Two-particle inclusive electroproduction

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We construct a Feynman diagram model to describe two-particle inclusive electroproduction processes. The cornerstone of the model is Regge behavior which is implemented via spin-J exchanges. The scaling laws for the structure functions are obtained.

I. INTRODUCTION

After the exhaustive experimental and theoretical analysis of inclusive electroproduction processes, attention has recently been devoted to one-particle inclusive processes, i.e., those in which one hadron is detected in coincidence with the outgoing lepton. Dynamical models have been constructed by a number of authors[1] to describe such processes. Of special interest to us here is the Feynman-diagram model discussed by Cheng and Zee[2] for the case of one-particle inclusive electroproduction processes, and by the present author for neutrino-induced reactions.[3] The cornerstone of the model is Regge behavior which is implemented by the procedure of Van Hove[4,5] through infinite spin-J exchanges. In the case of one-particle inclusive electroproduction, this is done by separating the detected hadron from the final state, allowing it to interact with the initial hadron and the vertex containing the undetected hadronic complex through the exchange of a spin-J particle. The amplitude defining the model is then obtained by summing over all possible values of J. The exchange of an infinite number of particles of increasing spin provides a convenient vehicle for implementing Regge behavior.

In this paper we consider an extension of the Feynman-diagram model to describe two-particle inclusive electroproduction processes,

$$e+h\to e+h'+h''+\text{anything}, \tag{1.1}$$

in the one-photon-exchange approximation (Fig. 1). Corresponding to the two momentum-transfer variables $t_1=(p-p')^2$ and $t_2=(p-p'')^2$, we are led to consider Regge exchanges in both of these channels, and hence to regard the amplitude describing the process (1.1) as given by the doubly infinite sum of two types of spin-J exchange graphs. These correspond to the incident hadron h interacting with hadron h' and the other with hadron h'' (Fig. 2). In the next section we carry out the detailed construction for the structure function $W^{\mu\nu}$ that enters in the description of the cross section for the reaction (1.1). We do this for the Regge region and, afterwards, consider the deep-inelastic Regge region where, by the usual arguments, the light cone dominates. With the aid of operator-product expansions near the light cone[6] we exhibit the behavior of the invariant structure functions in the deep-inelastic region. This is particularly transparent in the further limit in which a momentum transfer variable is allowed to approach infinity.

II. TWO-PARTICLE INCLUSIVE ELECTROPRODUCTION

We consider the reaction

$$\gamma(q)+h(p)\to h'(p')+h''(p'')+n\,, \tag{2.1}$$

where $\gamma(q)$ is a virtual photon of four-momentum q, and p, p', p'' are the four-momenta of the incident and the two detected hadrons, respectively. The undetected hadronic complex is denoted by n. We take the hadrons h, h', h'' to be spinless for simplicity. The process (2.1) is depicted in Fig. 1. We describe it by the set of doubly infinite sum of exchange graphs shown in Figs. 2(a) and 2(b). In these graphs J_1, J_1' denote bosons of spins J_1 and J_1', and our model for the process (2.1) is specified by summing over J_1 and J_1' from zero to infinity. The hadronic vertex in Fig. 1 is described by seven independent invariant variables that we choose to be

$$\nu=q\cdot p,\; q\cdot\Delta_1,\; q^2,\; \Delta_1{}^2,\; \Delta_2{}^2,\; \kappa=p\cdot p',\; \nu'=q\cdot p', \tag{2.2}$$

where

$$\Delta_1=\Delta_2-p',\;\; \Delta_2=p-p''.$$

In the analysis of the two-particle inclusive cross section there enters the tensor

$$W^{\mu\nu}=\sum_n (2\pi)^3\delta^4(p+q-p'-p''-p_n)\langle(\text{out}),n(p_n),p',p''|J^\mu(0)|p\rangle\langle(\text{out}),n(p_n),p',p''|J^\nu(0)|p\rangle^*. \tag{2.3}$$

In our model $W^{\mu\nu}$ is then described by a sum of four terms corresponding to the description of the scattering amplitude as a sum of two terms given by the graphs of Figs. 2(a) and 2(b). A typical term in the sum defining $W^{\mu\nu}$ is shown in Fig. 3. Since our aim here is to exhibit the general behavior of the structure functions that enter in the tensor decomposition of $W^{\mu\nu}$, we shall compute the contribution to $W^{\mu\nu}$ arising from "squaring" the amplitude corresponding to the exchange graphs of Fig. 2(a). We denote this contribution by $W^{\mu\nu}_{(1)}$. Evaluation of the remaining contributions proceeds along similar lines to that given below for $W^{\mu\nu}_{(1)}$. Thus we have

$$W^{\mu\nu}_{(1)} = \sum_{J_1, J_2, J_1', J_2' = 0}^{\infty} W^{\mu\nu}(J_1, J_2; J_1', J_2') , \qquad (2.4)$$

where $W^{\mu\nu}(J_1, J_2; J_1', J_2')$ is the contribution of Fig. 3 and is given by

$$\begin{aligned} W^{\mu\nu}(J_1, J_2; J_1', J_2') = {} & \mathcal{C}(J_2)_{\beta_1 \cdots \beta_{J2}} \mathcal{P}(J_2)^{\beta_1 \cdots \beta_{J2}; \beta_1' \cdots \beta_{J2}'} \mathcal{C}(J_1, J_2)_{\beta_1' \cdots \beta_{J2}'; \beta_1'' \cdots \beta_{J1}''} \mathcal{P}(J_1)^{\beta_1'' \cdots \beta_{J1}''; \alpha_1 \cdots \alpha_{J1}} \\ & \times W(J_1, J_1')^{\mu\nu}_{\alpha_1 \cdots \alpha_{J1}; \alpha_1' \cdots \alpha_{J'1}'} \mathcal{P}(J_1')^{\alpha_1' \cdots \alpha_{J'1}'; \alpha_1'' \cdots \alpha_{J'1}''} \mathcal{C}(J_1', J_2')_{\alpha_1'' \cdots \alpha_{J'1}''; \gamma_1 \cdots \gamma_{J'2}} \\ & \times \mathcal{P}(J_2')^{\gamma_1 \cdots \gamma_{J'2}; \gamma_1' \cdots \gamma_{J'2}'} \mathcal{C}(J_2')_{\gamma_1' \cdots \gamma_{J'2}'} . \end{aligned} \qquad (2.5)$$

In Eq. (2.5) $\mathcal{C}(J)$ describes the coupling of two spin-zero particles to a boson of spin J. $\mathcal{C}(J_1, J_2)$ denotes the coupling of a spin-zero particle to two bosons of spins J_1 and J_2. $\mathcal{P}(J)$ is the propagator of an off-shell spin-J boson, and

$$W(J_1, J_1')^{\mu\nu}_{\alpha_1 \cdots \alpha_{J1}; \alpha_1' \cdots \alpha_{J'1}'}$$

is the virtual Compton scattering amplitude. When the spin J_1 and spin J_1' particles are on shell,

$$W(J_1, J_1')^{\mu\nu}_{\alpha_1 \cdots \alpha_{J1}; \alpha_1' \cdots \alpha_{J'1}'}$$

is defined by

$$\epsilon^{*\alpha_1' \cdots \alpha_{J'1}'} W(J_1, J_1')^{\mu\nu}_{\alpha_1 \cdots \alpha_{J1}; \alpha_1' \cdots \alpha_{J'1}'} \epsilon^{\alpha_1 \cdots \alpha_{J1}} = \sum_n (2\pi)^3 \langle \Delta_1', \epsilon_{J_1'}' | J_\nu(0) | n \rangle \langle n | J_\mu(0) | \Delta_1, \epsilon_{J1} \rangle \delta^4(q + \Delta_1 - p_n) \qquad (2.6)$$

In Eq. (2.6) $\epsilon_J^{\alpha_1 \cdots \alpha_J}$ is the polarization vector of a spin-J boson, and $\Delta_1^2 = M^2(J_1)$, $\Delta_1'^2 = M^2(J_1')$. Eventually we shall assume that we can continue to the point $\Delta_1 = \Delta_1'$ with Δ_1^2 being small and negative.

We describe the spin-0–spin-0–spin-J vertex by the following effective Lagrangian:

$$\mathcal{L} = g(J)\, \psi^{\alpha_1 \cdots \alpha_J}(x)\, \partial_{\alpha_1} \cdots \partial_{\alpha_J} \phi_1(x)\, \phi_2(x) , \qquad (2.7)$$

where $\psi^{\alpha_1 \cdots \alpha_J}$, ϕ_1, and ϕ_2 are the fields of the spin-J particle and the two spinless ones. For the spin-0–spin-J_1–spin-J_2 vertex we write the following effective Lagrangian interaction:

$$\begin{aligned} \mathcal{L} = {} & h_0 \psi^{\alpha_1 \cdots \alpha_{J2}}(x)\, \psi^{\beta_1 \cdots \beta_{J1}}(x)\, g_{\alpha_1\beta_1} \cdots g_{\alpha_{J2}\beta_{J2}} \partial_{\beta_{J2+1}} \cdots \partial_{\beta_{J1}} \phi(x) \\ & + h_1 \psi^{\alpha_1 \cdots \alpha_{J2}}(x)\, \psi^{\beta_1 \cdots \beta_{J1}}(x)\, g_{\alpha_1\beta_1} \cdots g_{\alpha_{J2-1}\beta_{J2-1}} \partial_{\alpha_{J2}} \partial_{\beta_{J2}} \cdots \partial_{\beta_{J1}} \phi(x) \\ & + \cdots + h_{J2} \psi^{\alpha_1 \cdots \alpha_{J2}}(x)\, \psi^{\beta_1 \cdots \beta_{J1}}(x)\, \partial_{\alpha_1} \cdots \partial_{\alpha_{J2}} \partial_{\beta_1} \cdots \partial_{\beta_{J1}} \phi(x) , \end{aligned} \qquad (2.8)$$

where we have assumed that $J_1 > J_2$.

The off-shell spin-J boson propagator is given by[5,7]

$$\mathcal{P}(J)_{\alpha_1 \cdots \alpha_J; \beta_1 \cdots \beta_J} = \frac{(-)^J}{\Delta^2 - M^2(J)}\, i\Gamma^J_{\alpha_1 \cdots \alpha_J; \beta_1 \cdots \beta_J}(M^2) , \qquad (2.9)$$

with

$$\Gamma^J_{\alpha_1 \cdots \alpha_J; \beta_1 \cdots \beta_J}(M^2) = \sum_{r=0}^{[J/2]} (-)^r \frac{2^r (2J-2r)!}{(2J)!\,(J-r)!} \{ g_{\alpha_1\beta_1}(M^2) \cdots g_{\alpha_J\beta_J}(M^2) \}^{(J)}_{(r)} \qquad (2.10)$$

and

$$g_{\alpha\beta}(M^2) = g_{\alpha\beta} - \frac{\Delta_\alpha \Delta_\beta}{M^2} , \qquad (2.11)$$

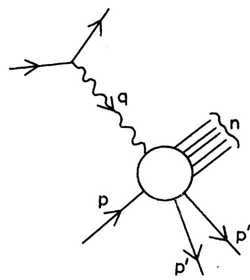

FIG. 1. Two-particle inclusive electroproduction in the one-photon-exchange approximation.

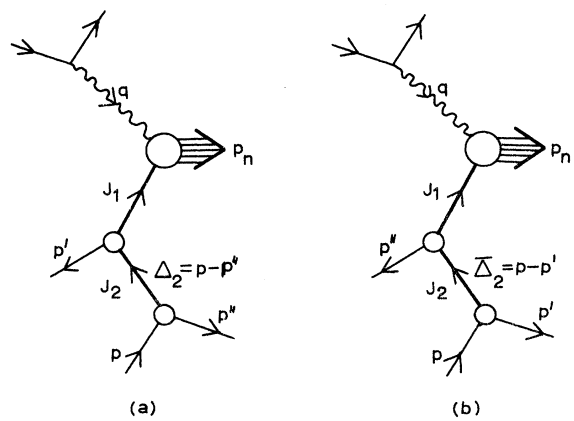

FIG. 2. Spin J_1 and J_2 exchanges in the description of the electroproduction process.

Δ being the four-momentum carried by the spin-J boson. In Eq. (2.10) $[J/2]$ denotes the maximum integer contained in $J/2$, and the symbol $\{\cdots\}\{^J_r\}$ is a product of J factors $g_{\alpha\beta}(M^2)$ completely symmetrized with respect to either the α's or the β's. In r distinct pairs of the $g_{\alpha\beta}(M^2)$ the α index of one is interchanged with the β index of the other.

We shall be interested in the kinematic region in which $\nu, \nu' \to \infty$ at fixed ratio $\eta = \nu'/\nu$. We shall also later allow the momentum transfer variable κ to grow very large. The remaining variables q^2, $q\cdot\Delta_1$, $\Delta_1^{\,2}$, and $\Delta_2^{\,2}$ are held finite. This then defines the Regge region for us. The leading behavior will consequently arise from the last term in Eq. (2.8) corresponding to the maximum number of derivatives. In Eq. (2.5) the following term enters:

$$\mathcal{C}(J_1,J_2)_{\beta';\beta''}\mathcal{P}(J_1)^{\beta'';\alpha}=(-i)^{J_1+J_2}i(-)^{J_1}h(J_1,J_2)p'_{\beta'_1}\cdots p'_{\beta'_{J_2}}p'_{\beta''_1}\cdots p'_{\beta''_{J_1}}\frac{1}{\Delta_1^{\,2}-M^2(J_1)}\Gamma^{J_1,\beta''_1\cdots\beta''_{J_1};\alpha}(M^2), \tag{2.12}$$

where $h(J_1, J_2)$ denotes the spin-J_1–spin-J_2–spin-0 coupling coefficient and the notation $\beta' = \{\beta'_1 \cdots \beta'_{J_1}\}$ has been used. In the contraction, only the term consisting of $g_{\beta''_i\alpha_j}$ factors with one index from the set β'' and the other index from the set α would give the dominant contribution for large ν'. The terms proportional to $\Delta_{1\beta''}\Delta_{1\alpha}$ in $g_{\beta''\alpha}(M^2)$ will eventually give rise to $q\cdot\Delta_1$ terms and in the Regge limit $q\cdot p' \gg q\cdot\Delta_1$ these can be neglected. Therefore we have

$$\mathcal{C}(J_1,J_2)_{\beta';\beta''}\mathcal{P}(J_1)^{\beta'';\alpha}=(-i)^{J_1+J_2}\frac{i(-)^{J_1}}{\Delta_1^{\,2}-M^2(J_1)}p'_{\beta'_1}\cdots p'_{\beta'_{J_2}}p'^{\alpha_1}\cdots p'^{\alpha_{J_1}}h(J_1,J_2)+\cdots, \tag{2.13}$$

where the dots signify terms that are nonleading in the Regge limit.

We can now write Eq. (2.5) as

$$\begin{aligned}W_{\mu\nu}(J_1,J_2;J'_1,J'_2)={}&(-i)^{J_1+J'_1}g(J_2)g(J'_2)h(J_1,J_2)h(J'_1,J'_2)\\&\times p^{\beta_1}\cdots p^{\beta_{J_2}}\frac{(-)^{J_2}}{\Delta_2^{\,2}-M^2(J_2)}\Gamma^{J_2}_{\beta_1\cdots\beta_{J_2};\beta'_1\cdots\beta'_{J_2}}(M^2)p'^{\beta'_1}\cdots p'^{\beta'_{J_2}}\\&\times\frac{(-)^{J_1}}{\Delta_1^{\,2}-M^2(J_1)}p'_{\alpha_1}\cdots p'_{\alpha_{J_1}}W(J_1,J'_1)^{\alpha_1\cdots\alpha_{J_1};\lambda_1\cdots\lambda'_{J_1}}_{\mu\nu}\frac{(-)^{J'_1}}{\Delta'^{\,2}_1-M^2(J'_1)}\\&\times p'_{\lambda_1}\cdots p'_{\lambda_{J'_1}}p'^{\gamma_1}\cdots p'^{\gamma_{J'_2}}\frac{(-)^{J'_2}}{\Delta'^{\,2}_2-M^2(J'_2)}\Gamma^{J'_2}_{\gamma_1\cdots\gamma_{J'_2};\gamma'_1\cdots\gamma'_{J'_2}}(M^2)p^{\gamma'_1}\cdots p^{\gamma'_{J'_2}},\end{aligned} \tag{2.14}$$

where we have suppressed the dependence of the coupling coefficients g and h on Δ_1^2 and Δ_2^2. Next we observe that[8]

$$p^{\alpha_1}\cdots p^{\alpha_{J2}}\Gamma^{J2}_{\alpha_1\cdots\alpha_{J2};\beta_1\cdots\beta_{J2}}(M^2)p'^{\beta_1}\cdots p'^{\beta_{J2}}$$

$$=(-)^{J_2}\frac{2^{J_2}\Gamma^2(J_2+1)}{\Gamma(2J_2+1)}(\bar{p}^2\bar{p}'^2)^{J_2/2}P_{J_2}(\bar{z}), \quad (2.15)$$

where

$$\bar{p}^2=-p^\mu g_{\mu\nu}(M^2)p^\nu , \quad (2.16)$$

$$\bar{p}'^2=-p'^\mu g_{\mu\nu}(M^2)p'^\nu , \quad (2.17)$$

with

$$g_{\mu\nu}(M^2)=g_{\mu\nu}-\frac{\Delta_{2\mu}\Delta_{2\nu}}{M^2(J_2)} ,$$

and

$$\bar{p}\bar{p}'\bar{z}=-p'^\mu g_{\mu\nu}(M^2)p^\nu=-p'\cdot p+\frac{p'\cdot\Delta_2 p\cdot\Delta_2}{M^2(J_2)} . \quad (2.18)$$

Equation (2.15) can then be used in the evaluation of the right-hand side of Eq. (2.14).

Going back to Eq. (2.6) we can write it as

$$\epsilon^{*\alpha_1'\cdots\alpha_{J'1}'}W_{\mu\nu}(J_1,J_1')_{\alpha_1\cdots\alpha_{J1};\alpha_1'\cdots\alpha_{J'1}'}\epsilon^{\alpha_1\cdots\alpha_{J1}}$$

$$=\frac{1}{2\pi}\int d^4x\, e^{iq\cdot x}\langle\Delta_1',\epsilon_{J_1'}|J_\nu(x)J_\mu(0)|\Delta_1,\epsilon_{J_1}\rangle . \quad (2.19)$$

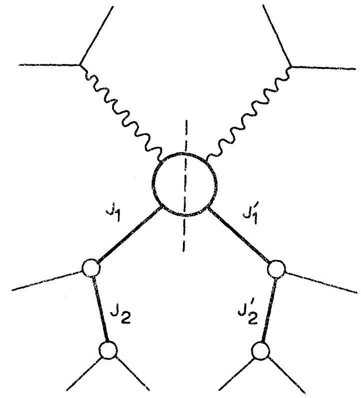

FIG. 3. Spin J_1, J_1', J_2, and J_2' contribution to the cross section.

We want to examine the deep-inelastic Regge limit defined by

$$-q^2,\ q\cdot\Delta_1\to\infty \text{ with } \omega=-\frac{q^2}{2q\cdot\Delta_1} \text{ fixed} . \quad (2.20)$$

We are thus concerned with the subdomain of the Regge limit defined earlier, in which the virtual photon mass and the hadronic mass grow very large.[2] The standard argument then tells us that the light cone dominates in the limit (2.20).[9] We use the operator-product expansion near the light cone for two electromagnetic currents,[6]

$$J_\mu(x)J_\nu(0)=(\partial_\mu\partial_\nu-g_{\mu\nu}\Box)E_1(x)Q_1(x,0)$$
$$+(g_{\mu\nu}\partial_\lambda\partial_\sigma-g_{\mu\lambda}\partial_\nu\partial_\sigma-g_{\nu\sigma}\partial_\mu\partial_\lambda+g_{\mu\lambda}g_{\nu\sigma}\Box)E_2(x)Q_2^{\lambda\sigma}(x,0)-i\epsilon_{\mu\nu\lambda\sigma}\partial^\lambda E_3(x)Q_3^\sigma(x,0) . \quad (2.21)$$

To proceed, we must define matrix elements of the operators $Q_i(x,0)$ appearing in Eq. (2.21). To that end we define the functions $M_{i\alpha;\beta}$ by

$$\langle\Delta_1',\epsilon_{J_1'}|Q_i(x,0)|\Delta_1,\epsilon_{J_1}\rangle=\epsilon^{*\alpha_1\cdots\alpha_{J'1}}M_{i,\alpha_1\cdots\alpha_{J'1};\beta_1\cdots\beta_{J1}}\epsilon^{\beta_1\cdots\beta_{J1}} . \quad (2.22)$$

We then have

$$M_{1,\alpha;\beta}=-i\tilde{f}^{(1)}_{J_1,J_1'}x_{\alpha_1}\cdots x_{\alpha_{J'1}}x_{\beta_1}\cdots x_{\beta_{J1}}$$
$$+i\sum_{k,l}\tilde{h}^{(1)}_{[kl];J_1,J_1'}g_{\alpha_k\beta_l}x_{\alpha_1}\cdots[x_{\alpha_k}]\cdots x_{J_1'}x_{\beta_1}\cdots[x_{\beta_l}]\cdots x_{\beta_{J1}}+\cdots , \quad (2.23)$$

where the square bracket around an x_α factor indicates that it is missing from the product. The dots indicate terms with products of two or more g factors. Next we have

$$M^{\lambda\sigma}_{2\alpha;\beta}=-i\tilde{f}^{(2)}_{J_1,J_1'}\Delta_1^\lambda\Delta_1^\sigma x_{\alpha_1}\cdots x_{\alpha_{J'1}}x_{\beta_1}\cdots x_{\beta_{J1}}+i\tilde{l}^{(2)}_{J_1,J_1'}g^{\lambda\sigma}x_{\alpha_1}\cdots x_{\alpha_{J'1}}x_{\beta_1}\cdots x_{\beta_{J1}}$$
$$+\tfrac{1}{2}\tilde{g}^{(2)}_{J_1,J_1'}\Big[\sum_{k=1}^{J_1'}(\Delta_1^\lambda g^\sigma_{\alpha_k}+\Delta_1^\sigma g^\lambda_{\alpha_k})x_{\alpha_1}\cdots[x_{\alpha_k}]\cdots x_{\alpha_{J'1}}x_{\beta_1}\cdots x_{\beta_{J1}}$$
$$+\sum_{l=1}^{J_1}(\Delta_1^\lambda g^\sigma_{\beta_l}+\Delta_1^\sigma g^\lambda_{\beta_l})x_{\alpha_1}\cdots x_{\alpha_{J'1}}x_{\beta_1}\cdots[x_{\beta_l}]\cdots x_{\beta_{J1}}\Big]$$
$$+\tfrac{1}{4}i\tilde{h}^{(2)}_{J_1,J_1'}\Big[\sum_{k\neq l}g^\lambda_{\alpha_k}g^\sigma_{\alpha_l}x_{\alpha_1}\cdots[x_{\alpha_k}]\cdots[x_{\alpha_l}]\cdots x_{\alpha_{J'1}}x_{\beta_1}\cdots x_{\beta_{J1}}$$
$$+\sum_{k\neq l}(g^\lambda_{\alpha_k}g^\sigma_{\beta_l}+g^\sigma_{\alpha_k}g^\lambda_{\beta_l})x_{\alpha_1}\cdots[x_{\alpha_k}]\cdots x_{\alpha_{J'1}}x_{\beta_1}\cdots[x_{\beta_l}]\cdots x_{\beta_{J1}}$$
$$+\sum_{k\neq l}g^\lambda_{\beta_k}g^\sigma_{\beta_l}x_{\alpha_1}\cdots x_{\alpha_{J'1}}x_{\beta_1}\cdots[x_{\beta_k}]\cdots[x_{\beta_l}]\cdots x_{\beta_{J1}}\Big]$$
$$+\cdots+(\Delta_1\leftrightarrow\Delta_1')\,. \tag{2.24}$$

The operator $Q_3^\sigma(x,0)$ does not contribute to spin-averaged deep-inelastic electroproduction. However, it contributes here and also in polarized electron-nucleon scattering. Its matrix element is defined by

$$M^\sigma_{3\alpha;\beta}=\tilde{f}^{(3)}_{J_1,J_1'}\epsilon^{\sigma\tau\gamma\delta}\Delta_{1\tau}x_\gamma\Big[\sum_{k=1}^{J_1'}g_{\alpha_k\delta}x_{\alpha_1}\cdots[x_{\alpha_k}]\cdots x_{\alpha_{J'1}}x_{\beta_1}\cdots x_{\beta_{J1}}$$
$$+\sum_{k=1}^{J_1}g_{\beta_k\delta}x_{\alpha_1}\cdots x_{\alpha_{J'1}}x_{\beta_1}\cdots[x_{\beta_k}]\cdots x_{\beta_{J1}}\Big]$$
$$+\cdots+(\Delta_1\leftrightarrow\Delta_1')\,. \tag{2.25}$$

The scalar functions that occur in the right-hand sides of Eqs. (2.23) to (2.25) are functions of $x\cdot\Delta_1$, $x\cdot\Delta_1'$, Δ_1^2, $\Delta_1'^2$, and $\Delta_1\cdot\Delta_1'$.

The singular functions $E_i(x)$ that appear in Eq. (2.21) are given by

$$E_1(x)=E_3(x)=\frac{1}{-x^2+i\epsilon x_0}, \tag{2.26}$$

$$E_2(x)=\ln(-x^2+i\epsilon x_0)\,. \tag{2.27}$$

Using Eqs. (2.15) to (2.27) we can now calculate $W_{\mu\nu}(J_1,J_2;J_1',J_2')$. The J_1,J_1',J_2,J_2' summations are then performed in the usual manner using the Sommerfeld-Watson transformation.[10] The calculations are quite lengthy and will not be reproduced here.[11] We recall that we are concerned with the deep-inelastic Regge limit. We also allow the momentum transfer variable κ to approach infinity. In this limit, one retains only the leading-pole contribution when performing the Sommerfeld-Watson transformation in the J_2 and J_2' planes. In performing the various summations, we take the leading pole, for simplicity, to be described by the same trajectory function. We also note that in the limit $\kappa\to\infty$ the contributions of the graphs of Fig. 2(b) are suppressed relative to those of Fig. 2(a), so that $W^{\mu\nu}_{(1)}$ describes the entire tensor structure function $W^{\mu\nu}$. We thus obtain finally

$$W_{\mu\nu}=\Delta_{2\mu}\Delta_{2\nu}W_1+(\Delta_{2\mu}\Delta_{1\nu}+\Delta_{2\nu}\Delta_{1\mu})W_2+\Delta_{1\mu}\Delta_{1\nu}W_3+(\Delta_{2\mu}q_\nu+\Delta_{2\nu}q_\mu)W_4+(\Delta_{1\mu}q_\nu+\Delta_{1\nu}q_\mu)W_5$$
$$+q_\mu q_\nu W_6+g_{\mu\nu}W_7+i(\Delta_{2\mu}\Delta_{1\nu}-\Delta_{2\nu}\Delta_{1\mu})W_8+i(q_\mu\Delta_{2\nu}-q_\nu\Delta_{2\mu})W_9+i(q_\mu\Delta_{1\nu}-q_\nu\Delta_{1\mu})W_{10}, \tag{2.28}$$

where, we recall, $\Delta_2=p-p''$ and $\Delta_1=\Delta_2-p'$.[12] The invariant structure functions W_i are given in the aforementioned limit by the following:

$$W_1\simeq G\alpha(\Delta_1^2)[2\alpha(\Delta_1^2)-1]\eta^{2\alpha(\Delta_1^2)-2}\left(\frac{\nu}{q\cdot\Delta_1}\right)^{2\alpha(\Delta_1^2)-2}\times(2q\cdot\Delta_1)^{-1}F_1,$$

$$W_2\simeq-2G\alpha(\Delta_1^2)\eta^{2\alpha(\Delta_1^2)-1}\left(\frac{\nu}{q\cdot\Delta_1}\right)^{2\alpha(\Delta_1^2)-1}(2q\cdot\Delta_1)^{-1}F_2,$$

$$W_3\simeq G\eta^{2\alpha(\Delta_1^2)}\left(\frac{\nu}{q\cdot\Delta_1}\right)^{2\alpha(\Delta_1^2)}(2q\cdot\Delta_1)^{-1}F_3,$$

$$W_4\simeq-G\alpha(\Delta_1^2)\eta^{2\alpha(\Delta_1^2)-1}\left(\frac{\nu}{q\cdot\Delta_1}\right)^{2\alpha(\Delta_1^2)-1}(2q\cdot\Delta_1)^{-1}F_4,$$

$$W_5\simeq G\eta^{2\alpha(\Delta_1^2)}\left(\frac{\nu}{q\cdot\Delta_1}\right)^{2\alpha(\Delta_1^2)}(2q\cdot\Delta_1)^{-1}F_5, \tag{2.29}$$

$$W_6 \simeq G\eta^{2\alpha(\Delta_1^2)}\left(\frac{\nu}{q\cdot\Delta_1}\right)^{2\alpha(\Delta_1^2)}(2q\cdot\Delta_1)^{-1}F_6,$$

$$W_7 \simeq G\eta^{2\alpha(\Delta_1^2)}\left(\frac{\nu}{q\cdot\Delta_1}\right)^{2\alpha(\Delta_1^2)}F_7,$$

$$W_8 \simeq 16G\alpha(\Delta_1^2)\eta^{2\alpha(\Delta_1^2)-1}\left(\frac{\nu}{q\cdot\Delta_1}\right)^{2\alpha(\Delta_1^2)-1}(2q\cdot\Delta_1)^{-1}F_8,$$

$$W_9 \simeq 16G\alpha(\Delta_1^2)\eta^{2\alpha(\Delta_1^2)-1}\left(\frac{\nu}{q\cdot\Delta_1}\right)^{2\alpha(\Delta_1^2)-1}(2q\cdot\Delta_1)^{-1}F_9,$$

$$W_{10} \simeq -16G\alpha(\Delta_1^2)\eta^{2\alpha(\Delta_1^2)}\left(\frac{\nu}{q\cdot\Delta_1}\right)^{2\alpha(\Delta_1^2)}(2q\cdot\Delta_1)^{-1}F_9, \tag{2.29}$$

where F_i are functions of ω and $\alpha(\Delta_1^2)$ only. The function G is given by

$$G=\frac{\pi^2}{2}\frac{g^2(\alpha(\Delta_1^2),\Delta_1^2)}{\sin^2\pi\alpha(\Delta_1^2)}\frac{h^2(\alpha(\Delta_1^2),\alpha(\Delta_2^2),\Delta_1^2,\Delta_2^2)}{\sin^2\pi\alpha(\Delta_2^2)}\left(\frac{d\alpha(\Delta_1^2)}{d\Delta_1^2}\right)^2\left(\frac{d\alpha(\Delta_2^2)}{d\Delta_2^2}\right)^2\frac{2^{2\alpha(\Delta_2^2)}\Gamma^2(\alpha(\Delta_2^2)+1)\,\Gamma^2(\alpha(\Delta_2^2)+\frac{1}{2})}{\Gamma^2(2\alpha(\Delta_2^2)+1)}(2\kappa)^{2\alpha(\Delta_1^2)}. \tag{2.30}$$

The nine functions $F_i(\omega, \alpha(\Delta_1^2))$ are given as linear combinations of integrals involving the scalar functions that enter in the definition of the matrix elements of the operators $Q_i(x,0)$ as indicated in Eqs. (2.23) to (2.25). They may be described as "Reggeon" structure functions.[2,3]

Equations (2.29) then describe the scaling laws for the structure functions in two-particle inclusive electroproduction. Note that as in the case of one-particle inclusive reactions induced by leptons[2,3] the characteristic factor $\nu/q\cdot\Delta_1$ appears in the expressions for the structure functions. A notable additional feature here is the explicit dependence on η and on κ as is evident from Eqs. (2.29) and (2.30).

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[9]Here we assume commutativity of the Regge and scaling limits. See H. D. I. Abarbanel, M. L. Goldberger, and S. B. Treiman, Phys. Rev. Lett. 22, 500 (1969); R. A. Brandt, *ibid.* 22, 1149 (1969); H. Harari, *ibid.* 1078 (1969).

[10]We make the reasonable assumption that the analytic structure of the coupling coefficients $g(J)$ and $h(J,J')$ is such that this is possible.

[11]Since we are concerned with leading behavior we keep, in the usual fashion, only the most singular terms in the operator-production expansion (2.21). Thus manifest current conservation is not maintained.

[12]The structure functions that appear multiplying the antisymmetric covariants do not contribute to the cross section upon contraction with the lepton tensor.