It follows from (4.4) that

$$S_a + S_b + S_c + S_d + S_e + S_f + S_g + S_h + S_i = 0 , \tag{4.6}$$

so that the total scattering contribution involving the b-field coupling vanishes. It is especially interesting to note that the so-called wave function renormalization terms in S_a, S_b, S_c, and S_d are exactly canceled by the vertex renormalization terms in S_e and S_f.

Thus, despite earlier misgivings,[4,5] we have demonstrated by explicit calculations for a specific process that a transformation of the Lagrangian density (4.1) does not affect even the renormalization constants.

*Work supported in part by the U. S. Atomic Energy Commission under Contract No. AT(11-1)-2302.

[1]S. N. Gupta, Proc. Phys. Soc. Lond. A64, 695 (1951).
[2]See, for instance, T. D. Lee, Nuovo Cimento 59A, 579 (1969); A. Salam and J. Strathdee, Phys. Rev. D 2, 2869 (1970).
[3]See, for instance, G. 't Hooft, Nucl. Phys. B35, 167 (1971).
[4]D. G. Boulware, Ann. Phys. (N.Y.) 56, 140 (1970).
[5]T. Appelquist and J. R. Primack, Phys. Rev. D 4, 2454 (1971).
[6]C. N. Yang and R. L. Mills, Phys. Rev. 96, 191 (1954).
[7]S. N. Gupta and W. H. Weihofen, Phys. Rev. D 3, 1957 (1971).
[8]This is closely related to the transformation applied by D. G. Boulware in Ref. 4.
[9]During the extensive simplifications required here, we did not employ any objectionable device such as shifting the origin in the k space.

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Higgs model with ghost-field formalism*

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The Higgs model for the interaction of vector and scalar fields is treated by means of the ghost-field formalism, which preserves the unitarity of the scattering operator as well as simplifies the renormalization procedure. Our ghost-field formalism has some similarity to the R_ξ formalism with $\xi = 1$, but it does not require the introduction of the so-called gauge-compensating terms. Applications to the second-order self-energy of a scalar particle and to the fourth-order scattering of vector particles are discussed.

I. INTRODUCTION

Although the ghost-field formalism for vector fields involves the use of an indefinite metric, it is possible to carry out transformation of the Lagrangian density in such a formalism without destroying the unitary property of the scattering operator.[1] This fact is of great importance from a practical point of view, because we shall show that it enables us to develop a formalism for vector fields which preserves the unitarity of the scattering operator as well as simplifies the renormalization procedure. We shall make use of only the familiar techniques of quantum field theory. Moreover, since the unitarity of the scattering operator is ensured in our formalism by the consistency of the supplementary condition, it will not be necessary to introduce either any gauge-compensating terms or an arbitrary gauge parameter.[2]

We shall here consider the simple Higgs model,[3] which is sufficient to bring out the essential features of our treatment. In order to clarify the relationship of our work with that of the earlier authors,[4,5] we shall also investigate the second-order self-energy of a scalar particle and the fourth-order scattering of two vector particles. Applications to more complex systems of physical interest will be described in subsequent papers.

II. GHOST-FIELD FORMALISM FOR THE HIGGS MODEL

The Lagrangian density for the Higgs model can be expressed in the conventional form as[3]

$$L=-\tfrac{1}{4}(\partial_\mu B_\nu-\partial_\nu B_\mu)^2-\tfrac{1}{2}m^2B_\mu{}^2-\tfrac{1}{2}(\partial_\mu\phi)^2-\tfrac{1}{2}\mu^2\phi^2$$
$$-\tfrac{1}{2}g^2B_\mu{}^2(2\lambda\phi+\phi^2)-h\lambda\phi^3-\tfrac{1}{4}h\phi^4, \qquad (2.1)$$

where m and μ are the masses of the real vector and scalar fields B_μ and ϕ, g is the coupling constant, and the additional constants h and λ are given by

$$h=g^2\mu^2/2m^2, \quad \lambda=m/g. \qquad (2.2)$$

We are not concerned here with the derivation or justification of the above Lagrangian density. Our aim is to show that by using the ghost-field formalism and applying the appropriate transformation, (2.1) can be converted into a more advantageous form for practical applications.

In the ghost-field formalism,[1] (2.1) is replaced by

$$L=-\tfrac{1}{2}(\partial_\mu A_\nu)^2-\tfrac{1}{2}m^2A_\mu{}^2-\tfrac{1}{2}(\partial_\mu\theta)^2-\tfrac{1}{2}m^2\theta^2-\tfrac{1}{2}(\partial_\mu\phi)^2-\tfrac{1}{2}\mu^2\phi^2-\tfrac{1}{2}g^2(A_\mu+m^{-1}\partial_\mu\theta)^2(2\lambda\phi+\phi^2)-h\lambda\phi^3-\tfrac{1}{4}h\phi^4, \qquad (2.3)$$

while the physical states of the system are subject to the supplementary condition

$$(\partial_\mu A_\mu+m\theta)^+\Psi=0. \qquad (2.4)$$

Further, let us transform θ and ϕ into two other real scalar fields χ and ψ by means of the relation

$$(\lambda+\phi)e^{i\theta/\lambda}=\lambda+\psi+i\chi, \qquad (2.5)$$

which evidently ensures that

$$\theta=\chi+O(g), \quad \phi=\psi+O(g). \qquad (2.6)$$

By equating the real and imaginary parts of (2.5), it is found that

$$\theta=\lambda\tan^{-1}\left(\frac{\chi}{\lambda+\psi}\right), \quad \phi=[(\lambda+\psi)^2+\chi^2]^{1/2}-\lambda, \qquad (2.7)$$

and consequently

$$\partial_\mu\theta=\frac{\lambda[(\lambda+\psi)\partial_\mu\chi-\chi\partial_\mu\psi]}{(\lambda+\psi)^2+\chi^2}, \quad \partial_\mu\phi=\frac{(\lambda+\psi)\partial_\mu\psi+\chi\partial_\mu\chi}{[(\lambda+\psi)^2+\chi^2]^{1/2}}. \qquad (2.8)$$

With the help of (2.7) and (2.8), the Lagrangian density (2.3) can be expressed in terms of the fields A_μ, χ, and ψ as

$$L=-\tfrac{1}{2}(\partial_\mu A_\nu)^2-\tfrac{1}{2}m^2A_\mu{}^2-\tfrac{1}{2}(\partial_\mu\chi)^2-\tfrac{1}{2}m^2\chi^2-\tfrac{1}{2}(\partial_\mu\psi)^2-\tfrac{1}{2}\mu^2\psi^2-gmA_\mu{}^2\psi+gA_\mu(\chi\partial_\mu\psi-\psi\partial_\mu\chi)-h\lambda\psi(\chi^2+\psi^2)$$
$$-\tfrac{1}{2}g^2A_\mu{}^2(\chi^2+\psi^2)-\tfrac{1}{4}h(\chi^2+\psi^2)^2+m(\partial_\mu A_\mu+m\chi)\left[\chi-\lambda\tan^{-1}\left(\frac{\chi}{\lambda+\psi}\right)\right]-\tfrac{1}{2}m^2\left[\chi-\lambda\tan^{-1}\left(\frac{\chi}{\lambda+\psi}\right)\right]^2, \qquad (2.9)$$

where some simplification has been achieved by using (2.2) and dropping four-divergences. The supplementary condition (2.4) now takes the form

$$\left[\partial_\mu A_\mu+m\lambda\tan^{-1}\left(\frac{\chi}{\lambda+\psi}\right)\right]^+\Psi=0, \qquad (2.10)$$

and maintains its consistency.[1]

It follows in the usual way that the effective interaction energy density in the interaction picture, resulting from (2.9), is given by

$$H_{\text{eff}}=gmA_\mu{}^2\psi-gA_\mu(\chi\partial_\mu\psi-\psi\partial_\mu\chi)+h\lambda\psi(\chi^2+\psi^2)$$
$$+\tfrac{1}{2}g^2A_\mu{}^2(\chi^2+\psi^2)+\tfrac{1}{4}h(\chi^2+\psi^2)^2$$
$$-m(\partial_\mu A_\mu+m\chi)\left[\chi-\lambda\tan^{-1}\left(\frac{\chi}{\lambda+\psi}\right)\right]$$
$$+\tfrac{1}{2}m^2\left[\chi-\lambda\tan^{-1}\left(\frac{\chi}{\lambda+\psi}\right)\right]^2, \qquad (2.11)$$

with the contractions

$$A_\alpha(x)^\cdot A_\beta(x')^\cdot=-i\delta_{\alpha\beta}\Delta_F(m;x-x'),$$
$$\chi(x)^\cdot\chi(x')^\cdot=-i\Delta_F(m;x-x'), \qquad (2.12)$$
$$\psi(x)^\cdot\psi(x')^\cdot=-i\Delta_F(\mu;x-x'),$$

and the effective contractions

$$\partial_\mu A_\alpha(x)^\cdot\partial'_\nu A_\beta(x')^\cdot=-i\delta_{\alpha\beta}\partial_\mu\partial'_\nu\Delta_F(m;x-x'),$$
$$\partial_\mu\chi(x)^\cdot\partial'_\nu\chi(x')^\cdot=-i\partial_\mu\partial'_\nu\Delta_F(m;x-x'), \qquad (2.13)$$
$$\partial_\mu\psi(x)^\cdot\partial'_\nu\psi(x')^\cdot=-i\partial_\mu\partial'_\nu\Delta_F(\mu;x-x'),$$

where

$$\Delta_F(m;x-x')=\lim_{\epsilon\to+0}\frac{1}{(2\pi)^4}\int dk\,e^{ik\cdot(x-x')}\frac{1}{k^2+m^2-i\epsilon}. \qquad (2.14)$$

It is interesting to observe that (2.11) does not involve any $\delta(0)$ term.

The effective interaction (2.11) is expressible in powers of the coupling constant g as

$$H_{\text{eff}} = gmA_\mu{}^2\psi - gA_\mu(\chi\partial_\mu\psi - \psi\partial_\mu\chi) + (g\mu^2/2m)\psi(\chi^2+\psi^2)$$
$$+\tfrac{1}{2}g^2A_\mu{}^2(\chi^2+\psi^2) + (g^2\mu^2/8m^2)(\chi^2+\psi^2)^2$$
$$-(\partial_\mu A_\mu + m\chi)[g\chi\psi + (g^2/m)(\tfrac{1}{3}\chi^3 - \chi\psi^2)] + \tfrac{1}{2}g^2\chi^2\psi^2$$
$$+O(g^3)\,, \tag{2.15}$$

where we have made use of the relations (2.2).

III. SIGNIFICANCE OF COUPLING TRANSFORMATION

The main virtue of the ghost-field formalism for vector fields is that it enables us to carry out certain transformations which cannot be carried out in the conventional formalism. The significance of the coupling transformation for the Higgs model becomes apparent when we compare (2.3) and (2.9). We find that while (2.3) contains a coupling term with two field derivatives, the transformed coupling terms in (2.9) contain at most one field derivative, which reduces the degree of divergence in individual interaction diagrams and greatly facilitates the treatment of renormalization. This observation shows that it should also be possible to carry out the desired coupling transformation by successive approximations as explained below.

Consider the transformation

$$\theta \to \theta + gu_1\,, \tag{3.1}$$

where u_1 is any product of field operators and their derivatives. Then,

$$-\tfrac{1}{2}(\partial_\mu\theta)^2 \to -\tfrac{1}{2}(\partial_\mu\theta)^2 - g\partial_\mu u_1\partial_\mu\theta + O(g^2)$$

or, after dropping a four-divergence,

$$-\tfrac{1}{2}(\partial_\mu\theta)^2 \to -\tfrac{1}{2}(\partial_\mu\theta)^2 + gu_1\partial^2\theta + O(g^2)\,. \tag{3.2}$$

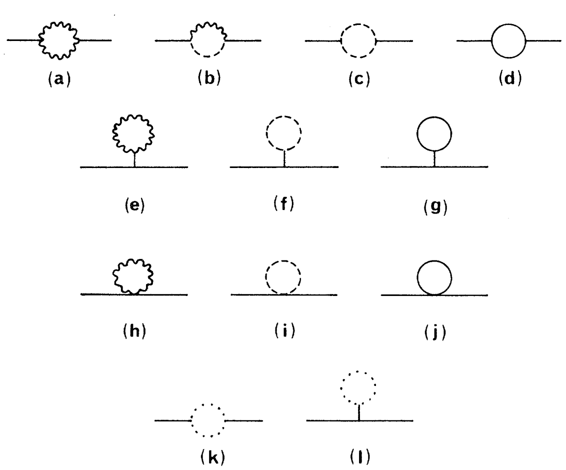

FIG. 1. Second-order self-energy of the scalar particle. The fields A_μ, χ, and ψ are represented by wavy, broken, and unbroken lines, respectively. The last two diagrams with dotted lines, which arise from gauge-compensating terms, should be ignored in our ghost-field formalism but included in the R_ξ formalism.

Thus, by applying a transformation of the form (3.1) to the Lagrangian density, we can always cancel first-order coupling terms involving $\partial^2\theta$. After this cancellation, another transformation of the form

$$\theta \to \theta + g^2u_2 \tag{3.3}$$

can be applied to eliminate second-order coupling terms involving $\partial^2\theta$, and so on. This procedure will yield a useful transformation of coupling terms whenever derivatives can be converted into ∂^2 by dropping four-divergences.

In the Lagrangian density given by (2.3) and (2.2), the first-order coupling term with two derivatives is

$$-\frac{1}{\lambda}\phi(\partial_\mu\theta)^2\,,$$

which can be converted, by dropping four-divergences, into

$$\frac{1}{\lambda}\theta\phi\partial^2\theta - \frac{1}{2\lambda}\theta^2\partial^2\phi\,,$$

and canceled by subjecting the Lagrangian density (2.3) to the transformation

$$\theta \to \theta - \frac{1}{\lambda}\theta\phi\,,$$
$$\phi \to \phi + \frac{1}{2\lambda}\theta^2\,. \tag{3.4}$$

Then, the second-order coupling terms with two derivatives in the transformed Lagrangian density are expressible, after dropping four-divergences, as

$$\frac{1}{\lambda^2}(\tfrac{1}{3}\theta^3 - \theta\phi^2)\partial^2\theta + \frac{1}{2\lambda^2}\theta^2\phi\partial^2\phi\,,$$

which can be eliminated by applying the transformation

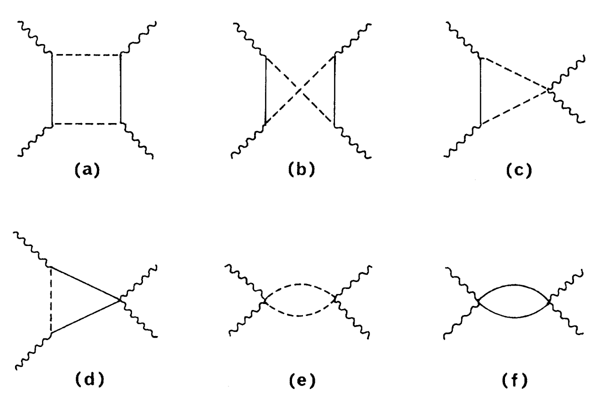

FIG. 2. Residual divergences in the fourth-order scattering of vector particles.

$$\theta \to \theta - \frac{1}{\lambda^2}(\tfrac{1}{3}\theta^3 - \theta\phi^2), \qquad (3.5)$$
$$\phi \to \phi - \frac{1}{2\lambda^2}\theta^2\phi.$$

The successive transformations (3.4) and (3.5) are equivalent to the single transformation

$$\theta \to \theta - \frac{1}{\lambda}\theta\phi - \frac{1}{\lambda^2}(\tfrac{1}{3}\theta^3 - \theta\phi^2) + O\left(\frac{1}{\lambda^3}\right), \qquad (3.6)$$
$$\phi \to \phi + \frac{1}{2\lambda}\theta^2 - \frac{1}{2\lambda^2}\theta^2\phi + O\left(\frac{1}{\lambda^3}\right),$$

which agrees with (2.7) on identifying the transformed θ and ϕ with χ and ψ, respectively.

IV. SELF-ENERGY OF SCALAR PARTICLE

The formalism described in Sec. II resembles the R_ξ formalism[6] with $\xi = 1$, but we do not find it necessary to introduce the gauge-compensating terms or demonstrate the ξ independence of results of physical interest. Since the absence of gauge-compensating terms in our formalism might seem surprising, we shall calculate the second-order self-energy of a scalar particle by our ghost-field formalism as well as by the $R_{\xi=1}$ formalism, and establish the equivalence of the two results.

The self-energy diagrams are shown in Fig. 1. For the present purpose, (2.15) can be reduced to

$$H_{\text{eff}} = gmA_\mu{}^2\psi + 2gA_\mu\psi\partial_\mu\chi + g(\mu^2/2m - m)\chi^2\psi + (g\mu^2/2m)\psi^3 + \tfrac{1}{2}g^2A_\mu{}^2\psi^2 + g^2(\mu^2/4m^2 + \tfrac{3}{2})\chi^2\psi^2 + (g^2\mu^2/8m^2)\psi^4, \qquad (4.1)$$

where we have dropped a four-divergence as well as ignored terms that do not contribute here. With the use of (4.1), the contribution of the scattering operator for diagrams a to j in Fig. 1 is found to be

$$S_a^{(2)} + S_b^{(2)} + \cdots + S_j^{(2)} = g^2\delta(p - p')\psi^-(p')\psi^+(p)$$
$$\times \int dk \left[\frac{6m^2 + \mu^4/2m^2 - 2\mu^2}{(k^2 + m^2)[(k-p)^2 + m^2]} + \frac{9\mu^4/2m^2}{(k^2 + \mu^2)[(k-p)^2 + \mu^2]} + \frac{6 + \mu^2/m^2}{k^2 + m^2} + \frac{3\mu^2/m^2}{k^2 + \mu^2}\right], \qquad (4.2)$$

where p is the propagation four-vector of the scalar particle.

On the other hand, according to the $R_{\xi=1}$ formalism, the effective interaction energy density for the Higgs model is[7]

$$\overline{H}_{\text{eff}} = gmA_\mu{}^2\psi - gA_\mu(\chi\partial_\mu\psi - \psi\partial_\mu\chi) + (g\mu^2/2m)\psi(\chi^2 + \psi^2) + \tfrac{1}{2}g^2A_\mu{}^2(\chi^2 + \psi^2) + (g^2\mu^2/8m^2)(\chi^2 + \psi^2)^2 + gm\psi\hat{C}^*\hat{C}, \qquad (4.3)$$

where $\hat{C}$ is a complex scalar Fermi field of mass m appearing only in closed loops, where its contribution is determined by the contraction

$$\hat{C}(x)^\cdot\hat{C}^*(x')^\cdot = -i\Delta_F(m; x - x'). \qquad (4.4)$$

We then obtain for the diagrams a to j

$$\overline{S}_a^{(2)} + \overline{S}_b^{(2)} + \cdots + \overline{S}_j^{(2)} = g^2\delta(p - p')\psi^-(p')\psi^+(p)$$
$$\times \int dk \left[\frac{7m^2 + \mu^4/2m^2 + 2p^2}{(k^2 + m^2)[(k-p)^2 + m^2]} + \frac{9\mu^4/2m^2}{(k^2 + \mu^2)[(k-p)^2 + \mu^2]} + \frac{9 + \mu^2/m^2}{k^2 + m^2} + \frac{3\mu^2/m^2}{k^2 + \mu^2}\right], \qquad (4.5)$$

while for the diagrams k and l involving the gauge-compensating terms

$$\overline{S}_k^{(2)} + \overline{S}_l^{(2)} = -g^2\delta(p - p')\psi^-(p')\psi^+(p)\int dk \left[\frac{m^2}{(k^2 + m^2)[(k-p)^2 + m^2]} + \frac{3}{k^2 + m^2}\right], \qquad (4.6)$$

so that the total contribution is

$$\overline{S}_a^{(2)} + \overline{S}_b^{(2)} + \cdots + \overline{S}_l^{(2)} = g^2\delta(p - p')\psi^-(p')\psi^+(p)$$
$$\times \int dk \left[\frac{6m^2 + \mu^4/2m^2 + 2p^2}{(k^2 + m^2)[(k-p)^2 + m^2]} + \frac{9\mu^4/2m^2}{(k^2 + \mu^2)[(k-p)^2 + \mu^2]} + \frac{6 + \mu^2/m^2}{k^2 + m^2} + \frac{3\mu^2/m^2}{k^2 + \mu^2}\right]. \qquad (4.7)$$

For a free scalar particle, in view of the relation $p^2=-\mu^2$, the results (4.2) and (4.7) become identical. Note that this equivalence holds only after the contribution of the gauge-compensating terms is taken into account in the $R_{\xi=1}$ formalism, while no such terms appear in our formalism.

V. SCATTERING OF VECTOR PARTICLES

As another application of our formalism, we shall consider the renormalization of the fourth-order scattering of two vector particles,[8] whose propagation four-vectors are p and q in the initial state and p' and q' in the final state. There is only one second-order interaction diagram for this scattering process, which represents ψ exchange and arises from the coupling term $gmA_\mu{}^2\psi$ in (2.15). There are, of course, many fourth-order interaction diagrams, but they can be divided into two categories. The diagrams belonging to the first category can be obtained by the insertion of self-energy and vertex parts in every possible way in the second-order diagram, and it can be readily shown by power-counting arguments that all divergences in such diagrams can be removed by renormalization of the masses m and μ and the coupling constant g. The second category contains diagrams that cannot be obtained by the above procedure, and the divergent diagrams belonging to this category are shown in Fig. 2, where it is understood that the propagation four-vectors p, q, p', and q' are to be associated with the external lines in every possible way. It only remains to be shown that the residual divergences in these diagrams are mutually canceled.

The contributions to the diagrams in Fig. 2 arise only from the coupling terms

$$-gA_\mu(\chi\partial_\mu\psi-\psi\partial_\mu\chi)+\tfrac{1}{2}g^2A_\mu{}^2(\chi^2+\psi^2)$$

in (2.15), and since they are only logarithmically divergent, the divergences can be isolated without regularization by setting all external propagation four-vectors equal to zero, and the calculations can be further simplified by dropping all masses. Then, the divergent parts of the contributions of these diagrams are expressible as

$$S_a^{(4)}=S_b^{(4)}=48g^4\delta(p+q-p'-q')A_\mu^-(p')A_\nu^-(q')A_\alpha^+(p)A_\beta^+(q)\int dk\frac{k_\mu k_\nu k_\alpha k_\beta}{k^8}\,, \tag{5.1}$$

$$S_c^{(4)}=S_d^{(4)}=-8g^4\delta(p+q-p'-q')A_\mu^-(p')A_\nu^-(q')A_\alpha^+(p)A_\beta^+(q)\times\int dk\,\frac{\delta_{\mu\nu}k_\alpha k_\beta+\delta_{\alpha\beta}k_\mu k_\nu+\delta_{\mu\alpha}k_\nu k_\beta+\delta_{\nu\beta}k_\mu k_\alpha+\delta_{\mu\beta}k_\nu k_\alpha+\delta_{\nu\alpha}k_\mu k_\beta}{k^6}\,, \tag{5.2}$$

$$S_e^{(4)}=S_f^{(4)}=2g^4\delta(p+q-p'-q')A_\mu^-(p')A_\nu^-(q')A_\alpha^+(p)A_\beta^+(q)\int dk\frac{\delta_{\mu\nu}\delta_{\alpha\beta}+\delta_{\mu\alpha}\delta_{\nu\beta}+\delta_{\mu\beta}\delta_{\nu\alpha}}{k^4}\,, \tag{5.3}$$

and since

$$\int dk\,k_\mu k_\nu f(k^2)=\tfrac{1}{4}\delta_{\mu\nu}\int dk\,k^2f(k^2)\,,$$
$$\int dk\,k_\mu k_\nu k_\alpha k_\beta f(k^2)=\tfrac{1}{24}(\delta_{\mu\nu}\delta_{\alpha\beta}+\delta_{\mu\alpha}\delta_{\nu\beta}+\delta_{\mu\beta}\delta_{\nu\alpha})\int dk\,k^4f(k^2)\,, \tag{5.4}$$

it follows that

$$S_a^{(4)}+S_b^{(4)}+S_c^{(4)}+S_d^{(4)}+S_e^{(4)}+S_f^{(4)}=0\,. \tag{5.5}$$

Our calculations again are no more difficult than those in the R_ξ formalism with $\xi=1$, and the use of our formalism automatically ensures the unitarity of the scattering operator.

*Work supported in part by the U. S. Atomic Energy Commission under Contract No. AT(11-1)-2302.

[1]D. Barua and S. N. Gupta, preceding paper, Phys. Rev. D 10, 3303 (1974).

[2]For an alternative formalism, which is based on the path-integral method and involves gauge-compensating terms and arbitrary gauge parameter, see the review article by E. S. Abers and B. W. Lee, Phys. Rep. 9C, 1 (1973).

[3]P. W. Higgs, Phys. Rev. 145, 1156 (1966).

[4]B. W. Lee, Phys. Rev. D 5, 823 (1972).

[5]T. Appelquist, J. Carazzone, T. Goldman, and H. R. Quinn, Phys. Rev. D 8, 1747 (1973).

[6]K. Fujikawa, B. W. Lee, and A. I. Sanda, Phys. Rev. D 6, 2923 (1972).

[7]For an investigation of the Higgs model with the use of

the general R_ξ formalism, see Ref. 5, where the second-order scalar-particle self-energy with an arbitrary value of ξ is also evaluated.

[8]This problem has been investigated, with the use of conventional formalism for the vector field, by T. Appelquist and H. Quinn, Phys. Lett. 39B, 229 (1972).

PHYSICAL REVIEW D VOLUME 10, NUMBER 10 15 NOVEMBER 1974

Infrared bootstrap for the electron mass in finite quantum electrodynamics

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This paper is concerned with the infrared structure of Johnson-Baker-Willey finite quantum electrodynamics. In this theory the insertion of $\overline{\psi}\psi$, the composite mass operator, into the electron propagator satisfies a homogeneous Bethe-Salpeter equation whose solution is conformally invariant at short distances with an anomalous dimension, $\gamma_{\overline{\psi}\psi}(\alpha)$. To determine whether the theory is forced to actually choose a nontrivial solution to this homogeneous equation we calculate the effective potential, $V(\langle\overline{\psi}\psi\rangle)$, using the dressed scalar vertex as input. We find that the infrared divergences of the theory cause the effective potential to develop a degenerate minimum away from the origin in classical field space. Thus dynamical γ_5 symmetry breaking takes place with the electron acquiring a mass $m \sim \langle\overline{\psi}\psi\rangle$. It is thought that this may be a general mechanism for generating masses in an otherwise conformal-invariant theory.

I. INTRODUCTION

This paper is concerned with the possibility that dynamical γ_5 symmetry breaking is the agency for introducing a scale into particle physics. In discussing theories in which all of the mass is to be dynamical we must start off with an underlying massless theory with dimensionless couplings. Such theories will exhibit conformal invariance with anomalous dimensions at short distances provided there is a renormalization-group fixed point.[1-3] We restrict ourselves to such theories only in this paper. These theories are generally regarded as being off-shell theories only without a sensible mass-shell limit. Thus the good ultraviolet limit is accompanied by a bad infrared limit. In perturbation theory we avoid but do not solve the infrared problem by renormalizing off the mass shell. Eventually, however, we have to sum the perturbation series and go to the mass shell, at which point we then have to face the infrared problem. The main point of this paper is that this infrared problem is then solved by dynamical γ_5 symmetry breaking, so that the fermions in the theory acquire masses by translating to the new vacuum. In this approach Wilson's skeleton theory[4] will be an exactly conformal-invariant renormalizable theory with either anomalous or canonical dimensions at short distances, depending on whether the ultraviolet-stable fixed point is nontrivial or at the origin; and all of the breaking of conformal invariance is achieved through the γ_5 degeneracy of the vacuum with no soft operators (or dilatons) being needed in the theory. This is the realization of an idea we suggested in a recent publication.[5]

The theory we analyze in detail in this paper is Johnson-Baker-Willey quantum electrodynamics[6-11] (finite QED) which possesses an explicit dynamical-symmetry-breaking solution. These authors have considered the Bethe-Salpeter equation (see Fig. 1)

$$m\tilde{\Gamma}_S(p,p,0) = m_0 Z_2 + \int \frac{d^4k}{(2\pi)^4}\, m\tilde{\Gamma}_S(k,k,0) i\tilde{S}(k) \times \tilde{K}(p,k,0) i\tilde{S}(k) \qquad (1)$$

for the insertion of the renormalized scalar operator, $\theta = \overline{\psi}\psi$, carrying zero momentum into the electron propagator. In the generalized Landau gauge where Z_2 is finite the electron propagator is canonical and the above equation admits of a solution

$$m\tilde{\Gamma}_S(p,p,0) = C(\alpha) m\left(\frac{-p^2}{m^2}\right)^{\gamma_\theta(\alpha)/2} \qquad (2)$$

for asymptotic p.[12] Thus if γ_θ, the anomalous dimension of θ, is negative the theory has a zero bare mass m_0 (in the limit of infinite cutoff).[13] Equation (1) then becomes a homogeneous bootstrap equation for the renormalized mass operator and admits of a nonvanishing physical mass. This