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Composite Higgs fields and finite symmetry breaking in gauge theories*

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Gauge models in which the symmetry breaking is dynamical, realized by certain composite Higgs fields acquiring a vacuum expectation value, are more constrained than corresponding models in which the symmetry breaking is implemented by auxiliary elementary scalar representations present at the Lagrangian level. In the former case, physical quantities which would otherwise be free parameters become computable. We illustrate this notable fact for the interesting case of the electron-muon mass ratio in Weinberg's chiral SU(3) model.

Multiplets of scalar fields play a central role in the current efforts to construct renormalizable models of the weak interactions based on a gauge principle.[1] It is through the symmetry-violating vacuum expectation value (VEV) of such multiplets that otherwise massless vector mesons and fermions acquire mass and that the excess of symmetry present in the Lagrangian is prevented from being communicated to the solutions of the theory.

In model-building activities, it has become customary to think of these multiplets as elementary and to associate with them sets of canonical fields in the Lagrangian whose couplings are regarded as more or less arbitrary. Following one's intuition, choices of different scalar representations can then be made to generate the desired pattern of VEV's and masses.

While this approach is certainly very useful in that it allows one to monitor the symmetry breaking, it is not aesthetically appealing in that it introduces a dependence on more parameters than one would expect in a fundamental theory. Furthermore,[2] the presence of canonical scalar fields in the Lagrangian does not appear to be a vital prerequisite for spontaneous symmetry breakdown. The role of elementary scalar fields in these phenomena could be assumed by dynamical bound states, similar in spirit to the fermion pairs of Nambu and Jona-Lasinio.[3] The fundamental world Lagrangian would then only involve elementary fermions and gauge fields.

Of course, whatever we gain by adopting this approach we pay for by giving up our controls on the theory (its solutions) and, short of acquiring a considerable amount of insight into nonperturbative effects, our general ability to perform even approximate calculations. What we gain includes very important advantages such as being able to compute, at least in principle, quantities which would otherwise be free parameters. Some such quantities may actually be computed using conventional perturbative techniques if certain conditions, dependent upon truly nonperturbative effects, are met. Within certain assumptions about the structure of the VEV's of dynamical bound states (which assumptions are testable by nonperturbative calculation), it is thus possible to test a gauge theory in which the symmetry breaking is postulated to be of purely dynamical origin.

In the present note, we shall illustrate the above assertions with reference to one of the long-standing problems of particle physics, the calculation of the electron-muon mass ratio. It is well known that attempts to solve this problem within the context of gauge theories with elementary Higgs fields have been faced by very serious difficulties.[4] After we rephrase these difficulties in our own language, it will become clear how an approach using dynamical symmetry breaking avoids them and makes it possible in principle to compute that ratio in models based on gauge groups for which we had previously been forced to regard the ratio as a free parameter. This is true, in particular, of Weinberg's SU(3)$\times$SU(3) model,[5] in the context of which the suggestion was originally made that the electron-muon mass ratio in gauge theories should be computable, and obstacles in the way to its implementation were later pointed out.[4] It is in terms of that particular model that we shall present our arguments, in spite of the fact that our considerations will manifestly possess a wider range of applicability.

In gauge models with elementary Higgs fields, general renormalization-theory arguments[6] indicate that a certain (lepton) mass will be calculable (in terms of other physical quantities) if and only if the gauge group and the group representations are such as to prevent VEV's from contributing to that mass. The absence of singlet contributions is automatically enforced in chiral gauge theories, such as Weinberg's SU(3)$\times$SU(3) model.[5] We remind the reader that in this model the leptons are arranged in a Konopinski-Mahmoud triplet (μ^+, ν, e^-) with left-handed and right-handed components transforming under the gauge group as a

$(1, \bar{3})$ and $(3, 1)$, respectively. The only meson representation which couples to the leptons by the gauge-invariant Yukawa coupling $(h\bar{\psi}_L\phi\psi_R + \text{H.c.})$ is a complex 3-by-3 elementary spinless matrix field transforming as a $(3, 3)$. Since the VEV of the latter is responsible for the zeroth-order lepton masses, to satisfy the condition that there be no (non-singlet) zeroth-order tadpole term contributing to the electron mass one must insist that the meson field in the array which undergoes a Yukawa coupling to the electron (ϕ_e) have, in the tree approximation, identically vanishing VEV.

Straightforward stability criteria imply that, to fulfill this requirement, the effective Lagrangian cannot possess terms linear in ϕ_e, which may arise whenever ϕ is "locked"[7] to other meson representations (χ) also acquiring a nonvanishing VEV. The presence of such "locking terms" as counterterms in the Lagrangian is sometimes forced by renormalizability, i.e., by the need to absorb divergences in amplitudes involving both ϕ and χ.

Unfortunately, this is precisely what happens in the case of Weinberg's model. If the electron in this model is to acquire its mass via radiative corrections involving the muon, it is necessary that there be a direct mass mixing between left-handed (W_L) and right-handed (W_R) gauge fields induced by some scalar-meson representation (χ). But then there exist superficially divergent diagrams (see Fig. 1) with four external elementary meson legs including one ϕ_e whose renormalization requires the introduction of the unwanted locking terms.[8] Thus, the choice of zeroth-order VEV which gives the muon a mass while keeping the electron massless turns out to be inconsistent with simple stability criteria; the VEV of ϕ_e is incalculable and so is the electron-muon mass ratio.

While there appear to be ways to finesse the impasse presented in the diagrams of Fig. 1 in models which still make use of elementary scalar representations,[4] it now seems unlikely that this goal can be achieved with an acceptable model, which is not too *ad hoc* and dependent upon a number of artificial parameters.

Now let us consider the case in which there are no elementary scalar fields. Several authors have appealed to the possibility that the Schwinger mechanism or some variant may provide for the cancellation of the unwanted vector-meson poles at zero momentum squared. We do wish to assume that the theory exists, as these authors do implicitly. However, rather than pole cancellations, we only assume that the theory generates gauge-group multiplets of scalar bound states. Given that assumption, it is necessary to follow the

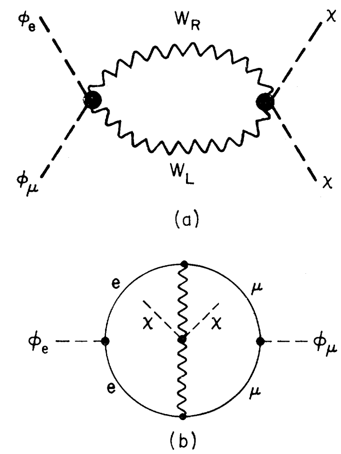

FIG. 1. Superficially divergent Feynman diagrams leading to the loss of the zeroth-order masslessness of the electron in Weinberg's $SU(3) \times SU(3)$ model with elementary Higgs fields.

observation of Coleman and E. Weinberg,[9] namely that the true vacuum state is obtained by minimizing a potential functional in the effective action which is defined as a functional Legendre transform of the usual action.[10] The effective action is a functional of properly defined classical fields, but in addition to fields corresponding to the elementary fields in the starting Lagrangian there are also fields corresponding to each of the bound states. The potential function to be minimized is expected to depend on the classical fields associated with the assumed scalar bound states. This dependence is calculable in principle, but we have no *a priori* knowledge as to the relative signs and magnitudes of the coefficients of the polynomial terms.[9]

If we assume that the minimum of the potential occurs at a nonzero VEV for some of the bound-state fields, then we return to a structure similar to that of a theory with Higgs scalars, but without the embarrassments of elementary scalars. The diagrams in Fig. 1 still exist, but are finite and calculable because of the appearance at the vertices of convergence factors arising from the structure of the bound states. The structure of the fermion mass contributions is displayed in Fig. 2, where we have replaced the classical fields associated with the scalar bound states by their VEV's everywhere in the effective action and ab-

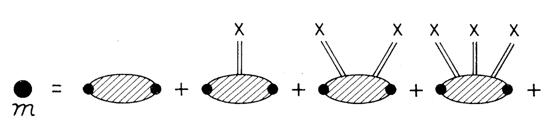

FIG. 2. Tadpole expansion for fermion self-energies.

stracted the terms proportional to the product of the fermion field and its Dirac adjoint and containing no derivatives. In the lepton model considered here the first term must still vanish, as no mass scale can appear in it, and the third term likewise will not contribute by virtue of its group structure. We may refer to such structure since we have postulated that symmetry breaking arises only via VEV's, and so the polynomial terms in the effective action must themselves be gauge invariant.[11] Thus, the terms that contribute to the electron mass, for instance, include single-tadpole terms and also terms with three tadpoles such as is elaborated in Fig. 3, namely terms of the structure proposed by Weinberg.

Note that all of the fermion masses are computable as all of the terms in Fig. 2 are finite (unrenormalized). However, given the present-day poverty of nonperturbative calculational techniques, this seems an impossible task practically. Indeed it would require nonperturbative calculation at least of the bound states and of the potential functional that generates the VEV's. On the other hand, if we are more modest in our goals, guided by our experience with elementary scalars we can assume the existence of, and an approximate form for, the VEV's, and attempt to examine the resulting theory for self-consistency and implications. In particular this may enable us to calculate the electron-muon mass ratio approximately within the assumptions, though not the electron and muon masses separately.

In Weinberg's chiral SU(3) model, we can reinstate in form the viability of his original conjecture as to the dominant contribution to the electron mass. Figure 3 is just that contribution with composite scalars and form factors replacing elementary scalars and vertices. We have already argued that the first and third terms in Fig. 2 do not contribute to the electron mass in this model. For the contribution represented by Fig. 3 to be dominant we must assume that the second term and the implicit terms of Fig. 2 are, for the electron, in comparison, negligibly small. This implies, in particular, that the one-tadpole term for the electron must be sufficiently small (say, at least one order of magnitude smaller than α) in the scale set by the corresponding tadpole term for the muon.

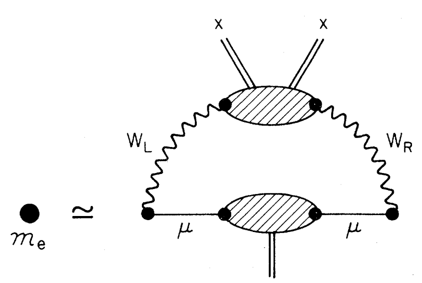

FIG. 3. Diagram for the electron mass.

To determine the self-consistency of this assumption we must consider terms in the effective action that are linear in ϕ_e and involve ϕ_μ (as well as any other fields), since these provide the kind of functional coupling that would tend to generate a nonvanishing VEV for ϕ_e once $\langle\phi_\mu\rangle \neq 0$, and argue that these terms are at least one order of magnitude smaller than α. One is tempted to try to examine this question by considering perturbation-theory-like diagrams. For example, the contributions coming from diagrams with the topological structure of Fig. 1(b) are expected to be negligible because (a) these diagrams involve one explicit power of α originating in the vector-meson–fermion vertices, and (b) the effective scalar-fermion coupling constants should be very small if the appearance of parity and strangeness nonconservation effects at intolerable levels in the hadron sector is to be avoided.[12] The contributions coming from diagrams with the topological structure of Fig. 1(a) are more difficult to dispose of. The magnitude of this contribution to $\langle\phi_e\rangle$ (as estimated by calculating with pointlike couplings) is of order $(M_1/M_\phi)^2\,\alpha\sin 2\theta\ln(M_1^2/M_2^2)\langle\phi_\mu\rangle$, where M_1 and M_2 are the masses of the physical (diagonal) gauge bosons, θ is the mixing angle relating these bosons to W_L and W_R, and M_ϕ is the mass of the ϕ_e bound state. Because of the factor $(M_1/M_\phi)^2$, this contribution has no "natural" order of magnitude and will violate our hypothesis that $\langle\phi_e\rangle \lesssim \frac{1}{10}\alpha\langle\phi_\mu\rangle$ unless $M_1 \lesssim \frac{1}{3}M_\phi$.

These two examples illustrate that no *definite* conclusions can be drawn from such naive analysis. The relative magnitude of $\langle\phi_e\rangle$ and $\langle\phi_\mu\rangle$ can really only be determined by nonperturbative calculations. Thus, if $\langle\phi_e\rangle \sim \alpha\langle\phi_\mu\rangle$, the understanding of m_e/m_μ must be completely nonperturbative. However, if it were true that $\langle\phi_e\rangle \lesssim \frac{1}{10}\alpha\langle\phi_\mu\rangle$, then one might retain a partial understanding of this ratio in terms of the perturbationlike diagram of Fig. 3.

We estimate the value of this diagram by using pole approximations[13] for the gauge-boson and muon propagators. The value of the loop integral derives mainly from the neighborhood of the gauge-boson masses. The contribution from asymptotic values of the loop momentum ($k \gg$ all physical masses) is shielded by an effective cutoff on the order of the gauge-boson masses and so should be insensitive to whether or not the theory is asymptotically free. Thus, one obtains

$$m_e/m_\mu \simeq (\alpha/2\pi)\,|\sin 2\theta\,\ln(M_1^2/M_2^2)|\ . \tag{1}$$

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[1]For a review see, for example, E. S. Abers and B. W. Lee, Phys. Rep. 9C, 1 (1973).
[2]R. Jackiw and K. Johnson, Phys. Rev. D 8, 2386 (1973); J. Cornwall and R. Norton, *ibid.* 8, 3338 (1973).
[3]Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961).
[4]H. Georgi and S. L. Glashow, Phys. Rev. D 7, 2457 (1973); 6, 2977 (1972).
[5]S. Weinberg, Phys. Rev. D 5, 1962 (1972).
[6]S. Weinberg, Phys. Rev. Lett. 29, 388 (1972); R. N. Mohapatra and P. Vinciarelli, Phys. Rev. D 8, 481 (1973).
[7]S. Weinberg, Phys. Rev. Lett. 29, 1698 (1972).
[8]This consistency problem was noted on the two-loop level by Georgi and Glashow (see Ref. 4). However, the problem already occurs at the one-loop level. We thank K. Lane for convincing us of the importance of the diagram in Fig. 1(a).
[9]S. Coleman and E. Weinberg, Phys. Rev. D 7, 1888 (1973).
[10]J. Rzewuski, *Field Theory*, revised English edition (Hafner, New York, 1969), Vol. 2.
[11]This is true for all terms not involving vector gauge mesons or derivatives, since such terms are not sensitive to the local character of the gauge group.
[12]R. N. Mohapatra, J. Pati, and P. Vinciarelli, Phys. Rev. D 8, 3652 (1973).
[13]P. Vinciarelli, Phys. Rev. D 9, 3456 (1974).

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Modified WKB approximation for phase shifts of an attractive singular potential

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The conventional WKB method for phase-shift calculations is known to fail for singular potentials at low incident energies. The modified WKB method can be applied successfully to singular potentials even at low energies since it is an extreme generalization of the conventional WKB method. As a special case, the modified WKB method can be reduced to the conventional WKB approximation and then agrees with the conventional WKB method in the high-energy region where the conventional WKB approximation is valid. Unlike the conventional WKB method, the modified WKB method can be applied to multi-turning-point problems without any difficulty. It is for these reasons that we present this method for determining the phase shifts of the attractive singular potential using a potential of the form $U(r) = -g^2 r^{-4}$ as an example. We restrict ourselves to the low-energy, nonzero-orbital-angular-momentum case where there are three classical turning points in order to demonstrate the ability of this method to handle many turning points. The phase shifts obtained by this method agree with the numerical results.

I. INTRODUCTION

The conventional WKB approximations for phase-shift calculations fails at low energies, while yielding accurate results at high energies. This was pointed out in a recent review article by Frank, Land, and Spector[1] on singular potentials.

Since the conventional WKB approximation is a special case of the more general modified WKB approximation, we believe that the modified WKB method can be applied over the entire energy range without being hampered by the turning-point problems of the conventional WKB method. We choose to illustrate the problem by an attractive r^{-4} potential so that comparisons can be made with the exact results. This is done to verify the accuracy of the modified WKB method in regions where the conventional WKB approximation is known to fail. We are not necessarily confining ourselves to such a potential, nor do we intend to produce better results than the existing ones. We selected the $-r^{-4}$ potential because of the availability of the exact results. The modification of the WKB method, in order to deal with many turning points over all energies, is our main concern.

In order to avoid the singularity at the origin, a simple truncation is introduced such that,

$$u(r) = \frac{-g^2}{r^4}\,\theta(r-d)\ ,$$

where θ is the unit-step function and $g^2 > 0$.

The modified WKB method[2] approaches the problem by formulating a model potential qualitatively similar to the actual potential and whose Schrödinger equation can be solved exactly. Using the exact solutions as the bases of the approximation, one can obtain an approximation of the wave functions for the actual potential. The differences