PHYSICAL REVIEW D VOLUME 10, NUMBER 10 15 NOVEMBER 1974

Ghost-field formalism for vector particles*

Debojit Barua and Suraj N. Gupta
Department of Physics, Wayne State University, Detroit, Michigan 48202

(Received 20 June 1974)

The quantization of vector fields with the use of an indefinite metric is examined. It is shown that a transformation of vector-field couplings in such a formalism will in general preserve the unitarity of the scattering operator for physical processes. Moreover, explicit calculations for a specific process seem to indicate that the coupling transformation does not affect even the renormalization constants.

I. INTRODUCTION

It was first shown[1] by one of us that the vector field can be quantized with the introduction of a scalar ghost field and the use of an indefinite metric. This formalism has been used by many authors,[2] and it has become even more popular[3] because of the recent interest in the formulation of renormalizable theories of weak interactions. The aim of this paper is to examine the role of the transformation of couplings in the ghost-field formalism, and we shall specifically investigate the following two problems:

1. It is of vital importance to find out whether a transformation of couplings in the ghost-field formalism will destroy the unitarity of the scattering operator for physical processes. We shall show that the unitarity of the scattering operator is indeed preserved even when the transformations involve field derivatives. In order to establish this fact, we shall provide a general proof, and then apply our reasoning to the ghost-field formalism for the nonlinear isotriplet vector field.

2. It would be interesting to clarify whether a transformation of the vector-field couplings affects the renormalization constants. For this purpose, we shall consider the special case of the coupling of neutral vector mesons with nucleons, where there seems to persist some doubt[4,5] as to whether the transforming away of the scalar field will affect the divergent renormalization constants. We shall show by explicit calculation of the fourth-order nucleon-nucleon scattering matrix elements that the scalar field does not make any contribution even to the renormalization constants.

We shall denote the space-time coordinates as $x_\mu = (x_1, x_2, x_3, ix_0)$, and take $c = \hbar = 1$.

II. EFFECT OF LAGRANGIAN TRANSFORMATION ON SUPPLEMENTARY CONDITION

In the ghost-field formalism[1] for vector particles, a supplementary condition is imposed on vectors representing the physical states of the system, and the consistency of the supplementary condition ensures the unitarity of the scattering operator for physical processes. We shall investigate how the Lagrangian transformation affects the supplementary condition.

Consider a system of interacting fields with the Lagrangian density

$$L = L(u^{(1)}, \ldots, u^{(n)}, \partial_\mu u^{(1)}, \ldots, \partial_\mu u^{(n)}), \tag{2.1}$$

which yields the field equations

$$\frac{\partial L}{\partial u^{(r)}} - \partial_\mu \left(\frac{\partial L}{\partial(\partial_\mu u^{(r)})}\right) = 0 \tag{2.2}$$

for $r = 1, 2, \ldots, n$. Let us apply to the Lagrangian density (2.1) the transformation

$$u^{(r)} \to \bar{u}^{(r)}, \tag{2.3}$$

where the $\bar{u}^{(r)}$ are of the form

$$\bar{u}^{(r)} = u^{(r)} + f^{(r)}(u^{(1)}, \ldots, u^{(n)}, \partial_\mu u^{(1)}, \ldots, \partial_\mu u^{(n)}), \tag{2.4}$$

and the $f^{(r)}$ vanish for vanishing values of the coupling constants. Then, (2.1) is transformed as

$$\bar{L} = L(\bar{u}^{(1)}, \ldots, \bar{u}^{(n)}, \partial_\mu \bar{u}^{(1)}, \ldots, \partial_\mu \bar{u}^{(n)}), \tag{2.5}$$

and by expressing the variation

$$\int dx\, \delta\bar{L}$$

in terms of the $\delta\bar{u}^{(r)}$, we arrive at the field equations

$$\frac{\partial \bar{L}}{\partial \bar{u}^{(r)}} - \partial_\mu \left(\frac{\partial \bar{L}}{\partial(\partial_\mu \bar{u}^{(r)})}\right) = 0. \tag{2.6}$$

The above result shows that a transformation of the Lagrangian density is equivalent to the application of the same transformation to the field equations. Moreover, since the consistency of the supplementary condition in the ghost-field formalism is a direct consequence of the field equations, it follows that this consistency can be maintained if the transformation of the Lagrangian density is accompanied by the application of the same transformation to the supplementary condition.

III. SELF-INTERACTING ISOTRIPLET VECTOR FIELD

Although the argument presented in Sec. II is quite general, it would be useful to illustrate by direct calculations that it is possible to maintain the consistency of the supplementary condition under a Lagrangian transformation. For this purpose, we shall consider the nonlinear isotriplet vector field with the Lagrangian density

$$\begin{aligned} L = &-\tfrac{1}{4}(\partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu)^2 - \tfrac{1}{2}m^2 \vec{\rho}_\mu{}^2 \\ &- \tfrac{1}{2}g(\partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu)\cdot(\vec{\rho}_\mu \times \vec{\rho}_\nu) \\ &- \tfrac{1}{4}g^2 \vec{\rho}_\mu{}^2 \vec{\rho}_\nu{}^2 + \tfrac{1}{4}g^2(\vec{\rho}_\mu \cdot \vec{\rho}_\nu)^2 , \end{aligned} \tag{3.1}$$

which can be obtained either by using the broken Yang-Mills symmetry[6] or by requiring the source function in the isotriplet vector-field equation to have a vanishing divergence.[7]

According to the ghost-field formalism,[1] we replace the conventional form of the Lagrangian density (3.1) by

$$\begin{aligned} L = &-\tfrac{1}{2}(\partial_\mu \vec{a}_\nu)^2 - \tfrac{1}{2}m^2 \vec{a}_\mu{}^2 - \tfrac{1}{2}(\partial_\mu \vec{b})^2 - \tfrac{1}{2}m^2 \vec{b}^2 \\ &-\tfrac{1}{2}g(\partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu)\cdot(\vec{\rho}_\mu \times \vec{\rho}_\nu) \\ &-\tfrac{1}{4}g^2 \vec{\rho}_\mu{}^2 \vec{\rho}_\nu{}^2 + \tfrac{1}{4}g^2(\vec{\rho}_\mu \cdot \vec{\rho}_\nu)^2 , \end{aligned} \tag{3.2}$$

with

$$\vec{\rho}_\mu \equiv \vec{a}_\mu + \frac{1}{m}\partial_\mu \vec{b} , \tag{3.3}$$

and impose on the physical states of the system the supplementary condition

$$(\partial_\mu \vec{a}_\mu + m\vec{b})^+ \Psi = 0 , \tag{3.4}$$

where the superscript + denotes the positive-frequency part. Since the coupling terms in (3.2) involve the fields $\vec{a}_\mu$ and $\vec{b}$ only in the combination (3.3), the resulting field equations are of the form

$$\begin{aligned} (\partial^2 - m^2)\vec{a}_\mu &= \vec{j}_\mu , \\ (\partial^2 - m^2)\vec{b} &= -\frac{1}{m}\partial_\mu \vec{j}_\mu , \end{aligned} \tag{3.5}$$

so that

$$(\partial^2 - m^2)(\partial_\mu \vec{a}_\mu + m\vec{b}) = 0 , \tag{3.6}$$

which establishes the consistency of the supplementary condition (3.4).

An interesting transformation[8] of the isotriplet vector field is given by

$$\vec{a}_\mu \to \vec{a}_\mu + \vec{f}_\mu , \tag{3.7}$$

with

$$\begin{aligned} \vec{f}_\mu = &\frac{g}{m}(\vec{a}_\mu \times \vec{b}) + \frac{g}{2!\,m^2}(\partial_\mu \vec{b} \times \vec{b}) \\ &+ \frac{g^2}{2!\,m^2}(\vec{a}_\mu \times \vec{b})\times \vec{b} + \frac{g^2}{3!\,m^3}(\partial_\mu \vec{b} \times \vec{b})\times \vec{b} + O(g^3) , \end{aligned} \tag{3.8}$$

which transforms the Lagrangian density (3.2) into

$$\begin{aligned} \bar{L} = &-\tfrac{1}{2}(\partial_\mu \vec{a}_\nu)^2 - \tfrac{1}{2}m^2 \vec{a}_\mu{}^2 - \tfrac{1}{2}(\partial_\mu \vec{b})^2 - \tfrac{1}{2}m^2 \vec{b}^2 - g(\partial_\mu \vec{a}_\nu \cdot \vec{a}_\mu \times \vec{a}_\nu + \tfrac{1}{2}\vec{a}_\mu \cdot \partial_\mu \vec{b} \times \vec{b}) - \frac{g}{m}\partial_\mu \vec{a}_\mu \cdot \vec{a}_\nu \times \partial_\nu \vec{b} - \frac{g}{2m^2}\partial_\mu \vec{a}_\mu \cdot \partial^2 \vec{b} \times \vec{b} \\ &- \tfrac{1}{4}g^2(\vec{a}_\mu{}^2 \vec{a}_\nu{}^2 - \vec{a}_\mu \cdot \vec{a}_\nu \vec{a}_\mu \cdot \vec{a}_\nu) - \frac{g^2}{3m}(\vec{a}_\mu \cdot \partial_\mu \vec{b}\vec{b}^2 - \vec{a}_\mu \cdot \vec{b}\partial_\mu \vec{b} \cdot \vec{b}) \\ &- \frac{g^2}{2m^2}(\partial_\mu \vec{a}_\nu \cdot \vec{b}\vec{a}_\mu \cdot \partial_\nu \vec{b} - \partial_\mu \vec{a}_\nu \cdot \partial_\nu \vec{b}\vec{a}_\mu \cdot \vec{b} + \vec{a}_\mu \cdot \vec{a}_\nu \partial_\mu \vec{b} \cdot \partial_\nu \vec{b} - \vec{a}_\mu \cdot \partial_\mu \vec{b}\vec{a}_\nu \cdot \partial_\nu \vec{b} + \tfrac{1}{4}\partial_\mu \vec{b} \cdot \partial_\mu \vec{b}\vec{b}^2 - \tfrac{1}{4}\partial_\mu \vec{b} \cdot \vec{b}\partial_\mu \vec{b} \cdot \vec{b}) \\ &- \frac{g^2}{m^3}(\vec{a}_\mu \cdot \partial_\nu \vec{b}\partial_\mu \vec{b} \cdot \partial_\nu \vec{b} - \vec{a}_\mu \cdot \partial_\mu \vec{b}\partial_\nu \vec{b} \cdot \partial_\nu \vec{b} + \partial_\mu \vec{a}_\nu \cdot \vec{b}\partial_\mu \vec{b} \cdot \partial_\nu \vec{b} - \tfrac{1}{2}\partial_\mu \vec{a}_\nu \cdot \partial_\nu \vec{b}\partial_\mu \vec{b} \cdot \vec{b} \\ &\qquad - \tfrac{1}{2}\partial_\mu \vec{a}_\nu \cdot \partial_\mu \vec{b}\vec{b} \cdot \partial_\nu \vec{b} + \tfrac{1}{3}\partial_\mu \vec{a}_\mu \cdot \partial_\nu \vec{b}\partial_\nu \vec{b} \cdot \vec{b} \\ &\qquad - \tfrac{1}{3}\partial_\mu \vec{a}_\mu \cdot \vec{b}\partial_\nu \vec{b} \cdot \partial_\nu \vec{b} + \tfrac{1}{3}\vec{b}^2 \partial_\mu \vec{a}_\mu \cdot \partial^2 \vec{b} - \tfrac{1}{3}\partial_\mu \vec{a}_\mu \cdot \vec{b}\vec{b} \cdot \partial^2 \vec{b} + \tfrac{1}{2}\vec{a}_\mu \cdot \vec{b}\partial_\mu \vec{b} \cdot \partial^2 \vec{b} - \tfrac{1}{2}\vec{a}_\mu \cdot \partial_\mu \vec{b}\vec{b} \cdot \partial^2 \vec{b}) \\ &- \frac{g^2}{8m^4}(\vec{b}^2 \partial^2 \vec{b} \cdot \partial^2 \vec{b} - \vec{b} \cdot \partial^2 \vec{b}\vec{b} \cdot \partial^2 \vec{b}) + O(g^3) , \end{aligned} \tag{3.9}$$

where we have dropped some four-divergences to convert all second-order derivatives into ∂^2.

The field equations for $\vec{a}_\mu$ and $\vec{b}$ can be obtained up to g^2 terms by substituting (3.9) into

$$\frac{\partial \bar{L}}{\partial a_\mu^i} - \partial_\nu\left(\frac{\partial \bar{L}}{\partial(\partial_\nu a_\mu^i)}\right) = 0\,,$$
$$\frac{\partial \bar{L}}{\partial b^i} - \partial_\mu\left(\frac{\partial \bar{L}}{\partial(\partial_\mu b^i)}\right) + \partial_\mu\partial_\nu\left(\frac{\partial \bar{L}}{\partial(\partial_\mu\partial_\nu b^i)}\right) = 0\,. \tag{3.10}$$

After evaluating (3.10) and carrying out simplification and rearrangement of terms, we find that it is possible to express the field equations in the form

$$(\partial^2 - m^2)(\vec{a}_\mu + \vec{f}_\mu) = \bar{\vec{j}}_\mu\,,$$
$$(\partial^2 - m^2)\vec{b} = -\frac{1}{m}\partial_\mu \bar{\vec{j}}_\mu\,, \tag{3.11}$$

so that

$$(\partial^2 - m^2)(\partial_\mu \vec{a}_\mu + m\vec{b} + \partial_\mu \vec{f}_\mu) = 0\,, \tag{3.12}$$

which ensures the consistency of the supplementary condition

$$(\partial_\mu \vec{a}_\mu + m\vec{b} + \partial_\mu \vec{f}_\mu)^+ \Psi = 0\,. \tag{3.13}$$

In the absence of interaction, (3.13) is of course equivalent to (3.4). Therefore, as explained in Ref. 1, the supplementary condition allows only the transverse and longitudinal vector particles to exist in the initial and final states in scattering processes.

IV. INTERACTION OF NEUTRAL VECTOR MESONS WITH NUCLEONS

In order to examine the effect of the transformation of vector-meson couplings on renormalization constants, we shall consider an interacting system of neutral vector mesons and nucleons with the Lagrangian density

$$L = -\tfrac{1}{2}(\partial_\mu a_\nu)^2 - \tfrac{1}{2}m^2 a_\mu{}^2 - \tfrac{1}{2}(\partial_\mu b)^2 - \tfrac{1}{2}m^2 b^2 - \bar{\psi}\gamma_\mu\partial_\mu\psi - M\bar{\psi}\psi + ig(a_\mu + m^{-1}\partial_\mu b)\bar{\psi}\gamma_\mu\psi\,, \tag{4.1}$$

where the use of the ghost-field formalism for neutral vector mesons requires the supplementary condition

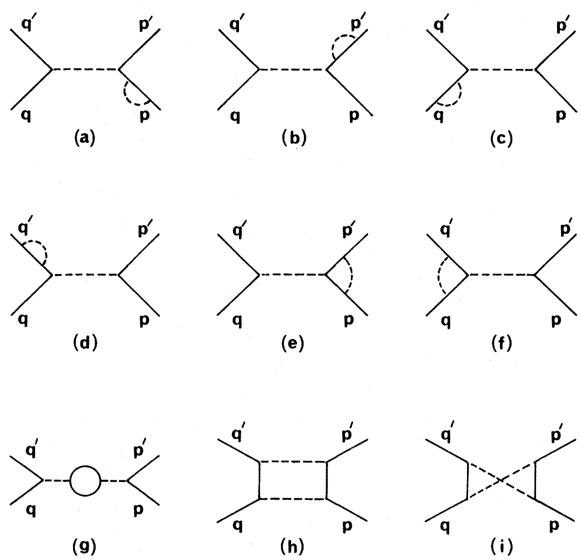

FIG. 1. Fourth-order nucleon-nucleon scattering.

$$(\partial_\mu a_\mu + mb)^+ \Psi = 0\,. \tag{4.2}$$

It is well known that the b-field coupling in (4.1) can be made to vanish by means of the transformation

$$\psi \to e^{(ig/m)b}\,\psi\,. \tag{4.3}$$

We shall, however, use the Lagrangian density in the form (4.1), and explicitly calculate the matrix elements for the fourth-order nucleon-nucleon scattering to determine the effect of the b-field coupling.

The interaction diagrams for the process under consideration are shown in Fig. 1, where the unbroken lines represent the nucleons. The contribution of each diagram consists of the pure a_μ terms, the pure b terms, and the mixed terms. When the pure a_μ terms are dropped but all other terms are retained, the contributions of various diagrams can be simplified and put in the form[9]

$$S_a = S_b = S_c = S_d = -\tfrac{1}{2}S_e = -\tfrac{1}{2}S_f\,,$$
$$S_g = 0\,, \tag{4.4}$$
$$S_h = -S_i\,,$$

where

$$S_a = \frac{g^4}{2m^2}\delta(p+q-p'-q')\frac{\bar{\psi}^-(\vec{q}\,')\gamma_\mu\psi^+(\vec{q})\bar{\psi}^-(\vec{p}\,')\gamma_\mu\psi^+(\vec{p})}{(p-p')^2+m^2}\int dk\,\frac{1}{k^2+m^2}\,,$$
$$S_h = (g^4/m^4)\delta(p+q-p'-q')\bar{\psi}^-(\vec{q}\,')\gamma_\mu\psi^+(\vec{q})\bar{\psi}^-(\vec{p}\,')\gamma_\nu\psi^+(\vec{p})\int dk\,\frac{2m^2\delta_{\mu\nu} + (k_\mu + q_\mu - q'_\mu)(k_\nu + p'_\nu - p_\nu)}{(k^2+m^2)[(k+p'-p)^2+m^2]}\,. \tag{4.5}$$

It follows from (4.4) that

$$S_a + S_b + S_c + S_d + S_e + S_f + S_g + S_h + S_i = 0\,, \tag{4.6}$$

so that the total scattering contribution involving the b-field coupling vanishes. It is especially interesting to note that the so-called wave function renormalization terms in S_a, S_b, S_c, and S_d are exactly canceled by the vertex renormalization terms in S_e and S_f.

Thus, despite earlier misgivings,[4,5] we have demonstrated by explicit calculations for a specific process that a transformation of the Lagrangian density (4.1) does not affect even the renormalization constants.

*Work supported in part by the U. S. Atomic Energy Commission under Contract No. AT(11-1)-2302.

[1]S. N. Gupta, Proc. Phys. Soc. Lond. A64, 695 (1951).
[2]See, for instance, T. D. Lee, Nuovo Cimento 59A, 579 (1969); A. Salam and J. Strathdee, Phys. Rev. D 2, 2869 (1970).
[3]See, for instance, G. 't Hooft, Nucl. Phys. B35, 167 (1971).
[4]D. G. Boulware, Ann. Phys. (N.Y.) 56, 140 (1970).
[5]T. Appelquist and J. R. Primack, Phys. Rev. D 4, 2454 (1971).
[6]C. N. Yang and R. L. Mills, Phys. Rev. 96, 191 (1954).
[7]S. N. Gupta and W. H. Weihofen, Phys. Rev. D 3, 1957 (1971).
[8]This is closely related to the transformation applied by D. G. Boulware in Ref. 4.
[9]During the extensive simplifications required here, we did not employ any objectionable device such as shifting the origin in the k space.

PHYSICAL REVIEW D VOLUME 10, NUMBER 10 15 NOVEMBER 1974

Higgs model with ghost-field formalism*

Debojit Barua and Suraj N. Gupta
Department of Physics, Wayne State University, Detroit, Michigan 48202
(Received 20 June 1974)

The Higgs model for the interaction of vector and scalar fields is treated by means of the ghost-field formalism, which preserves the unitarity of the scattering operator as well as simplifies the renormalization procedure. Our ghost-field formalism has some similarity to the R_ξ formalism with $\xi = 1$, but it does not require the introduction of the so-called gauge-compensating terms. Applications to the second-order self-energy of a scalar particle and to the fourth-order scattering of vector particles are discussed.

I. INTRODUCTION

Although the ghost-field formalism for vector fields involves the use of an indefinite metric, it is possible to carry out transformation of the Lagrangian density in such a formalism without destroying the unitary property of the scattering operator.[1] This fact is of great importance from a practical point of view, because we shall show that it enables us to develop a formalism for vector fields which preserves the unitarity of the scattering operator as well as simplifies the renormalization procedure. We shall make use of only the familiar techniques of quantum field theory. Moreover, since the unitarity of the scattering operator is ensured in our formalism by the consistency of the supplementary condition, it will not be necessary to introduce either any gauge-compensating terms or an arbitrary gauge parameter.[2]

We shall here consider the simple Higgs model,[3] which is sufficient to bring out the essential features of our treatment. In order to clarify the relationship of our work with that of the earlier authors,[4,5] we shall also investigate the second-order self-energy of a scalar particle and the fourth-order scattering of two vector particles. Applications to more complex systems of physical interest will be described in subsequent papers.

II. GHOST-FIELD FORMALISM FOR THE HIGGS MODEL

The Lagrangian density for the Higgs model can be expressed in the conventional form as[3]