Inelastic electron scattering in the quark model and in the quark-diquark model

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An extended version of an earlier model where the electroproduction processes were calculated using a modified Woods-Saxon potential in the quark model and in the quark-diquark model is presented in this paper. The dependence of the cross section on the quark mass is investigated and the ratio $R=\sigma_L/\sigma_T$ is calculated.

Many authors have treated the processes $ep\to eN^*$ in the nonrelativistic quark model.[1-3] Thornber calculated the nonrelativistic form factors in the N^* rest frame with a harmonic-oscillator potential[1] (HOP) and a $1/r$ potential[2] as a binding potential. However, none of these results is in complete agreement with the experimental data. In this frame the scaling law for the elastic nucleon form factors is not well satisfied. If they are calculated in the Breit frame (BF) or in the least-velocity frame[4] (LVF), then it is well satisfied. Le Yaouanc *et al.*[3] pointed out that if the form factors are calculated in these frames the HOP gives correct values for $(d\sigma/d\Omega)(N^*)/(d\sigma/d\Omega)(\text{elastic})$. However, $(d\sigma/d\Omega)(N^*)$ and $(d\sigma/d\Omega)(\text{elastic})$ separately still strongly disagree with the experimental data.

In a previous paper[5] the present author pointed out that if the form factors are calculated in these frames, a modified Woods-Saxon potential (MWP)

$$V(r)=-V_0\frac{(r/b)+1}{(r/b)+e^{(r-R)/a}} \tag{1}$$

gives greatly improved values not only for $(d\sigma/d\Omega)(N^*)/(d\sigma/d\Omega)(\text{elastic})$, but also for $(d\sigma/d\Omega)(N^*)$ and $(d\sigma/d\Omega)(\text{elastic})$.

Similar results were obtained in the quark-diquark model[6] proposed by the present author[7] and Lichtenberg.[8] We feel that the following comments regarding these models are in order at this stage.

(1) Corrections of order $1/m_q$ and of order $1/m_q^2$ in the quark model. In our earlier works[5,6] the masses of the constituent particles are assumed to be infinity. In the quark model, many authors have studied the case $m_q=m_p/2.793$, i.e., $g_q=1$. If we expand the nonrelativistic current operator[9] in powers of $1/m_q$, retaining terms up to $O(1/m_q^2)$,

$$\begin{aligned}\rho(\vec{x})&=\sum_i Q(i)\delta(\vec{x}-\vec{r}_i)\\&\quad+\frac{q^2}{8m_q^2}(1-2\mu_q)\sum_i Q(i)\delta(\vec{x}-\vec{r}_i),\\ \vec{J}(\vec{x})&=\sum_{j=1}^{3}\frac{Q(j)}{2im_q}[\delta(\vec{x}-\vec{r}_j)\vec{\nabla}+\vec{\nabla}\delta(\vec{x}-\vec{r}_j)]\\&\quad+\vec{\nabla}\times\sum_{i=1}^{3}Q(i)\mu_q\delta(\vec{x}-\vec{r}_i)\vec{\sigma}(i).\end{aligned} \tag{2}$$

It is well known that $\mu_q=\mu_p\approx-\frac{3}{2}\mu_n$. We use the same notation as Ref. 5. The second term in $\rho(\vec{x})$ is the well-known Darwin term. The dependence on the quark mass m_q is shown in Fig. 1. The nonrelativistic form factors are calculated in the BF and LVF. As can be seen in this figure, if we retain only the term of order $1/m_q$ then the dependence on the mass m_q of the cross section is small. However, if we include the Darwin term, then the cross section strongly depends on the quark mass. Therefore, for the light quark mass $m_q=m_p/2.793$ these nonrelativistic approximations become poor.

Furthermore, if the Darwin term is included, then the scaling law ($\mu_pG_E/G_M=1$) is strongly violated:

$$\begin{aligned}\frac{\mu_pG_E}{G_M}&=1+\frac{q^2}{8m_q^2}(1-2\mu_q)\quad\text{for BF and LVF}\\&=\left[1+\frac{q^2}{8m_q^2}(1-2\mu_q)\right]\Big/\left(1+\frac{q^2}{4m_p^2}\right)\quad\text{for the } N^* \text{ rest frame and the lab frame}.\end{aligned} \tag{3}$$

(2) The Roper resonance: $P_{11}(1470)$. In our earlier work[5,6] we did not take into account the contribution due to the Roper resonance. It is known that the cross section for producing this resonance obtained with the HOP in the nonrelativistic quark model is too high.[3] [In the relativistic quark model of Feynman, Kislinger, and Ravndal[10,11] the $P_{11}(1470)$ contribution is small.]

To make matters worse, the cross sections obtained with the MWP are still larger. In Fig. 2 the contribution of the $P_{11}(1470)$ predicted by the MWP in the quark model is compared with the cross section for the second peak.

In the quark-diquark model, if this resonance is assigned to ${}^2S'_{1/2}$ (radially excited state), the predicted cross section is identical with that in the quark model. On the contrary, if this resonance is assigned to ${}^2D_{1/2}$, then the contribution of this resonance becomes small enough, as is shown in Fig. 2.

In this way, as long as this resonance is assigned to the radially excited state it causes a difficulty in the nonrelativistic quark model and in the nonrelativistic quark-diquark model with the MWP. Of course, if the range R of the MWP becomes large, the contribution of this resonance becomes small; however, the contributions of other resonances corresponding to the second and third peaks fall off more rapidly. Therefore this change causes another difficulty.

(3) The ratio $R=\sigma_L/\sigma_T$. Brasse *et al.*[12] found the following limits for R:

$$R \leq 0.2 \quad \text{for } 0.5 \leq q^2 \leq 2.0\ (\text{GeV}/c)^2, \tag{4}$$

$$R \leq 0.35 \quad \text{for } 2.0 \leq q^2 \leq 4.0\ (\text{GeV}/c)^2, \tag{5}$$

while Miller *et al.*[13] obtained

$$R < 0.5 \quad \text{for } 1.5 \leq q^2 \leq 21\ (\text{GeV}/c)^2. \tag{6}$$

If it is assumed that the ratio R does not depend on q^2 in this range, then

$$R = 0.18 \pm 0.10 \quad \text{for } 1.5 \leq q^2 \leq 21\ (\text{GeV}/c)^2. \tag{7}$$

In the quark model it has been known that if the nonrelativistic form factors are calculated in the N^* rest frame, the predictions obtained with the HOP with the light quark mass $m_q \approx \frac{1}{3}m_p$ for the ratio R exceed the limits of Brasse *et al.*[12] However, as was shown by Abdullah,[14] if the form factors are calculated in the BF (the LVF gives results nearly identical with those in the BF), the HOP gives a ratio which does not exceed the limit. It is interesting to note that for the light quark mass $m_q \approx \frac{1}{3}m_p$ the nonrelativistic approxima-

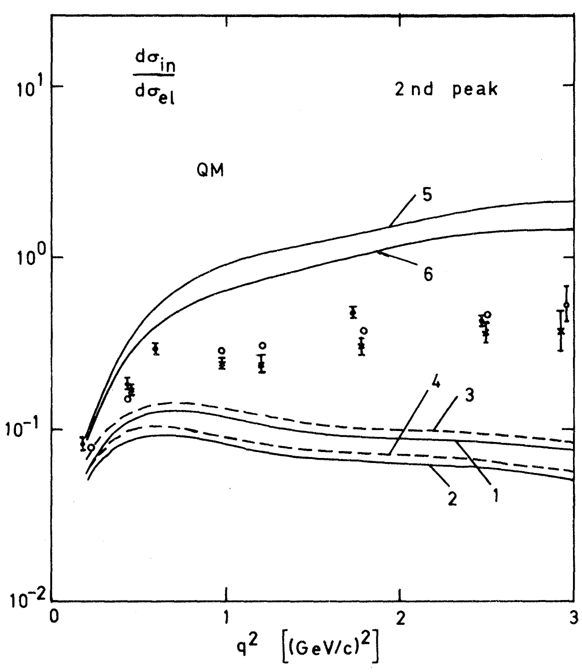

FIG. 1. The ratio of the inelastic cross section for the second peak to the elastic cross section obtained with the MWP in the quark model. Curves 1 and 2 are given by assuming $m_q = \infty$ in the BF and the LVF, respectively. Curves 3 and 4 are given by assuming $m_q = m_p/2.793$ and not including the Darwin term in the BF and the LVF, respectively. Curves 5 and 6 are given by assuming $m_q = m_p/2.793$ and including the Darwin term in the BF and the LVF, respectively. The same experimental data as in Refs. 5 and 6 are used.

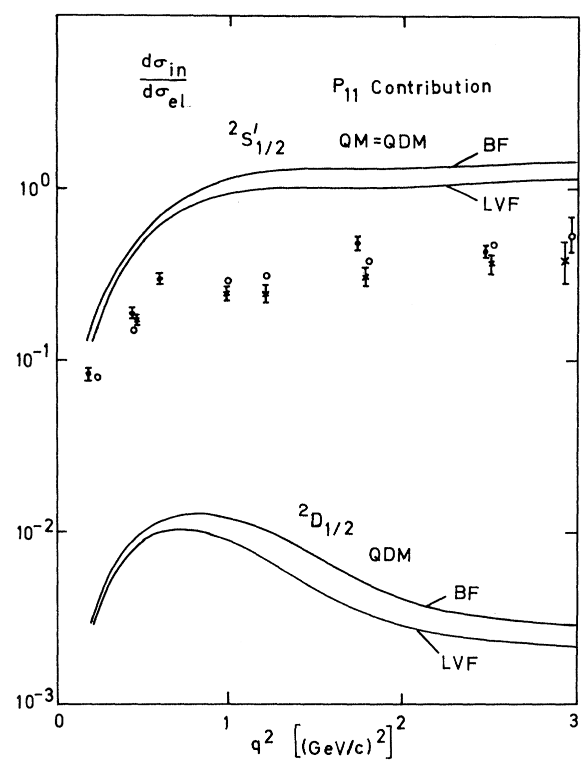

FIG. 2. The contribution of the Roper resonance in the quark model and in the quark-diquark model is compared with the cross section for the second peak. If this resonance is assigned to ${}^2S'_{1/2}$, then the predicted value of the quark model is equal to that of the quark-diquark model.

tion is not good, and if the Darwin term, for example, is included, then the above results change considerably.

In the following, we will briefly summarize the results predicted with the heavy quark mass $m_q = \infty$ in the quark model and $m_q = m_d = \infty$ for the quark-diquark model. The transverse and longitudinal cross sections for resonance production are given by

$$\sigma_L = \frac{4\pi e^2}{K}\frac{m^2}{(\alpha E+\beta E')^2}\frac{q^2}{\vec{q}^2}|f_c|^2 \times \frac{\Gamma/2\pi}{(E^*+q_0^*-M)^2+\frac{1}{4}\Gamma^2}, \quad (8)$$

$$\sigma_T = \frac{4\pi e^2}{K}\frac{m^2}{(\alpha E+\beta E')^2}\frac{|f_+|^2+|f_-|^2}{2} \times \frac{\Gamma/2\pi}{(E^*+q_0^*-M)^2+\frac{1}{4}\Gamma^2}, \quad (9)$$

$$K = (M^2 - m^2)/2m, \quad e^2 = 1/137.03, \quad (10)$$

where E, E', and $\vec{q}^2$ refer to the frame in which the nonrelativistic form factors are calculated, and E^* and q_0^* refer to the N^* rest frame (see Refs. 5 and 6). Since

$$R = \frac{2q^2|f_c|^2}{\vec{q}^2(|f_+|^2+|f_-|^2)}, \quad (11)$$

$$|f_c|^2 \propto I_{fi}^2, \qquad |f_+|^2+|f_-|^2 \propto I_{fi}^2, \quad (12)$$

where I_{fi} is defined in Refs. 5 and 6, the ratio R does not depend on the form of the binding potential.

The value of R for each resonance in the quark model is shown in Fig. 3 and the values summed over all the resonances in each peak for the quark model and for the quark-diquark model are shown in Fig. 4. The form factors are calculated in the LVF. For the first peak only $P_{33}(1236)$ contributes and we get $R = 0$ for the quark model, which agrees with the experimental data. For the second peak, $D_{33}(1525)$ and $S_{11}(1550)$ contribute. If we assume that $P_{11}(1470)$ is included in this peak, it dominates in this peak and the predicted ratio R for it slightly exceeds the experimental limit for small q^2.

For the third resonance $F_{15}(1688)$, $S_{31}(1655)$, and $D_{33}(1685)$ contribute and $F_{15}(1688)$ is dominant. Although the ratios R for $S_{31}(1655)$ and $D_{33}(1685)$ are too high, their contributions to this peak are

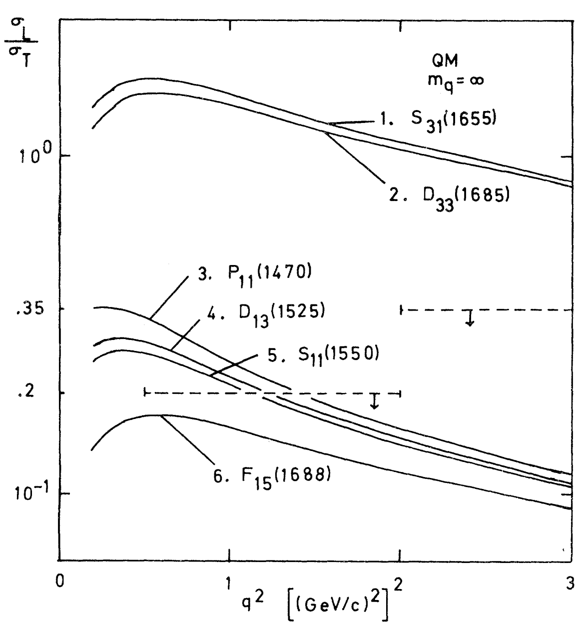

FIG. 3. The ratio of longitudinal to transverse virtual-photon absorption cross section in the quark model for the following resonances: 1. $S_{31}(1650)$; 2. $D_{33}(1670)$; 3. $P_{11}(1470)$; 4. $D_{13}(1525)$; 5. $S_{11}(1550)$; and 6. $F_{15}(1688)$.

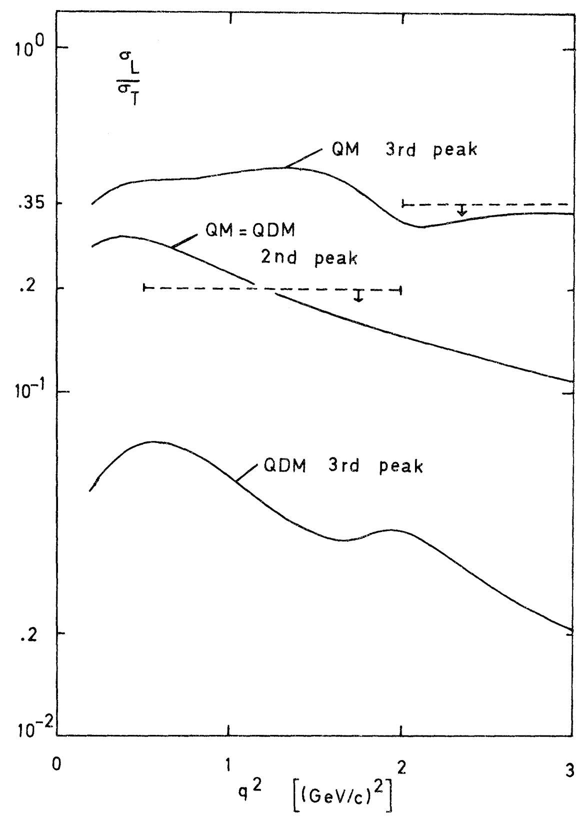

FIG. 4. The ratio R in the second and third peaks summed over all the resonances for the quark model and quark-diquark model. The nonrelativistic form factors are calculated in the LVF. (1) Second peak: $D_{13}(1525)$, $S_{11}(1550)$. (2) Third peak: $D_{15}(1680)$, $D_{13}(1675)$, $F_{15}(1690)$, $S_{11}(1710)$, $S_{31}(1640)$, $D_{33}(1690)$.

small. Therefore, the summed ratios do not exceed the experimental limit seriously (see Fig. 4).

Next we study the quark-diquark model. The predicted ratios are closely related to those of the quark model owing to the relations of Eq. (12). If we assume that the resonances $P_{11}(1470)$, $D_{13}(1525)$, $S_{11}(1550)$, and $F_{15}(1688)$ are assigned to ${}^2S'_{1/2}$ (the first radially excited state), ${}^2P_{3/2}$, ${}^2P_{1/2}$, and ${}^2D_{5/2}$, respectively, then the predicted values of R for these resonances calculated in the quark model and in the quark-diquark model are identical. As for the resonances $S_{31}(1655)$ and $D_{33}(1685)$, if these are assigned to ${}^4P_{1/2}$ and ${}^4P_{3/2}$, then this model predicts vanishing values for R. Although the cross sections for the resonances $D_{15}(1670)$ and $S_{11}(1700)$ vanish in the quark model owing to the Moorhouse selection rule,[15] in the quark-diquark model these cross sections do not vanish. However, the ratios R due to these resonances turn out to be zero. Therefore, the summed value of R for the third peak becomes very small (see Fig. 4) and is consistent with the experimental data.

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