ singlets.

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