Regge trajectories and the quark-gluon coupling constant*

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It is argued that vector gluons interacting with quarks produce logarithmic behavior of Regge trajectories at large momentum transfer, in agreement with proton-proton scattering data. It is found that $g^2/4\pi = 14$, where g is the gluonic charge of the quarks. Some suggestive relations between quark-gluon dynamics, duality, magnetic charge, and Jacobi's imaginary transformation are discussed.

Recently, Coon, Sukhatme, and Tran Thanh Van[1] (called CST hereafter) showed that the non-Pomeranchukon part of the elastic proton-proton cross section can be fitted remarkably well over a very wide range of energy s and momentum transfer $-t$ using two logarithmic trajectories of the form

$$\alpha(t) = \frac{1}{\ln q}\ln(b - at) \quad (0 < q < 1) . \tag{1}$$

The logarithmic trajectory is associated with a modified version of the nonlinear dual amplitude proposed earlier by Baker and Coon.[2] For the purpose of the present note however, we can regard (1) as a purely phenomenological fit and try to deduce general implications of the logarithmic dependence of $\alpha(t)$ for $t \to -\infty$,

$$\alpha(t) \sim \frac{1}{\ln q}\ln(-t) \quad (-t \to \infty) . \tag{2}$$

Specifically, we show that (2) is consistent with the vector-gluon model[3] and further that the magnitude of the gluonic charge of the quarks can be determined from the parameter q. The parametrization obtained by CST gives

$$q = 0.8 \tag{3}$$

for each of the two trajectories.

Quite generally, we may assume that $\alpha(t)$ satisfies a once subtracted dispersion relation

$$\alpha(t) = \alpha(t_1) + \frac{(t - t_1)}{\pi}\int_{t_0}^{\infty} dt' \frac{\mathrm{Im}\alpha(t')}{(t' - t_1)(t' - t)} . \tag{4}$$

The condition for the logarithmic behavior (2) for $t \to -\infty$ is

$$\mathrm{Im}\alpha(t) \to \text{const} \quad (t \to \infty)$$

and in such a case we have

$$\frac{1}{|\ln q|} = \frac{1}{\pi}\mathrm{Im}\alpha(\infty) . \tag{5}$$

The asymptotically constant $\mathrm{Im}\alpha(\infty)$ is in fact, realized in the case of the positronium trajectory. Using an eikonal approximation, Lévy and Sucher[4] inferred that the electron-positron Coulomb scattering amplitude would be given by

$$T = \frac{1}{\lambda^2}\frac{\Gamma(1 - f(s))}{\Gamma(f(s))}\left(\frac{-t}{4\lambda^2}\right)^{f(s)-1} , \tag{6}$$

where t is the momentum transfer, λ is the photon mass, and

$$f(s) = \frac{e^2}{8\pi}\frac{2s - 4m^2}{[s(4m^2 - s)]^{1/2}} . \tag{7}$$

No electron loop contributions are included here.

As pointed out by Lévy and Sucher, there are different ways of identifying the positronium trajectory which lead to very small differences in the positronium spectrum. For larger e^2 and s far from $4m^2$ these differences become important. If we identify $f(s) - 1$ with the trajectory function as in potential scattering we have an unphysical left-hand cut starting at $s = 0$. This identification would also imply that no bound state could have a mass less than $\sqrt{2}m$, which would be disastrous for the quark-vector gluon model. We observe that within the eikonal framework it is natural to make the identification

$$f(s) = \alpha(s) - \alpha(4m^2 - s) , \tag{8}$$

where $\alpha(4m^2 - s)$ enters as the small t approximation to $\alpha(u)$. Thus, s-u crossing is satisfied at $t = 0$. Note that $e^2 \to -e^2$ under s-u crossing since the u-channel interaction is repulsive. This identification is possible because $f(s)$ is an antisymmetric function of $s - 2m^2$. We next observe that the problem of the left-hand cut is immediately resolved, since the right-hand cut in $\alpha(s)$ will produce the left- and right-hand cuts in $f(s)$.

If we determine the right-hand cut of $\alpha(s)$ from

(8) we find

$$\operatorname{Im}\alpha(\infty)=\frac{e^2}{4\pi}\,, \tag{9}$$

and upon dispersing $\operatorname{Im}\alpha(s)$ we obtain

$$\alpha(s)=\alpha(2m^2)+\frac{e^2}{4\pi^2}\frac{s-2m^2}{[-s(4m^2-s)]^{1/2}}\times\ln\left\{\frac{[-s(4m^2-s)]^{1/2}-s+2m^2}{2m^2}\right\}. \tag{10}$$

This expression appears to have a branch point at $s=0$, but it is easy to verify that it does not. As $s\to-\infty$

$$\alpha(s)\sim-\frac{e^2}{4\pi^2}\ln(-s/m^2)\,. \tag{11}$$

A possibly more rigorous treatment of high-energy electron-positron scattering is due to Cheng and Wu,[5] who obtained Eq. (6) with $f(s)$ replaced by its asymptotic form $ie^2/4\pi$. This result together with the assumption of no left-hand cut in α is sufficient to give the logarithmic behavior (11).

The above result (11) is valid for massive as well as massless photons. However, the spin of the photon is crucial. According to Lévy and Sucher,[4] $\operatorname{Im}f(s)\to 0$ as $s\to\infty$ for a scalar photon. The difference is due to the absence of the factor $2s-4m^2$ in Eq. (7). This factor is just the $t\to 0$ limit of $s-u$ which is the characteristic vertex factor of a vector exchange.

We next equate the coefficient of the phenomenological, non-Pomeronchukon hadronic trajectory (2) with the coefficient of a quark-antiquark trajectory of the form (11). Calling the vector-gluon coupling constant g and using Eq. (3), we find

$$\frac{g^2}{4\pi}=-\frac{\pi}{\ln q}=14\,, \tag{12}$$

which is a large and reasonable coupling constant for the vector gluon. This is in contrast with the perplexingly small value $g^2/4\pi\approx 0.3$ obtained by Cheng, Walker, and Wu[6] in their fit to the diffractive component of proton-proton scattering.

If our analysis is correct, we should expect a universal large-$(-t)$ behavior given by (2) in any scattering dominated by quark-antiquark exchange. For elastic scattering, one must be able to separate the diffractive component (as in the CST fit) in order to observe this universal behavior

We note that the identification

$$\ln q=-\frac{4\pi^2}{g^2} \tag{13}$$

relates q and g^2 in a way which makes sense over their entire range. In the dual model[2] $0<q\leq 1$, which is related by (13) to $0<g^2<\infty$. The Veneziano model with linear trajectories is obtained[2] in a limit in which $q\to 1$ and a threshold point approaches infinity. We interpret this as the limit in which g^2 and the quark mass both go to infinity.[3]

We also observe that the parameter q in the dual model of Ref. 2 has a universal character in that all trajectories which participate in the same N-point function must have the same q. This can be understood as a simple consequence of hadrons having zero color so that in an N-meson quark duality graph all quarks have the same magnitude of gluonic charge.

The mathematical functions which are associated with logarithmic trajectory dual models are the basic hypergeometric functions[7] and the elliptic ϑ functions[8] in which q is a parameter. One of the transformation properties of these functions involves $q\to 1/q$.[9] We observe that this corresponds to a reversal of sign of one of the charges in Eq. (13). Another transformation involving q is Jacobi's imaginary transformation[8] which relates parameters q and q' of two elliptic ϑ functions as follows[10]:

$$(\ln q)(\ln q')=4\pi^2\,. \tag{14}$$

Using Eq. (13) we can try to interpret (14) as a relation between two charges g and e. We find

$$(g^2/4\pi)(e^2/4\pi)=\tfrac{1}{4}\,, \tag{15}$$

which is Dirac's relation[11] between the magnetic monopole charge and the electric charge. For unit electric charge, the magnetic charge $g^2/4\pi=137/4=34$, which is larger than our experimentally determined $g^2/4\pi=14$. (The value 34 would correspond to a more linear trajectory since $g^2=\infty$ implies linearity.) The magnitude agreement and the existence of relation (15) are not so far from suggesting gluon = photon and quark = magnetic monopole.

An interpretation of Jacobi's imaginary transformation involving strings and dual models has been given by Brink and Nielsen.[12]

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Comments on CP violation through phase angles in weak currents, the $\Delta I=\frac{1}{2}$ rule, and the relation $\eta_{+-}=\eta_{00}$

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It is shown explicitly in a relativistic quark model that the approximate $\Delta I=\frac{1}{2}$ rule requires that the phase angles ϕ and ξ of the axial-vector weak currents, which introduce CP violation, be opposite.

The equality between the CP-violation parameters $\eta_{+-}=\eta_{00}$ has been shown recently[1] to follow from the isospin transformation properties of the weak currents, if their phase angles are opposite, $\phi=-\xi$. Since the soft-pion limit is not used, the authors stress that the result does not involve the $\Delta I=\frac{1}{2}$ rule. In fact, from such simple symmetry arguments, one cannot draw definite conclusions about the $\Delta I=\frac{1}{2}$ rule, as pointed out by Lee.[2]

We shall show, by means of explicit dynamical computations in a relativistic quark model,[3] that the relation $\phi=-\xi$ not only implies $\eta_{+-}=\eta_{00}$ (as proven in Ref. 1), but it is required by the approximate $\Delta I=\frac{1}{2}$ rule. We show, moreover, that $\eta_{+-}=\eta_{00}$ holds even if the rule is broken to allow, for instance, for $K^+\to\pi^+\pi^0$ decay.

In the relativistic quark model that we use, the mesons are bound states of spin-$\frac{1}{2}$ quark-antiquark pairs ($q\bar{q}$), described by the Bethe-Salpeter (BS) equation with an harmonic oscillator kernel. When symmetry breaking is introduced through quark mass differences, one can explain all the K-meson decays, including the $\Delta I=\frac{1}{2}$ rule and its deviations.[4] The BS amplitude for the pseudoscalar mesons reads

$$\chi(P,r)=\frac{4\pi}{(3\beta)^{1/2}}\left(1+\frac{\not{P}}{M}\right)\gamma_5\exp\left(-\frac{r_E^{\,2}}{2\sqrt{\beta}}\right)|q\bar{q}\rangle. \quad (1)$$

$2\sqrt{\beta}\simeq 1\ \mathrm{GeV}^2$ is the inverse of the Regge slope, P is the meson momentum, and M is the quark mass, which is related to the K decay constant by

$$F_K=\frac{4\sqrt{\beta}}{\pi\sqrt{3}\,M}\,. \quad (2)$$

$|q\bar{q}\rangle$ is the SU(3) wave function and r_E is the Wick-rotated relative momentum of the $q\bar{q}$ inside the meson.

The relevant diagrams for the $K\to 2\pi$ decays are shown in Figs. 1, where the circles are BS amplitudes[5] and the triangles stand for the effective weak current

$$J_q^\alpha=\bar{\mathcal{P}}\gamma^\alpha(1+e^{i\phi}\gamma_5)\mathfrak{N}\cos\theta_C+\bar{\mathcal{P}}(\gamma^\alpha(1+e^{i\xi}\gamma_5)+\delta k^\alpha)\lambda\sin\theta_C, \quad (3)$$
$$\delta=-\frac{3}{4M}\,\frac{m_K^{\,2}-m_\pi^{\,2}}{m_{K^*}^{\,2}}\,.$$

The diagrams Figs. 1(a) and 1(b) vanish identically for $K^+\to\pi^+\pi^0$. The diagram Fig. 1(c) does not contribute to $K^0\to\pi^0\pi^0$, whereas the one in Fig. 1(d) does not contribute to $K^0\to\pi^+\pi^-$. The