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Gauge invariance of the scalar-vector mass ratio in the Coleman-Weinberg model*

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The scalar-meson to vector-meson mass ratio, due to spontaneous symmetry breakdown, is calculated in the two-loop approximation of massless scalar quantum electrodynamics. Although the effective potential is gauge-dependent, this mass ratio is found to be gauge-independent. This strongly supports the interpretation given by Coleman and Weinberg that the radiative corrections drive the spontaneous symmetry breakdown in this theory. The mass ratio is found to be $m_S^2/m_V^2 = \frac{3}{2}e^2/4\pi^2 - \frac{61}{48}(e^2/4\pi^2)^2$, where e^2 is the physical coupling constant.

I. INTRODUCTION

Spontaneously broken symmetries play an important role in elementary-particle physics. Typically this is realized in field theory when the mass term of the Lagrangian becomes negative, so that the ground-state vacuum is not invariant under the symmetry group of the Lagrangian and the symmetry is spontaneously broken. If the theory does not involve the vector fields there exist massless scalar mesons called Goldstone bosons.[1] In the presence of gauge fields, however, the situation is more complex. In this case a Higgs phenomenon[2] is possible and some or all of the would-be Goldstone bosons become the longitudinal components of vector mesons and these vector mesons are in fact massive, which allows the construction of renormalizable massive Yang-Mills theories.[3]

Recently Coleman and Weinberg[4] proposed that the symmetry is broken spontaneously in massless scalar quantum electrodynamics. They observed that radiative corrections become the driving force for the symmetry breaking, so that the symmetry is *dynamically* broken. To argue this point they calculated the effective potential in the one-loop approximation, and found that the minimum of the effective potential indicates that $\langle\phi\rangle \neq 0$ and the symmetric vacuum is not the ground state of the theory. An interesting phenomenon called "dimensional transmutation" also takes place, whereby some of the coupling constants are determined in terms of others. This does not mean that the number of independent parameters is reduced, but rather that one trades some dimensionless parameters for dimensional ones expressed in terms of $\langle\phi\rangle$. Consequently the mass ratio of the scalar and the vector mesons was calculated in the lowest nontrivial order as a function of the gauge coupling. However, it was emphasized by Jackiw[5] that the effective potential is not gauge-invariant, which posed a question about the Coleman-Weinberg program and the usefulness of the effective potential. As a result of symmetry breaking, particles become massive and the effective potential, which is the generating functional for one-particle-irreducible Green's functions with all external lines carrying zero momenta, no longer describes *on-shell* amplitudes. Thus it may well be gauge-dependent. Nevertheless all physical quantities such as the S-matrix elements and mass ratios are expected to be gauge-invariant if the theory is to make sense. When we convert the loop expansion of the theory into ordinary perturbation theory Coleman and Weinberg show that the effective potential and the scalar-vector mass ratio is gauge-independent to lowest order. Therefore any possible gauge dependence will first appear in higher-order calculations. In the unlikely event that the mass ratio is gauge-dependent, the theory would be in serious trouble. On the other hand, if the ratio is gauge-invariant, then the interpretation given by Coleman and Weinberg has a firmer basis; spontaneous symmetry breaking can occur dynamically as a result of higher-order radiative corrections. Therefore the calculation of the next leading term of the particle mass ratio provides us a good test of the model as a successful example of radiatively driven symmetry breaking.

The purpose of this paper is to calculate the mass ratio of the scalar mesons to the vector mesons in the first two leading orders in the gauge coupling. To this end we need to calculate the effective potential in the two-loop approximation. We also need to know the particle propagators in the one-loop approximation, because the particle masses are no longer given by the curvature of the effective potential at its minimum. We find the mass ratio is indeed gauge-invariant as expected, which strongly supports the idea of the radiative corrections as the origin of spontaneous symmetry breaking in this model.

The plan of the paper is as follows: In Sec. II we review the concept of the effective potential as derived from the generating functional for the one-particle-irreducible Green's functions. We also discuss massless scalar QED and the parameters essential for a calculation of the particle mass ratio. In Sec. III the renormalized effective potential is calculated in the two-loop approximation. The propagators of the scalar and the vector mesons are computed in the one-loop approximation in Sec. IV. In this section it is shown that the mass ratio is gauge-independent. Section V presents a discussion of the result. Finally in Appendix A we derive the effective propagators by the Feynman diagram method, and confirm that they are equivalent to those obtained by the algebraic method. Appendix B contains some basic formulas for the two-loop integrations which are needed in our work.

II. PRELIMINARIES

A. Effective potentials

It is increasingly in fashion to study the spontaneously broken symmetry by searching the minima of the effective potential.[4-6] We define the effective potential as the generating functional of the 1PI (one-particle-irreducible) vertices with vanishing external momenta. In general the generating functional of the 1PI vertices, $\Gamma[\phi]$, can be most readily obtained from the Legendre transformation of the connected part of the vacuum transition amplitude in the presence of external c-number sources.[7] If we know all the 1PI vertices $\Gamma^{(n)}(x_1, x_2, \ldots, x_n)$, however, $\Gamma[\phi]$ can be also evaluated by directly summing up the series

$$\Gamma[\phi]=\sum_n \frac{1}{n!}\int d^4x_1\cdots d^4x_n \sum_{i_1,\ldots,i_n} \Gamma^{(n)}_{i_1\cdots i_n}(x_1,\ldots,x_n)\,\phi_{i_1}(x_1)\cdots\phi_{i_n}(x_n)\,. \tag{2.1}$$

Here $i_1, \ldots, i_n$ refer to any internal indices that the fields might have as a result of internal symmetry. They also refer to the Lorentz or the spinor indices if the corresponding fields have spin one or one half, respectively. [It is to be noted that $-\Gamma^{(2)}$ is the inverse propagator of the theory.]

Fourier transforms $\tilde{\Gamma}^{(n)}(p_1, \ldots, p_n)$ of the 1PI vertices $\Gamma^{(n)}(x_1, \ldots, x_n)$ can be expanded as a power series of momenta in momentum space, and we define the effective potential $V[\phi]$ as contribution of the lowest-order terms $\tilde{\Gamma}^{(n)}(0, \ldots, 0)$ to $i\Gamma[\phi]$ up to a constant factor $\int d^4x$. This demonstrates that the effective potential is a function of *space-time independent* c-number fields ϕ_i only, and we have

$$V[\phi]=i\sum_n \frac{1}{n!}\sum_{i_1,\ldots,i_n}\tilde{\Gamma}_{i_1\cdots i_n}(0,\ldots,0)\,\phi_{i_1}\cdots\phi_{i_n}\,. \tag{2.2}$$

Spontaneous symmetry breaking occurs when the effective potential has a local minimum at a value of ϕ which does not have the symmetry of the Lagrangian

$$\left.\frac{\partial V[\phi]}{\partial\phi}\right|_{\phi=\langle\phi\rangle}=0\,. \tag{2.3}$$

The *absolute* minimum of $V[\phi]$ is to be taken as the true ground state of the theory and the perturbation expansion is carried out near this point. If the vacuum as determined by Eq. (2.3) is symmetric in the lowest approximation of $V[\phi]$ but radiative corrections to the effective potential give rise to an asymmetric ground state, then we call the symmetry *dynamically* broken.

There are several systematic ways of calculating $V[\phi]$.[4-6] One involves the direct consideration of Feynman diagrams with all appropriate combinatoric factors taken into account.[4] Another is an algebraic method which can be formally extended to the arbitrary higher order.[5] A third involves a perturbation solution of the functional equations satisfied by $\Gamma[\phi]$.[6] Of course these methods are all equivalent. We now give a brief description of the algebraic method, leaving the details of its proof to Ref. 5. Let us consider a theory described by a Lagrangian $\mathcal{L}[\varphi]$. The first step is to rescale $\mathcal{L}[\varphi]$,

$$\mathcal{L}[\varphi]\to\frac{1}{h}\mathcal{L}[\varphi]\,. \tag{2.4}$$

It is well known that the perturbation expansion in h is equivalent to the loop expansion.[8] The second step is to shift the fields $\varphi(x)$,

$$\varphi(x)\to\varphi(x)+\bar{\phi}\,, \tag{2.5}$$

where $\bar{\phi}$ is a constant field (although in general it need not be). Then remove the linear terms in φ from the resulting Lagrangian by the replacement

$$\mathcal{L}[\varphi(x)]\to\bar{\mathcal{L}}[\varphi(x),\bar{\phi}]=\mathcal{L}[\varphi(x)+\bar{\phi}]-\mathcal{L}[\bar{\phi}]-\int d^4y\left.\frac{\delta\mathcal{L}}{\delta\varphi(y)}\right|_{\varphi=\bar{\phi}}\varphi(y)\,. \tag{2.6}$$

The removal of $\mathcal{L}[\bar{\phi}]$ is possible since it only affects the vacuum renormalization. Discarding the linear terms in $\varphi(x)$ is equivalent to ignoring all the tadpole diagrams, whose role is to shift the vacuum state, which we may do since the vacuum is to be obtained from Eq. (2.3). Note that the quadratic terms in $\bar{\mathfrak{L}}[\varphi, \bar{\phi}]$ define new "effective propagators" $D[\bar{\phi}]$, and the cubic and quartic terms specify new "effective vertices." The effective propagators as well as the effective couplings depend upon the constant external field $\bar{\phi}$. The effective potential $V[\phi]$ is given by

$$V[\phi] = -\mathfrak{L}[\phi] - \tfrac{1}{2} i h \ln \det i D^{-1}[\phi] + i h \left\langle\!\!\left\langle 0 \left| T \exp\left\{ \frac{i}{h} \int d^4x\, \bar{\mathfrak{L}}_{\rm int}[\varphi(x), \phi] \right\} \right| 0 \right\rangle\!\!\right\rangle , \tag{2.7}$$

where the double bracket $\langle\langle\ \rangle\rangle$ indicates that only 1PI graphs are retained and h only counts the number of loops and can be set equal to one in the end.

We shall show in Appendix A that the direct diagram method gives the same result in massless scalar QED. There we shall also demonstrate that the effective propagators represent the propagators of the original Lagrangian when an arbitrary number of external lines with zero momenta are attached to them. Similarly one obtains the effective vertices by considering all possible insertions of external lines with vanishing momenta.

The two-point Green's functions are obtained by functional differentiation of $\Gamma[\phi]$, which gives in momentum space

$$-G_{ij}^{(2)}(p)^{-1} = \sum_{n=0}^{\infty} \frac{1}{n!} \tilde{\Gamma}_{ij\,i_1 \cdots i_n}^{(n+2)}(p, -p, 0, \ldots, 0) \times \langle \phi_{i_1} \rangle \cdots \langle \phi_{i_n} \rangle . \tag{2.8}$$

Therefore one can also use the effective propagators to calculate the Green's functions.

B. Massless scalar QED

We now consider massless scalar quantum electrodynamics described by the Lagrangian

$$\begin{aligned} \mathfrak{L} = &-\tfrac{1}{4} F^{\mu\nu} F_{\mu\nu} + \tfrac{1}{2}(\partial_\mu \varphi_1^0 + e_0 A_\mu^0 \varphi_2^0)^2 \\ &+ \tfrac{1}{2}(\partial_\mu \varphi_2^0 - e_0 A_\mu^0 \varphi_1^0)^2 \\ &- \tfrac{1}{8}\lambda_0[(\varphi_1^0)^2 + (\varphi_2^0)^2]^2 , \end{aligned} \tag{2.9}$$

where

$$F_{\mu\nu} = \partial_\nu A_\mu^0 - \partial_\mu A_\nu^0 ,$$

and the fields and couplings in $\mathfrak{L}$ are bare quantities. Because the quadratic part of $\mathfrak{L}$ is singular, we have to introduce a gauge-defining term, which we shall choose as $-(1/2\xi_0)(\partial_\mu A_0^\mu)^2$. Moreover counterterms must be added for the theory to be finite. Therefore we consider the Lagrangian

$$\mathfrak{L} \to \mathfrak{L} - \frac{1}{2\xi_0}(\partial_\mu A_0^\mu)^2 - \tfrac{1}{2}\delta m^2 \varphi_0^2 , \tag{2.10}$$

with renormalization constants defined as

$$\begin{aligned} \varphi_i^0 &= Z_\phi^{1/2} \varphi_i , \\ A_\mu^0 &= Z_A^{1/2} A_\mu , \\ e_0 &= \frac{Z_e}{Z_\phi Z_A^{1/2}} e , \\ \lambda_0 &= \frac{Z_\lambda}{Z_\phi^2} \lambda , \\ \xi_0 &= Z_A \xi . \end{aligned} \tag{2.11}$$

All the renormalization constants are equal to one in the tree approximation (zero-loop approximation). The higher-order contributions are calculated from the relevant Feynman diagrams with

$$Z_e = Z_\phi ,$$

as dictated by the Ward identities.

Since we are renormalizing a gauge theory we are to adopt a gauge-invariant regularization. We find the dimensional regularization[9] most convenient in our work. In this method Feynman diagrams are calculated in n-dimensional Euclidean space, and the ultraviolet divergences associated with four-dimensional space appear as poles when n approaches four and are to be removed by appropriate counterterms. The one-loop diagrams in Fig. 1 are calculated in $(4-2\epsilon)$-dimensional space, which imply

$$\begin{aligned} Z_e &= Z_\phi = 1 + \frac{h}{16\pi^2} \frac{1}{\epsilon}(3-\xi) e^2 , \\ \delta m^2 &= \frac{h}{16\pi^2} \delta \bar{m}^2 , \\ Z_A &= 1 - \frac{h}{16\pi^2} \frac{1}{3\epsilon} e^2 , \\ \lambda Z_\lambda &= \lambda + \frac{h}{16\pi^2}\left[\frac{1}{\epsilon}(5\lambda^2 - 2\xi\lambda e^2 + 6e^4) - 4e^4\right] . \end{aligned} \tag{2.13}$$

Renormalization points are not specified in Eq. (2.13). We still need to have an intermediate finite renormalization from Eq. (2.13) to the renormalization constants defined on the mass shell of the scalar and vector mesons, which turns out to be nonvanishing. This can be done by adding finite terms to Eq. (2.13) and determining them according to the renormalization conditions. For our purpose, however, we do not have to renormalize *all* couplings on the mass shell. The ratio

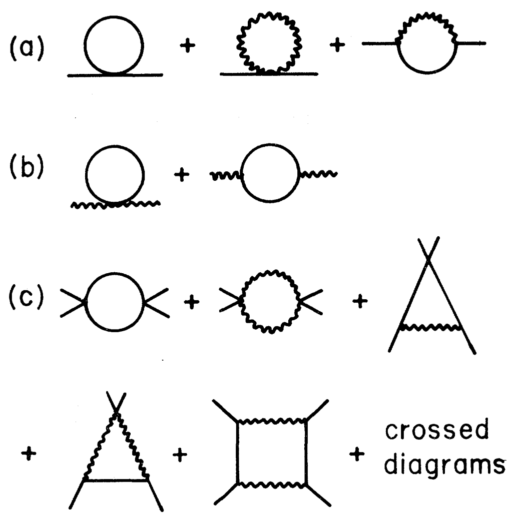

FIG. 1. One-loop Feynman diagrams for the calculation of (a) Z_ϕ and δm^2, (b) Z_A, (c) Z_λ.

of particle masses is dimensionless, and thus should depend upon λ and e only. But λ is expressed in terms of e through dimensional transmutation and the ratio is a function of e alone. Therefore it is sufficient for only e to be renormalized on the mass shell in the calculation of the mass ratio.

The finite term of $-4e^4$ is inserted in the definition of Z_λ for later convenience in the calculation of the one-loop effective potential. It is also to be noted that δm^2 is not divergent in the dimensional regularlization and $\delta \bar{m}^2$ is finite. $\delta \bar{m}^2$ is to be determined from the condition that the renormalized mass parameter vanishes, which implies the following constraint:

$$\frac{\partial^2 V}{\partial \phi^2} = 0 \text{ at } \phi = 0 \ . \tag{2.14}$$

This is not to be interpreted as requiring the *physical* mass of the theory to be zero (it does imply that the inverse propagators vanish at $p^2 = 0$). This condition only relates to the physical mass if the origin of the effective potential is the absolute minimum. When a nonvanishing vacuum expectation value $\langle \phi \rangle \neq 0$ appears, the particle masses should be calculated with respect to this physical vacuum.

C. Particle mass ratio

We shall now consider the general form of the mass ratio to $O(e^4)$. The first term which is of $O(e^2)$ has been already calculated by Coleman and Weinberg, and is found to be gauge-independent. The first gauge-dependent term of the effective potential is observed to be of $O(e^6)$ when the loop expansion is converted to an ordinary perturbation expansion. Therefore the gauge dependence of the mass ratio, if any, will show up first in $O(e^4)$. In order to examine this possibility we need to know the effective potential up to the two-loop term and the two-point Green's functions up to the one-loop approximation. One-loop Green's functions are necessary because the particle masses are defined as the pole of the Green's functions, and they are no longer given by the second derivatives of the effective potential evaluated at the minimum of the potential.

Let us assume the following form of the effective potential:

$$V[\phi] = \tfrac{1}{8}\lambda\phi^4 + \frac{1}{16\pi^2}\,\tfrac{1}{8}(a_1\lambda^2 + a_2\lambda e^2 + a_3 e^4)\,\phi^4\left(\ln\frac{\phi^2}{M^2} - \tfrac{3}{2}\right)$$
$$+\left(\frac{1}{16\pi^2}\right)^2 \frac{1}{8}\Big[(b_1\lambda^3 + b_2\lambda^2 e^2 + b_3\lambda e^4 + b_4 e^6)\,\phi^4 + (c_1\lambda^3 + c_2\lambda^2 e^2 + c_3\lambda e^4 + c_4 e^6)\,\phi^4 \ln\frac{\phi^2}{M^2}$$
$$+(d_1\lambda^3 + d_2\lambda^2 e^2 + d_3\lambda e^4 + d_4 e^6)\phi^4 \ln^2\frac{\phi^2}{M^2}\Big] \ . \tag{2.15}$$

The first term comes from the zero-loop effective potential ($O(h^0)$ term of $V_0[\phi]$). It is obvious as we can see in Sec. III A. The second and the third terms are the $O(h^1)$ and $O(h^2)$ terms of the effective potential in the one-loop and two-loop approximations, respectively. They are homogeneous polynomials in λ and e^2. The specific form, $(\ln \phi^2/M^2 - \frac{3}{2})$, of the second term will become apparent from Sec. III B. The mass parameter M is arbitrary. One fixed M refers to a certain renormalization point, which we shall not bother with. All we need to know is the relation of e^2 at this point to ${e_{\text{phy}}}^2$ defined on the mass shell.

The scalar field develops a nonvanishing vacuum expectation value when $V[\phi]$ given by Eq. (2.3) has an absolute minimum for $\phi \neq 0$, which then gives λ as a function of e^2. Solving for λ perturbatively, one obtains

$$\lambda=\frac{1}{16\pi^2}a_3e^4\left(1-\ln\frac{\langle\phi\rangle^2}{M^2}\right)+\left(\frac{1}{16\pi^2}\right)^2e^6\left[(a_2a_3-b_4-\tfrac{1}{2}c_4)-(2a_2a_3+c_4+d_4)\ln\frac{\langle\phi\rangle^2}{M^2}+(a_2a_3-d_4)\ln^2\frac{\langle\phi\rangle^2}{M^2}\right]. \tag{2.16}$$

Keeping in mind that λ is of $O(e^4)$ in Eq. (2.16) we assume the following forms for the vector-meson and the scalar-meson propagators:

$$G^{\mu\nu}(p^2)^{-1}=ig^{\mu\nu}\left[p^2-e^2\langle\phi\rangle^2-\frac{1}{16\pi^2}e^2p^2\left(y_1+y_2\ln\frac{\langle\phi\rangle^2}{M^2}\right)-\frac{1}{16\pi^2}e^4\langle\phi\rangle^2\left(z_1+z_2\ln\frac{\langle\phi\rangle^2}{M^2}\right)+O(\lambda p^2,e^2p^4)\right]$$
$$+p^\mu p^\nu \text{ terms },$$
$$G_{ij}(p^2)^{-1}=-i\eta_i\eta_j\left[p^2-\frac{1}{16\pi^2}e^2p^2\left(x_1+x_2\ln\frac{\langle\phi\rangle^2}{M^2}\right)-\frac{\partial^2V}{\partial\phi^2}\bigg|_{\phi=\langle\phi\rangle}+O(\lambda p^2,e^2p^4)\right]+(\delta_{ij}-\eta_i\eta_j)\text{ terms}. \tag{2.17}$$

The terms not explicitly shown are not necessary for our calculation of the particle masses. In particular the transverse part of the scalar fields, $(\delta_{ij}-\eta_i\eta_j)$ terms, will become the longitudinal part of the vector fields as in the Abelian Higgs model. The particle masses are to be calculated as the zeros of Eqs. (2.17) as functions of p^2. Solving Eq. (2.17) perturbatively one finds that p^2 (the particle mass) is of $O(e^2)$ and $O(e^4)$ for the vector and scalar mesons, respectively. As a result, $O(\lambda p^2, e^2p^4)$ terms are of $O(e^6)$ for the vector mesons and of $O(e^8)$ for the scalar mesons. These will contribute to the third nontrivial order of the gauge coupling, which we are not interested in in this work. Therefore we can systematically neglect $O(\lambda p^2, e^2p^4)$ terms in our calculation. Furthermore one notes from Sec. IV that the $\ln p^2$ terms are absent when one expands the Green's functions around the origin of momentum space. Therefore the zeros of Eqs. (2.17) become

$$m_V^2=e^2\langle\phi\rangle^2+\frac{1}{16\pi^2}e^4\langle\phi\rangle^2\left[(y_1+z_1)+(y_2+z_2)\ln\frac{\langle\phi\rangle^2}{M^2}\right], \tag{2.18}$$

$$m_S^2=\frac{1}{16\pi^2}a_3e^4\langle\phi\rangle^2+\left(\frac{1}{16\pi^2}\right)^2e^6\langle\phi\rangle^2\left[(c_4+d_4+a_3x_1+a_2a_3)+(2d_4-a_2a_3+a_3x_2)\ln\frac{\langle\phi\rangle^2}{M^2}\right].$$

Now we have to relate e to the physical coupling e_{phy}. From Eqs. (2.11) and (2.12) it follows that

$$e_0^2=\frac{1}{Z_A}e^2=\frac{1}{\tilde Z_A}e_{\text{phy}}^2, \tag{2.19}$$

where $\tilde Z_A$ is defined as

$$G^{\mu\nu}(p^2)^{-1}\underset{p^2\to m_V^2}{\sim}ig^{\mu\nu}(p^2-m_V^2)+p^\mu p^\nu\text{ terms}. \tag{2.20}$$

Therefore we find from Eq. (2.17) (and Sec. IVA) that

$$\tilde Z_A=1+\frac{1}{16\pi^2}e_{\text{phy}}^2\left[y_1+y_2\left(-\frac{1}{\epsilon}+\ln\frac{\langle\phi\rangle^2}{M^2}\right)\right], \tag{2.21}$$

and thus

$$e^2=e_{\text{phy}}^2\left[1-\frac{1}{16\pi^2}e_{\text{phy}}^2\left(y_1+y_2\ln\frac{\langle\phi\rangle^2}{M^2}\right)\right]. \tag{2.22}$$

Finally the mass ratio becomes

$$\frac{m_S^2}{m_V^2}=\frac{1}{16\pi^2}a_3e_{\text{phy}}^2+\left(\frac{1}{16\pi^2}\right)^2e_{\text{phy}}^4\left\{c_4+d_4+a_3(x_1-2y_1-z_1+a_2)+[2d_4+a_3(x_2-2y_2-z_2-a_2)]\ln\frac{\langle\phi\rangle^2}{M^2}\right\}. \tag{2.23}$$

It is obvious from Eq. (2.23) that we do not have to calculate all the terms of the two-loop effective potential in order to obtain the mass ratio. Only two terms, c_4 and d_4, as well as a_2 and a_3 from the one-loop effective potentials are sufficient for our purposes.

III. EFFECTIVE POTENTIALS

The first step in the calculation of the effective potentials is to determine the effective propagators and the effective vertices obtained by shifting the fields as in Eq. (2.6). Since the vector fields, due to Lorentz invariance, do not develop nonvanishing vacuum expectations values, it is most convenient to shift the scalar fields alone,

$$\varphi(x) \rightarrow \varphi(x) + \phi,\ A^\mu(x) \rightarrow A^\mu(x)\ . \tag{3.1}$$

The inverse propagators are readily obtained from the quadratic part of the shifted Lagrangian $\bar{\mathfrak{L}}_0$,

$$\bar{\mathfrak{L}}_0 = \tfrac{1}{2}(\varphi_i,\ A_\mu)\begin{pmatrix} Z_\phi^{1/2} & 0 \\ 0 & Z_A^{1/2}\end{pmatrix}\Big(iD_0^{-1}[\phi]\Big)\begin{pmatrix} Z_\phi^{1/2} & 0 \\ 0 & Z_A^{1/2}\end{pmatrix}\begin{pmatrix}\varphi_j \\ A_\nu\end{pmatrix},$$

where

$$iD_0^{-1}[\phi] = \begin{pmatrix} (k^2-m_2^2)(\delta_{ij}-\eta_i\eta_j)+(k^2-m_1^2)\eta_i\eta_j & +ie_0\epsilon_{ii'}\phi_{i'}k_\nu \\ -ie_0\epsilon_{jj'}\phi_{j'}k_\mu\,, & -(k^2-\mu^2)(g^{\mu\nu}-k^\mu k^\nu/k^2)-\dfrac{1}{\xi_0}(k^2-\xi_0\mu^2)\dfrac{k^\mu k^\nu}{k^2} \end{pmatrix},$$

and

$$\begin{aligned} m_1^2 &= \tfrac{3}{2}\lambda_0\phi_0^2 + \delta m^2\ , \\ m_2^2 &= \tfrac{1}{2}\lambda_0\phi_0^2 + \delta m^2\ , \\ \mu^2 &= e_0^2\phi_0^2\ , \\ \eta_i &= \frac{1}{\sqrt{\phi^2}}\ \phi_i\ . \end{aligned} \tag{3.2}$$

The Feynman rules for the propagators are obtained by inverting $iD_0^{-1}[\phi]$,[10] and are shown in Fig. 2 along with the effective vertices. All the parameters in Fig. 2 are the renormalized ones, since we need to know only the lowest-order expression for the internal lines and the vertices in our calculation.

We shall expand the effective potential $V[\phi]$ in number of loops L and calculate $V_L[\phi]$ up to $L=2$.

A. Zero-loop effective potential

This is trivial. It is simply the negative sum of all nonderivative terms in $\mathfrak{L}[\phi, 0]$:

$$\begin{aligned} V_0[\phi] &= \tfrac{1}{8}\lambda_0\phi_0^4 + \tfrac{1}{2}\delta m^2\phi_0^2 \\ &= \tfrac{1}{8}\lambda\phi^4 + \frac{h}{16\pi^2}\left\{\tfrac{1}{2}\delta\bar{m}^2\phi^2 + \tfrac{1}{8}\phi^4\left[\frac{1}{\epsilon}(5\lambda^2 - 2\xi\lambda e^2 + 6e^4) - 4e^4\right]\right\} \\ &\quad + \cdots\ , \end{aligned} \tag{3.3}$$

where we have used Eqs. (2.11)–(2.13) to reexpress $V_0[\phi]$ in terms of renormalized quantities. Here, and for the rest of this work, the three dots represent $O(h^2)$ counterterms which are polynomial in ϕ^2. These can be explicitly calculated from the two-loop diagrams and will be removed by the corresponding divergences of $V_1[\phi]$ and $V_2[\phi]$. The finite part, after the cancellation, does not play any role in the calculation of the mass ratio [see

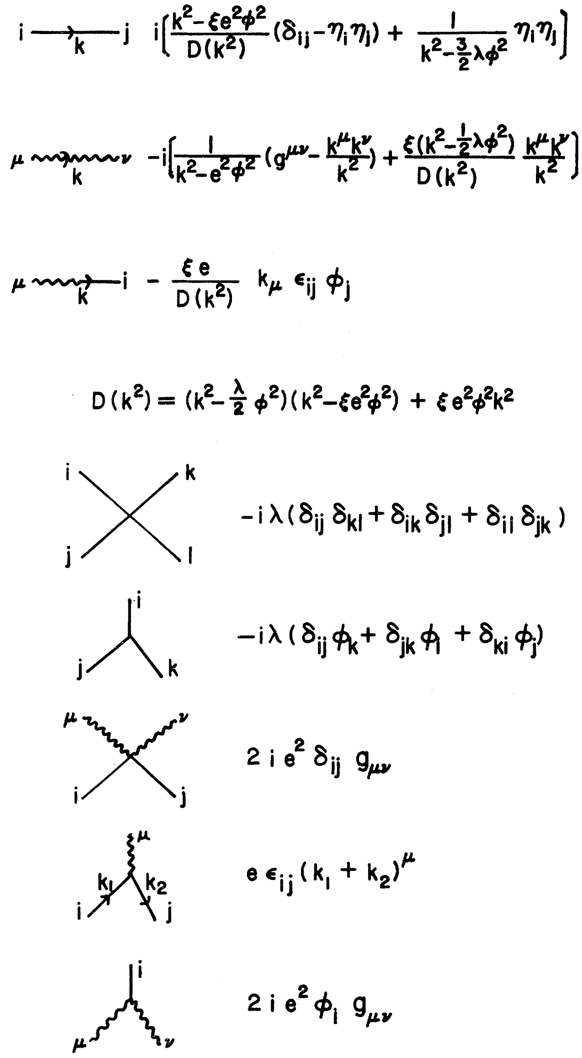

FIG. 2. Feynman rules for the effective propagators and effective vertices.

Eq. (2.23)]. Therefore we shall ignore any $O(h^2)$ polynomial terms from now on. It is trivial to note that $O(h^0)$ approximation of $V[\phi]$, $\frac{1}{8}\lambda\phi^4$, has the symmetric vacuum $\phi=0$ and therefore there is no symmetry breaking in the lowest order.

B. One-loop effective potential

This has been already evaluated by Jackiw.[5] Nevertheless we shall also calculate the divergences of $V_1[\phi]$ to demonstrate explicitly that $O(h)$ divergences cancel those in $V_0[\phi]$, and $O(h^2)$ divergences are removed by those of $V_2[\phi]$. From Eq. (2.7) it follows that

$$V_1[\phi]=-\tfrac{1}{2}ih\int\frac{d^4k}{(2\pi)^4}\ln\det|iD_0^{-1}[\phi]|\ , \tag{3.4}$$

where ϕ-independent divergent terms are subtracted away because they have nothing to do with any physically interesting properties of $V[\phi]$. By making use of the property

$$\det\begin{vmatrix}A & B\\ C & D\end{vmatrix}=\det|A|\,\det|D-CA^{-1}B|,\ (\det|A|\neq 0)\ , \tag{3.5}$$

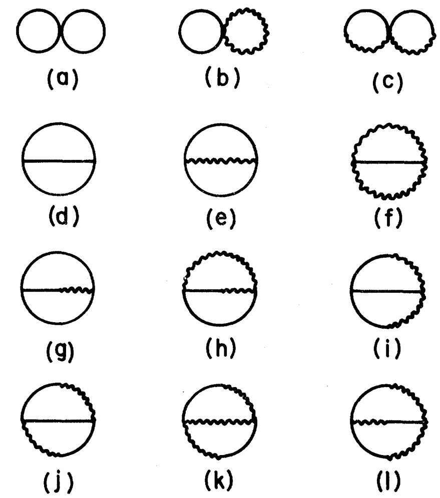

FIG. 3. Two-loop diagrams contributing to the effective potential $V_2[\phi]$.

$\det|iD_0^{-1}[\phi]|$ can be evaluated straightforwardly. Rotating the integral into n-dimensional Euclidean space and carrying out the integration for $n=4-2\epsilon$, we have

$$\begin{aligned}V_1[\phi]=\frac{h}{16\pi^2}\,\frac{1}{4}\Big\{&\Big(-\frac{1}{\epsilon}-\frac{3}{2}+\epsilon\times\text{const}\Big)[m_1^4+(3-2\epsilon)\,\mu^4+\beta_1^4+\beta_2^4]+m_1^4\Big[(1+\tfrac{3}{2}\epsilon)\ln\frac{m_1^2}{M^2}-\tfrac{1}{2}\epsilon\ln^2\frac{m_1^2}{M^2}\Big]\\ &+(3-2\epsilon)\,\mu^4\Big[(1+\tfrac{3}{2}\epsilon)\ln\frac{\mu^2}{M^2}-\tfrac{1}{2}\epsilon\ln^2\frac{\mu^2}{M^2}\Big]+\beta_1^4\Big[(1+\tfrac{3}{2}\epsilon)\ln\frac{\beta_1^2}{M^2}-\tfrac{1}{2}\epsilon\ln^2\frac{\beta_1^2}{M^2}\Big]\\ &+\beta_2^4\Big[(1+\tfrac{3}{2}\epsilon)\ln\frac{\beta_2^2}{M^2}-\tfrac{1}{2}\epsilon\ln^2\frac{\beta_2^2}{M^2}\Big]+O(\epsilon^2)\Big\}\ ,\end{aligned} \tag{3.6}$$

where

$$\beta_1^2+\beta_2^2=m_2^2,\quad \beta_1^2\beta_2^2=\xi_0\mu^2m_2^2\ .$$

It is important to keep terms of $O(\epsilon)$ in $V_1[\phi]$. Although these terms can be neglected in the $O(h)$ terms of $V_1[\phi]$, they give finite contribution to the $O(h^2)$ terms due to the fact that the renormalization constants have $1/\epsilon$ behavior. We are not concerned with the unspecified constant in Eq. (3.6). It affects the quadratic polynomial in ϕ^2 only in the $O(h^2)$ approximation, which does not matter in the calculation of the mass ratio.

The $O(h)$ and $O(h^2)$ terms of $V_1[\phi]$ are to be found by expanding Eq. (3.6) in a power series in h, i.e.,

$$\begin{aligned}V_1[\phi]=\frac{h}{16\pi^2}\,\frac{1}{8}\,\phi^4&\Big[\Big(-\frac{1}{\epsilon}+\ln\frac{\phi^2}{M^2}-\frac{3}{2}\Big)(5\lambda^2-2\xi\lambda e^2+6e^4)+4e^4\Big]\\ +\Big(\frac{h}{16\pi^2}\Big)^2\frac{1}{8}\,\phi^4&\Big\{\Big[\Big(\frac{1}{\epsilon}+1\Big)\ln\frac{\phi^2}{M^2}-\frac{1}{2}\ln^2\frac{\phi^2}{M^2}\Big][50\lambda^3-(30+20\xi)\,\lambda^2e^2+(60+4\xi^2)\,\lambda e^4+(40-24\xi)\,e^6]\\ &-\ln\frac{\phi^2}{M^2}[-40\lambda e^4+(\tfrac{80}{3}-16\xi)\,e^6]\Big\}+\cdots\ .\end{aligned} \tag{3.7}$$

We have omitted the logarithms of coupling constants. They are absorbed in the renormalization constants as shown by Coleman and Weinberg.[11]

We observe that the $O(h)$ divergences of $V_1[\phi]$ cancel those of $V_0[\phi]$ as they should. Furthermore the renormalization condition, Eq. (2.14), as applied to the $O(h)$ approximation of $V[\phi]$, dictates

$$\delta\bar{m}^2=0\ . \tag{3.8}$$

Therefore the effective potential up to the one-loop level is

$$V_0[\phi]+V_1[\phi]=\tfrac{1}{8}\lambda\phi^4+\frac{h}{16\pi^2}\frac{1}{8}\phi^4\left(\ln\frac{\phi^2}{M^2}-\frac{3}{2}\right)(5\lambda^2-2\xi\lambda e^2+6e^4)$$
$$+\left(\frac{h}{16\pi^2}\right)^2\frac{1}{8}\phi^4\left\{\left[\left(\frac{1}{\epsilon}+1\right)\ln\frac{\phi^2}{M^2}-\frac{1}{2}\ln^2\frac{\phi^2}{M^2}\right][50\lambda^3-(30+20\xi)\lambda^2e^2+(60+4\xi^2)\lambda e^4+(40-24\xi)e^6]\right.$$
$$\left.-\ln\frac{\phi^2}{M^2}[-40\lambda e^4+(\tfrac{80}{3}-16\xi)e^6]\right\}+\cdots, \tag{3.9}$$

which is finite up to $O(h)$; $O(h^2)$ divergences are to be removed by $V_2[\phi]$. Dynamical symmetry breaking happens in the $O(h)$ approximation as follows from the argument of Coleman and Weinberg.

C. Two-loop effective potential

There are twelve diagrams to be calculated for $V_2[\phi]$, shown in Fig. 3. Since we require the $O(e^6)$ terms of $V_2[\phi]$ for the mass ratio (see Sec. II C) we shall calculate all the diagrams in this order. It is straightforward to integrate the individual diagrams of Fig. 3, with the basic formulas of the two-loop integrals presented in Appendix B. By making use of these results it follows that

$$\begin{aligned}
V_2^{(a)}[\phi]&=\cdots\cdots,\\
V_2^{(b)}[\phi]&=\left(\frac{h}{16\pi^2}\right)^2e^6\phi^4\left[-3\xi\left(-\frac{1}{\epsilon}\ln\frac{\phi^2}{M^2}+\ln^2\frac{\phi^2}{M^2}\right)+4\xi\ln\frac{\phi^2}{M^2}\right]+\cdots\cdots,\\
V_2^{(c)}[\phi]&=0,\\
V_2^{(d)}[\phi]&=\cdots\cdots,\\
V_2^{(e)}[\phi]&=\left(\frac{h}{16\pi^2}\right)^2e^6\phi^4\left[(\tfrac{1}{2}+\tfrac{3}{2}\xi)\left(-\frac{1}{\epsilon}\ln\frac{\phi^2}{M^2}+\ln^2\frac{\phi^2}{M^2}\right)-(\tfrac{3}{2}+\tfrac{7}{2}\xi)\ln\frac{\phi^2}{M^2}\right]+\cdots\cdots,\\
V_2^{(f)}[\phi]&=\left(\frac{h}{16\pi^2}\right)^2e^6\phi^4\left[(\tfrac{9}{2}+\tfrac{3}{2}\xi)\left(-\frac{1}{\epsilon}\ln\frac{\phi^2}{M^2}+\ln^2\frac{\phi^2}{M^2}\right)-(\tfrac{21}{2}+\tfrac{7}{2}\xi)\ln\frac{\phi^2}{M^2}\right]+\cdots\cdots,\\
V_2^{(g)}[\phi]&=\cdots\cdots,\\
V_2^{(h)}[\phi]&=\left(\frac{h}{16\pi^2}\right)^2e^6\phi^4\left[-3\xi\left(-\frac{1}{\epsilon}\ln\frac{\phi^2}{M^2}+\ln^2\frac{\phi^2}{M^2}\right)+7\xi\ln\frac{\phi^2}{M^2}\right]+\cdots\cdots,\\
V_2^{(i)}[\phi]&=\cdots\cdots,\\
V_2^{(j)}[\phi]&=\cdots\cdots,\\
V_2^{(k)}[\phi]&=V_2^{(l)}[\phi]=0.
\end{aligned} \tag{3.10}$$

Here the five dots mean the omission of terms of $O(\lambda e^4, \lambda^2e^2, \lambda^3)$ as well as the quadratic polynomial in ϕ^2. Summing up all the two-loop diagrams we finally have

$$V_2[\phi]=\left(\frac{h}{16\pi^2}\right)^2e^6\phi^4\left[(5-3\xi)\left(-\frac{1}{\epsilon}\ln\frac{\phi^2}{M^2}+\ln^2\frac{\phi^2}{M^2}\right)-(12-4\xi)\ln\frac{\phi^2}{M^2}\right]+\cdots\cdots. \tag{3.11}$$

Adding $V_2[\phi]$ to $V_0[\phi]+V_1[\phi]$ [Eq. (3.9)] we observe that the $(1/\epsilon)\ln\phi^2/M^2$ terms vanish and the final expression is finite as it must be. Since the divergences cannot be removed by adding counterterms to the Lagrangian, they have to cancel among themselves.

Setting $h=1$ we now find the effective potential in the two-loop approximation to be

$$\begin{aligned}
V[\phi]&=V_0[\phi]+V_1[\phi]+V_2[\phi]\\
&=\tfrac{1}{8}\lambda\phi^4+\frac{1}{16\pi^2}\tfrac{1}{8}(5\lambda^2-2\xi\lambda e^2+6e^4)\phi^4\left(\ln\frac{\phi^2}{M^2}-\frac{3}{2}\right)\\
&\quad+\left(\frac{1}{16\pi^2}\right)^2\tfrac{1}{8}e^6\phi^4\left[(-\tfrac{248}{3}+24\xi)\ln\frac{\phi^2}{M^2}+(20-12\xi)\ln^2\frac{\phi^2}{M^2}\right]+\cdots\cdots,
\end{aligned} \tag{3.12}$$

with gauge dependence evident in the $O(\lambda e^2)$ and $O(e^6)$ terms. The values of the various coefficients needed for the calculation of the particle mass ratio are found to be

$$\begin{aligned} &a_2 = -2\xi, \quad a_3 = 6\ , \\ &c_4 = -\tfrac{248}{3} + 24\xi, \quad d_4 = 20 - 12\xi\ . \end{aligned} \tag{3.13}$$

IV. PARTICLE MASS RATIO

A. Vector-meson propagators

Now we shall calculate the vector-meson propagators in the one-loop approximation. From Eq. (2.8) it follows that

$$G^{\mu\nu}(p^2)^{-1} = G_0^{\mu\nu}(p^2)^{-1} - \Sigma^{\mu\nu}(p^2)\ , \tag{4.1}$$

where $G_0^{\mu\nu}(p^2)$ is the free particle propagator and $\Sigma^{\mu\nu}(p^2)$ is the one-loop self-energy correction. $G_0^{\mu\nu}$ is simply obtained from the Feynman rules

$$\begin{aligned} G_0^{\mu\nu}(p^2)^{-1} &= ig^{\mu\nu} Z_A(p^2 - \mu^2) + p^\mu p^\nu \text{ terms} \\ &= ig^{\mu\nu}\left\{p^2 - e^2\langle\phi\rangle^2 - \frac{h}{16\pi^2}\frac{1}{\epsilon}\left[\tfrac{1}{3}e^2p^2 + (3-\xi)e^4\phi^2\right]\right\} + p^\mu p^\nu \text{ terms}\ . \end{aligned} \tag{4.2}$$

Five diagrams contribute to $\Sigma^{\mu\nu}(p^2)$ as shown in Fig. 4. The straightforward calculation shows that

$$\begin{aligned} \Sigma_{\mu\nu}^{(a)}(p^2) &= ig_{\mu\nu}\frac{h}{16\pi^2}\left[(-2\lambda e^2 + \xi e^4)\langle\phi\rangle^2 + (2\lambda e^2 - \xi e^4)\langle\phi\rangle^2\left(-\frac{1}{\epsilon} + \ln\frac{\langle\phi\rangle^2}{M^2}\right)\right] + p^\mu p^\nu \text{ terms}\ , \\ \Sigma_{\mu\nu}^{(b)}(p^2) &= ig_{\mu\nu}\frac{h}{16\pi^2}\left\{(3\lambda e^2 - \tfrac{3}{2}\xi e^4)\langle\phi\rangle^2 + \tfrac{31}{18}e^2p^2 + \left[(-2\lambda e^2 + \xi e^4)\langle\phi\rangle^2 + \tfrac{1}{3}e^2p^2\right]\left(-\frac{1}{\epsilon} + \ln\frac{\langle\phi\rangle^2}{M^2}\right)\right. \\ &\qquad \left. + O(\lambda p^2, e^2p^4)\right\} + p^\mu p^\nu \text{ terms}\ , \\ \Sigma_{\mu\nu}^{(c)}(p^2) &= ig_{\mu\nu}\frac{h}{16\pi^2}\left[-(\tfrac{5}{2} + \tfrac{3}{2}\xi)e^4\langle\phi\rangle^2 + (3+\xi)e^4\langle\phi\rangle^2\left(-\frac{1}{\epsilon} + \ln\frac{\langle\phi\rangle^2}{M^2}\right) + O(\lambda p^2, e^2p^4)\right] + p^\mu p^\nu \text{ terms}\ , \\ \Sigma_{\mu\nu}^{(d)}(p^2) &= ig_{\mu\nu}\frac{h}{16\pi^2}\left[3\xi e^4\langle\phi\rangle^2 - 4e^2p^2 - 2\xi e^4\left(-\frac{1}{\epsilon} + \ln\frac{\langle\phi\rangle^2}{M^2}\right) + O(\lambda p^2, e^2p^4)\right] + p^\mu p^\nu \text{ terms}\ , \\ \Sigma_{\mu\nu}^{(e)}(p^2) &= 0\ . \end{aligned} \tag{4.3}$$

Each integral of (4.3) is expanded in a power series in p^2, keeping only the linear terms in p^2 which are of $O(e^2)$. Since the particle masses of the vector and scalar mesons are zero in the tree approximation, their masses computed from higher-order corrections are small, and it is sufficient to expand the inverse Green's functions around $p^2 = 0$ for the calculation of these masses.

Terms of $O(\lambda p^2, e^2p^4)$ would give us still higher-order corrections and are not necessary in our approximation. Adding all the terms in Eq. (4.3) we have

$$\begin{aligned} \Sigma_{\mu\nu}(p^2) = ig_{\mu\nu}\frac{h}{16\pi^2}&\left\{\left[\lambda e^2 - (\tfrac{5}{2} - \xi)e^4\right]\langle\phi\rangle^2 - \tfrac{41}{18}e^2p^2\right. \\ &\left. + \left[(3-\xi)e^4\langle\phi\rangle^4 + \tfrac{1}{3}e^2p^2\right]\left(-\frac{1}{\epsilon} + \ln\frac{\langle\phi\rangle^2}{M^2}\right) + O(\lambda p^2, e^2p^4)\right\} + p^\mu p^\nu \text{ terms}\ . \end{aligned} \tag{4.4}$$

The divergent terms of $G_0^{\mu\nu}(p^2)$ cancel those of $\Sigma^{\mu\nu}(p^2)$ as they should. Setting $h = 1$ we have

$$\begin{aligned} G^{\mu\nu}(p^2)^{-1} = ig^{\mu\nu}&\left\{p^2 - e^2\langle\phi\rangle^2 - \frac{1}{16\pi^2}e^2p^2\left[-\tfrac{41}{18} + \tfrac{1}{3}\ln\frac{\langle\phi\rangle^2}{M^2}\right]\right. \\ &\left. - \frac{1}{16\pi^2}e^4\langle\phi\rangle^2\left[-\tfrac{5}{2} + \xi + (3-\xi)\ln\frac{\langle\phi\rangle^2}{M^2}\right] + O(\lambda e^2\langle\phi\rangle^2, \lambda p^2, e^2p^4)\right\} + p^\mu p^\nu \text{ terms}\ , \end{aligned} \tag{4.5}$$

with the gauge dependence still in evidence. Therefore we obtain the following values for the y's and z's in Eq. (2.17):

$$y_1=-\tfrac{41}{18},\quad y_2=\tfrac{1}{3}\ ,$$
$$z_1=-\tfrac{5}{2}+\xi,\quad z_2=3-\xi\ . \tag{4.6}$$

B. Scalar-meson propagators

Similarly one can also calculate the scalar-meson propagators. The inverse Green's function is

$$G_{ij}(p^2)^{-1}=G^0_{ij}(p^2)^{-1}-\Sigma_{ij}(p^2)\ , \tag{4.7}$$

where $G^0_{ij}(p^2)$ is the free propagator,

$$\begin{aligned}G^0_{ij}(p^2)^{-1}&=-i\eta_i\eta_j Z_\phi(p^2-m_1{}^2)+(\delta_{ij}-\eta_i\eta_j)\ \text{terms}\\ &=-i\eta_i\eta_j\left\{p^2-\tfrac{3}{2}\lambda\langle\phi\rangle^2+\frac{h}{16\pi^2}\left[\frac{1}{\epsilon}(3-\xi)e^2p^2-\frac{3}{2\epsilon}\langle\phi\rangle^2(5\lambda^2-2\xi\lambda e^2+6e^4)+6e^4\langle\phi\rangle^2\right]\right\}\\ &\quad+(\delta_{ij}-\eta_i\eta_j)\ \text{terms}\ .\end{aligned} \tag{4.8}$$

The divergent terms of $G^0_{ij}(p^2)^{-1}$ are removed by the one-loop corrections $\Sigma_{ij}(p^2)$. The nine diagrams contributing to $\Sigma_{ij}(p^2)$ are shown in Fig. 5, with contributions of the individual diagram as follows:

$$\begin{aligned}
\Sigma^{(a)}_{ij}(p^2)&=-i\eta_i\eta_j\frac{h}{16\pi^2}\left[(-\tfrac{5}{2}\lambda^2+\tfrac{1}{2}\xi\lambda e^2)\langle\phi\rangle^2+(\tfrac{5}{2}\lambda^2-\tfrac{1}{2}\xi\lambda e^2)\langle\phi\rangle^2\left(-\frac{1}{\epsilon}+\ln\frac{\langle\phi\rangle^2}{M^2}\right)\right]+\cdots\cdots\ ,\\
\Sigma^{(b)}_{ij}(p^2)&=-i\eta_i\eta_j\frac{h}{16\pi^2}\left[-e^4\langle\phi\rangle^2+3e^4\langle\phi\rangle^2\left(-\frac{1}{\epsilon}+\ln\frac{\langle\phi\rangle^2}{M^2}\right)\right]+\cdots\cdots\ ,\\
\Sigma^{(c)}_{ij}(p^2)&=-i\eta_i\eta_j\frac{h}{16\pi^2}\left[\frac{4\xi^2\lambda e^4}{\lambda-8\xi e^2}\langle\phi\rangle^2+5\lambda^2\langle\phi\rangle^2\left(-\frac{1}{\epsilon}+\ln\frac{\langle\phi\rangle^2}{M^2}\right)\right]+\cdots\cdots\ ,\\
\Sigma^{(d)}_{ij}(p^2)&=-i\eta_i\eta_j\frac{h}{16\pi^2}\left[\left(\tfrac{1}{2}\xi\lambda e^2+\frac{4\xi^3e^6}{\lambda-8\xi e^2}\right)\langle\phi\rangle^2+(\tfrac{5}{2}-\tfrac{1}{48}\xi)e^2p^2\right.\\
&\qquad\left.+[(-\tfrac{1}{2}\xi\lambda e^2+\xi^2e^4)\langle\phi\rangle^2-(3-\xi)e^2p^2]\left(-\frac{1}{\epsilon}+\ln\frac{\langle\phi\rangle^2}{M^2}\right)\right]+\cdots\cdots\ ,\\
\Sigma^{(e)}_{ij}(p^2)&=-i\eta_i\eta_j\frac{h}{16\pi^2}\left[\left(4e^4+\frac{8\xi^3e^6}{\lambda-8\xi e^2}\right)\langle\phi\rangle^2-(\tfrac{5}{2}+\tfrac{5}{4}\xi)e^2p^2+(6+2\xi^2)e^4\langle\phi\rangle^2\left(-\frac{1}{\epsilon}+\ln\frac{\langle\phi\rangle^2}{M^2}\right)\right]+\cdots\cdots\ ,\\
\Sigma^{(f)}_{ij}(p^2)&=-i\eta_i\eta_j\frac{h}{16\pi^2}\left[-\frac{4\xi^2\lambda e^4}{\lambda-8\xi e^2}\langle\phi\rangle^2+\tfrac{7}{3}\xi e^2p^2-2\xi\lambda e^2\langle\phi\rangle^2\left(-\frac{1}{\epsilon}+\ln\frac{\langle\phi\rangle^2}{M^2}\right)\right]+\cdots\cdots\ ,\\
\Sigma^{(g)}_{ij}(p^2)&=-i\eta_i\eta_j\frac{h}{16\pi^2}\left[\frac{4\xi^2\lambda e^4-16\xi^3e^6}{\lambda-8\xi e^2}\langle\phi\rangle^2+\tfrac{2}{3}\xi e^2p^2-4\xi^2e^4\langle\phi\rangle^2\left(-\frac{1}{\epsilon}+\ln\frac{\langle\phi\rangle^2}{M^2}\right)\right]+\cdots\cdots\ ,\\
\Sigma^{(h)}_{ij}(p^2)&=-i\eta_i\eta_j\frac{h}{16\pi^2}\left(-\frac{4\xi^2\lambda e^4}{\lambda-8\xi e^2}\langle\phi\rangle^2-\tfrac{4}{3}\xi e^2p^2\right)+\cdots\cdots\ ,\\
\Sigma^{(i)}_{ij}(p^2)&=-i\eta_i\eta_j\frac{h}{16\pi^2}\left[\frac{4\xi^3e^6}{\lambda-8\xi e^2}\langle\phi\rangle^2+\tfrac{29}{48}\xi e^2p^2+\xi^2e^4\langle\phi\rangle^2\left(-\frac{1}{\epsilon}+\ln\frac{\langle\phi\rangle^2}{M^2}\right)\right]+\cdots\cdots\ .
\end{aligned} \tag{4.9}$$

The five dots represent terms of $O(\lambda p^2, e^2p^4)$ as well as $(\delta_{ij}-\eta_i\eta_j)$ terms. It should be noted that although the $\langle\phi\rangle^2$ terms are rational functions of λ and e^2 in the individual diagrams, they add up to a simple polynomial in λ and e^2. Summing all the diagrams in Eq. (4.9) we obtain

$$\begin{aligned}\Sigma_{ij}(p^2)&=-i\eta_i\eta_j\frac{h}{16\pi^2}\left[-\tfrac{1}{2}(5\lambda^2-2\xi\lambda e^2-6e^4)\langle\phi\rangle^2+\xi e^2p^2\right.\\ &\qquad\left.+[\tfrac{3}{2}(5\lambda^2-2\xi\lambda e^4+6e^4)\langle\phi\rangle^2-(3-\xi)e^2p^2]\left(-\frac{1}{\epsilon}+\ln\frac{\langle\phi\rangle^2}{M^2}\right)\right]+\cdots\cdots\ .\end{aligned} \tag{4.10}$$

Once again we verify the cancellation of the ultraviolet divergences. The scalar propagator now becomes

$$\begin{aligned}G_{ij}(p^2)&=-i\eta_i\eta_j\left\{p^2-\tfrac{3}{2}\lambda\langle\phi\rangle^2+\frac{1}{16\pi^2}\left[-\xi e^2p^2+(3-\xi)e^2p^2\ln\frac{\langle\phi\rangle^2}{M^2}-(5\lambda^2-2\xi\lambda e^2+6e^4)\left(\tfrac{3}{2}\ln\frac{\langle\phi\rangle^2}{M^2}-\frac{1}{2}\right)\langle\phi\rangle^2\right]\right\}\\ &\quad+\cdots\cdots\ .\end{aligned} \tag{4.11}$$

The expression inside the curly brackets with $p^2=0$ should be identical to the second derivative of the effective potential $-V_0[\phi]-V_1[\phi]$, which can be verified from Eq. (3.9). The values of x's needed in Eq. (2.17) are found to be

$$x_1=\xi,\ x_2=-(3-\xi)\ . \tag{4.12}$$

C. Particle mass ratio

We have now collected all the necessary information to calculate the mass ratio. From Eqs. (2.23), (3.13), (4.6), and (4.12) the ratio of the scalar-meson mass to the vector-meson mass is

$$\frac{m_S^2}{m_V^2}=\frac{3}{2}\left(\frac{e^2}{4\pi^2}\right)_{\text{phy}}-\frac{61}{48}\left(\frac{e^2}{4\pi^2}\right)_{\text{phy}}^2\ . \tag{4.13}$$

Therefore we find the mass ratio to be gauge-independent, as expected from general principles.

V. DISCUSSION

It was first suggested by Coleman and Weinberg that radiative corrections *may* be the main driving force for spontaneous symmetry breakdown in massless theories. The absolute minimum of the effective potential no longer occurs for zero external field, and the scalar field develops a nonvanishing vacuum expectation value. Furthermore this implies a dimensional transmutation in which some of the coupling constants are determined in terms of others. The fact that the effective potential is gauge-dependent in gauge theories raises some questions as to the power of the method although it is not a defect in itself, since the effective potentials, the vacuum expectation values, and the propagators are not the quantities measured physically, and are thus allowed to be gauge-dependent. However, physical quantities such as the S-matrix elements and particle mass ratios should be independent of gauge in gauge theories with symmetry breakdown. We have calculated the first two leading terms of the mass ratio in scalar QED and found it to be gauge-invariant. This strongly supports the interpretation given by Coleman and Weinberg of the radiative correction as the origin of the spontaneously broken symmetry in massless gauge theories.

The negative sign of Eq. (4.13) shows that the expression is not valid for large coupling ($e^2/4\pi \gtrsim 3.7$). However, this is to be interpreted as the failure of the perturbation expansion. Thus Eq. (4.13) is perfectly consistent in the framework of perturbation theory.

The same calculation can be carried out for non-Abelian gauge models. It simply involves more diagrams in the two-loop effective potential and the one-loop propagators. However, if the theory is asymptotically free, the calculation will merely be a formal exercise without the physical meaning of the Coleman-Weinberg model, since these non-Abelian gauge theories are *not* infrared-stable. Hence issues such as spontaneous symmetry breakdown, etc., will involve strong *effective* coupling constants, which is outside the domain of validity of a perturbation or loop expansion.

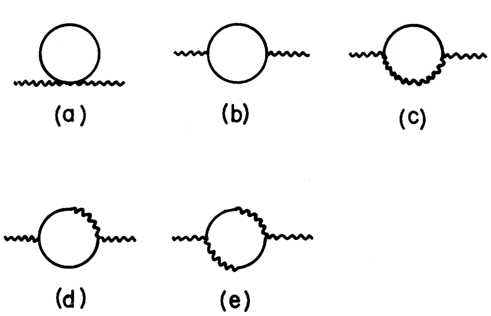

FIG. 4. One-loop diagrams contributing to the vector-meson propagator. The crossed diagram corresponding to (d) is not shown.

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APPENDIX A

In this appendix we discuss the Feynman rules for the propagators in the presence of external fields. The following argument can be extended to an arbitrary theory, but here we confine our-

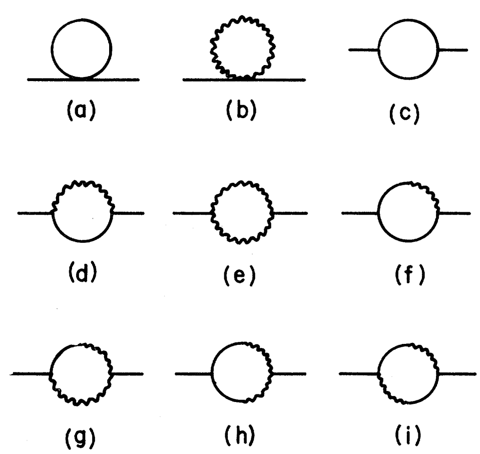

FIG. 5. One-loop diagrams contributing to the scalar-meson propagator. The crossed diagrams corresponding to (f), (g), and (h) are not shown.

selves to massless QED. Since we know the effective potential is a function of the fields only in the combination $\phi_1^2+\phi_2^2$ due to charge conservation, it is sufficient to consider all those diagrams with only ϕ_1 external lines. Given any Feynman diagram we define its semiprototype and prototype diagram as follows: The semiprototype diagram is obtained from the original diagram by removing all four-point vertices with two *external* lines (we also remove the corresponding external lines). The prototype diagram is obtained from the semiprototype diagram by removing all three-point vertices with one *external* line. An example is shown in Figs. 6(a), 6(b), and 6(c), respectively. It is to be noted that all the diagrams in Figs. 3, 4, and 5 are prototype. Conversely, given a prototype diagram, one can construct all other Feynman diagrams by attaching three- and four-point vertices.

Now consider the sum of all diagrams of the same semiprototype. It was observed by Coleman and Weinberg[11] that summing all these diagrams is equivalent to making the following substitutions:

$$\Delta_i(k)\to\Delta_i^{\text{semi}}(k)=\Delta_i(k)\,\frac{1}{1-\Delta_i(k)\,V_i\,\frac{1}{2}\phi^2}\,. \qquad \text{(A1)}$$

V_i is the four-point vertex of i lines with two ϕ_1 lines. The factor of one-half comes from Bose symmetry. Therefore we have

$$\phi_1\text{ line}:\ \frac{1}{k^2}\to\frac{1}{k^2-\frac{3}{2}\lambda\phi^2}\ ,$$

$$\phi_2\text{ line}:\ \frac{1}{k^2}\to\frac{1}{k^2-\frac{1}{2}\lambda\phi^2}\ ,$$

$$\begin{aligned}A_\mu\text{ line}:\ &\frac{1}{k^2}\left(g^{\mu\nu}-\frac{k^\mu k^\nu}{k^2}\right)+\frac{\xi}{k^2}\,\frac{k^\mu k^\nu}{k^2} \qquad \text{(A2)}\\ &\to\frac{1}{k^2-e^2\phi^2}\left(g^{\mu\nu}-\frac{k^\mu k^\nu}{k^2}\right)\\ &\quad+\frac{\xi}{k^2-\xi e^2\phi^2}\,\frac{k^\mu k^\nu}{k^2}\,.\end{aligned}$$

Now let us sum all the semiprototype diagrams of the same prototype. In this case the substitution rule becomes

$$\begin{aligned}\Delta^{\text{semi}}_{\alpha\beta}(k)&\to\Delta^{\text{proto}}_{\alpha\beta}(k)\\ &=\Delta^{\text{semi}}_{\alpha\gamma}(k)\,\frac{1}{1-V_{\gamma\delta}\Delta^{\text{semi}}_{\delta\epsilon}(k)V_{\epsilon\eta}\Delta^{\text{semi}}_{\eta\beta}(k)\phi^2}\,. \qquad \text{(A3)}\end{aligned}$$

Here $\alpha,\beta,\ldots$ refer to either i indices for ϕ lines or space-time μ indices for A^μ lines, and $V_{\alpha\beta}$ is the three-point vertex of α,β lines with ϕ_1 lines. This gives the following rules:

$$\begin{aligned}&\phi_1\text{ line}:\ i\,\frac{1}{k^2-\frac{3}{2}\lambda\phi^2}\ ,\\ &\phi_2\text{ line}:\ i\,\frac{k^2-\xi e^2\phi^2}{D(k^2)}\ ,\\ &A_\mu\text{ line}:\ -i\,\frac{1}{k^2-e^2\phi^2}\left(g^{\mu\nu}-\frac{k^\mu k^\nu}{k^2}\right)\\ &\qquad\qquad -i\,\frac{\xi(k^2-\frac{1}{2}\lambda\phi^2)}{D(k^2)}\,\frac{k^\mu k^\nu}{k^2}\ ,\\ &A_\mu-\phi_1\text{ transition}:\ 0\ , \qquad \text{(A4)}\\ &A_\mu-\phi_2\text{ transition}:\ \xi e\phi\,\frac{k^\mu}{D(k^2)}\ ,\end{aligned}$$

where

$$D(k^2)=(k^2-\tfrac{1}{2}\lambda\phi^2)(k^2-\xi e^2\phi^2)+\xi e^2\phi^2k^2\ .$$

This rule is the same as the effective Feynman propagators given in Fig. 2 with $\phi_2=0$.

In the process of substituting effective propagators for prototype diagrams, the combinatoric factors come out correctly to guarantee the validity of this substitution, since one obtains a geometric series *unless* the diagrams have additional symmetry factors such as those appearing in one-loop diagrams. This is why the one-loop diagrams require separate treatment.

APPENDIX B

Two-loop integrals of $V_2[\phi]$ are calculated in Euclidean space by the method of dimensional regularization.[9] This is one way of regularizing divergences while preserving gauge invariance, Ward identities, and unitarity. In this method

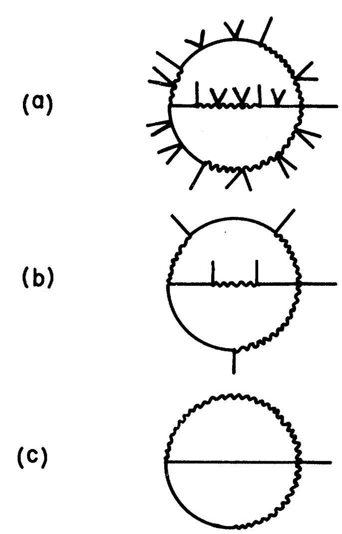

FIG. 6. An example of semiprototype diagram (b) and prototype diagram (c) corresponding to a given Feynman diagram (a).

Feynman diagrams are evaluated in n-dimensional space with loop integrals replaced as follows:

$$\int \frac{d^4k}{(2\pi)^4} \to \int\left(\frac{dk}{2\pi}\right)_n \equiv \frac{M^{4-n}}{16\pi^{2+n/2}\Gamma(3-\frac{1}{2}n)} \int d^n k \,. \tag{B1}$$

The scale factor M^{4-n} is inserted to maintain the same mass scale as in four-dimensional space and M is arbitrary which is to be fixed by the renormalization condition. The other factors are to eliminate unessential complication in the result of integration. When we go to four-dimensional space eventually, ultraviolet divergences appear as poles in the n space and these should be removed by the process of renormalization.

It is straightforward to derive the following formulas:

$$\int \left(\frac{dk_1}{2\pi}\right)_{4-2\epsilon} \int \left(\frac{dk_2}{2\pi}\right)_{4-2\epsilon} \frac{1}{k_1{}^2+\gamma\phi^2} = 0 \,, \tag{B2}$$

$$\int \left(\frac{dk_1}{2\pi}\right)_{4-2\epsilon} \int \left(\frac{dk_2}{2\pi}\right)_{4-2\epsilon} \frac{1}{(k_1+k_2)^2+\gamma\phi^2} = 0 \,, \tag{B3}$$

$$\int \left(\frac{dk_1}{2\pi}\right)_{4-2\epsilon} \int \left(\frac{dk_2}{2\pi}\right)_{4-2\epsilon} \frac{1}{k_1{}^2+\gamma_1\phi^2}\,\frac{1}{k_2{}^2+\gamma_2\phi^2} = \left(\frac{1}{16\pi^2}\right)^2 2\gamma_1\gamma_2\phi^4\left(-\frac{1}{\epsilon}\ln\frac{\phi^2}{M^2} - 2\ln\frac{\phi^2}{M^2} + \ln^2\frac{\phi^2}{M^2}\right) + \cdots \,, \tag{B4}$$

$$\int \left(\frac{dk_1}{2\pi}\right)_{4-2\epsilon} \int \left(\frac{dk_2}{2\pi}\right)_{4-2\epsilon} \frac{1}{k_1{}^2+\gamma_1\phi^2}\,\frac{1}{(k_1+k_2)^2+\gamma_2\phi^2} = \left(\frac{1}{16\pi^2}\right)^2 2\gamma_1\gamma_2\phi^4\left(-\frac{1}{\epsilon}\ln\frac{\phi^2}{M^2} - 2\ln\frac{\phi^2}{M^2} + \ln^2\frac{\phi^2}{M^2}\right) + \cdots \,, \tag{B5}$$

$$\int \left(\frac{dk_1}{2\pi}\right)_{4-2\epsilon} \int \left(\frac{dk_2}{2\pi}\right)_{4-2\epsilon} \frac{1}{k_1{}^2+\gamma_1\phi^2}\,\frac{1}{k_2{}^2+\gamma_2\phi^2}\,\frac{1}{(k_1+k_2)^2+\gamma_3\phi^2} = \left(\frac{1}{16\pi^2}\right)^2 (\gamma_1+\gamma_2+\gamma_3)\phi^2\left(\frac{1}{\epsilon}\ln\frac{\phi^2}{M^2} + 3\ln\frac{\phi^2}{M^2} - \ln^2\frac{\phi^2}{M^2}\right) + \cdots \,, \tag{B6}$$

$$\int \left(\frac{dk_1}{2\pi}\right)_{4-2\epsilon} \int \left(\frac{dk_2}{2\pi}\right)_{4-2\epsilon} \frac{1}{k_1{}^2+\gamma_1\phi^2}\,\frac{k_2{}^2}{(k_1+k_2)^2+\gamma_2\phi^2} = \left(\frac{1}{16\pi^2}\right)^2 2\gamma_1\gamma_2(\gamma_1+\gamma_2)\phi^6\left(\frac{1}{\epsilon}\ln\frac{\phi^2}{M^2} + 2\ln\frac{\phi^2}{M^2} - \ln^2\frac{\phi^2}{M^2}\right) + \cdots \,. \tag{B7}$$

Here three dots refer to the polynomial terms in ϕ^2 with coefficients which are functions of γ_i. Terms of $O(\epsilon)$ are also suppressed. We apply Eqs. (B2)–(B7) to two-loop integration of all diagrams in Fig. 3.

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[10]Alternatively we obtain the same Feynman rules by taking the diagonal blocks only as the inverse propagators and treating off-diagonal blocks as interaction terms.

[11]See Appendix A of Coleman and Weinberg, Ref. 4.