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Bounds and threshold constraints for nucleon structure functions*

M. S. K. Razmi†
Institute of Theoretical Science, University of Oregon, Eugene, Oregon 97403
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Based on assumptions suggested by the experimental data we examine the ratios of the nucleon structure functions using the framework of the light-cone algebra of Fritzsch and Gell-Mann. We find that

(i) the ratio $(F_2^{en}/F_2^{ep}) \leq 1$;

(ii) $0 \leq R_2 = F_2^{\nu p}/F_2^{\nu n} \leq 1$, $0 \leq R_3 = F_3^{\nu p}/F_3^{\nu n} \leq 1$;

(iii) at threshold, $R_2 = R_3 = 0$, $R = (F_2^{\nu p} + F_2^{\nu n})/(F_2^{ep} + F_2^{en}) = \frac{18}{5}$ and $F_2 = 2F_1 = |F_3|$.

Positivity conditions[1] on the weak and electromagnetic scale functions of the nucleon lead to bounds for a number of quantities of interest. Particular mention should be made of the ratios $R(\xi) \equiv (F_2^{\nu p} + F_2^{\nu n})/(F_2^{ep} + F_2^{en})$ and $R_1(\xi) \equiv F_2^{en}/F_2^{ep}$ which satisfy the bounds

$$0 \leq R(\xi) \leq \tfrac{18}{5}$$

and

$$\tfrac{1}{4} \leq R_1(\xi) \leq 4 .$$

The range of values allowed is so big that one wonders whether these bounds can really serve as good tests of the theory. It is important to investigate the possibility of improving these and related bounds. Recently Lipkin and Paschos[2] have studied the ratio $R(\xi)$ in some detail and they give arguments to show that the lower limit for $R(\xi)$ is 3 instead of 0. In the present note, we investigate the consequences of this by making use of the light-cone algebra of Fritzsch and Gell-Mann[3] and some assumptions suggested by the experimental data. Our main conclusions are the following:

(i) $R_1(\xi)$ is bounded above by unity.

(ii) With the definitions $R_2(\xi) \equiv F_2^{\nu p}/F_2^{\nu n}$ and $R_3(\xi) \equiv F_3^{\nu p}/F_3^{\nu n}$,

$$0 \leq R_2(\xi) \leq 1 \quad \text{and} \quad 0 \leq R_3(\xi) \leq 1 .$$

(iii) At threshold, i.e., at $\xi = 1$,

$$R = \tfrac{18}{5}, \quad R_2 = 0, \quad \text{and } R_3 = 0 .$$

(iv) At threshold, $F_2^{\nu N} = 2F_1^{\nu N} = |F_3^{\nu N}|$, where N stands for a proton or a neutron.

Let us now turn to the derivation of these results.

The inelastic nucleon form factors are given by

$$F_2^{ep}(\xi) = \tfrac{1}{6}\xi[2(\tfrac{2}{3})^{1/2}A^0(\xi) + A^3(\xi) + (1/\sqrt{3})A^8(\xi)] , \tag{1a}$$

$$F_2^{en}(\xi) = \tfrac{1}{6}\xi[2(\tfrac{2}{3})^{1/2}A^0(\xi) - A^3(\xi) + (1/\sqrt{3})A^8(\xi)] , \tag{1b}$$

$$F_2^{\nu p}(\xi) = \xi[(\tfrac{2}{3})^{1/2}A^0(\xi) - S^3(\xi) + (1/\sqrt{3})A^8(\xi)] , \tag{1c}$$

$$F_2^{\nu n}(\xi) = \xi[(\tfrac{2}{3})^{1/2}A^0(\xi) + S^3(\xi) + (1/\sqrt{3})A^8(\xi)] , \tag{1d}$$

$$F_3^{\nu p}(\xi) = -(\tfrac{2}{3})^{1/2}S^0(\xi) + A^3(\xi) - (1/\sqrt{3})S^8(\xi) , \tag{1e}$$

$$F_3^{\nu n}(\xi) = -(\tfrac{2}{3})^{1/2}S^0(\xi) - A^3(\xi) - (1/\sqrt{3})S^8(\xi) , \tag{1f}$$

where the notation is standard.[3] The functions $A^i(\xi)$ $(i = 0, 3, 8)$ satisfy the following well-known positivity conditions[1]:

$$(\tfrac{2}{3})^{1/2}A^0(\xi) + A^3(\xi) + (1/\sqrt{3})A^8(\xi) \geq 0 , \tag{2a}$$

$$(\tfrac{2}{3})^{1/2}A^0(\xi) - A^3(\xi) + (1/\sqrt{3})A^8(\xi) \geq 0 , \tag{2b}$$

$$(\tfrac{2}{3})^{1/2}A^0(\xi) - (2/\sqrt{3})A^8(\xi) \geq 0 . \tag{2c}$$

Let $\chi_j(\xi)$ denote the contribution of the jth-type current quark to the functions $A^i(\xi)$.[4] Here j stands for the $\mathcal{P}$, $\mathfrak{N}$, or λ type. In terms of $\chi_j(\xi)$, we have

$$A^0(\xi) = (\tfrac{2}{3})^{1/2}[\chi_{\mathcal{P}}(\xi) + \chi_{\mathfrak{N}}(\xi) + \chi_\lambda(\xi)] , \tag{3a}$$

$$A^3(\xi) = \chi_{\mathcal{P}}(\xi) - \chi_{\mathfrak{N}}(\xi) , \tag{3b}$$

$$A^8(\xi) = (1/\sqrt{3})[\chi_{\mathcal{P}}(\xi) + \chi_{\mathfrak{N}}(\xi) - 2\chi_\lambda(\xi)] . \tag{3c}$$

Then the positivity conditions (2) immediately give

$$\chi_j(\xi) \geq 0 , \quad j = \mathcal{P}, \mathfrak{N}, \lambda . \tag{4}$$

To begin with, we consider the ratio $R(\xi)$ defined earlier. Using Eqs. (3) and (1a)–(1d), we have

$$R(\xi) = \frac{18}{5 + 2\chi_\lambda(\xi)/[\chi_{\mathcal{P}}(\xi) + \chi_{\mathfrak{N}}(\xi)]} . \tag{5}$$

The bounds $0 \leq R(\xi) \leq \frac{18}{5}$ are evident from (5). As mentioned earlier, Lipkin and Paschos[2] have improved the lower bound of $R(\xi)$ considerably so that the new lower limit is 3 instead of 0. The crux of their arguments lies in the assumption that in the quark-parton language the sum of the λ and $\bar{\lambda}$ quark-parton densities is bounded above by the sum of $\mathcal{P}$ and $\bar{\mathcal{P}}$ quark-parton densities. In our language, this corresponds to the bound

$$\chi_\lambda(\xi) \leq \tfrac{1}{2}[\chi_{\mathcal{P}}(\xi) + \chi_{\mathfrak{N}}(\xi)] . \tag{6}$$

Let us now examine some consequences of (6).

Since the question of a lower bound for the ratio $R(\xi)$ is intimately connected with the relative sizes of $I=0$ and $I=1$ contributions to the sum $(F_2^{ep}+F_2^{en})$, let us write these in the following. Denote the $I=0$ and $I=1$ contributions by $F(0)$ and $F(1)$, respectively. It is then easy to check that

$$F(0)=\tfrac{1}{6}\xi[(\tfrac{2}{3})^{1/2}A^0(\xi)-(1/\sqrt{3})A^8(\xi)]$$
$$=\tfrac{1}{18}\xi[\chi_{\mathcal{P}}(\xi)+\chi_{\mathfrak{N}}(\xi)+4\chi_\lambda(\xi)]\,, \tag{7}$$

$$F(1)=\tfrac{1}{2}\xi[(\tfrac{2}{3})^{1/2}A^0(\xi)+(1/\sqrt{3})A^8(\xi)]$$
$$=\tfrac{1}{2}\xi[\chi_{\mathcal{P}}(\xi)+\chi_{\mathfrak{N}}(\xi)]\,. \tag{8}$$

Using (6), one finds

$$\frac{1}{9}\leqslant\frac{F(0)}{F(1)}\leqslant\frac{1}{3}\,, \tag{9}$$

i.e., the isoscalar part of the photon gives a contribution to $(F_2^{ep}+F_2^{en})$ of at least 10% and at most 25%.

Even more important, condition (6) gives

$$A^8(\xi)\geqslant 0\,. \tag{10}$$

The positivity conditions (4) already guarantee that $A^0(\xi)\geqslant 0$, but what can we say about $A^3(\xi)$?

To answer this question, we seek guidance from the behavior of the functions $S^i(\xi)$. These are related to the proton matrix elements of baryon number, isotopic spin, and hypercharge currents. Indeed, in terms of expansions at the tip of the light cone, we have

$$S^i(\xi)=s^i\delta(\xi)+\cdots\,,$$

where $s^0=2\sqrt{6}$, $s^3=2$, $s^8=2\sqrt{3}$. This gives

$$S^3(\xi)=\frac{1}{\sqrt{3}}S^8(\xi)\,, \tag{11}$$

so that $S^3(\xi)/S^8(\xi)>0$. Now the functions $S^i(\xi)$ and $A^i(\xi)$ are inverse Fourier transforms of the proton matrix elements of the bilocal operators $J_\sigma^i(z,0)\pm J_\sigma^i(0,z)$, which suggests that they may behave similarly as far as the SU(3) properties are concerned. Whether or not the A's satisfy a precise relation like (11) does not concern us here. We will instead make use of the much less restrictive assumption

$$A^3(\xi)/A^8(\xi)>0\,. \tag{12}$$

This seems to be supported by the following experimental estimates. Recent CERN data[5] give

$$\int_0^1 d\xi(F_2^{\nu p}+F_2^{\nu n})=1.00\pm0.04\,,$$

while the SLAC data[6,7] give

$$\int_0^1 d\xi(F_2^{ep}+F_2^{en})=0.28\pm0.01\,.$$

Using Eqs. (1a)–(1d), we find that

$$(\tfrac{2}{3})^{1/2}\int_0^1 d\xi\,\xi A^0(\xi)=0.34\pm0.03\,,$$
$$\frac{1}{\sqrt{3}}\int_0^1 d\xi\,\xi A^8(\xi)=0.16\pm0.05\,. \tag{13}$$

On the other hand, we have the estimate[6,7]

$$\int_0^1 d\xi\,\xi A^3(\xi)=3\int_0^1 d\xi(F_2^{ep}-F_2^{en})$$
$$=0.13\pm0.01\,. \tag{14}$$

The equality of (13) and (14) within experimental errors lends support to the relation (12).

Let us examine relation (12) a bit further. If h_a and h_s denote the f- and d-type reduced matrix elements, we have

$$A^3=-\tfrac{1}{2}(h_a-h_s)\,,$$
$$\frac{1}{\sqrt{3}}A^8=-\tfrac{1}{2}(h_a+\tfrac{1}{3}h_s)\,,$$

i.e.,

$$\frac{A^8}{\sqrt{3}A^3}=\frac{h_a+\frac{1}{3}h_s}{h_a-h_s}\,. \tag{15}$$

This ratio is positive if $|h_a|>\frac{1}{3}|h_s|$. To get some idea of the relative magnitudes of h_a and h_s, we assume that these two quantities have the same ξ dependence. This is, of course, a reasonable assumption. We then write $h_s(\xi)=ch_a(\xi)$, where c is a constant. Using the experimental data quoted above, we find that $c=0.15\pm0.25$. It is, therefore, evident that the ratio (15) is almost certainly positive. For it to be negative, $|h_s|$ would have to be more than three times bigger than $|h_a|$, so that (since c is at worst 0.40) c would have to be about 7.5 times bigger than its value determined above. Barring such a pathological situation, it follows that

$$A^3(\xi)\geqslant 0 \tag{16}$$

or, equivalently,

$$R_1(\xi)=[F_2^{en}(\xi)/F_2^{ep}(\xi)]\leqslant 1\,, \tag{17}$$

in agreement with all presently available experimental data.[7] In fact, the ratio $R_1(\xi)$ shows an empirical trend to unity for the lowest experimentally accessible ξ values.

Another interesting experimental phenomenon is the threshold behavior of $R_1(\xi)$. Experimental data[7] seem to suggest that $R_1(\xi)\to\frac{1}{4}$ as $\xi\to1$. We shall assume that this is indeed the case. Then from the expression $R_1(\xi)$ in terms of $\chi_J(\xi)$,

$$R_1(\xi)=\frac{\chi_{\mathcal{P}}(\xi)+4\chi_{\mathfrak{N}}(\xi)+\chi_\lambda(\xi)}{4\chi_{\mathcal{P}}(\xi)+\chi_{\mathfrak{N}}(\xi)+\chi_\lambda(\xi)}\,, \tag{18}$$

it is easy to see that $\chi_{\mathfrak{N}}(\xi)$ and $\chi_\lambda(\xi) \to 0$ as $\xi \to 1$. If $\chi_{\mathcal{P}}(\xi) \to 0$ as $\xi \to 1$, it must do so slower than $\chi_{\mathfrak{N}}(\xi)$ and $\chi_\lambda(\xi)$; otherwise the ratio $R_1(\xi)$ will not go to $\frac{1}{4}$, as can be verified from (18). Thus $\chi_{\mathfrak{N}}(\xi)$ and $\chi_\lambda(\xi)$ would behave something like $(1-\xi)^\alpha \chi_{\mathcal{P}}(\xi)$ near $\xi = 1$, where α is positive and need not be the same for $\chi_{\mathfrak{N}}$ and χ_λ. Note, however, that we do not require the vanishing of $\chi_{\mathcal{P}}(\xi)$ in the following.

One consequence of the vanishing of $\chi_{\mathfrak{N}}(\xi)$ and $\chi_\lambda(\xi)$ near the threshold is that

$$R(\xi) \to \tfrac{18}{5} \text{ as } \xi \to 1. \tag{19}$$

As a second consequence, we can determine the threshold contribution to $(F_2^{ep} + F_2^{en})$ of the isoscalar part of the photon. From Eqs. (7) and (8), it is evident that this contribution is only 10% as $\xi \to 1$. A third consequence concerns the scale functions $F_3^{\nu p}$ and $F_3^{\nu n}$. The positivity of $A^3(\xi)$ gives

$$F_3^{\nu p} - F_3^{\nu n} \geq 0. \tag{20}$$

Since the functions $S^i(\xi)$ $(i = 0, 3, 8)$ are positive, we see from Eq. (1f) that

$$F_3^{\nu n}(\xi) \leq 0 \tag{21}$$

so that

$$R_3(\xi) = (F_3^{\nu p}/F_3^{\nu n}) \leq 1. \tag{22}$$

A lower limit for the ratio $R_3(\xi)$ will be of great interest. We can obtain such a limit if we know the relative sizes of $[(\frac{2}{3})^{1/2}S^0 + (\frac{1}{3})^{1/2}S^8]$ and A^3. We shall assume that

$$(\tfrac{2}{3})^{1/2}S^0(\xi) + (1/\sqrt{3})S^8(\xi) \geq A^3(\xi). \tag{23}$$

Before proceeding further, we detail evidence in support of (23).

(i) Integrating the two sides of (23), we find that

$$\int_0^1 d\xi\,[(\tfrac{2}{3})^{1/2}S^0(\xi) + (1/\sqrt{3})S^8(\xi)] = 3, \tag{24}$$

whereas[9]

$$\int_0^1 d\xi\, A^3(\xi) = 1.0 \pm 0.2. \tag{25}$$

(ii) A recent experimental estimate gives[5]

$$\int_0^1 d\xi\, \xi(F_3^{\nu p} + F_3^{\nu n}) = -0.88 \pm 0.04.$$

Then, the first integrated moment of the left-hand side of (23) is

$$\int_0^1 d\xi\, \xi[(\tfrac{2}{3})^{1/2}S^0 + (1/\sqrt{3})S^8] = 0.44 \pm 0.02. \tag{26}$$

The corresponding integrated moment of the right-hand side is 0.13 ± 0.01 [cf. Eq. (14)]. The ratio of the two is 3.4 ± 0.3. The corresponding ratio for the zeroth integrated moments, Eqs. (24) and (25), is 3.0 ± 0.6. The approximate equality of the two ratios may imply that the two sides in (23) have the same functional dependence on ξ.

(iii) Consider the positivity conditions[1,8]

$$F_2^{\nu N}(\xi) \geq 2\xi F_1^{\nu N}(\xi) \geq |\xi F_3^{\nu N}(\xi)|, \tag{27}$$

where N stands for a proton or a neutron. Relations (27) are quite general and not dependent on any particular model. Even so it is interesting to note that in the parton model with spin-$\frac{1}{2}$ partons only, one obtains

$$F_2^{\nu N}(\xi) = 2\xi F_1^{\nu N}(\xi) = |\xi F_3^{\nu N}(\xi)|,$$

i.e., only equalities obtain in (27). It is also worthwhile to remember that the equalities in (27) are satisfied experimentally to within 10–15%. This fact also supports (23) as we see below. Using Eqs. (1c)–(1f) and assuming (23), the condition (27) is cast into the form

$$(\tfrac{2}{3})^{1/2}S^0 + (1/\sqrt{3})S^8 \leq (\tfrac{2}{3})^{1/2}A^0 + (1/\sqrt{3})A^8. \tag{28}$$

Had we assumed the converse of (23), the condition (27) would have yielded

$$A^3 \leq (\tfrac{2}{3})^{1/2}A^0 + (1/\sqrt{3})A^8. \tag{29}$$

While (29) is certainly correct, we should expect the equality sign to be satisfied to within 10–15% since (29) is only a recast of (27), with the converse of (23) assumed. Multiplying with ξ and integrating, we find that the right-hand side gives 0.50 ± 0.02 while the left-hand side yields a mere 0.13 ± 0.01, so that the equality sign in (29) is badly violated. This again confirms the conclusion reached in (i) and (ii), namely that the converse of (23) cannot be true. Let us now examine (28), which is a recast of (27), with (23) assumed. Multiplying with ξ and integrating, we find that

$$\frac{\int_0^1 d\xi\, \xi((\frac{2}{3})^{1/2}S^0 + (1/\sqrt{3})S^8)}{\int_0^1 d\xi\, \xi((\frac{2}{3})^{1/2}A^0 + (1/\sqrt{3})A^8)} = 0.88 \pm 0.05.$$

Similarly, multiplying with ξ^2 and integrating,

$$\frac{\int_0^1 d\xi\, \xi^2((\frac{2}{3})^{1/2}S^0 + (1/\sqrt{3})S^8)}{\int_0^1 d\xi\, \xi^2((\frac{2}{3})^{1/2}A^0 + (1/\sqrt{3})A^8)} = 0.87 \pm 0.08.$$

Here we have made use of the experimental estimate[5] that

$$\frac{\int_0^1 d\xi\, \xi^2(F_3^{\nu p} + F_3^{\nu n})}{\int_0^1 d\xi\, \xi(F_2^{\nu p} + F_2^{\nu n})} = -0.87 \pm 0.08.$$

Thus the equality sign in (28) is satisfied to the desired degree. These considerations lead us to the conclusion that (23) is a correct assumption.

Combining (23) and (28), we have

$$(\tfrac{2}{3})^{1/2}A^0 + (1/\sqrt{3})A^8 \geq (\tfrac{2}{3})^{1/2}S^0 + (1/\sqrt{3})S^8 \geq A^3. \tag{30}$$

We now see that

$$F_3^{\nu p}(\xi) \le 0 \tag{31}$$

and, therefore,

$$0 \le R_3(\xi) \le 1 . \tag{32}$$

Also,

$$-\frac{F_3^{\nu p}(\xi) - F_3^{\nu n}(\xi)}{F_3^{\nu p}(\xi) + F_3^{\nu n}(\xi)} = \frac{A^3(\xi)}{(\frac{2}{3})^{1/2}S^0(\xi) + (1/\sqrt{3})S^8(\xi)}$$

$$\ge \frac{A^3(\xi)}{(\frac{2}{3})^{1/2}A^0(\xi) + (1/\sqrt{3})A^8(\xi)}$$

$$= \frac{\chi_{\mathcal{P}}(\xi) - \chi_{\mathfrak{N}}(\xi)}{\chi_{\mathcal{P}}(\xi) + \chi_{\mathfrak{N}}(\xi)} .$$

Let us take $\xi \to 1$. Then

$$F_3^{\nu p}(1) - F_3^{\nu n}(1) \ge - F_3^{\nu p}(1) - F_3^{\nu n}(1) ,$$

i.e.,

$$F_3^{\nu p}(1) \ge 0 .$$

Combining this result with (31), we find that

$$F_3^{\nu p}(1) = 0 . \tag{33}$$

One further consequence of (30) can be derived as follows. Write

$$(\tfrac{2}{3})^{1/2}A^0(\xi) + (1/\sqrt{3})A^8(\xi) = (\tfrac{2}{3})^{1/2}S^0(\xi) + (1/\sqrt{3})S^8(\xi) + \eta(\xi) ,$$

where $\eta(\xi) \ge 0$. Using this expression in (1e), we find that

$$F_3^{\nu p}(\xi) = -2\chi_{\mathfrak{N}}(\xi) + \eta(\xi) .$$

Since $F_3^{\nu p}(\xi) \to 0$ as $\xi \to 1$, we have the result that $\eta(\xi) \to 0$ as $\xi \to 1$. This means that at threshold, the equality signs in (30) hold. Thus, not only is (33) true, but also the relation

$$[F_3^{\nu p}(1)/F_3^{\nu n}(1)] = 0 . \tag{34}$$

Further, the positivity condition $F_2^{\nu p} \ge |\xi F_3^{\nu p}|$ implies that at threshold $A^3 \ge S^3$, while the condition $F_2^{\nu n} \ge |\xi F_3^{\nu n}|$ implies that $S^3 \ge A^3$. Hence, we get

$$S^3(1) = A^3(1) , \tag{35}$$

i.e., at threshold, the equality signs in (27) hold. As a consequence of (35) follows the important result that

$$[F_2^{\nu p}(1)/F_2^{\nu n}(1)] = 0 . \tag{36}$$

Of course, the positivity of $S^3(\xi)$ also gives

$$R_2(\xi) = [F_2^{\nu p}(\xi)/F_2^{\nu n}(\xi)] \le 1 . \tag{37}$$

Finally, note that because of the positivity conditions (27)

$$\int_0^1 d\xi\, F_2^{\nu p}(\xi) \ge \int_0^1 d\xi\, \xi((\tfrac{2}{3})^{1/2}S^0 + (1/\sqrt{3})S^8 - A^3)$$

$$= 0.31 \pm 0.02 \tag{38}$$

and

$$\int_0^1 d\xi\, F_2^{\nu n}(\xi) \ge 0.57 \pm 0.02 . \tag{39}$$

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†On leave from the Department of Physics, University of Islamabad, Pakistan.

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[4]The $A^i(\xi)$ are introduced by the definition

$$\langle p|J_\sigma^i(z,0) - J_\sigma^i(0,z)|p\rangle = p_\sigma \int_{-1}^{+1} d\xi\, e^{-i\xi p\cdot z} A^i(\xi) + \text{trace terms},$$

where the nucleon spin summation is understood, and $J_\sigma^i(z,0)$ is the current $i\bar{q}(z)(\lambda_i/2)\gamma_\sigma q(0)$. For a given λ matrix, the current J_σ can be expressed as a sum of contributions from the $\mathcal{P}$, $\mathfrak{N}$, and λ-type quark current densities. The functions $A^i(\xi)$, being inverse Fourier transforms of

$$\langle p|J_\sigma^i(z,0) - J_\sigma^i(0,z)|p\rangle ,$$

can likewise be expressed as linear combinations of the contributions $\chi_j(\xi)$, where j runs over $\mathcal{P}$, $\mathfrak{N}$, and λ.

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[9]Actually in a quark-parton model one has the sum rule

$$\int_0^1 \frac{d\xi}{\xi}(F_2^{ep} - F_2^{en}) = \tfrac{1}{3} ,$$

a result which can be derived by assuming that the infinite parton sea acts the same in the presence of proton or neutron valence quarks. See J. D. Bjorken and E. Paschos, Phys. Rev. 185, 1975 (1969). This sum rule can also be obtained by assuming exact exchange degeneracy for t-channel meson exchanges in hadronic reactions. See P. M. Fishbane and D. Z. Freedman, Phys. Rev. D 5, 2582 (1972). Since the exchange degeneracy is good to about $\pm 20\%$ in the t channel for hadronic reactions, the right-hand side in the above sum rule should perhaps be 0.33 ± 0.07. In the experimental evaluation of the left-hand side, one obtains $0.28 \pm$ unknown error. The errors are unknown because the experimental data available are from $\xi = 1$ to $\xi = 0.05$ only. From $\xi = 0.05$ to $\xi = 0$, one uses Regge extrapolation with the attendant uncertainty about the accuracy of this procedure. For details, see Ref. 7.